Modeling the interaction of biomimetical slender structures with a fluid flow.

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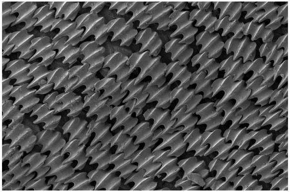
Motivations

Aim of the work is to explore numerically how different structural parameters (mass, bending stiffness and permeability) affect the dynamics of biological tissues when exposed to fluid flows.

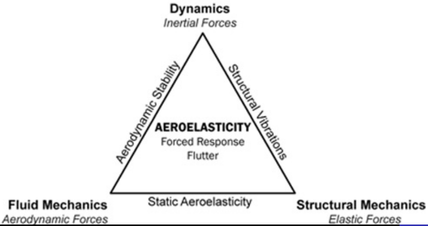


Based on a biomimetic approach, we seek new control strategies based on the introduction of new surface coatings inspired by the natural world.

Aeroelasticity

. . . is the study of the mutual interaction that takes place within the triangle of the inertial, elastic, and aerodynamic forces acting on structural members exposed to an airstream.

A. R. Collar, 1947



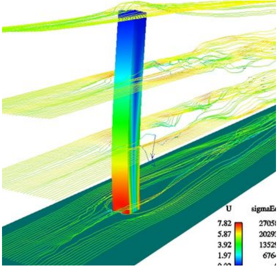


Figure : Deformation of a cantilever subject to a uniform flow, Hrvoje Jasak©

Flutter

Flutter is a dynamic instability of an elastic structure subject to a fluid flow, caused by resonance between the body's deflection and the forcing exerted by the fluid flow.

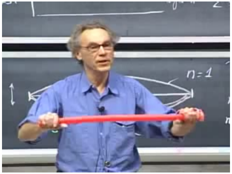
A wide spectrum of forcing frequencies are applied by the flow to the body, which begins to resonate at its own natural frequency.

In system dynamics, flutter is interpreted as a limit cycle.

Flutter

Flutter can be a destructive phoenomena when applied to materials since it leads to failure due to large deformations or to fatigue.

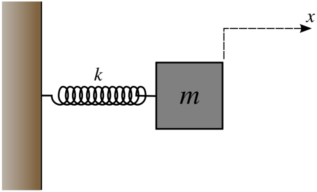
However, resonance does not always bring to destructive phoenomena: for instance, it enables musical instruments to work!



Resonance, an example

Unforced system

$$m\ddot{x} = -kx$$



$$x = A cos(\omega_N t)$$

$$\ddot{x} = -A\omega_N^2 cos(\omega_N t)$$

$$-Am\omega^2 cos(\omega_N t) = -kA cos(\omega_N t)$$

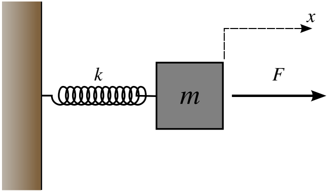
$$m\omega_N^2 = k$$

$$\omega_N = \sqrt{\frac{k}{m}}$$

Resonance, an example

Forced system

$$m\ddot{x} = F - kx$$



$$F = F_0 cos(\omega t)$$

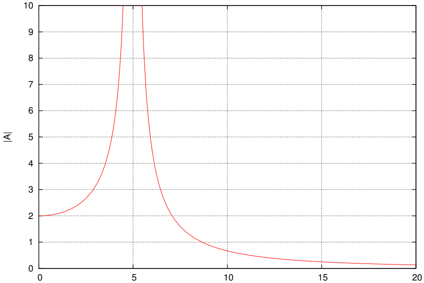
$$x = Acos(\omega t)$$

$$\ddot{x} = -A\omega^2 cos(\omega t)$$

$$-Am\omega^2 cos(\omega t) = F_0 cos(\omega t) - kAcos(\omega t)$$

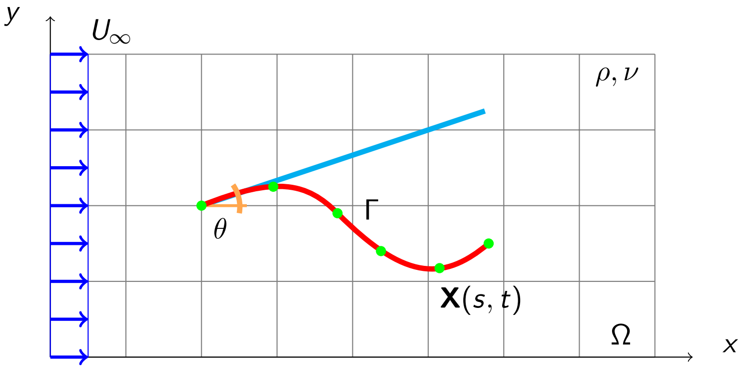
$$-Am\omega^2 = F_0 - kA$$

$$A = \frac{F_0}{(k - m\omega^2)} = \frac{F_0}{m(\omega_N^2 - \omega^2)}$$



The flag-in-the-wind problem

The model



Ruling equations

For the viscous incompressible fluid

$$\begin{cases} \dfrac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \dfrac{1}{Re} \nabla^2 \mathbf{u} \\ \nabla \cdot \mathbf{u} = 0 \end{cases} ,$$

for the slender 1D structure

$$\rho_1 \frac{\partial^2 \mathbf{X}}{\partial t^2} = \frac{\partial}{\partial s}\left(T \frac{\partial \mathbf{X}}{\partial s} \right) - \frac{\partial^2}{\partial s^2}\left(K_b \frac{\partial^2 \mathbf{X}}{\partial s^2} \right) + \rho_1 \mathbf{g} - \mathbf{F}$$

