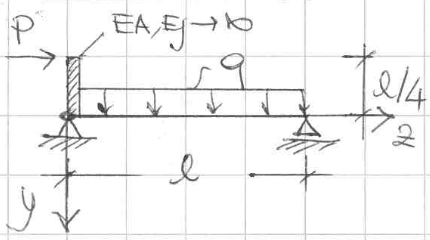
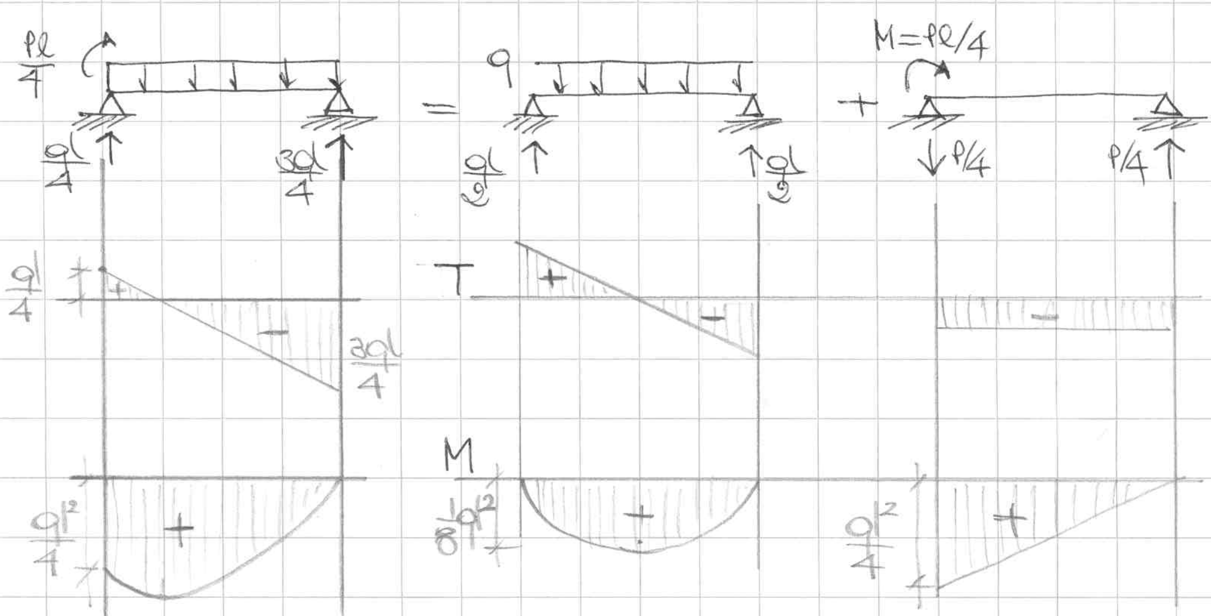
SOLUZIONE FILA A

Esercizio n°1



$P = ql$

risoluzione del sistema isostatico:



$$M(z) = -\frac{qz^2}{2} + \frac{qlz}{4} + \frac{ql^2}{4}$$

Condizioni di vincolo

$v(0)=0 \qquad v(l)=0$

Equazione della linea elastica: $\dfrac{d^2v}{dz^2} = -\dfrac{M(z)}{EJ}$

Dunque si ha:

$$\frac{d^2v}{dz^2} = -\frac{M(z)}{EJ} = \frac{1}{EJ}\left(\frac{qz^2}{2} - \frac{qlz}{4} - \frac{ql^2}{4}\right)$$

Integrando:

$$v(z) = \frac{q}{24EJ}z^4 - \frac{ql}{24EJ}z^3 - \frac{ql^2}{8EJ}z^2 + Az + B$$

$$v(0) = 0 \longrightarrow B = 0$$

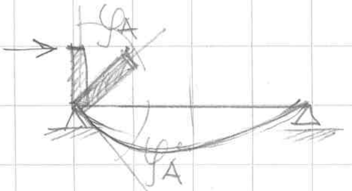
$$v(l) = 0 \longrightarrow A = \frac{ql^3}{8EJ}$$

