Optimal control of a thin-airfoil wake using a Riccati-less approach

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Motivation and outline

- Classical optimal-control theory requires the solution of a matrix Riccati equation, intractable for large fluid problems.
- As an alternative to model reduction, for the last few years we have been developing Riccati-less solutions:
  - Minimal-control-energy (MCE) stabilization: only requires knowledge of the direct and adjoint unstable modes.
  - Adjoint of the direct-adjoint (ADA): only requires iterations of the direct and adjoint problem.
- Both have been successful on the cylinder wake (only one complex conjugate pair of unstable eigenvalues).
- As a problem with more than one unstable eigenvalue, we are now applying these techniques to the wake of a thin airfoil.

The standard, linear, optimal control problem

- Full state information is assumed;
- a dual estimation problem can always be solved separately.

The classical full-state-information control problem is formulated as follows: to find the control $\mathbf{u}$ that minimizes the cost function

$$J = \frac{1}{2} \int_0^T [\mathbf{x}^H Q \mathbf{x} + l^2 \mathbf{u}^H R \mathbf{u}] \, dt,$$

where l is a penalty on the control energy, and the state $\mathbf{x}$ and the control $\mathbf{u}$ are related via the state equation

$$\frac{\partial \mathbf{x}}{\partial t} = A\mathbf{x} + B\mathbf{u} \quad \text{on} \quad 0 < t < T, \quad \text{with} \quad \mathbf{x} = \mathbf{x}_0 \quad \text{at} \quad t = 0.$$

The solution depends on: $\mathbf{x}_0$, T, Q, R and l.

Optimization

The adjoint variable $\mathbf{p}$ is introduced as a Lagrange multiplier. The augmented cost function is written

$$J = \int_0^T \frac{1}{2}[\mathbf{x}^H Q\mathbf{x} + l^2 \mathbf{u}^H R\mathbf{u}] - \mathbf{p}^H[\frac{\partial \mathbf{x}}{\partial t} - A\mathbf{x} - B\mathbf{u}]\, dt.$$

Integration by parts and $\delta J = 0$ give

$$0 = \int_0^T \delta\mathbf{u}^H[\underbrace{B\mathbf{p} + l^2 R\mathbf{u}}_{=0}] + \delta\mathbf{x}^H[\underbrace{\frac{\partial \mathbf{p}}{\partial t} + A^H\mathbf{p} + Q\mathbf{x}}_{=0}]\, dt + [\delta\mathbf{x}^H\mathbf{p}]_0^T,$$

adjoint equations

$$\frac{\partial \mathbf{p}}{\partial t} = -A^H\mathbf{p} - Q\mathbf{x}, \quad \text{with} \quad \mathbf{p}(t = T) = 0,$$

and optimality condition

$$\mathbf{u} = -\frac{1}{l^2} R^{-1} B^H \mathbf{p}.$$

Two-point boundary value problem

The direct and adjoint equations can be combined in a block matrix form

$$\frac{d\mathbf{z}}{dt} = Z\mathbf{z} \quad \text{where} \quad Z = Z_{2n\times 2n} = \begin{bmatrix} A & -l^{-2}BR^{-1}B^H \\ -Q & -A^H \end{bmatrix} \qquad (1)$$

$$\mathbf{z} = \begin{bmatrix} \mathbf{x} \\ \mathbf{p} \end{bmatrix}, \quad \text{and} \quad \begin{cases} \mathbf{x} = \mathbf{x}_0 & \text{at} \quad t = 0, \\ \mathbf{p} = 0 & \text{at} \quad t = T. \end{cases}$$

(Z has a Hamiltonian symmetry, such that eigenvalues appear in pairs of equal imaginary and opposite real part.)

This linear ODE is a two-point boundary value problem and may be solved using a linear relationship between the state vector $\mathbf{x}(t)$ and adjoint vector $\mathbf{p}(t)$ vi a matrix $X(T)$ such that $\mathbf{p} = X\mathbf{x}$, and inserting this solution ansatz into (1) to eliminate $\mathbf{p}$.

