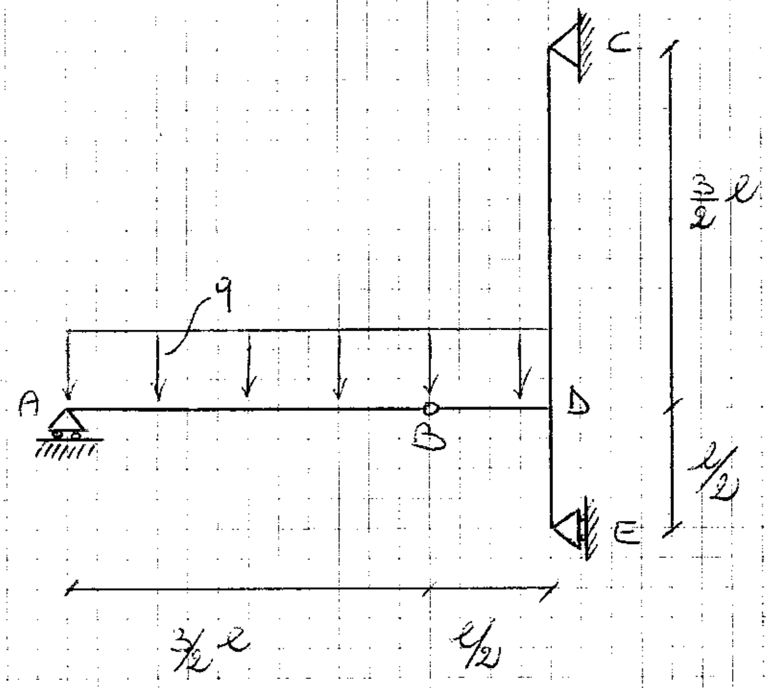
FILA (B)

ESERCIZIO N° 1



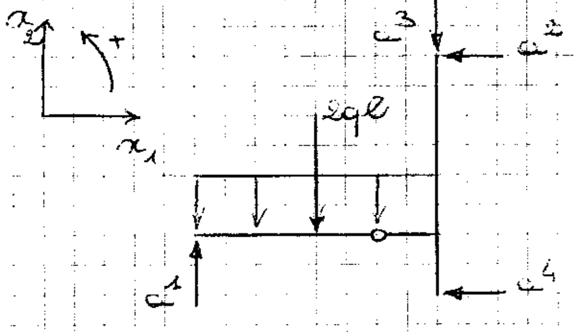
4 = N° vincoli semplici

1 = N° sconnessioni

$S = 3 + 1 = 4$

Il sistema è ISOSTATICO

- EQUAZIONI CARDINALI DELLA STATICA



$$\begin{cases} x_1: & -a^2 - a^4 = 0 \\ x_2: & a^1 - 2ql - a^3 = 0 \\ M(C): & -a^1 \cdot 2l + 2ql \cdot l - a^4 \cdot 2l = 0 \end{cases}$$