Da cui:

$$\boxed{v(z) = \frac{q}{24EJ}z^4 - \frac{ql}{24EJ}z^3 - \frac{ql^2}{8EJ}z^2 + \frac{ql^3}{8EJ}z}$$

per definizione $\varphi(z) = -\frac{dv}{dz}$ da cui:

$$\boxed{\varphi_A = \varphi(0) = -\frac{ql^3}{8EJ}}$$



deformata qualitativa

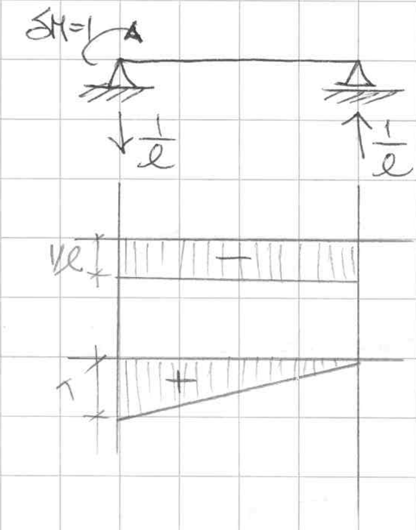
per piccole rotazioni

$$\eta_C = |\varphi_A| \cdot \frac{1}{4}l = \frac{ql^3}{8EJ}\frac{l}{4} = \boxed{\frac{ql^4}{32EJ} = \eta_C}$$

• Verifica di φ_A col Principio dei lavori virtuali.

sistema effettivo

sistema virtuale



$$\delta\mathcal{L}_e = \delta M \cdot \varphi_A$$

$$\delta\mathcal{L}_i = \int_0^l M\,\delta\chi\,dz = \int_0^l \left(-\frac{qz^2}{2} + \frac{qlz}{4} + \frac{ql^2}{4}\right)\left(1-\frac{z}{l}\right)\frac{1}{EJ}\,dz$$

considerando solo il contributo flessionale

$$\delta\chi = \frac{\delta M}{EJ}$$

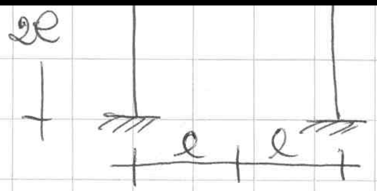
$$\delta\mathcal{L}_i = \frac{1}{8}\frac{ql^3}{EJ}$$

$$\delta\mathcal{L}_e = \delta\mathcal{L}_i$$

$$\delta M \cdot \varphi_A = 1 \cdot \boxed{\varphi_A = \frac{1}{8}\frac{ql^3}{EJ}}$$

CONCORDE COL VERSO DEL MOMENTO VIRTUALE SCELTO

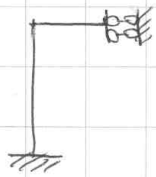
[redacted]



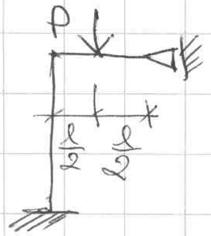
EJ costante e uguale in ogni tratto

struttura simmetrica caricata in modo simmetrico.

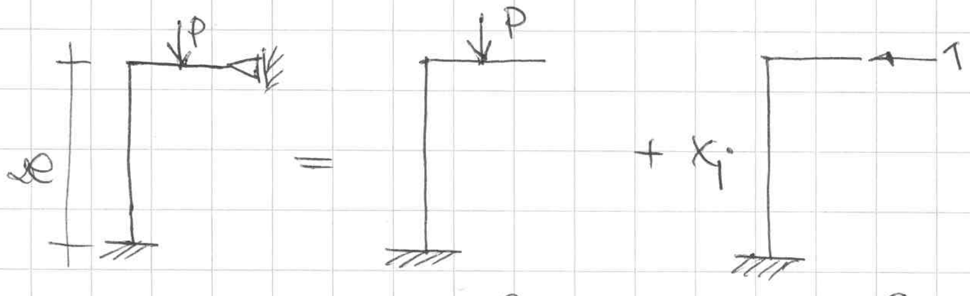
per la simmetria posso ricondurmi a



, considerando poi anche la cornice interna;