Dimensionless parameter

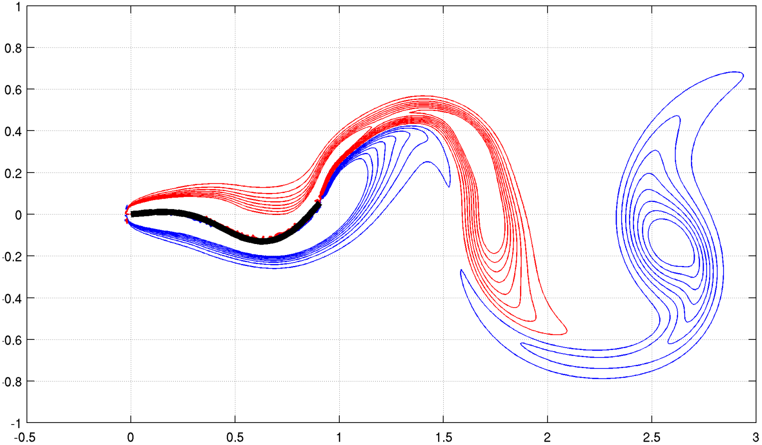
The ruling equations can be non-dimensionalized with the following characteristic scales:

$$x^* = \frac{x}{L}, \mathbf{u}^* = \frac{\mathbf{u}}{U_\infty}, \mathbf{f}^* = \frac{\mathbf{f}L}{\rho_0 U_\infty^2}, \mathbf{F}^* = \frac{\mathbf{F}L}{\rho_1 U_\infty^2}$$

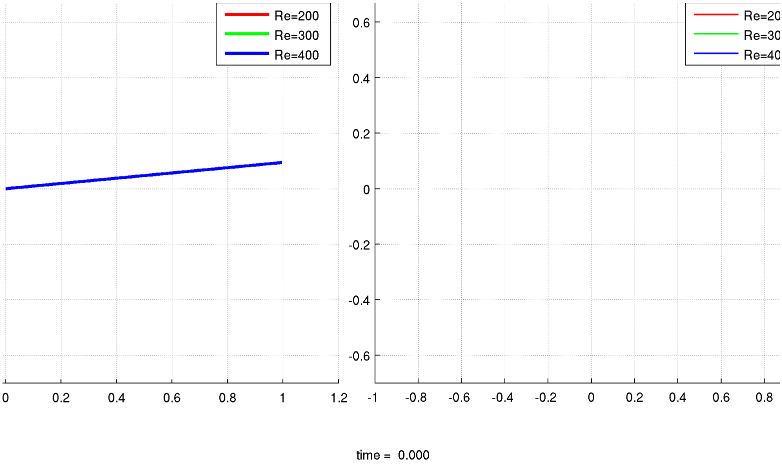
Doing so, several dimensionless parameters arises:

$$Re = \frac{U_\infty L}{\nu}, \quad Fr = \frac{gL}{U_\infty^2}, \quad \rho = \frac{\rho_1}{\rho_0 L}, \quad \gamma = \frac{K_b}{\rho_1 U_\infty^2 L^2}$$