- EQUAZIONE AUSILIARIA $M(B) = 0$

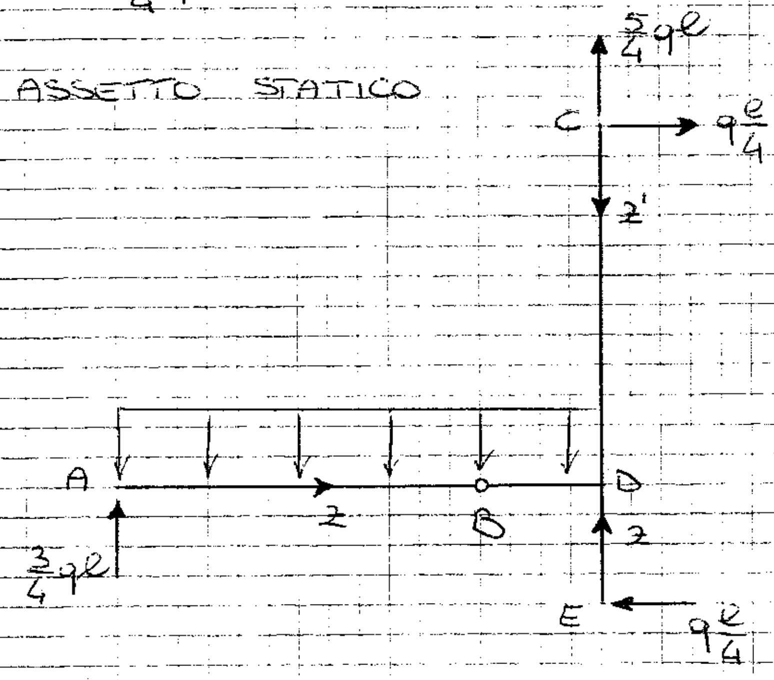
$$-a^1 \cdot \frac{3}{\cancel{2}} \cancel{l} + q \cdot \frac{3 \cancel{l}}{\cancel{2}} \cdot \frac{3}{4} l = 0 \qquad \text{(parte di sinistra)}$$

Sistema di 4 equazioni in 4 incognite

$$\begin{cases} -a^2 - a^4 = 0 \\ a^1 - 2ql - a^3 = 0 \\ -a^1 \cancel{2l} + \cancel{2}ql^{\cancel{2}} - a^4 \cdot \cancel{2l} = 0 \\ -a^1 + \frac{3}{4} ql = 0 \end{cases} \qquad \begin{cases} a^2 = -a^4 \\ a^3 = a^1 - 2ql = \frac{3}{4} ql - 2ql \\ a^4 = -a^1 + ql = -\frac{3}{4} ql + ql \\ a^1 = \frac{3}{4} ql \end{cases}$$

$$\begin{cases} a^2 = -q\dfrac{\ell}{4} \\ a^3 = -\dfrac{5}{4}q\ell \\ a^4 = q\dfrac{\ell}{4} \\ a^1 = \dfrac{3}{4}q\ell \end{cases}$$

- ASSETTO STATICO



- TRAVE AD

$N = \text{cost} = 0$

$T(z) = \frac{3}{4}q\ell - qz$ carico distribuito $\rightarrow$ T lineare

$T(z=0) = \frac{3}{4}q\ell$

$T(z = \frac{3}{2}\ell) = \frac{3}{4}q\ell - \frac{3}{2}q\ell = -\frac{3}{4}q\ell$

$T(z = 2\ell) = \frac{3}{4}q\ell - 2q\ell = -\frac{5}{4}q\ell$

$T = 0 = \frac{3}{4}q\ell - qz \rightarrow z = \frac{3}{4}\ell$

$M(z) = \frac{3}{4}q\ell z - q\frac{z^2}{2}$ T lineare $\rightarrow$ M parabolico

$M(z=0) = 0$

$M(z = \frac{3}{2}\ell) = M_B = \frac{9}{8}q\ell^2 - \frac{9}{8}q\ell^2 = 0$ ok!

$M(z = 2\ell) = \frac{3}{2}q\ell^2 - 2q\ell = -q\frac{\ell^2}{2}$

Quindi per $z = \frac{3}{4}\ell$ $(T=0)$ il momento ha un punto d'estremo con tangente orizzontale

$$M\left(z=\tfrac{3}{4}\ell\right) = \frac{9}{16} q\ell^2 - \frac{9}{32} q\ell^2 = \frac{9}{32} q\ell^2$$