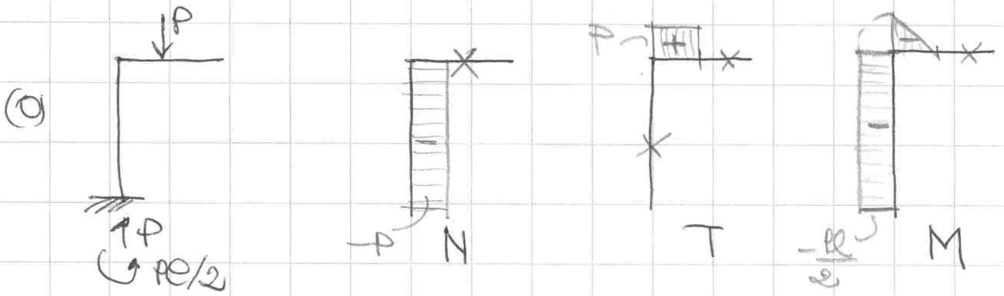


struttura 1 volta iperstatica



sistema (0)

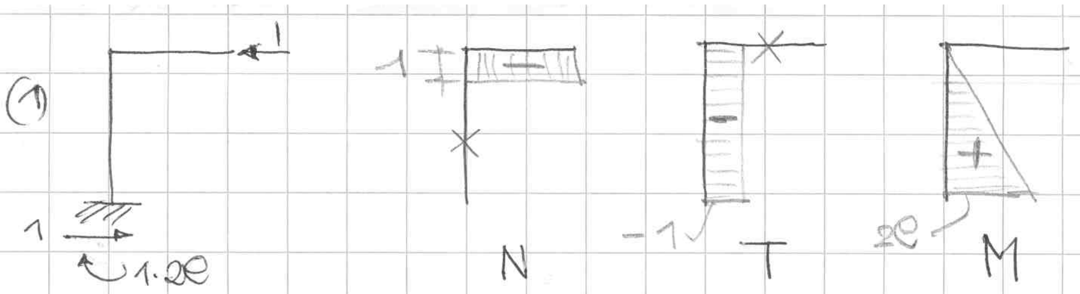
sistema (1)



→ contributo fornito da questo sistema

$$\eta_{10} = -\frac{Pl^3}{EJ}$$

(1)

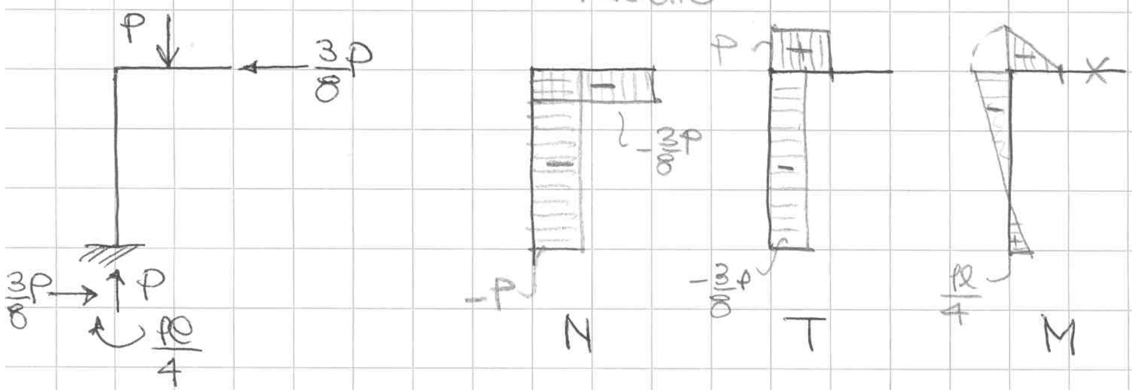


→ contributo fornito da questo schema

$$\eta_{11} = \frac{8}{3}\frac{l^3}{EJ}$$

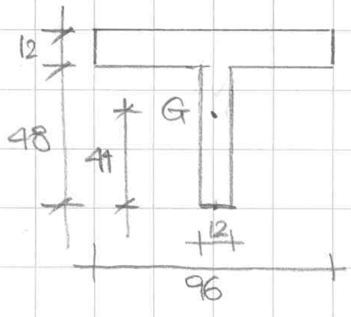
In definitiva:

