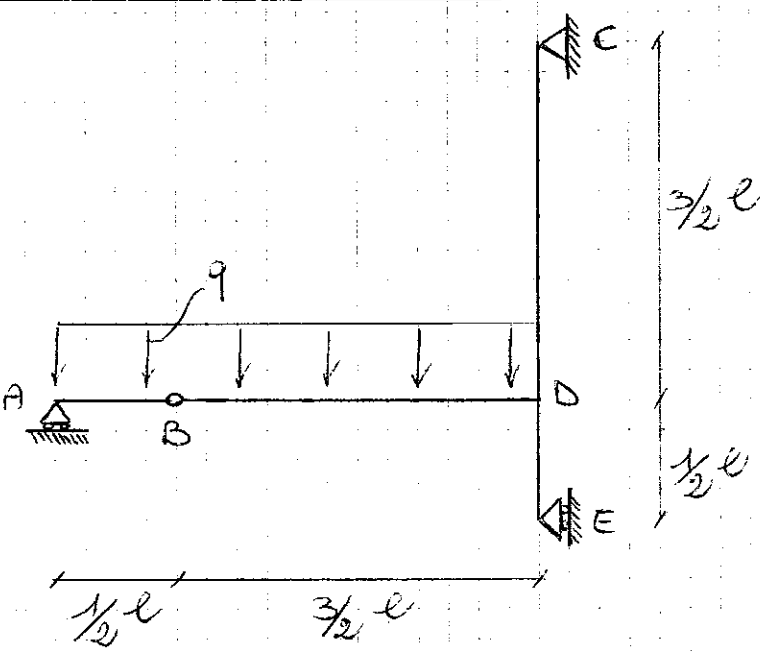
I COMPITINO DI SCIENZA DELLE COSTRUZIONI - CORREZIONE

FILA A

ESERCIZIO N° 1



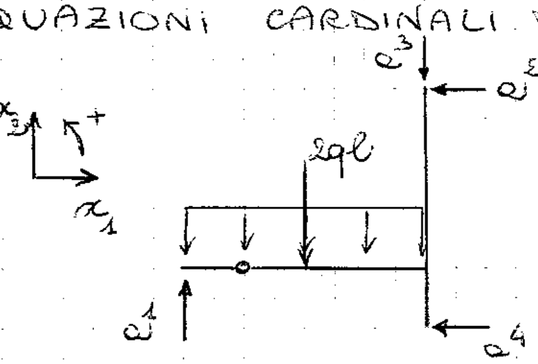
4 = N° vincoli semplici

1 = N° sconnessioni

$s = 3 + 1 = 4$

Il sistema è ISOSTATICO

- EQUAZIONI CARDINALI DELLA STATICA



$$
\begin{cases}
x_1: & -a^2 - a^4 = 0 \\
x_2: & a^1 - a^3 - 2ql = 0 \\
M(C): & -a^1 \cdot 2l + 2ql \cdot l - a^4 \cdot 2l = 0
\end{cases}
$$