The Riccati equation

It follows that matrix X obeys the differential Riccati equation

$$-\frac{dX}{dt} = A^H X + XA - X l^{-2} B R^{-1} B^H X + Q \quad \text{with} \quad X(T) = 0. \quad (2)$$

Once X is known, the optimal value of $\mathbf{u}$ may then be written in the form of a feedback control rule such that

$$\mathbf{u} = K\mathbf{x} \quad \text{where} \quad K = -l^{-2} R^{-1} B^H X.$$

Finally, if the system is time invariant (LTI) and we take the limit that $T \to \infty$, the matrix X in (2) may be marched to steady state. This steady state solution for X satisfies the continuous-time algebraic Riccati equation

$$0 = A^H X + XA - X l^{-2} B R^{-1} B^H X + Q,$$

where additionally X is constrained such that $A + BK$ is stable.

The classical way of solution

A linear time-invariant system (LTI) can be solved using its eigenvectors. Assume that an eigenvector decomposition of the $2n \times 2n$ matrix Z is available such that

$$Z = V \Lambda_c V^{-1} \quad \text{where} \quad V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \quad \text{and} \quad \mathbf{z} = \begin{bmatrix} \mathbf{x} \\ \mathbf{p} \end{bmatrix}$$

and the eigenvalues of Z appearing in the diagonal matrix Λ_c are enumerated in order of increasing real part. Since

$$\mathbf{z} = V e^{\Lambda_c t} V^{-1} \mathbf{z}_0$$

the solutions $\mathbf{z}$ that obey the boundary conditions at $t \to \infty$ are spanned by the first n columns of V. The direct ($\mathbf{x}$) and adjoint ($\mathbf{p}$) parts of the these columns are related as $\mathbf{p} = X\mathbf{x}$, where

$$[\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_n] = X[\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n] \quad \to \quad X = V_{21} V_{11}^{-1}$$

Minimal-control-energy stabilization

In the limit that $l^2 \to \infty$ we consider

$$J = \int_0^T \frac{1}{2}[l^{-2}\mathbf{x}^H Q \mathbf{x} + \mathbf{u}^H R \mathbf{u}]$$

With this defintion the same derivation as before leads to

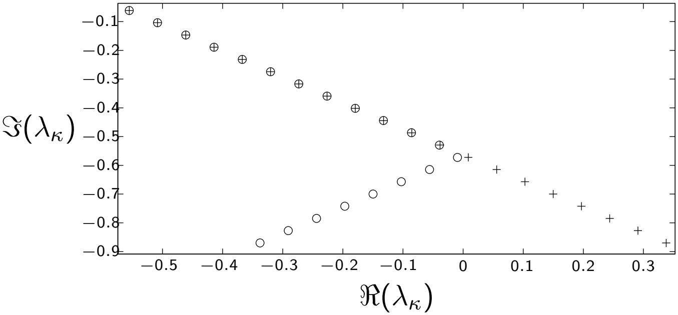
$$\frac{d\mathbf{z}}{dt} = Z\mathbf{z} \quad \text{where} \quad Z = Z_{2n\times 2n} = \begin{bmatrix} A & -BR^{-1}B^H \\ \underbrace{-l^{-2}Q}_{\to 0} & -A^H \end{bmatrix}$$

Z becomes block triangular. The direct and adjoint equations are

$$\frac{\partial \mathbf{x}}{\partial t} = A\mathbf{x} + B\mathbf{u}, \qquad \mathbf{u} = -R^{-1}B^H\mathbf{p}, \qquad \frac{\partial \mathbf{p}}{\partial t} = -A^H\mathbf{p} + 0$$

Minimal-control-energy stabilization

The eigenvalue spectrum of this system is given by the union of the eigenvalues of A and the eigenvalues of $-A^H$.