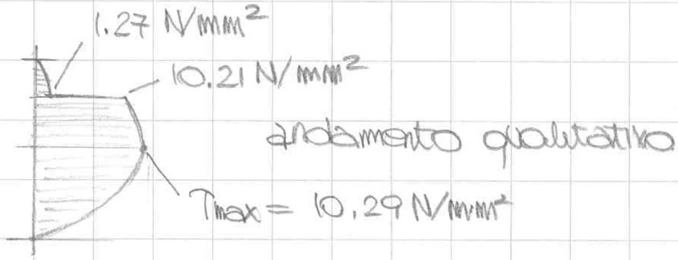
$$-\frac{Pl^3}{EJ} + X_1 \cdot \frac{8}{3}\frac{l^3}{EJ} \underbrace{= 0}_{\text{VINCOLO PERFETTO}} \Longrightarrow \boxed{X_1 = \frac{3}{8}P}$$



$$M_{max} = -\frac{Pl}{2} = -15 \text{ kN}\cdot\text{m}$$

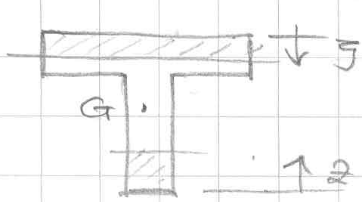
Esercizio n°3





$$\tau_{zy} = \frac{T_y \, S_\Omega}{J_x \cdot b}$$

$\tau_{zy\,max}$ => si realizza nella corda baricentrica



$$S_\Omega(\xi \in (0,12)) = (96 \cdot \xi) \cdot \left(16 - \frac{\xi}{2}\right)$$

$$\tau_{zy}(\xi \in (0,12)) = \frac{T_y \, S_\Omega}{J_x \cdot b} = \frac{5000}{470016 \cdot 96} \cdot 96 \xi \left(16 - \frac{\xi}{2}\right) = 0{,}0106 \, \xi \left(16 - \frac{\xi}{2}\right)$$

$$S_\Omega(z \in (0,48)) = (12 \cdot z)\left(44 - \frac{z}{2}\right)$$

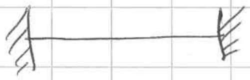
$$\tau_{zy}(z \in (0,48)) = \frac{5000}{470016 \cdot 12} (12 \cdot z)\left(44 - \frac{z}{2}\right) = 0{,}0106 \cdot z \cdot \left(44 - \frac{z}{2}\right)$$

$$\tau_{zy\,max} = \tau_{zy}(z = 44) = 10{,}29 \ \mathrm{N/mm^2}$$

Esercizio n°4

$$w(r) = A\,(a^2 - r^2)^2$$

condizioni di vincolo : incastro



$w(r=a) = 0$
$w_{,r}(r=a) = 0$

condizioni di carico : carico uniforme distribuito

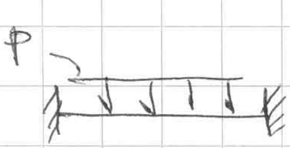
Verificabile tramite le:

$$M_r = -D\left(w_{,rr} + \frac{\nu}{r} w_{,r}\right)$$

$$M_\theta = -D\left(\frac{1}{r} w_{,r} + \nu w_{,rr}\right)$$

$$T_r = -D\left(w_{,rr} + \frac{1}{r} w_{,r}\right)_{,r}$$

$$D\Delta\Delta w = f_z \qquad D = \frac{E h^3}{12(1-\nu^2)}$$



$$w(r) = \frac{p}{64D}(a^2 - r^2)^2$$

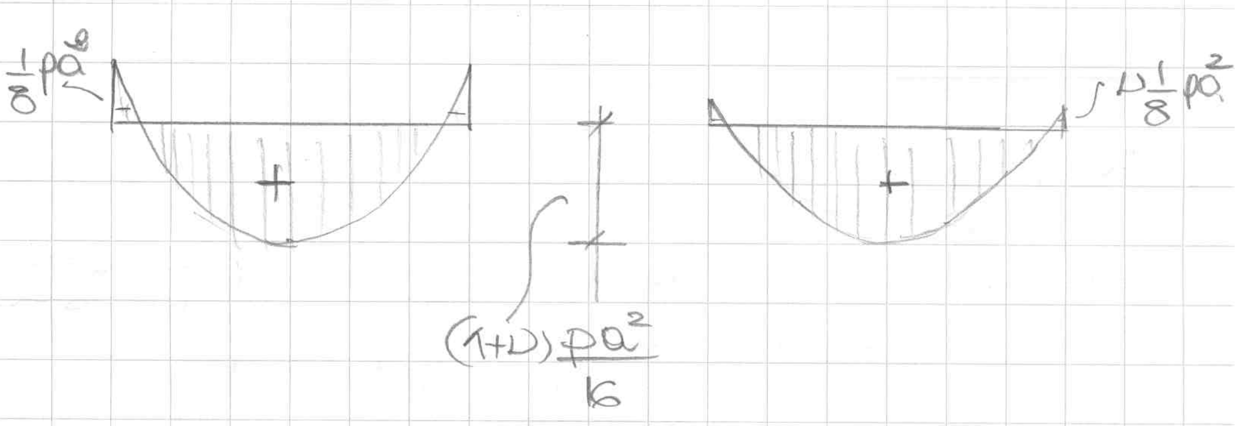
$$\Rightarrow A = \frac{p}{64D} \xrightarrow[\text{carico unitario}]{} A = \frac{1}{64D}$$

$$w_{max} = w(r=0) = A a^4 = \frac{a^4}{64D}$$

$$T_r = -\frac{pr}{2}$$

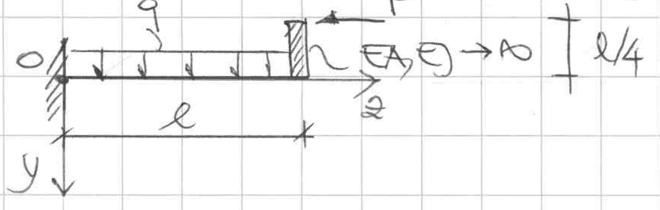
$$M_r = \frac{p}{16}\left[(1+\nu)a^2 - (3+\nu)r^2\right]$$