- EQUAZIONE AUSILIARIA $M(B) = 0$

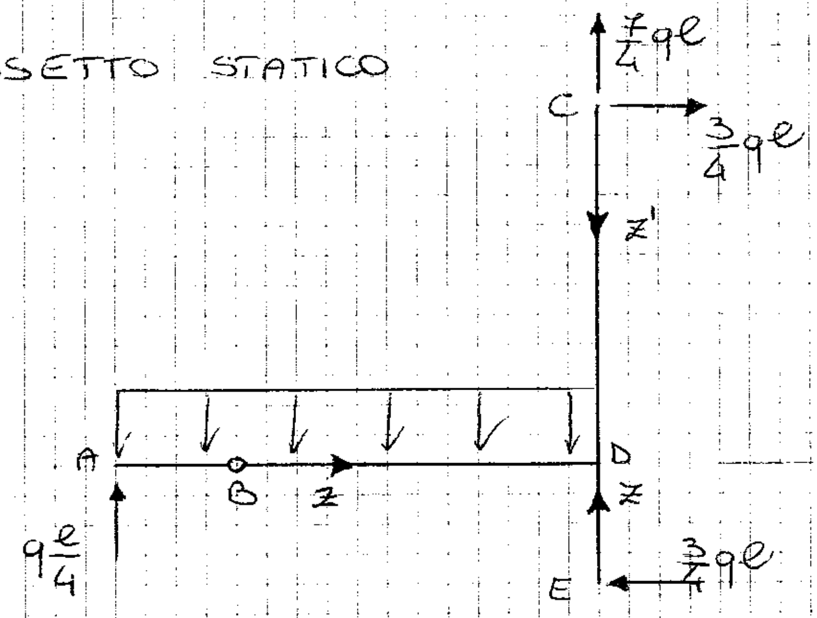
$$-a^1 \cdot \cancel{\frac{l}{2}} + q\frac{l}{\cancel{2}} \cdot \frac{l}{4} = 0 \qquad \text{(parte di sinistra)}$$

Sistema di 4 equazioni in 4 incognite

$$
\begin{cases}
-a^2 - a^4 = 0 \\
a^1 - a^3 - 2ql = 0 \\
-a^1 \cdot \cancel{2l} + \cancel{2}ql^{\cancel{2}} - a^4 \cdot \cancel{2l} = 0 \\
-a^1 + q\frac{l}{4} = 0
\end{cases}
\qquad
\begin{cases}
a^2 = -a^4 \\
a^3 = a^1 - 2ql = q\frac{l}{4} - 2ql \\
a^4 = -a^1 + ql = -q\frac{l}{4} + ql \\
a^1 = q\frac{l}{4}
\end{cases}
$$

$$\begin{cases} a^2 = -\frac{3}{4} q\ell \\ a^3 = -\frac{7}{4} q\ell \\ a^4 = \frac{3}{4} q\ell \\ a^1 = q\frac{\ell}{4} \end{cases}$$

- ASSETTO STATICO



- TRAVE AD

$N = \text{cost} = 0$

$T(z) = q\frac{\ell}{4} - qz$ carico distribuito → T lineare

$T(z=0) = q\ell/4$

$T(z=\ell/2) = q\frac{\ell}{4} - q\frac{\ell}{2} = -q\frac{\ell}{4}$

$T(z=2\ell) = q\frac{\ell}{4} - 2q\ell = -\frac{7}{4}q\ell$

$T = 0 = q\frac{\ell}{4} - qz \longrightarrow z = \frac{\ell}{4}$

$M(z) = q\frac{\ell}{4}z - q\frac{z^2}{2}$ T lineare → M parabolico

$M(z=0) = 0$

$M(z=\ell/2) = M_B = q\frac{\ell^2}{8} - q\frac{\ell^2}{8} = 0$ ok!

$M(z=2\ell) = q\frac{\ell}{4}\cdot 2\ell - q\frac{4\ell^2}{2} = -\frac{3}{2}q\ell^2$

Inoltre per $z = \frac{\ell}{4}$ $(T=0)$ il momento ha un punto d'estremo con tangente orizzontale.

$$M\left(z=\frac{\ell}{4}\right) = q\frac{\ell^2}{16} - q\frac{\ell^2}{32} = \frac{q\ell^2}{32}$$

- TRAVE ED

  - $N = \text{cost} = 0$
  - $T(z) = \frac{3}{4}q\ell = \text{cost}$
  - $M(z) = \frac{3}{4}q\ell z$ $\quad$ T costante $\rightarrow$ M lineare