- TRAVE ED

— $N = \text{cost} = 0$

— $T(z) = \text{cost} = q\frac{\ell}{4}$

— $M(z) = q\frac{\ell}{4} z$

T costante → M lineare

$$M(z=0) = 0$$

$$M\left(z=\ell/2\right) = q\frac{\ell^2}{8}$$

- TRAVE CD

$N = \text{cost} = \frac{5}{4} q\ell$

$T(z') = \text{cost} = q\frac{\ell}{4}$

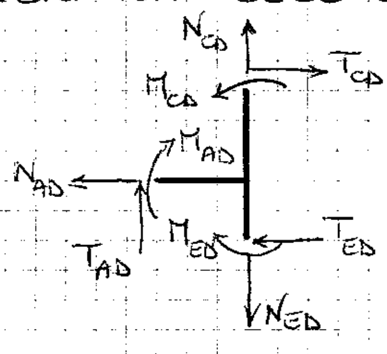
$M(z') = -q\frac{\ell}{4} z'$

T costante → M lineare

$$M(z'=0) = 0 = M_C$$

$$M\left(z' = \tfrac{3}{2}\ell\right) = -\frac{3}{8} q\ell^2$$

- VERIFICA DELL'EQUILIBRIO DEL NODO

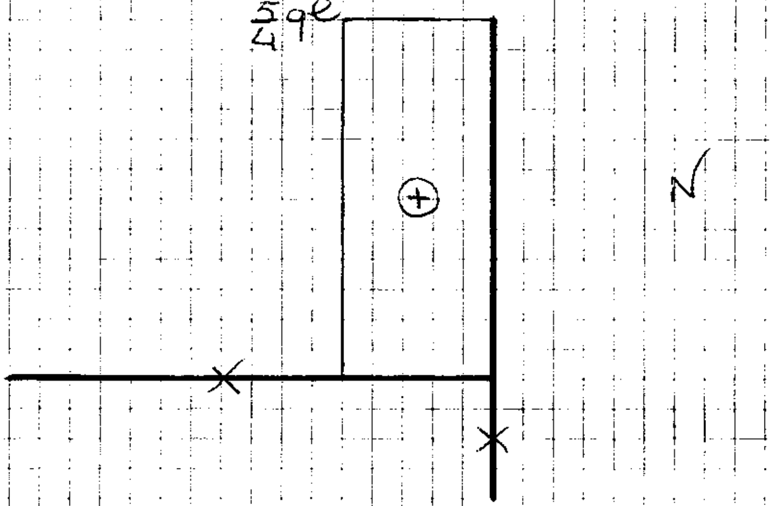


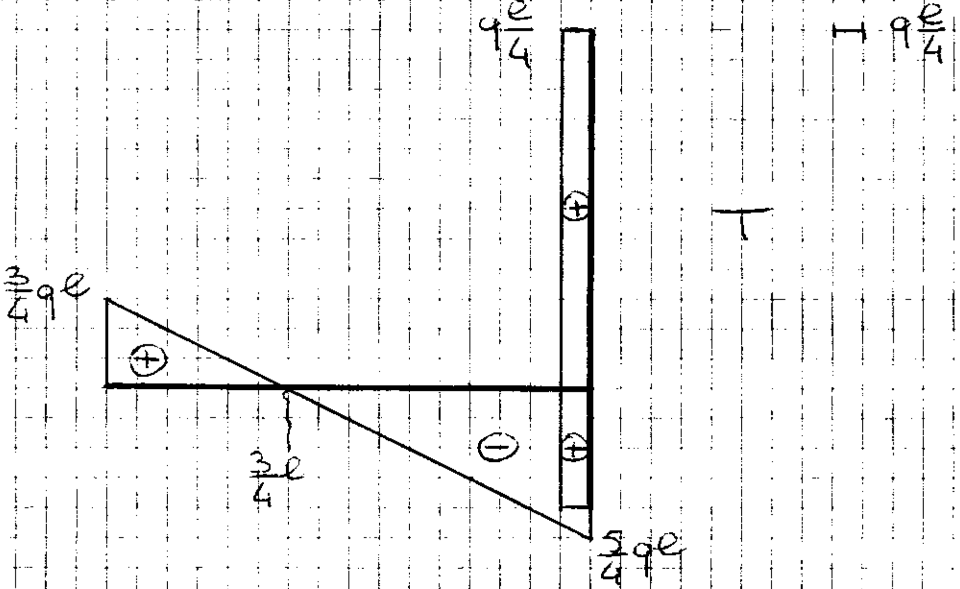
$$\begin{cases} N_{CD} + T_{AD} - N_{ED} \overset{?}{=} 0 \\ \frac{5}{4} q\ell - \frac{5}{4} q\ell + 0 = 0 \quad \text{ok!} \end{cases}$$