$$M_\theta = \frac{p}{16}\left[(1+\nu)a^2 - (1+3\nu)r^2\right]$$

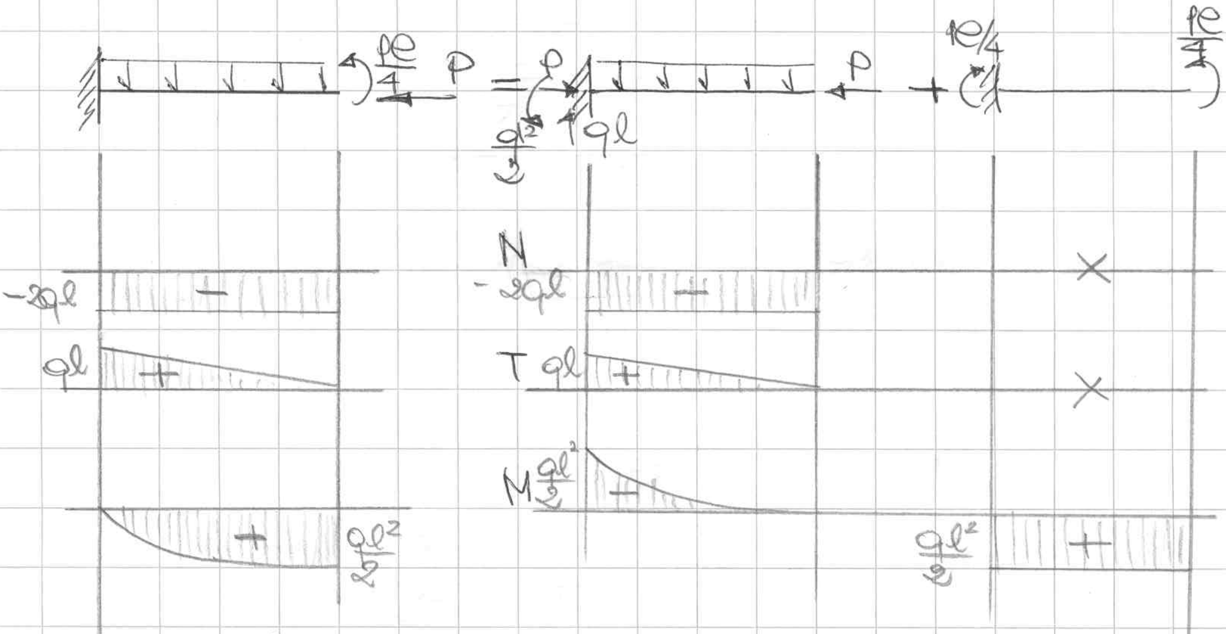

$\frac{1}{8}pa^2$
$+$
$(1+\nu)\frac{pa^2}{16}$
$+$
$\nu\frac{1}{8}pa^2$

SOLUZIONE FILA B

Esercizio n°1



$P = 2ql$



$$M(z) = -\frac{qz^2}{2} + qlz$$

Condizioni di vincolo $\quad v(0) = 0$

$\varphi(0) = 0$

Equ. curva elastica $\quad \frac{d^2v}{dz^2} = -\frac{M(z)}{EJ}$

$$\Rightarrow \frac{d^2v}{dz^2} = \frac{1}{EJ}\left(\frac{qz^2}{2} - qlz\right)$$

integrando: $v(z) = \frac{q}{24EJ} z^4 - \frac{ql}{6EJ} z^3 + Az + B$

$v(0)=0 \rightarrow B=0$

$\varphi(0)=0 \rightarrow A=0$

$$\boxed{v(z) = \frac{qz^4}{24EJ} - \frac{q\ell}{6EJ} z^3}$$

$$\varphi(z) = -\frac{dv}{dz}$$

quindi: $\boxed{\varphi_B = \varphi(z=\ell) = \frac{q\ell^3}{3EJ}}$

contributo della deformazione assiale:

$$\frac{dN}{dz} + p(z) = 0$$

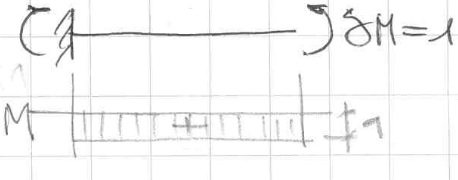
$$N(z) = \frac{pz}{EA} \qquad N(\ell) = \frac{2q\ell^2}{EA}$$

$$\eta_C = \frac{2q\ell^2}{EA} + \frac{1}{4}\ell \cdot \frac{q\ell^3}{3EJ} = \frac{2q\ell^2}{EA} + \frac{q\ell^4}{12EJ}$$

- verifica di φ_B col principio dei lavori virtuali:

schema effettivo

schema virtuale

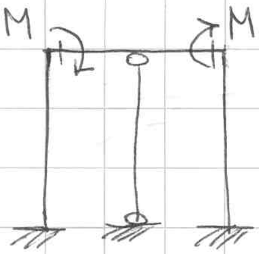


$$\delta L_e = \delta M \cdot \varphi_B = 1 \cdot \varphi_B = \varphi_B$$

$$\delta L_i = \int_0^\ell M \cdot \delta\chi \, dz = \int_0^\ell \left(-\frac{qz^2}{2} + q\ell z\right)\frac{1}{EJ}\,dz = \frac{q\ell^3}{3EJ}$$

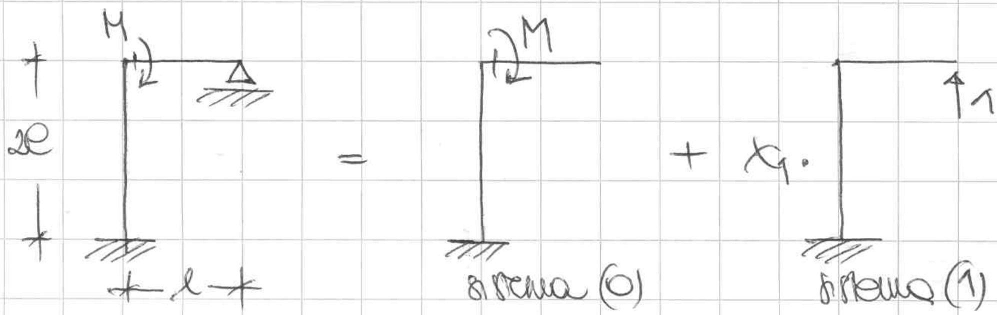
$$\delta L_e = \delta L_i \Rightarrow \varphi_B = q\ell^3/3EJ$$

