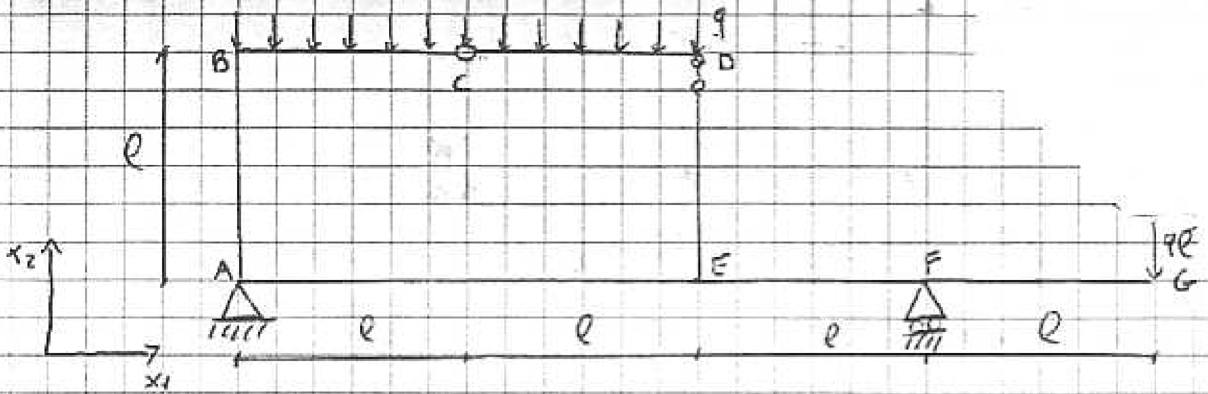


CONDIZIONE NECESSARIA PER L'EQUILIBRIO MA NON SUFFICIENTE:

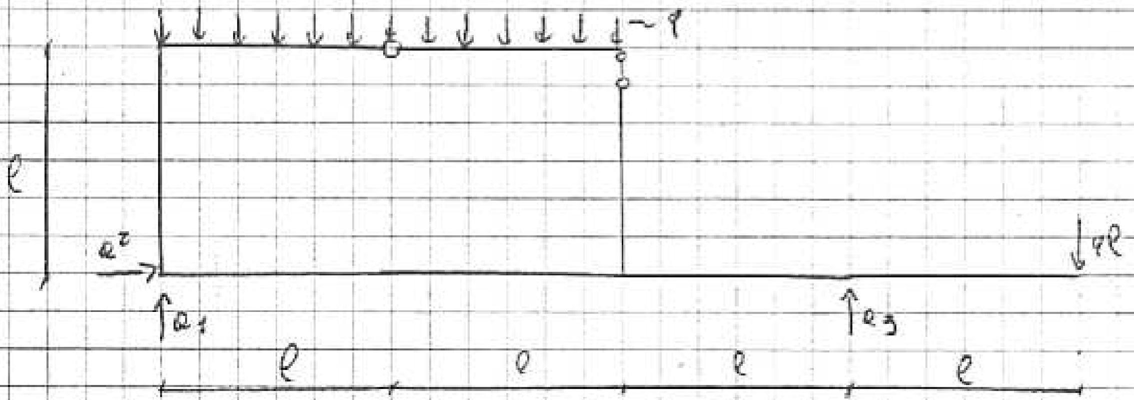
$s + 3(m-1) \geq 3 + n_g$

$s = 3$

$m = 2$ $\quad\quad 3+3(2-1) \geq 3+3$

$n_g = 3$ $\quad\quad 6 = 6$ la condizione necessaria è verificata

Sostituisco ai vincoli le reazioni vincolari incognite e le risolvo senza aprire la travatura



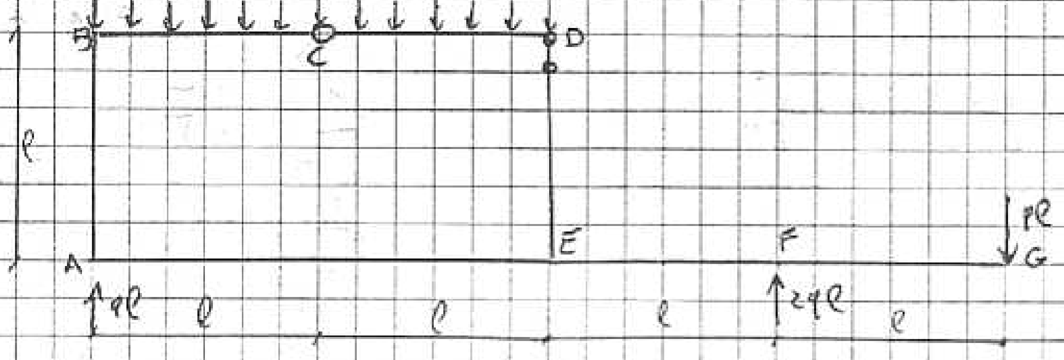
$$\begin{cases} a_2 = 0 \\ a_1 + a_3 - q2l - ql = 0 \\ A)\ -q2l\cancel{l} - ql4\cancel{l} + a_3\, 3\cancel{l} = 0 \end{cases} \quad\quad \begin{cases} a_2 = 0 \\ a_1 = ql \\ a_3 = \frac{\cancel{6}^{2} ql}{\cancel{3}} = 2ql \end{cases}$$

⇒ Come già affermava il testo, la travatura è ISOSTATICA ESTERNAMENTE SE NON CI SONO LABILITÀ NELLA PORZIONE CHIUSA

VERIFICA DIMENSIONALE:

$a_1 = ql \rightarrow \frac{[F]}{\cancel{[L]}} \cdot \cancel{[L]}$ OK $\quad\quad a_3 = 2ql \quad \frac{[F]}{\cancel{[L]}} \cdot \cancel{[L]} = [F]$ OK