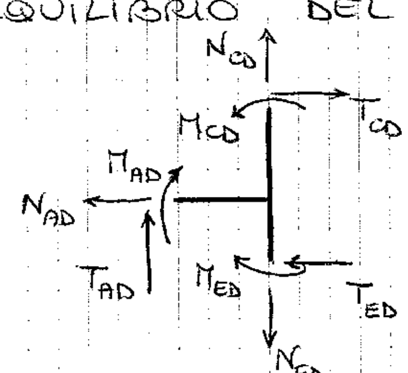
$$M(z=0) = 0 = M_E$$
$$M\left(z=\ell/2\right) = \frac{3}{8}q\ell^2$$

- TRAVE CD

  - $N = \text{cost} = \frac{7}{4}q\ell$
  - $T(z') = \text{cost} = \frac{3}{4}q\ell$
  - $M(z') = -\frac{3}{4}q\ell z'$ $\quad$ T costante $\rightarrow$ M lineare

$$M(z'=0) = 0 = M_C$$
$$M\left(z'=\frac{3}{2}\ell\right) = -\frac{9}{8}q\ell^2$$

- EQUILIBRIO DEL NODO - VERIFICA

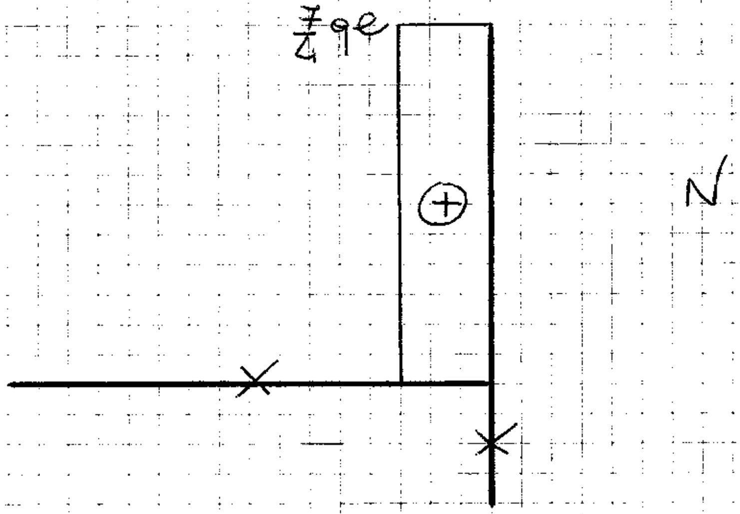


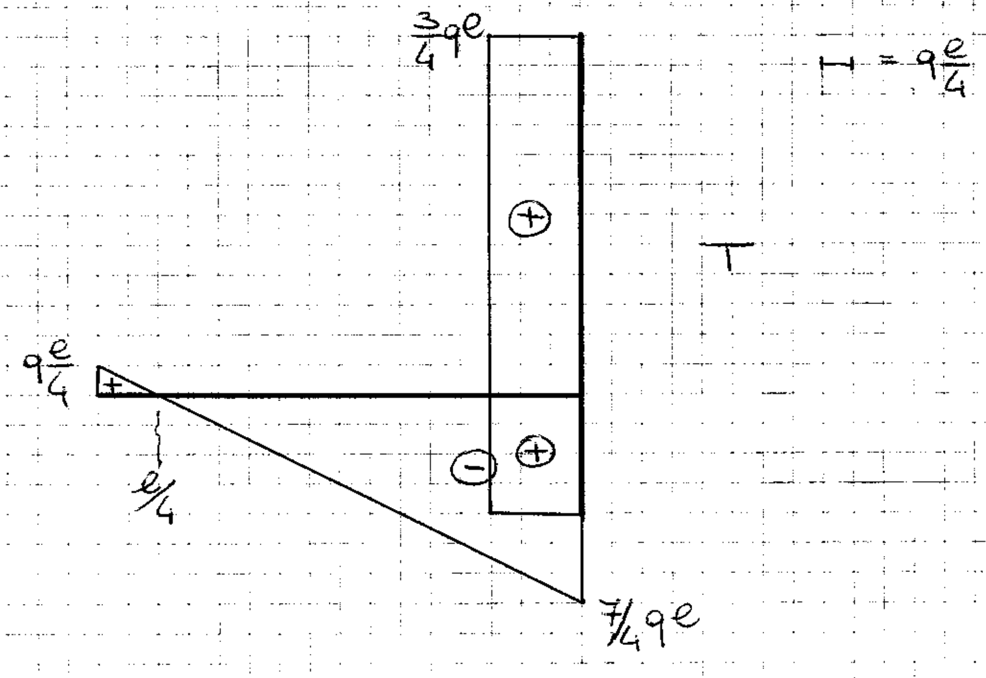
$$\begin{cases} N_{CD} + T_{AD} - N_{ED} \overset{?}{=} 0 \\ \frac{7}{4}q\ell - \frac{7}{4}q\ell + 0 = 0 \quad \text{ok!} \end{cases}$$