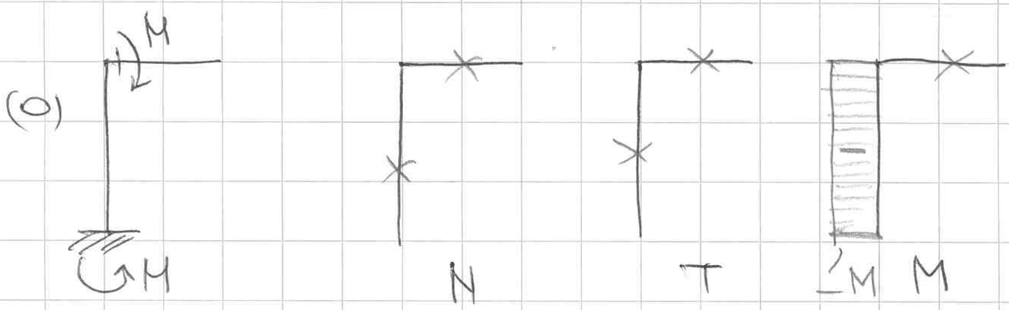
Esercizio n°2



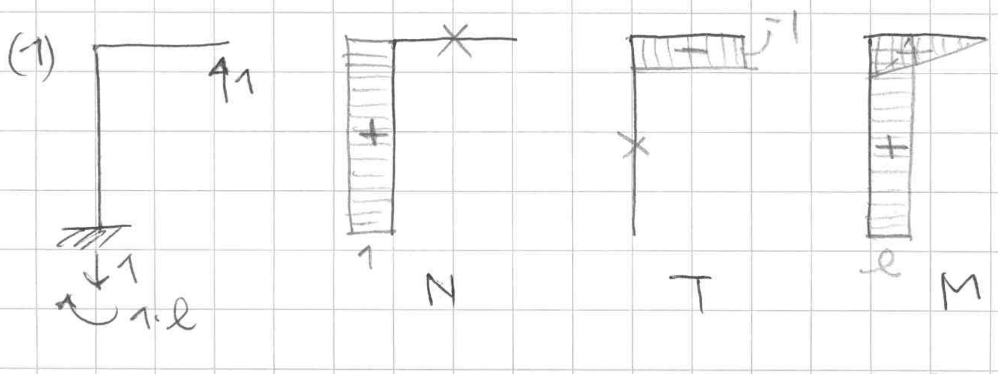
EA, GK → ∞ , EJ costante e uguale in ogni tratto

struttura simmetrica caricata in modo emisimmetrico.





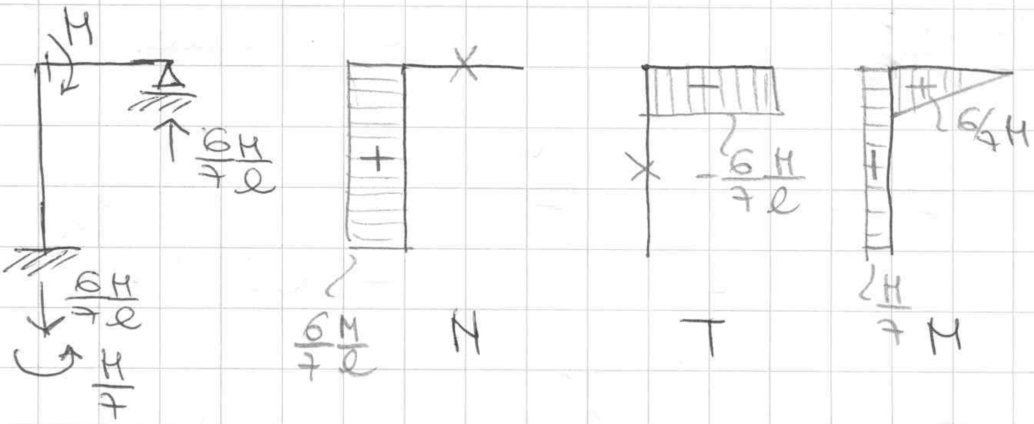
$\Longrightarrow$ contributo fornito da questo sistema $-\frac{2Ml^2}{EJ}$ (tratto 2l)