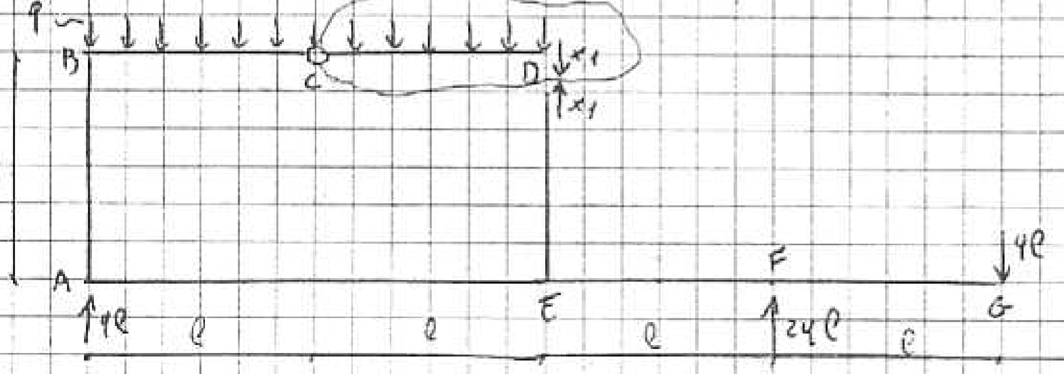
ASSETTO STATICO PER LA TRAVATURA CHIUSA:



VERIFICA ASSETTO STATICO:

$$\begin{cases} 0 = 0 \\ q\ell - q\ell + 2q\ell - 2q\ell = 0 \quad \text{ok} \\ G)\ -q\ell \cdot 4\ell - 2q\ell\ell + 2q\ell \cdot 3\ell = -6q\ell^2 + 6q\ell^2 = 0 \quad \text{ok} \end{cases}$$

Apro la travatura in prossimità della biella in D



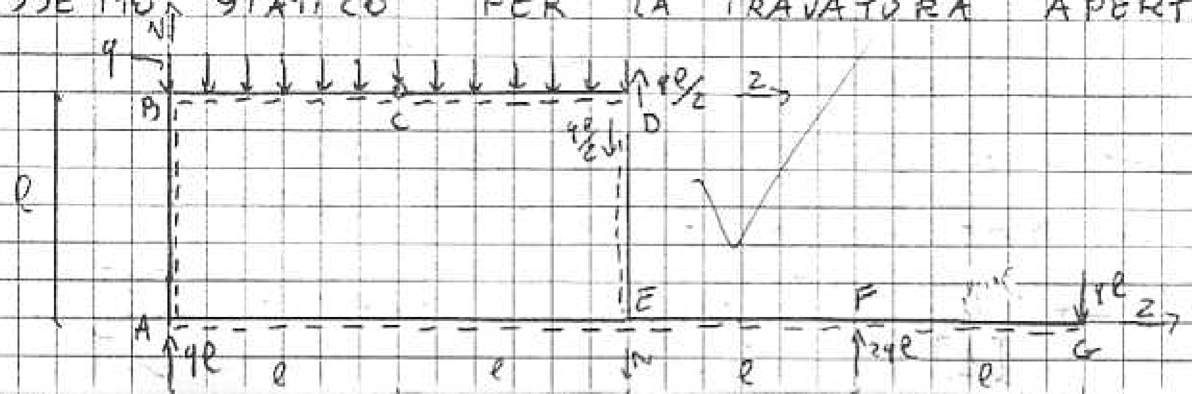
Ho già usato 2 equazioni ausiliarie me ne rimane ancora 1 calcolo il momento in C

$$M_C^{(2)} = 0 \quad \left\{ -X_1 \cancel{\ell} - q\ell \frac{\ell}{2} = 0 \qquad X_1 = -\frac{q\ell}{2} \right.$$

VERIFICA: (prendo l'altra parte di travatura)

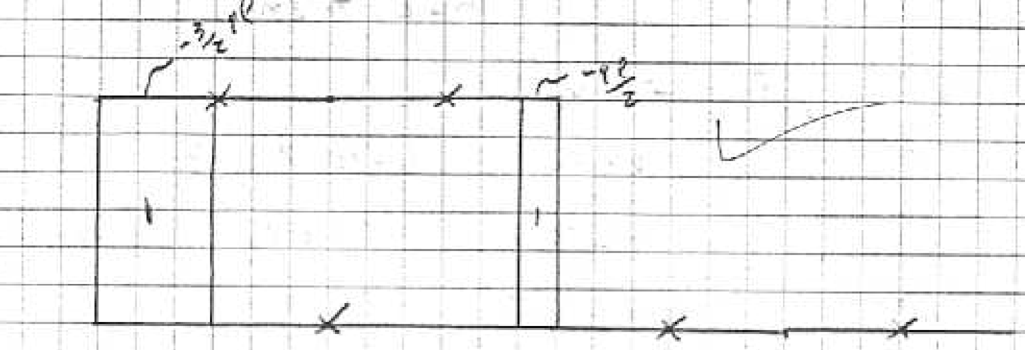
$$M_C^{(1)} = 0 \qquad \cancel{q\ell\frac{\ell}{2}} - \cancel{q\ell\ell} + \cancel{2q\ell}\,2\ell - \cancel{q\ell\cdot 3\ell} - \cancel{q\ell\frac{\ell}{2}} = 0 \qquad \text{ok}$$

ASSETTO STATICO PER LA TRAVATURA APERTA:

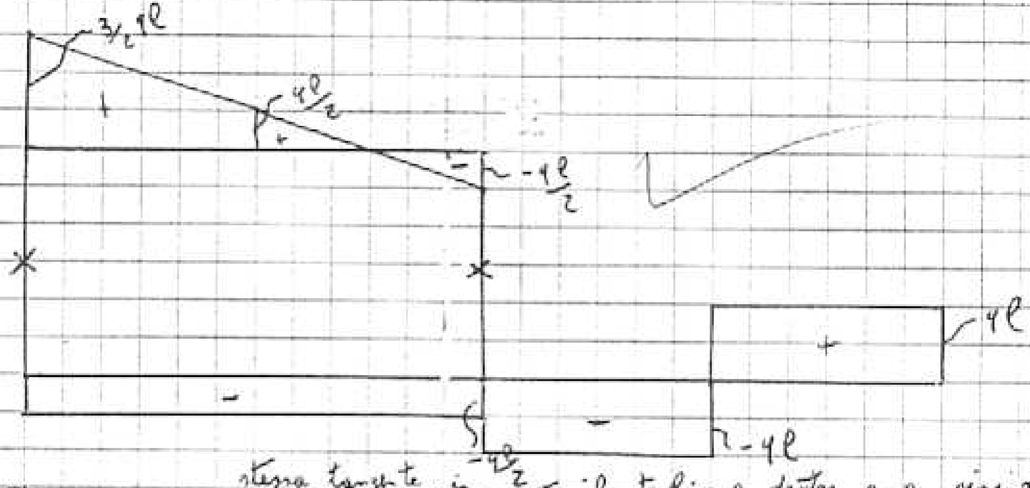


DIAGRAMMI DELLE CARATTERISTICHE DI SOLLECITAZIONE:

$N(z)$

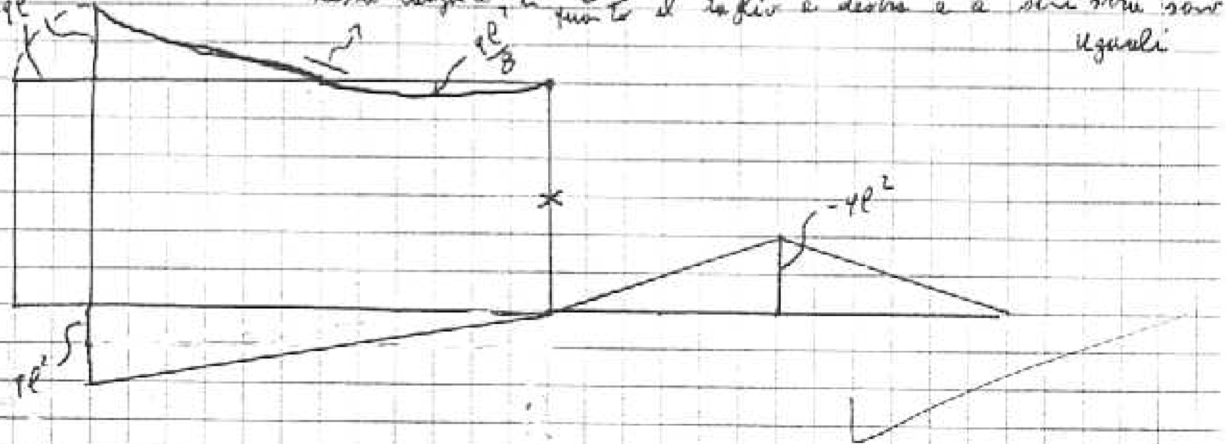


$T(z)$

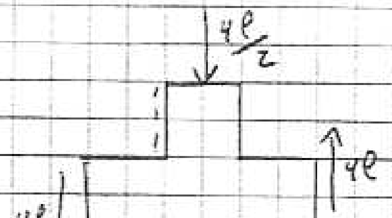


stessa tangente, in quanto il taglio a destra e a sinistra sono uguali

$M(z)$

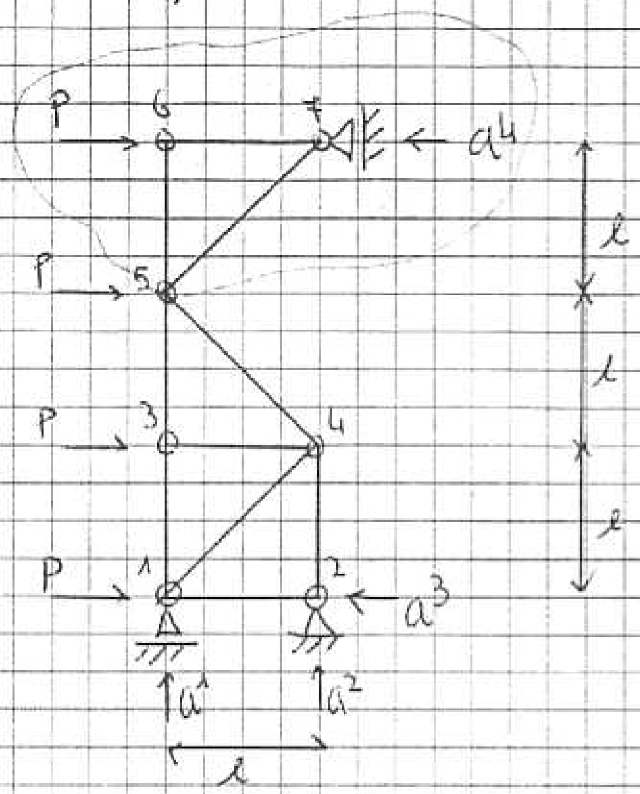


VERIFICA AL NODO E:



$$ql - \frac{ql}{2} - \frac{ql}{2} = 0 \qquad \text{OK}$$

Problema 2)



1)

- $s = 4$ (numero vincoli semplici)

  $n = 7$ (numero nodi)

  $a = 10$ (numero aste)

  $s + a \geq 2n$

  $4 + 10 \geq 2 \cdot 7 \rightarrow 14 = 14$

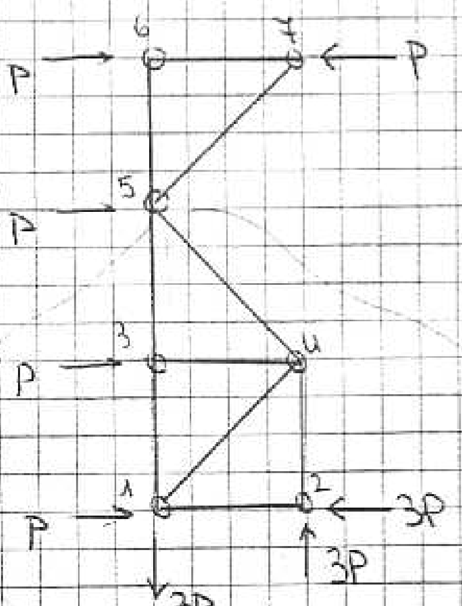
  Condizione necessaria per l'equilibrio è soddisfatta

- Determino le reazioni vincolari tramite le tre equazioni cardinali della statica più l'equazione ausiliaria relativa alla cerniera in 5

$$
(1)\quad \begin{cases} a^1 + a^2 = 0 \\ a^3 + a^4 - P - P - P - P = 0 \\ +a^2 \cdot \cancel{l} - P \cdot \cancel{l} - P \cdot 2\cancel{l} - P \cdot 3\cancel{l} + a^4 \cdot 3\cancel{l} = 0 \\ M_5^{\ominus} = 0 \quad + a^4 \cdot \cancel{l} - P \cdot \cancel{l} = 0 \end{cases}
$$