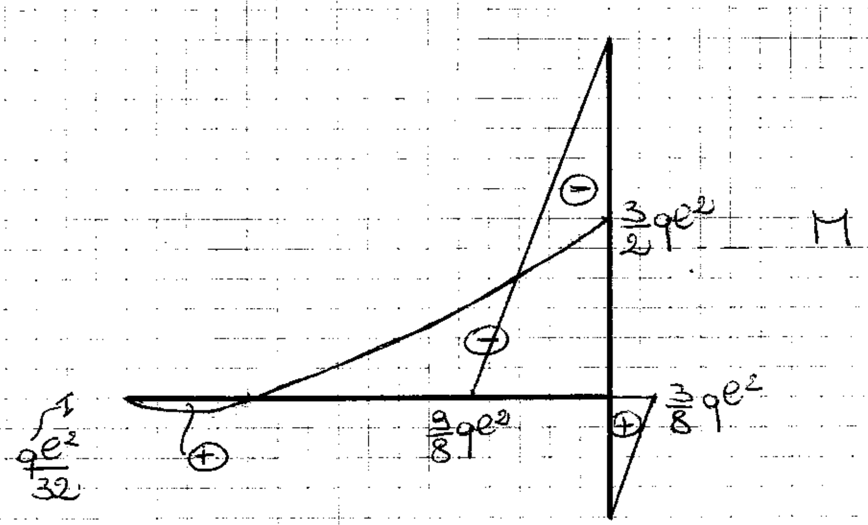
$$\begin{cases} -N_{AD} + T_{CD} - T_{ED} \overset{?}{=} 0 \\ 0 + \frac{3}{4}q\ell - \frac{3}{4}q\ell = 0 \quad \text{ok!} \end{cases}$$

$$\begin{cases} M_{CD} - M_{AD} - M_{ED} \overset{?}{=} 0 \\ -\frac{9}{8}q\ell^2 - \left(-\frac{3}{2}q\ell^2\right) - \frac{3}{8}q\ell^2 = 0 \quad \text{ok!} \end{cases}$$