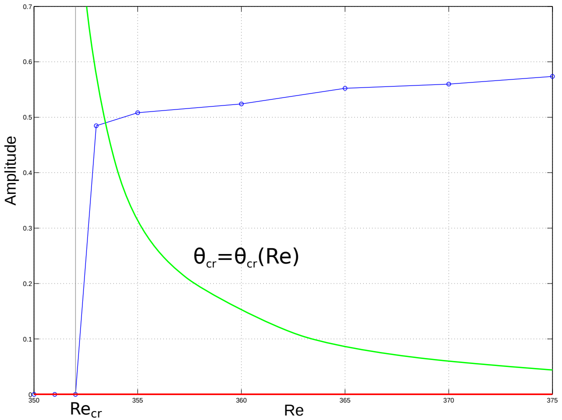
Impermeable filament



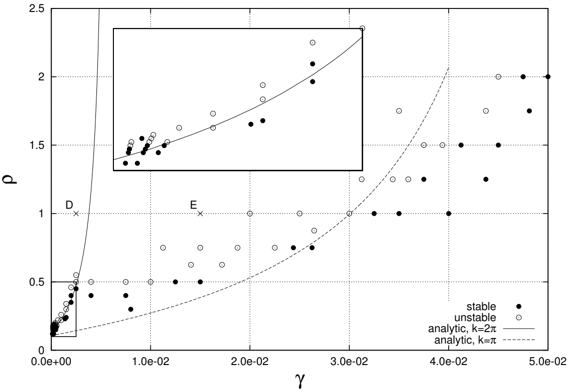
Bistabiilty



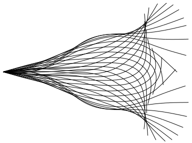
Bistabiilty



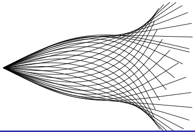
Linear stability analysis



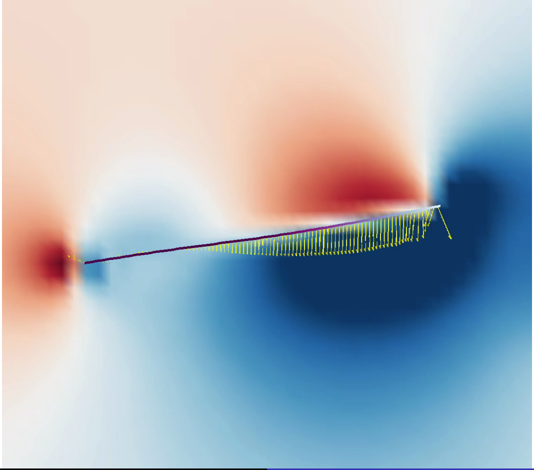
Case D



Case E



Permeable filament



How to model porosity?

Velocity-based approach by allowing a relative slip in the normal direction between the IB and the surrounding flow[1], given by:

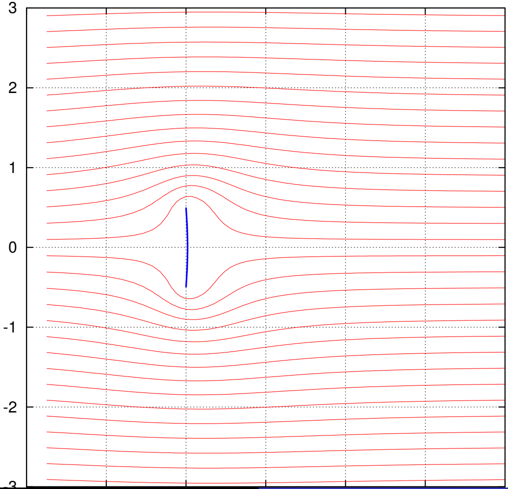
$$\mathbf{u}_p(s,t) = \mathbf{u}_{\mathbf{int}}(s,t) + \lambda(\mathbf{F}(s,t)\cdot\mathbf{n})\mathbf{n}$$

Force-based approach by decreasing the force extert on the structure in the normal direction:

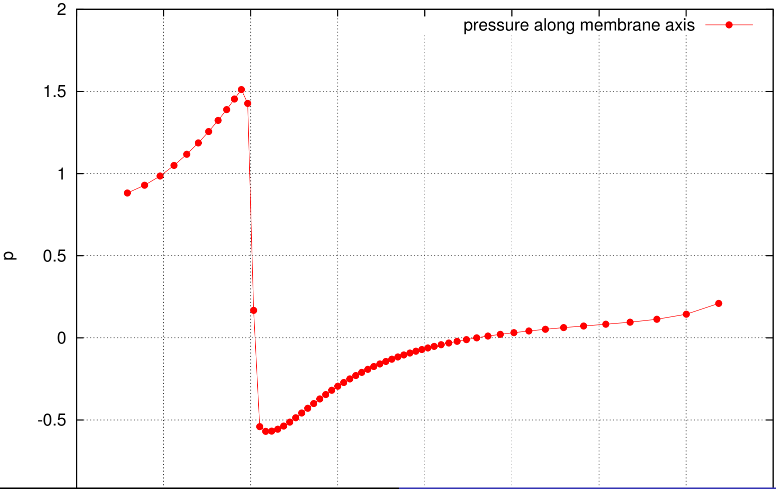
$$\mathbf{F}_p(s,t) = (1-\lambda)(\mathbf{F}(s,t)\cdot\mathbf{n})\mathbf{n} + (\mathbf{F}(s,t)\cdot\boldsymbol{\tau})\boldsymbol{\tau}$$

[1] Kim, Y., and Peskin, C. S., in *SIAM JSC*, **28** (6), 2294-2312 (2006).