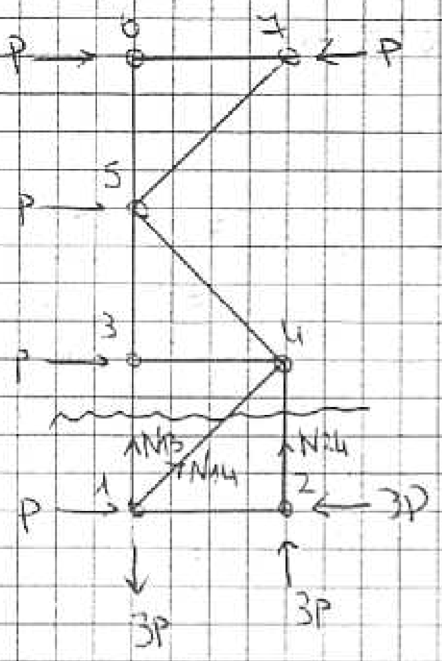
$$
\begin{cases} a^1 = -a^2 = -3P \\ a^3 = 4P - a^4 = 3P \\ a^2 = 6P - 3a^4 = 3P \\ a^4 = P \end{cases}
$$

- Assetto statico e verifiche



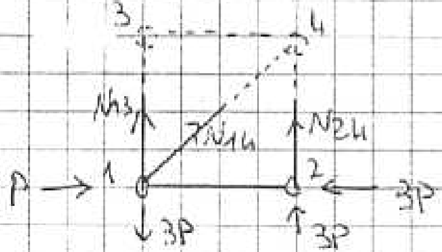
$$
(2)\quad \begin{cases} 3P - 3P = 0 \\ 4P - P - 3P = 0 \\ P \cdot 3l + 3P\,l - P\,l - P \cdot 2l - P \cdot 3l = 0 \\ M_5^{\ominus} = 0 \quad P\,l + P \cdot 2l + 3P \cdot l - 3P \cdot 2l = 0 \end{cases}
$$

2)



asta	N
N_{24}	$-3P$
N_{13}	P
N_{14}	$2\sqrt{2}P$
N_{35}	P
N_{34}	$-P$
N_{67}	$-P$
N_{75}	0
N_{65}	0
N_{54}	$-\sqrt{2}P$
N_{12}	$-3P$

Sezione di Ritter sulle aste 13, 14, 24



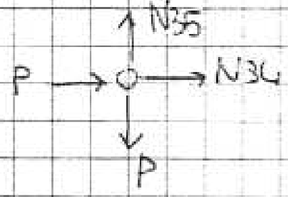
Impongo le tre equazioni sui momenti nei nodi:

$$
\begin{array}{l}
(1) \\ (4) \\ \uparrow\infty
\end{array}
\begin{cases}
3P\cancel{l} + N_{24}\cdot\cancel{l} = 0 \\
-N_{13}\cdot\cancel{l} + P\cdot\cancel{l} + \cancel{3P\cdot l} - \cancel{3P\cdot l} = 0 \\
P - 3P + N_{14}\frac{\sqrt{2}}{2} = 0
\end{cases}
\rightarrow
\begin{cases}
N_{24} = -3P \\
N_{13} = P \\
N_{14} = 2\sqrt{2}P
\end{cases}
$$

↳ equivale alla traslazione orizzontale

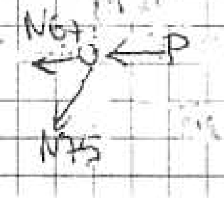
Continuo la risoluzione della travatura utilizzando il metodo dei nodi canonici.

Nodo (3)



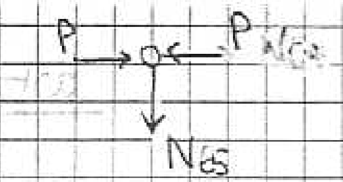
$$\begin{cases} N_{35} = P \\ N_{34} = -P \end{cases}$$

Nodo (7)



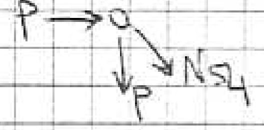
$$\begin{cases} P + N_{67} + \frac{\sqrt{2}}{2}N_{75} = 0 \\ \frac{\sqrt{2}}{2}N_{75} = 0 \end{cases} \rightarrow \begin{cases} N_{67} = -P \\ N_{75} = 0 \end{cases}$$

Nodo (6)



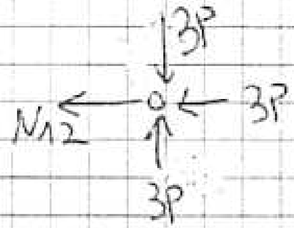
$$\begin{cases} P - P = 0 \quad \text{ok} \\ N_{65} = 0 \end{cases}$$

Nodo (5)



$$\begin{cases} P + \frac{\sqrt{2}}{2} N_{54} = 0 \\ P + \frac{\sqrt{2}}{2} N_{54} = 0 \end{cases} \rightarrow N_{54} = -\sqrt{2}P$$

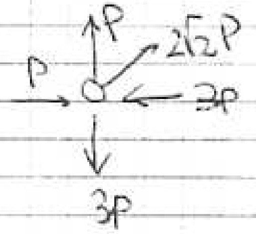
Nodo (2)



$$\begin{cases} 3P = 3P \quad \text{ok} \\ 3P + N_{12} = 0 \rightarrow N_{12} = -3P \end{cases}$$

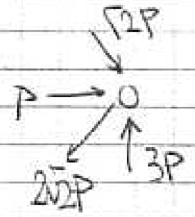
Studio, come verifica, il nodo (1) e il nodo (4)

Nodo (1)



$$\begin{cases} P - 3P + 2\cancel{\sqrt{2}} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} P = 0 \\ P - 3P + 2\cancel{\sqrt{2}} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} P = 0 \end{cases} \quad \text{ok}$$

Nodo (4)