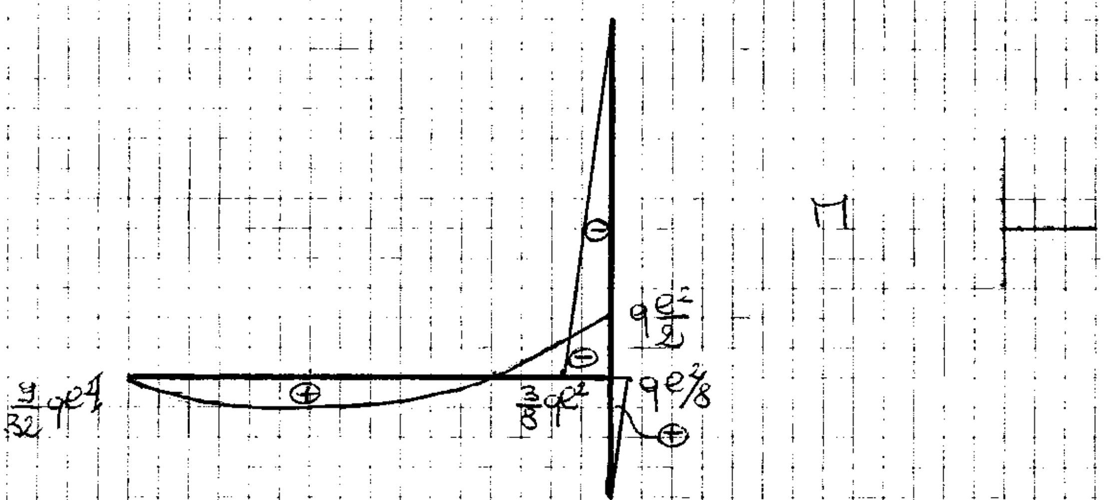
$$\begin{cases} -N_{AD} + T_{CD} - T_{ED} \overset{?}{=} 0 \\ 0 + q\frac{\ell}{4} - q\frac{\ell}{4} = 0 \quad \text{ok!} \end{cases}$$

$$\begin{cases} M_{CD} - M_{AD} - M_{ED} \overset{?}{=} 0 \\ -\frac{3}{8} q\ell^2 - \left(-q\frac{\ell}{2}\right) - q\frac{\ell^2}{8} = 0 \quad \text{ok!} \end{cases}$$