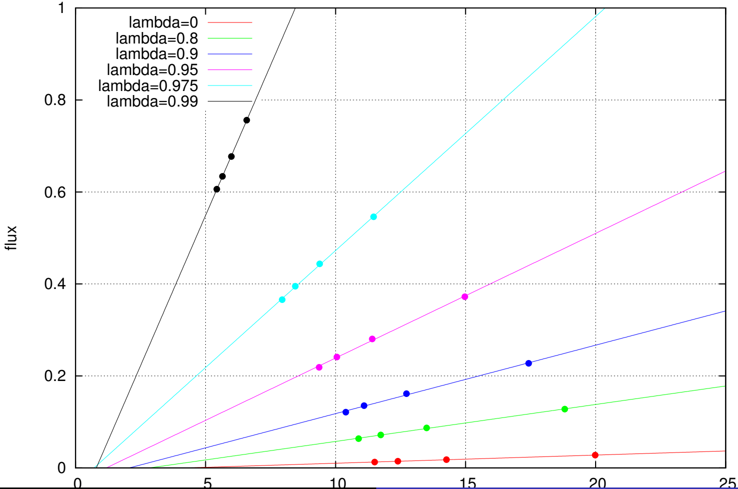
Darcy's benchmark



Darcy's benchmark



Darcy's benchmark



k-λ mapping

Darcy's law $\mathbf{U} - \mathbf{U}_{ib} = -k\nabla p$

Goldstein's feedback law $\mathbf{F} = \beta(\mathbf{U} - \mathbf{U}_{ib})$

$$\Rightarrow \frac{F}{\beta} = -k\nabla p$$

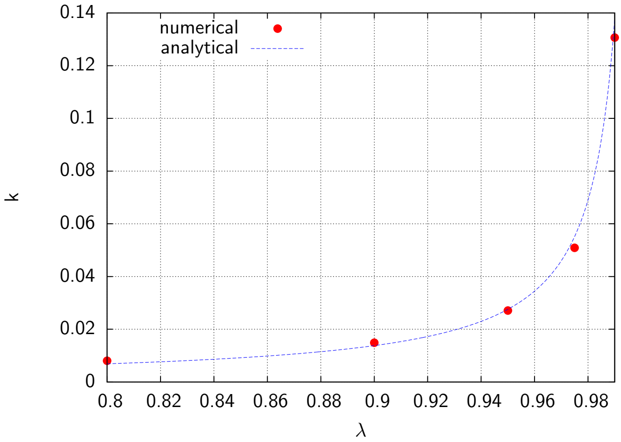
In our case

$$\frac{\partial p}{\partial x} \sim \frac{F_P}{\delta} = \frac{F(1-\lambda)}{\delta}$$

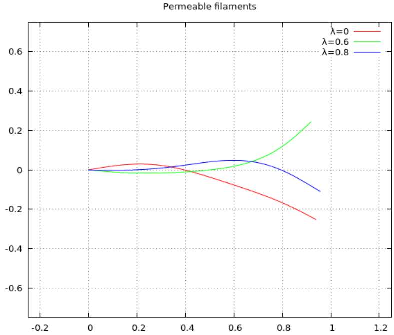
and by exchanging the normal direction $\mathbf{n}$ with the x-direction one obtains

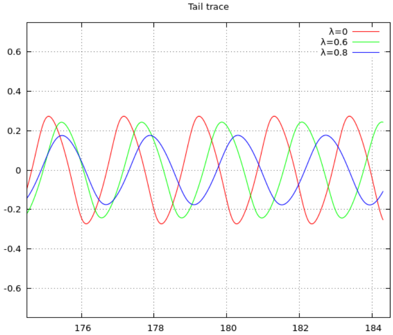
$$k = -\frac{\delta}{\beta(1-\lambda)}$$

k-λ mapping

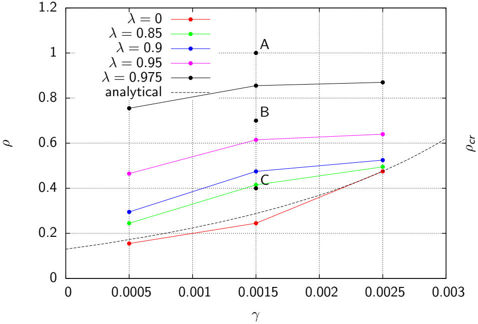


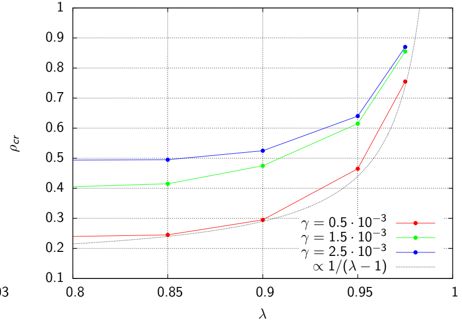
Permeable filament





Permeable filament





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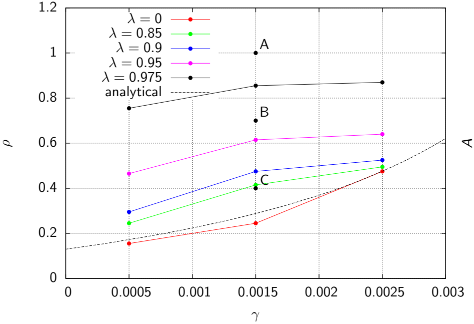
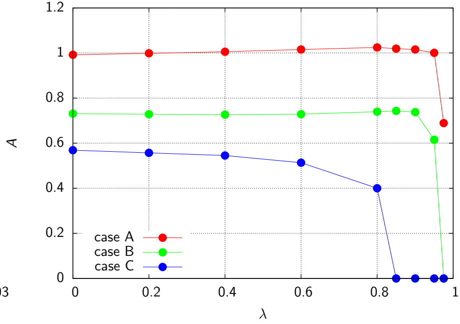
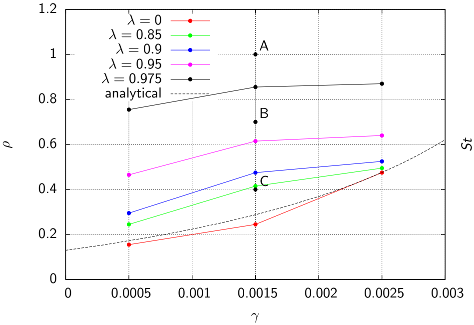
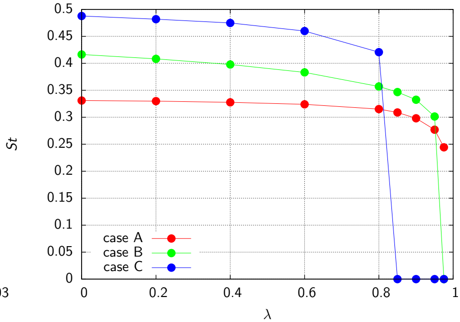