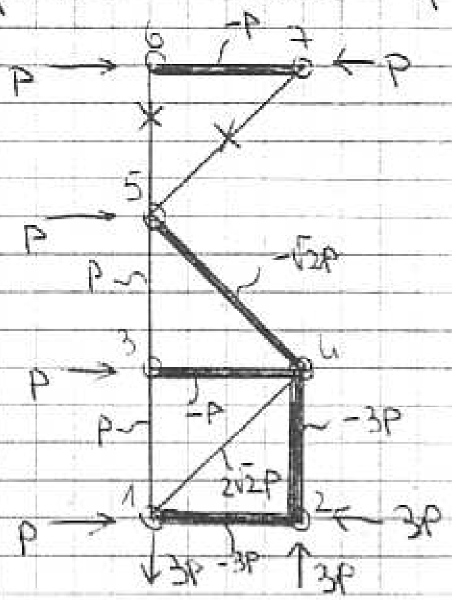


$$\begin{cases} P + \cancel{\sqrt{2}}P \cdot \frac{\cancel{\sqrt{2}}}{2} - 2\cancel{\sqrt{2}}P \frac{\cancel{\sqrt{2}}}{\cancel{2}} = 0 \\ 3P - 2\cancel{\sqrt{2}}P \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} - \cancel{\sqrt{2}}P \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} = 0 \end{cases} \quad \text{ok}$$

3) Diagrammi delle caratteristiche di sollecitazione (N)

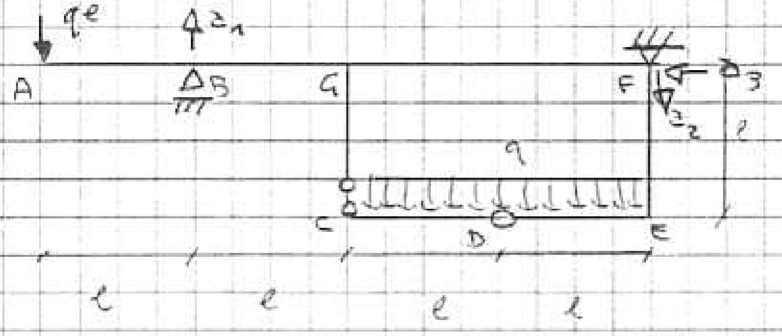
N positivo → tratto sottile → TIRANTI

N negativo → tratto spesso → PUNTONI



Problema (1)

Travatura chiusa



$s + 3(m-1) \geq 3 + m_s$

$s = 3$

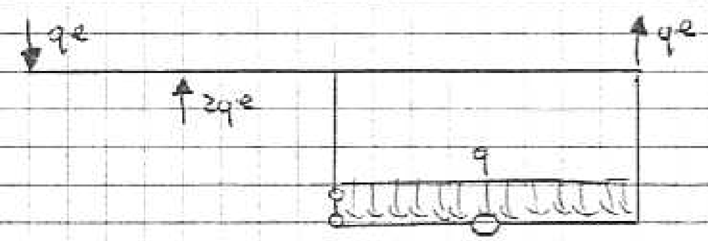
$m = 2$ $\rightarrow$ $6 = 6$ condiz necess per l'equilibrio (isostatico)

$m_s = 3$ $(1+2)$ ✓ soddisfatta

Riesco a determinare le reaz vincolari $(s=3)$

$$\begin{cases} a_1 - ql - 2ql - a_2 = 0 \\ -a_3 = 0 \\ (F)\ 2ql\,l - a_1 3l + ql\,4l = 0 \end{cases} \Rightarrow \begin{cases} a_2 = -ql \ ✓ \\ a_3 = 0 \ ✓ \\ a_1 = \frac{6}{3}ql = 2ql \ ✓ \end{cases}$$

Assetto statico esterno



Verifiche DIM $2ql = \frac{F}{K}K$ ✓ ✓

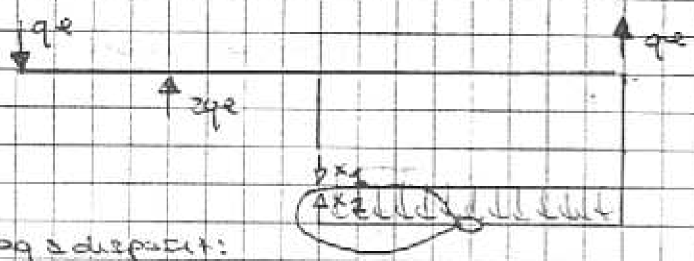
$ql = \frac{FL}{K}$ ✓

$$✓ \begin{cases} -ql + 2ql + ql - 2ql = 0 \ ✓ \\ 0 = 0 \ ✓ \\ (A)\ 2ql\,l - 2ql\,3l + ql\,4l = 0 \ ✓ \end{cases}$$

ok

(1)

Apro la travatura in C (sconnessione doppia) con $(m-1)$ tagli



ultima eq a disposiz:

$$M_D^{(sx)} = 0 \qquad -X_1 \ell + \frac{q\ell^2}{2} = 0 \;\to\; X_1 = \frac{q\ell}{2} \quad ✓$$

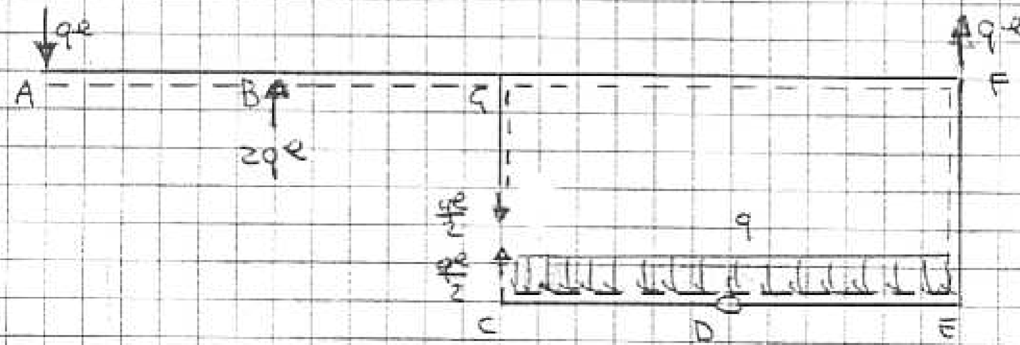
✓ Verifica dall'altra parte della travatura:

$$M_D = 0 \qquad -\frac{q\ell^2}{2} + q\ell\,\ell - 2q\ell\,2\ell + q\ell\,3\ell + X_1\ell = 0$$