$\Longrightarrow$ contributo fornito da questo sistema $\underbrace{\frac{2l^3}{EJ}}_{\text{tratto }2l} + \underbrace{\frac{l^3}{3EJ}}_{\text{tratto }l} = \frac{7}{3}\frac{l^3}{EJ}$

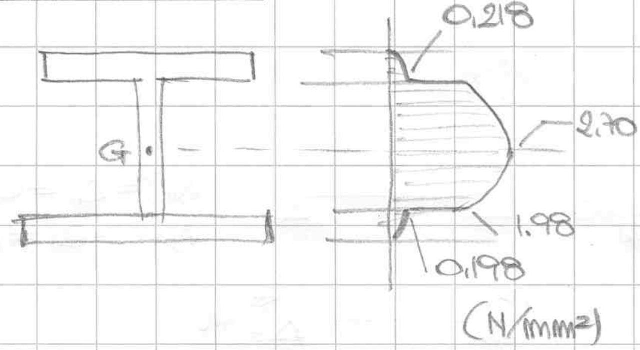
In definitiva:

$$-\frac{2Ml^2}{EJ} + X_1 \cdot \frac{7}{3}\frac{l^3}{EJ} = 0 \implies X_1 = \frac{6}{7}\frac{M}{l}$$

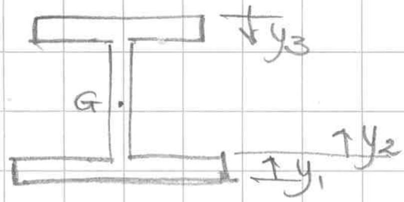


$$M_{max} = \frac{6}{7}M = \frac{6}{7} \cdot 10\ kN \cdot m = 8.57\ kN \cdot m$$

Esercizio n°3



andamento qualitativo



$$S_\Omega\left(y_1 \in (0,12)\right) = (120 \cdot y_1)\left(y_G - \frac{y_1}{2}\right) = (120 \cdot y_1)\left(105 - \frac{y_1}{2}\right)$$

$$S_\Omega\left(y_2 \in (0,196)\right) = (120 \cdot 12)\left(105 - \frac{12}{2}\right) + (y_2 \cdot 12)\left(105 - 12 - \frac{y_2}{2}\right)$$

$$S_\Omega\left(y_3 \in (0,12)\right) = (100 \cdot 12)\left(220 - 105 - \frac{y_3}{2}\right)$$

$$\tau = \frac{T_y \cdot S_\Omega}{J_x \cdot b}$$

τ_{max} = si realizza nella corda baricentrica =

$$= \frac{6000}{35990656 \cdot 12} \cdot \left[(120 \cdot 12)\left(105 - \frac{12}{2}\right) + (98 \cdot 12)\left(105 - 12 - \frac{98}{2}\right)\right] =$$

$$= 2.70 \text{ N/mm}^2$$

Esercizio n°4

$$w(r) = C(a^2 - r^2)$$

condizioni di vincolo: appoggio  $w(r=a)=0$, $w_{,r}(r=a)\neq 0$

condizioni di carico: 

verifichiamo tramite eq.:

$$M_r = -D\left(w_{,rr} + \frac{\nu}{r} w_{,r}\right)$$

$$M_\theta = -D\left(\frac{1}{r} w_{,r} + \nu w_{,rr}\right)$$

$$T_r = -D\left(w_{,rr} + \frac{1}{r} w_{,r}\right)_{,r}$$

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

$$M_r = 2CD(1+\nu) = m \qquad T_r = 0$$

$$M_\theta = 2CD(1+\nu) = m$$

carico unitario: $1 = 2CD(1+\nu) \implies C = \dfrac{1}{2D(1+\nu)}$

$$w_{max} = w(r=0) = C \cdot a^2 = \frac{a^2}{2D(1+\nu)}$$