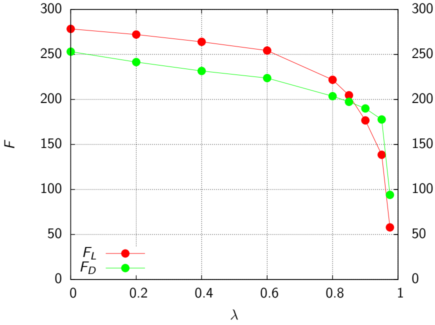


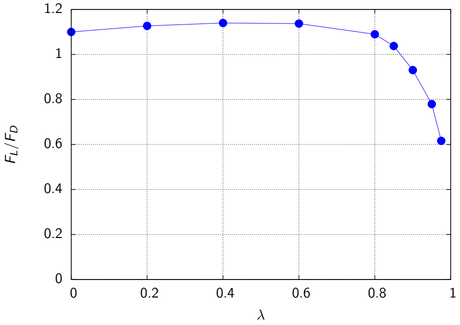
Permeable filament





Permeable filament





Permeable time-scale

We define the permeable time as the characteristic time needed by mass to cross the membrane of thickness δ. Following Darcy's empirical law $\mathbf{U} - \mathbf{U}_{ib} = -k\nabla p$,

$$\tau_{por} = \frac{\delta}{k\nabla p} = \frac{\delta^2}{k\Delta p}$$

In order to give a quantitative value for the pressure difference across the membrane, we resort to the slender body theory

$$\Delta p = \rho_a \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial s} \right)^2 h \simeq \rho_a \left(\frac{U}{L} \right)^2 h$$

where $\rho_a = m_a/\rho_0 L$. Thus

$$\tau_{por} = -\frac{\delta L^2 \beta(1-\lambda)}{\rho_a U^2 h}$$

Permeable time-scale

We estimate also the hydrodynamical time-scale as $\tau_{hdr} = L/U$, in order to assess λ critical value to have resonance between permeability and hydrodynamics:

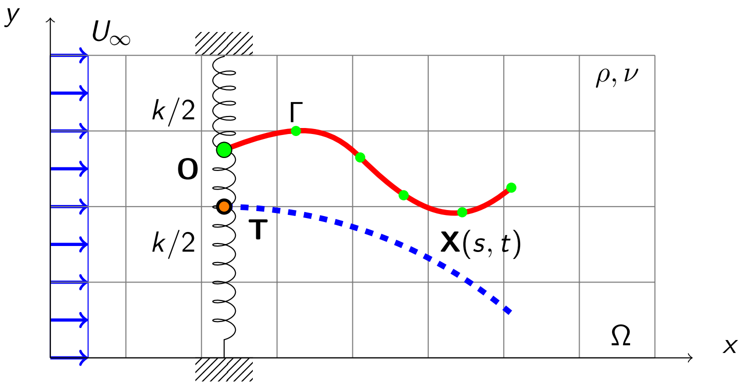
$$\frac{\tau_{por}}{\tau_{hdr}} = -\frac{\delta L^2 \beta(1-\lambda)}{\rho_a U^2 h}\frac{U}{L} = -\frac{\delta L \beta(1-\lambda)}{\rho_a U h} \simeq 1.$$

From this expression we can derive a critical value of λ as

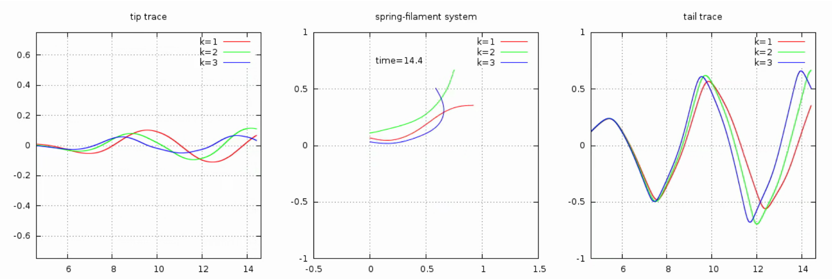
$$\lambda_{crt} \simeq 1 + \frac{\rho_a U h}{\delta L \beta}.$$

If we use the parameters given here the critical value of λ is $\simeq 0.98$.