$$\downarrow$$

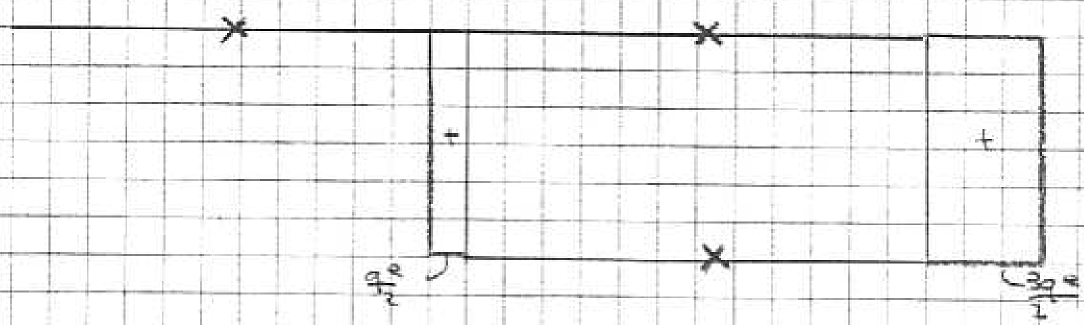
$$X_1 = -3q\ell + 4q\ell - q\ell + \frac{q\ell}{2} \quad ✓ \qquad \frac{F}{K}\ (2)\ ✓\ ✓$$

Assetto statico finale
✓



Diagrammi

(N) Normale ✓



Tutto il tratto $\overline{AG}$ non è soggetto a forza normale (c'è solamente solo a taglio in B)

Il tratto $\overline{CG}$ ha $N = \text{cost}$ $(q_z(z) = 0)$ $= \frac{q\ell}{2}$ → PERCHÉ? VA SPIEGATO

EQUIL? ⊗

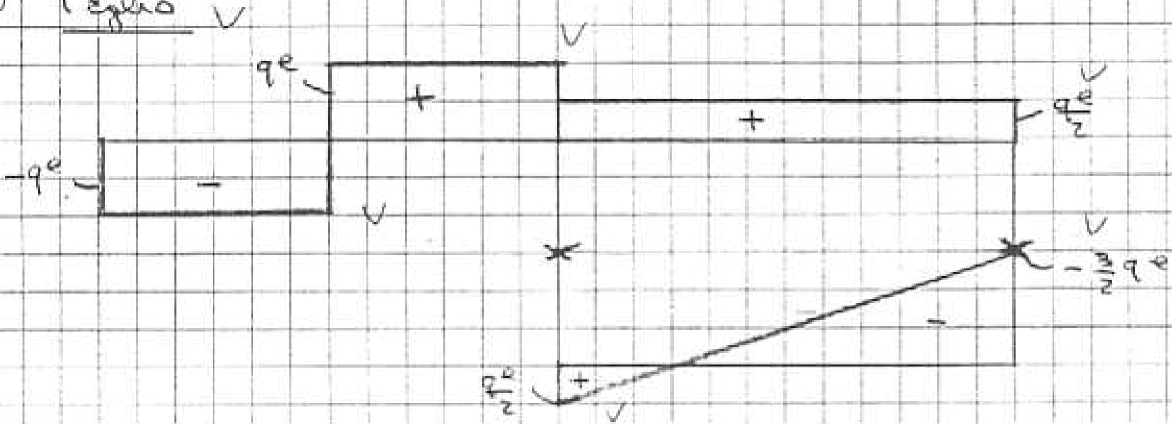
Il tratto $\overline{GF}$ non è nuovamente soggetto a forza Normale

Il tratto $\overline{EF}$ ha forza N = costante

$$N = q\ell - \frac{q\ell}{2} + 2q\ell - q\ell = \frac{3}{2}q\ell$$

I tratti CD e ED non sono soggetti a forza normale

(T) Taglio



$\overline{AB}$ $q_y(z) = 0 \rightarrow T$ cost $T = -q\ell$ ⟶ PERCHÉ? VA SPIEGATO

$\overline{BG}$ $q_y(z) = 0 \rightarrow T$ cost $T = -q\ell + 2q\ell = q\ell$

$\overline{CG}$ $T = 0$ (per la sconnessione)

$\overline{GF}$ $q_y(z) = 0 \rightarrow T$ cost $T = -q\ell + 2q\ell - \frac{q\ell}{2} = \frac{q\ell}{2}$

$\overline{FE}$ $q_y(z) = 0 \rightarrow$ Il tratto $\overline{FE}$ NON è soggetto a Taglio

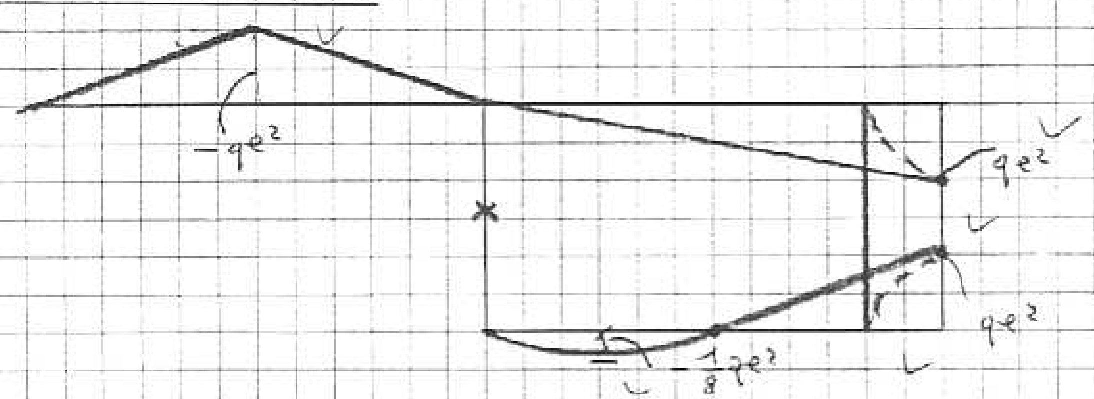
$\overline{CE}$ (la cerniera D non rappresenta una discontinuità a taglio)