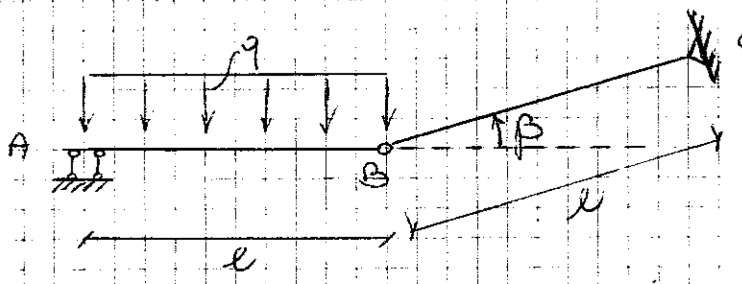
- DIAGRAMMI DELLE CARATTERISTICHE DI SOLLECITAZIONE

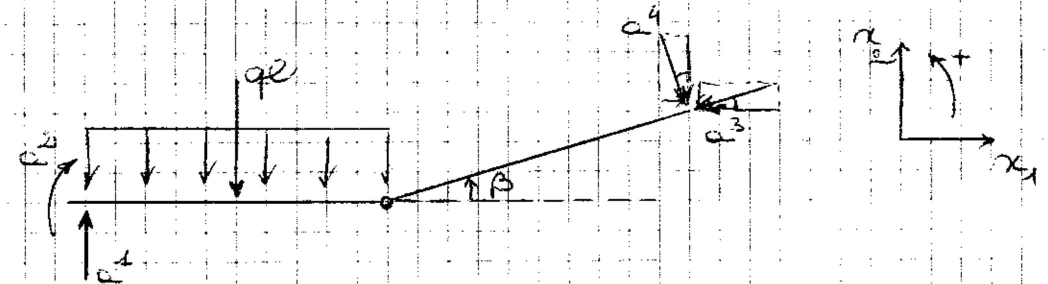






ESERCIZIO N° 2





- EQUAZIONI CARDINALI DELLA STATICA

$$\begin{cases} x_1: & a^4 \operatorname{sen}\beta - a^3 \cos\beta = 0 \\ x_2: & a^1 - ql - a^3 \operatorname{sen}\beta - a^4 \cos\beta = 0 \\ M(C): & -a^2 - a^1 (l + l\cos\beta) + ql\left(\frac{l}{2} + l\cos\beta\right) = 0 \end{cases}$$

- EQUAZIONE AUSILIARIA $\quad M(B) = 0$

$-a^4 l = 0 \qquad$ (parte di destra)

$a^4 = 0$

NOTA: alternativamente si poteva subito osservare che la trave BC è una bella e tranne solo sforzo normali $\rightarrow$ $a^4 = 0$

Riscrivo il sistema:

$$\begin{cases} -a^3 \cos\beta = 0 \\ a^1 - ql - a^3 \operatorname{sen}\beta = \\ -a^2 - a^1 (l + l\cos\beta) + ql\left(\frac{l}{2} + l\cos\beta\right) = 0 \end{cases}$$

In forma matriciale:

$$\underbrace{\begin{pmatrix} 0 & 0 & -\cos\beta \\ 1 & 0 & -\operatorname{sen}\beta \\ -(\ell+\ell\cos\beta) & -1 & 0 \end{pmatrix}}_{B} \begin{bmatrix} a^1 \\ a^2 \\ a^3 \end{bmatrix} = \begin{bmatrix} 0 \\ q\ell \\ -q\ell\left(\frac{\ell}{2}+\ell\cos\beta\right) \end{bmatrix}$$

$\det(B) = -\cos\beta\,(-1) = \cos\beta$

$\det(B) = 0 \iff \cos\beta = 0 \iff \beta = \frac{\pi}{2}, \frac{3}{2}\pi$

- Se $\beta = \frac{\pi}{2}, \frac{3}{2}\pi$, $\det(B) = 0$ $\quad \rho(B) = 2$

Il sistema diventa $(\beta = \pi/2) \rightarrow$ Il sistema è IMPOSSIBILE per una generica condizione di carico

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ -\ell & -1 & 0 \end{pmatrix} \begin{bmatrix} a^1 \\ a^2 \\ a^3 \end{bmatrix} = \begin{bmatrix} 0 \\ q\ell \\ -q\frac{\ell^2}{2} \end{bmatrix}$$

$\rho(B|-\tau) = 2$ perché la prima riga è sempre nulla

$\rho(B) = \rho(B|-\tau) = 2 < 3$

Il sistema è INDETERMINATO (∞^{3-2}) per la particolare condizione di carico

- Se $\beta \neq \frac{\pi}{2}, \frac{3}{2}\pi \longrightarrow \rho(B) = \rho(B|-\tau) = 3$

Il sistema è ISOSTATICO