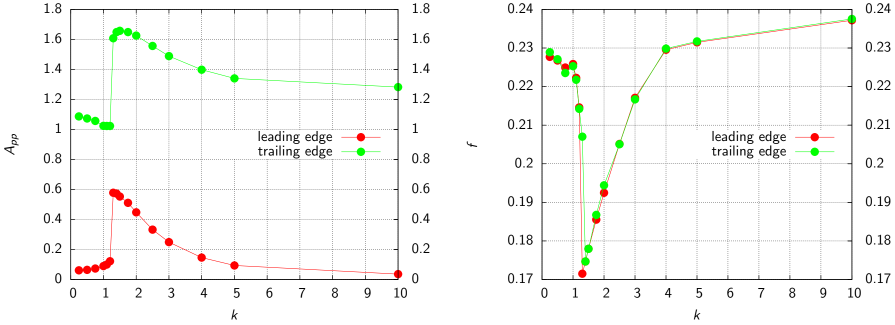
The model



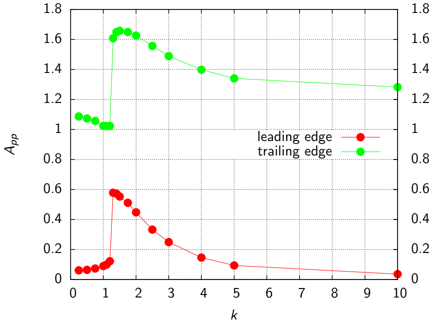
The spring-filament system

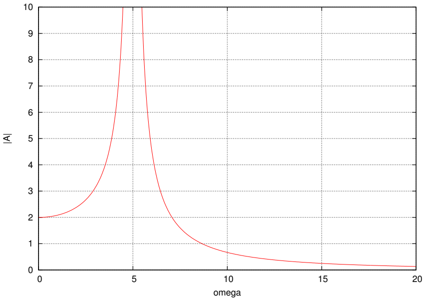


Leading edge oscillations ($Re = 200$)

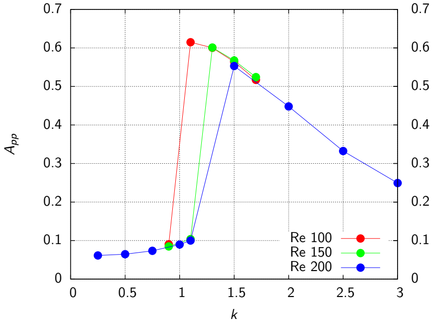


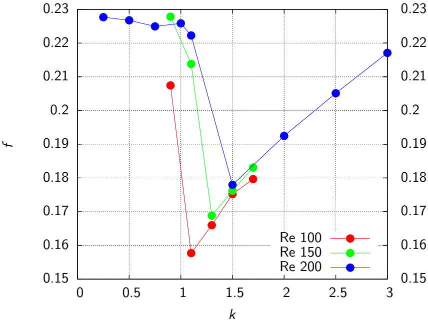
Leading edge oscillations ($Re = 200$)





Leading edge oscillations (different *Re*)





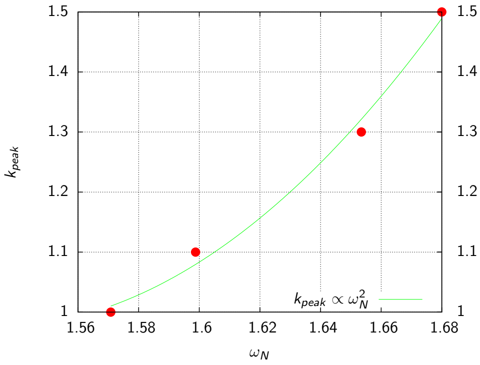
Who said "resonance"?!

Natural pulsatance $\omega_N = \sqrt{\frac{k}{m}} \Rightarrow k \propto \omega_N^2$

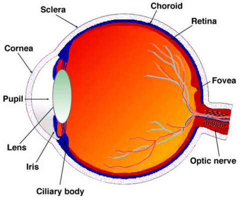
Oscillation amplitude $A = \frac{F_0}{m(\omega_N^2 - \omega_F^2)} \Rightarrow \lim_{\omega_N \to \omega_F} |A| \to \infty$

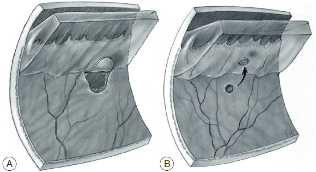
Re	$\omega_F = \omega_N$	k_{peak}
100	1.571	1
125	1.599	1.1
150	1.653	1.3
200	1.680	1.5

Who said “resonance”?!