$q_y(z) = $ cost ; T = lineare

$T(E) = -\cancel{q\ell} + \frac{q\ell}{2} - 2q\ell + \cancel{q\ell} = -\frac{3}{2}q\ell$

$T(C) = \frac{q\ell}{2}$

(M) Momento flettente



$\overline{AB}$ T costante $\rightarrow$ M lineare $M(A) = 0$ $M(B) = -q\ell^2$

$\overline{BG}$ T cost $\to$ M lineare $\quad M(B) = -q\ell^2 \quad M(G) = -q\ell 2\ell + 2q\ell\ell = 0$

$\overline{CG}$ $T = 0 \to M$ cost $\quad M = 0$

$\overline{GF}$ $T =$ cost $\to$ M lineare $\quad M(G) = 0$

$$M(F) = -q\ell 4\ell + 2q\ell 3\ell - \frac{q\ell}{\cancel{2}}\cancel{2}\ell = q\ell^2$$

$\overline{FE}$ $T = 0 \to M$ cost $\quad M(F) = q\ell^2$

$\overline{EC}$ $T =$ lineare $\to M =$ quadratica

$$M(z) = -q\ell z - \frac{q\ell}{2}(2\ell - z) + 2q\ell(3\ell - z) - q\ell(4\ell - z) + \frac{qz^2}{2}$$

$$= -q\ell z - q\ell^2 + \frac{q\ell z}{2} + 6q\ell^2 - 2q\ell z - 4q\ell^2 + q\ell z + \frac{qz^2}{2}$$

$$M(z) = \frac{qz^2}{2} - \frac{3}{2}q\ell z + q\ell^2 \quad \checkmark$$

$$M(E) = \frac{q\ell^2}{2}$$

$M(D) = 0$ CERNIERA ✓ ✓

$M(C) = 0$ ✓

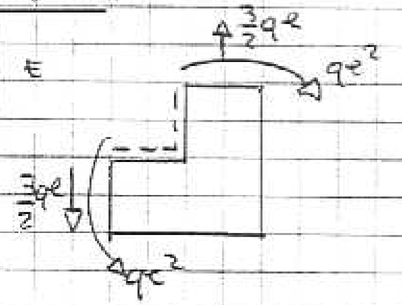
$M'(z) = qz - \frac{3}{2}q\ell = 0 \to z = \frac{3}{2}\ell$ ✓ MIN MAX FLESSO

$M''(z) = q > 0 \Rightarrow$ MIN ✓

$$M\left(z = \frac{3}{2}\ell\right) = \frac{q}{2}\frac{9}{4}\ell^2 - \frac{3}{2}q\ell\frac{3}{2}\ell + q\ell^2 = q\ell^2\left(\frac{9}{8} - \frac{9}{4} + 1\right) = -\frac{1}{8}q\ell^2 \quad \checkmark$$

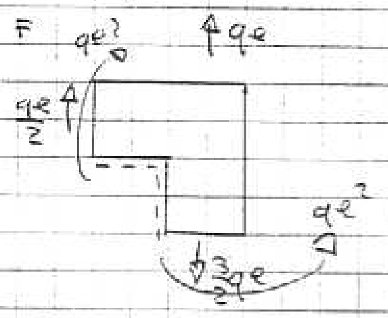
Verifiche:

NODO E



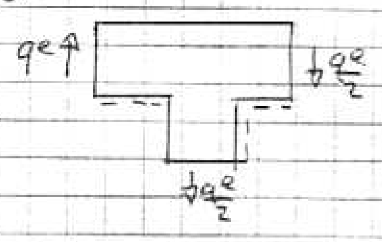
$$\begin{cases} \frac{3}{2}q\ell - \frac{3}{2}q\ell = 0 \ \checkmark \\ 0 = 0 \ \checkmark \\ q\ell^2 - q\ell^2 = 0 \ \checkmark \end{cases}$$ ok

NODO F



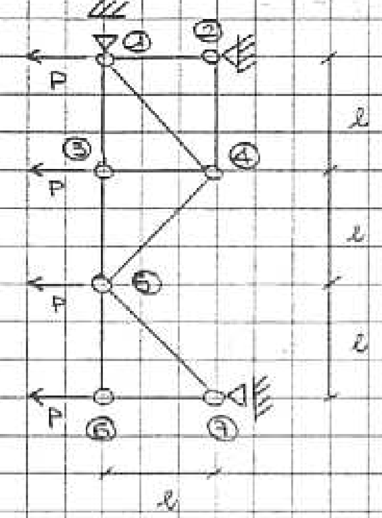
$$\begin{cases} q\ell + \frac{q\ell}{2} - \frac{3}{2}q\ell = 0 \ \checkmark \\ q\ell^2 - q\ell^2 = 0 \ \checkmark \\ 0 = 0 \ \checkmark \end{cases}$$ ok

NODO G



$$\begin{cases} 0 = 0 \ \checkmark \\ q\ell - \frac{q\ell}{2} - \frac{q\ell}{2} = 0 \ \checkmark \\ 0 = 0 \ \checkmark \end{cases}$$ ok

Problema 2



a = n° aste = 10

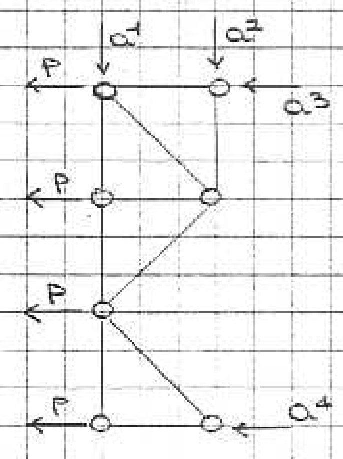
n = n° nodi = 7

s = n° vincoli esterni = 4

- la struttura è una travatura reticolare, perciò verifichiamo la condizione (nec. ma non suff.) di esistenza dell'equilibrio: $s + a \geq 2n$

$4 + 10 \geq 2 \cdot 7 \Rightarrow 7 = 7$

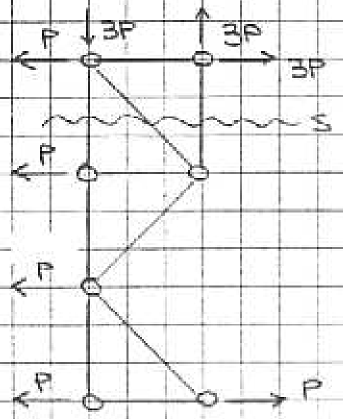
la condizione necessaria ma non sufficiente per l'esistenza dell'equilibrio ISOSTATICO è soddisfatta. le incognite determinate dalle azioni esercitate dai vincoli sono 4. Posso provare a trovarle ugualmente notando che in 5 ho una cerniera da cui posso ricavare una equazione ausiliaria.