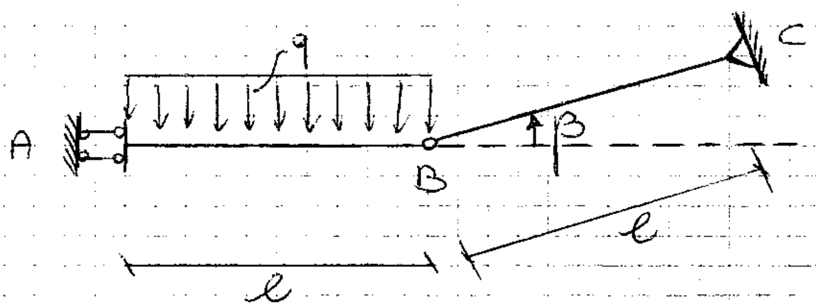
- DIAGRAMMI DELLE CARATTERISTICHE DI SOLLECITAZIONE

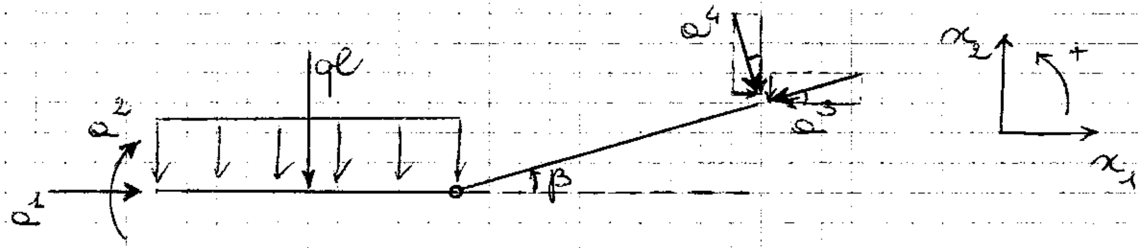






ESERCIZIO N° 2





- EQUAZIONI CARDINALI DELLA STATICA

$$
\begin{cases}
x_1: & a^1 + a^4 \operatorname{sen}\beta - a^3 \cos\beta = 0 \\
x_2: & -q\ell - a^4 \cos\beta - a^3 \operatorname{sen}\beta = 0 \\
M(C): & a^1 \cdot \ell \operatorname{sen}\beta - a^2 + q\ell\left(\frac{\ell}{2} + \ell\cos\beta\right) = 0
\end{cases}
$$

- EQUAZIONE AUSILIARIA $M(B) = 0$

$-a^4 \cdot \ell = 0$ (parte di destra)

$a^4 = 0$

NOTA: alternativamente si poteva osservare che la trave BC è una biella e trasmette solo sforzo normale $\rightarrow a^4 = 0$

Riscrivo il sistema:

$$
\begin{cases}
a^1 - a^3 \cos\beta = 0 \\
+q\ell + a^3 \operatorname{sen}\beta = 0 \\
a^1 \ell \operatorname{sen}\beta - a^2 + q\frac{\ell^2}{2} + q\ell^2\cos\beta = 0
\end{cases}
$$

In forma matriciale:

$$\begin{pmatrix} 1 & 0 & -\cos\beta \\ 0 & 0 & \operatorname{sen}\beta \\ l\operatorname{sen}\beta & 1 & 0 \end{pmatrix} \begin{bmatrix} a^1 \\ a^2 \\ a^3 \end{bmatrix} = \begin{bmatrix} 0 \\ -ql \\ -q\frac{l^2}{2} - ql^2\cos\beta \end{bmatrix}$$

$$B$$

$$\det(B) = 1\,(-\operatorname{sen}\beta) + l\operatorname{sen}\beta\,(\cdot 0) = -\operatorname{sen}\beta$$

$$\det(B) = 0 \iff \operatorname{sen}\beta = 0 \iff \beta = 0, \pi$$

- Se $\beta = 0, \pi$, $\det(B) = 0$ $\quad \rho(B) = 2$

Il sistema diventa: $(\beta = 0) \rightarrow$ il sistema è IMPOSSIBILE per una generica condizione di carico

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{bmatrix} a^1 \\ a^2 \\ a^3 \end{bmatrix} = \begin{pmatrix} 0 \\ ql \\ -\frac{3}{2}ql^2 \end{pmatrix}$$

$\rho(B|-\tau) = 3$ poiché isolando per esempio la prima, seconda colonna di B e aggiungendo la colonna dei termini noti, $\det \neq 0$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & ql \\ 0 & 1 & -\frac{3}{2}ql^2 \end{vmatrix} = -ql \neq 0 \rightarrow \rho(B|-\tau) = 3$$

Se $\underline{\beta = 0, \pi} \longrightarrow \rho(B) \neq \rho(B|-\tau)$

Per tale condizione di carico il sistema è IMPOSSIBILE

- Se $\underline{\beta \neq 0, \pi} \longrightarrow \rho(B) = 3 = \rho(B|-\tau)$

Il sistema è ISOSTATICO