The retinal detachment





The model

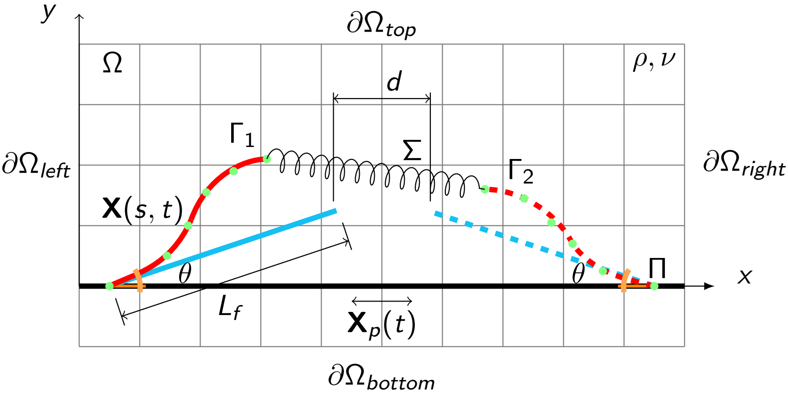
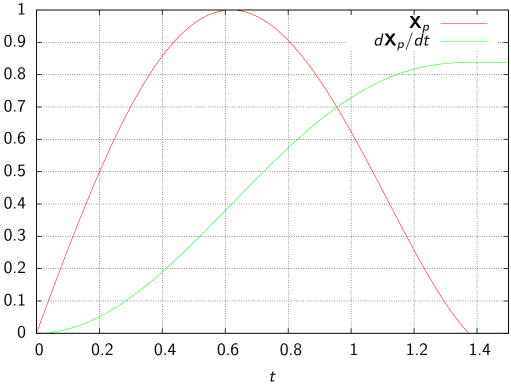


Plate imposed motion



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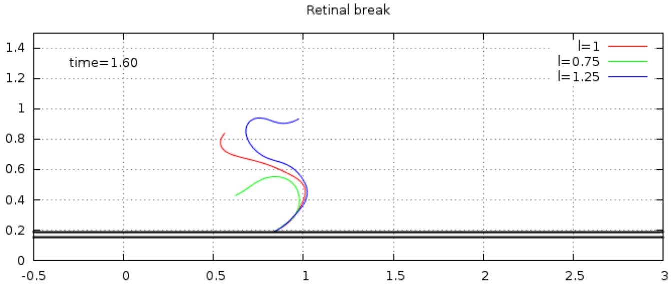
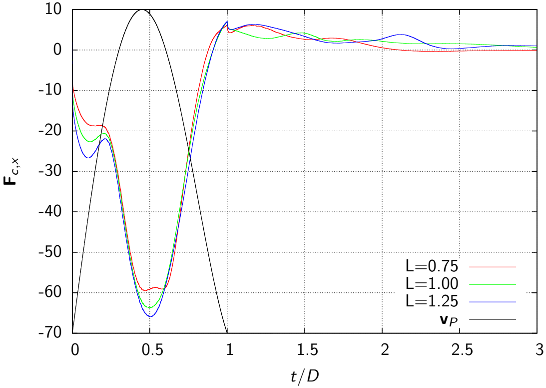
[2]Repetto, R., Stocchino, A., Cafferata, C., in *Phys. Med. Biol.*, **50**, 4729-4743 (2005)