$$\begin{cases} a^1 + a^2 = 0 \\ a^3 + a^4 + 4P = 0 \\ M_1 = 0 \quad +a^2\cancel{l} + P\cancel{l} + P2\cancel{l} + P3\cancel{l} + a^4 3\cancel{l} = 0 \\ M_5^s = 0 \quad P\cancel{l} + a^4\cancel{l} = 0 \end{cases}$$

$$\begin{cases} a^1 = -a^3 = 3P \\ a^3 = -4P - a^4 = -4P + P = -3P \\ a^2 = -6P - 3a^4 = -6P + 3P = -3P \\ a^4 = -P \end{cases}$$

→ ASSETTO STATICO TRAVATURA ✓



Verifiche ✓

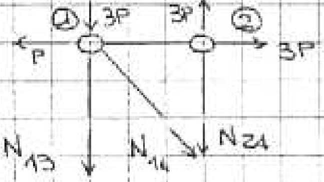
1) An. dimensionale:

$$[a^1]=[a^2]=[a^3]=[a^4]=[P]=[F] \quad ✓$$

2) Equilibrio: ✓

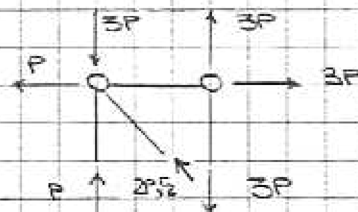
$$\begin{cases} P+P+P+P-3P-P=0 & ✓ \\ 3P-3P=0 & ✓ \\ M_6=0 \quad P\ell+2P\ell+3P\ell-\underbrace{3P\,3\ell}_{-9P\ell}+3P\ell=0 & ✓ \\ M^\circ_5=0 \quad P\ell+2P\ell+3P\ell-\underbrace{3P\,2\ell}_{-6P}=0 & ✓ \end{cases}$$

- Taglio con una sezione di Ritter le aste 13, 24, 14



$$\begin{cases} M_4=0 \quad -\cancel{3P\ell}+\cancel{3P\ell}+P\cancel{\ell}+N_{13}\cancel{\ell}=0 \\ M_1=0 \quad -N_{24}\cancel{\ell}+3P\cancel{\ell}=0 \\ M\downarrow_\infty \quad N_{14}\frac{\sqrt{2}}{2}+3P-P=0 \end{cases}$$

$$\begin{cases} N_{24}=3P & ✓ \\ N_{14}=-2P\sqrt{2} & ✓ \\ N_{13}=-P & ✓ \end{cases}$$

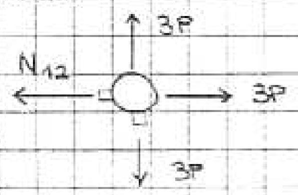


Verifiche:

$$\begin{cases} 3P+P+2P-3P-3P=0 & ✓ \\ 2P+P-3P=0 & ✓ \end{cases}$$

- Analisi dei nodi:

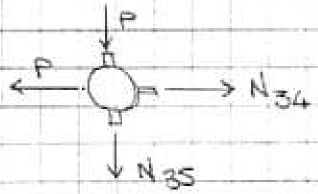
Nodo 2



$\Rightarrow \quad N_{12}=3P$

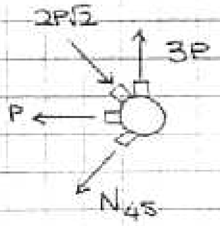
aste	N	
12	3P	tirante
24	3P	tirante
14	$-2P\sqrt{2}$	puntone
13	$-P$	puntone
34	P	tirante
35	$-P$	puntone
45	$P\sqrt{2}$	tirante
56	0	
57	0	
67	P	tirante

Nodo 3



$$\begin{cases} N_{34} = P \\ N_{35} = -P \end{cases}$$

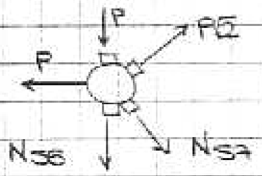
Nodo 4



$$3P - 2P - N_{45}\frac{\sqrt{2}}{2} = 0$$

$$\Rightarrow N_{45} = P\,\frac{2}{\sqrt{2}}\,\frac{\sqrt{2}}{\sqrt{2}} = P\sqrt{2}$$

Nodo 5

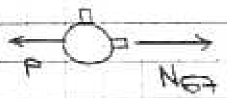


$$\begin{cases} \cancel{P} - \cancel{P} - N_{57}\frac{\sqrt{2}}{2} - N_{56} = 0 \\ N_{57}\frac{\sqrt{2}}{2} + \cancel{P} - \cancel{P} = 0 \end{cases}$$

$$\Downarrow$$

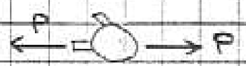
$$\begin{cases} N_{57} = 0 \\ N_{56} = 0 \end{cases}$$

Nodo 6



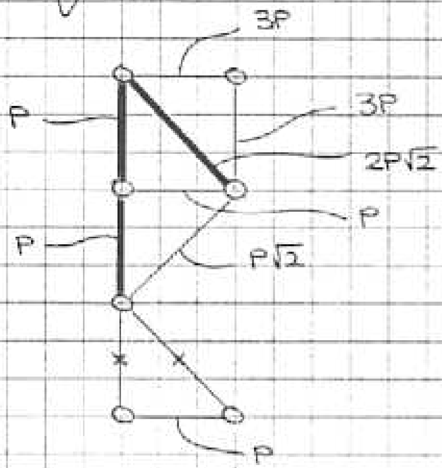
$$N_{67} = P$$

• Verifico i miei risultati con l'equilibrio del nodo 7:

Nodo 7: 

$P - P = 0$ ✓

• Disegno il diagramma della caratteristica di sollecitazione Normale: ✓



Avendo tutte aste, su di esse Momento e Taglio saranno nulli.