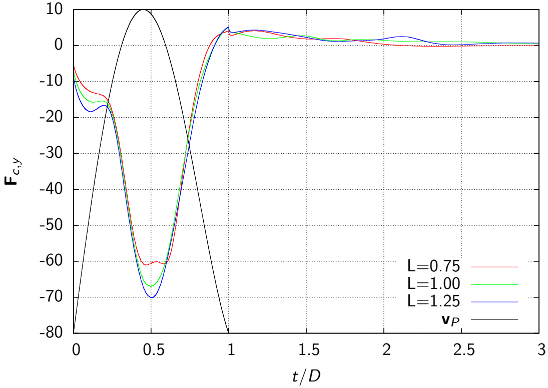
The “tear” case



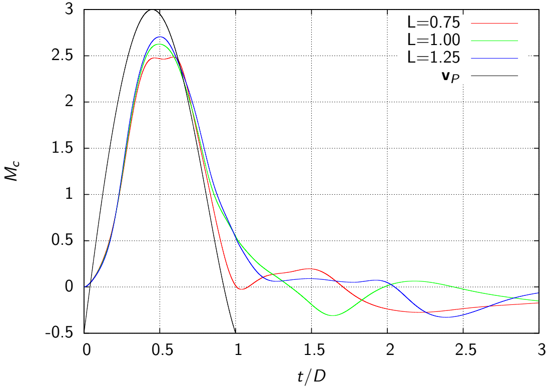
x-component of the clamping force



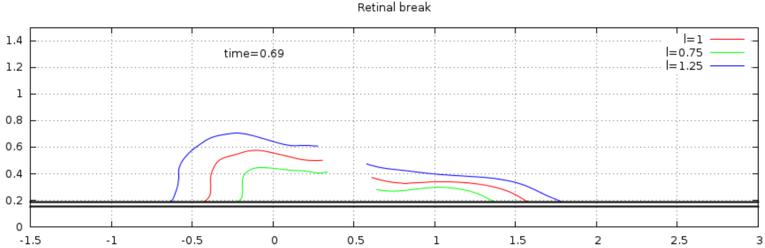
y-component of the clamping force



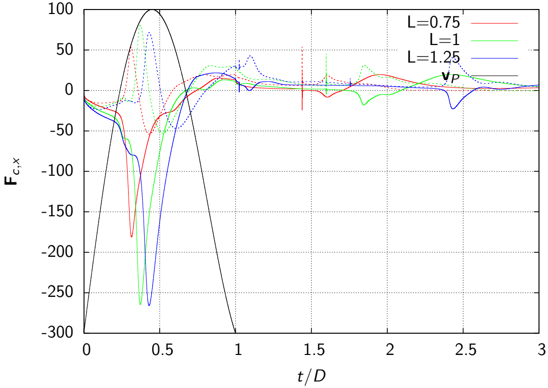
clamping torque



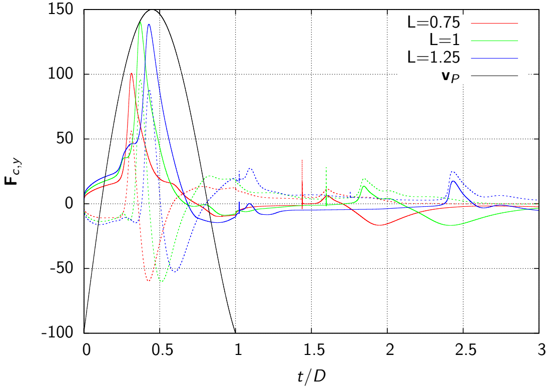
The "hole" case



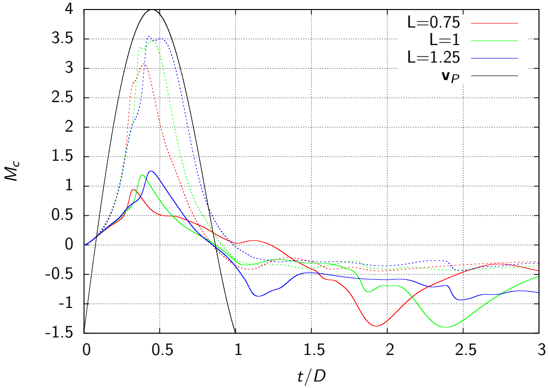
x-component of the clamping force



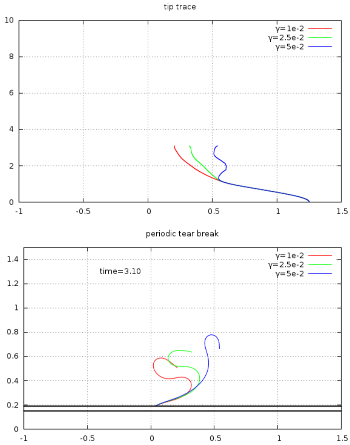
y-component of the clamping force



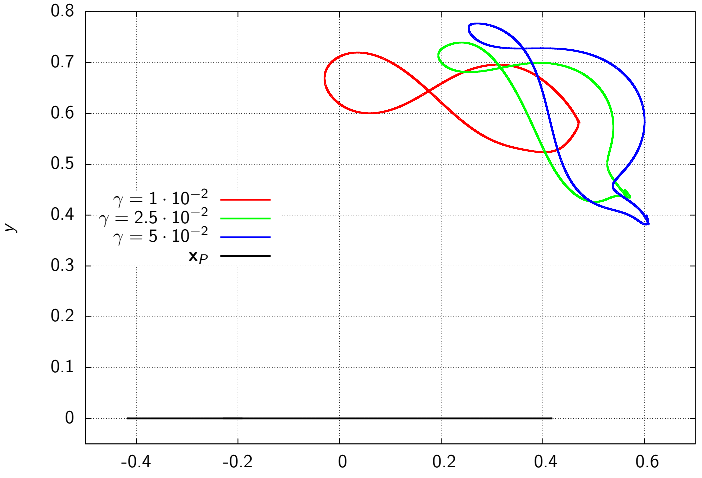
clamping torque



The "periodic tear" case



The "bullwhip" effect



The "bullwhip" effect

You have just seen...

- an attempt to shed light on how different structural parameters of biological surfaces play a role on its overall fluid-dynamical behavior,
- a work inherently related to fluid-structure interaction, the two-way coupling between fluid and structure in terms of both forces and displacements,
- a numerical investigation carried out through a finite volume code developed in the Matlab© environment with an immersed boundary approach,

You have just seen...

- a review of several methodologies so far proposed in the literature with the original contribution of an innovative and numerically stable way to include permeability along with the other parameters,
- a study of permeability as a new control strategy of the fluid-structure interaction by allowing a mass flux and thus the modification of the pressure distribution on the surface,
- a bio-engineering application as the simulation of a common disease of the human eye,
- an optimal design of a spring-filament system inspired by devices used to harvest energy.

What you *might* see in the future...

- addition of the 3-dimensional direction, even if this step would involve a plan to overcome computational limits.,
- implementation of 2-dimensional structures in a 2-dimensional flow by following an Immersed Boundary approach,
- migrate from DNS to LES or RANS with the implementation of suitable turbulence models.


“That's all Folks!”