The eigenvalues of (+) the discretized open-loop system, and (o) the closed-loop system $A + BK$ after minimal-energy control is applied.

Minimal-control-energy feedback

Denoting:

- $\mathbf{x}^i$ and λ^i the i-th right eigenvector and eigenvalue of A,
- $\mathbf{y}^i$ and $-\lambda^{i*}$ the i-th right eigenvector and eigenvalue of $-A^H$,
- $\mathbf{y}^{i*}$ is left eigenvector of A,

we see that the stable eigenvectors of

$$\frac{\partial \mathbf{x}}{\partial t} = A\mathbf{x} + B\mathbf{u}, \qquad \mathbf{u} = -R^{-1}B^H\mathbf{p}, \qquad \frac{\partial \mathbf{p}}{\partial t} = -A^H\mathbf{p}$$

are of two possible types:

$$\begin{array}{lll} \mathbf{p} = 0,\ \mathbf{x} = \mathbf{x}^i & \text{if} \quad \Re(\lambda^i) < 0 & \text{(stable)} \\ \mathbf{p} = \mathbf{y}^i,\ \mathbf{x} = (\lambda^{i*} + A)^{-1}BR^{-1}B^H\mathbf{y}^i & \text{if} \quad \Re(\lambda^i) > 0 & \text{(unstable)} \end{array}$$

We now project an arbitrary initial condition $\mathbf{x}_0$ onto these modes,

$$\mathbf{x}_0 = \sum_{\text{stable}} d_j \mathbf{x}^j + \sum_{\text{unstable}} f_j (\lambda^{j*} + A)^{-1} B R^{-1} B^H \mathbf{y}^j \qquad (3)$$

and note that in order to reconstruct $\mathbf{p}$ we only need the f_j's, because the stable modes have $\mathbf{p} = 0$. The coefficients d_j can be eliminated from (3) by projecting the left eigenvectors:

$$\mathbf{y}^{i*}\mathbf{x}_0 = \mathbf{y}^{i*} \sum_{\text{unstable}} f_j (\lambda^{j*} + A)^{-1} B R^{-1} B^H \mathbf{y}^j = \sum_{\text{unstable}} c_{ij} f_j$$

where, since $\mathbf{y}^{i*}$ is also a left eigenvector of $(\lambda^{j*} + A)^{-1}$,

$$c_{ij} = \frac{\mathbf{y}^{i*} B R^{-1} B^H \mathbf{y}^j}{\lambda^i + \lambda^{j*}}$$

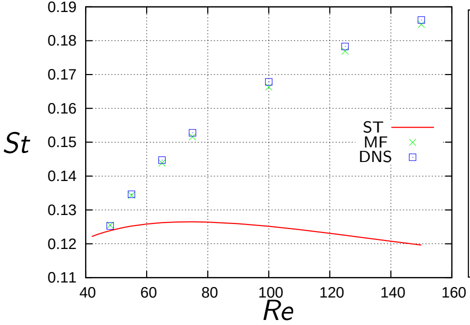
Only the unstable eigenvalues and left eigenvectors are needed.

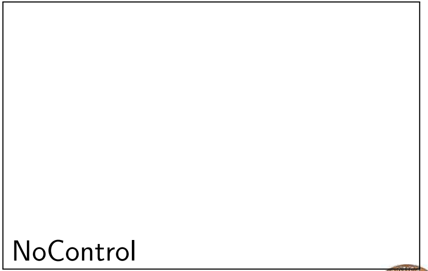
Application to the cylinder wake

The linear feedback matrix K which suppresses vortex shedding from a circular cylinder has been computed using:

Full state information, Actuator: angular oscillation, $Re = UD/\nu$

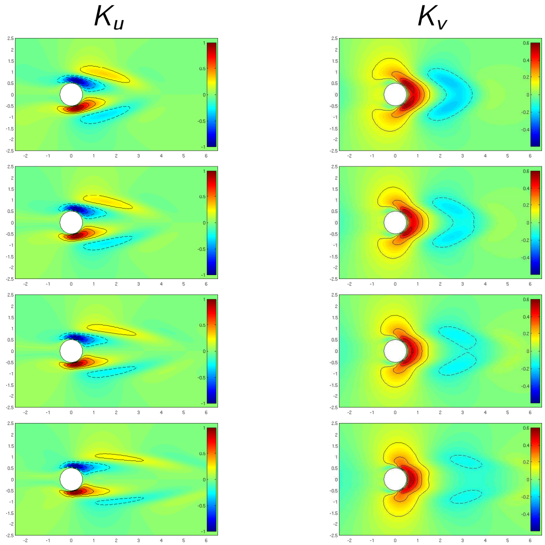
Dimension of control $\mathbf{u}$ is $m = 1$





The feedback matrix K ($\mathbf{u} = K\mathbf{x}$)

- $Re = 55$
- $Re = 75$
- $Re = 100$
- $Re = 150$

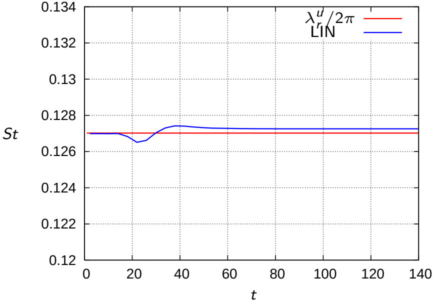


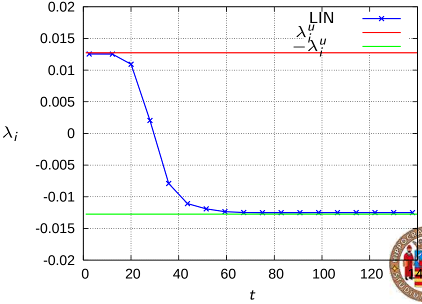
Results: linearized N-S equations

The temporal evolution of the frequency and growth rate is compared with the eigenvalue λ

- The Strouhal number: $St = fD/U$ compared to $St = \lambda_r/2\pi$
- The growth rate: $\sigma = \frac{d}{dt} log(u(t))$ compared to λ_i

Test case: $Re = 55$, control is turned on at $t = 18$





Control of nonlinear vortex shedding: $Re = 55$

WithControl

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WithControl

Adjoint of the direct-adjoint

The aim is to compute the solution for K, which is independent of x_0 and time invariant. This can be solved using an iterative procedure to "try" different $\mathbf{x}_0$ (computationally expensive).

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ALTERNATIVELY

For a converged solution at $t = 0$ we can write

$$\mathbf{u} = K\mathbf{x}_0 = -\frac{1}{l^2} R^{-1} B^H \mathbf{p}_0.$$

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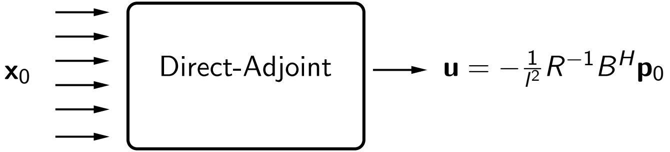
ALTERNATIVELY

For a converged solution at $t = 0$ we can write

$$\mathbf{u} = K\mathbf{x}_0 = -\frac{1}{l^2} R^{-1} B^H \mathbf{p}_0.$$

This is a linear relation between the input $\mathbf{x}_0$ and output $\mathbf{u}$.

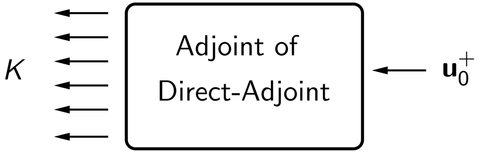


The input has a large dimension and the output a small dimension.

Adjoint of the direct-adjoint

Such a problem is efficiently solved using the adjoint equations.

The adjoint input has a small dimension and the output a large dimension.



K is obtained from the solution of the adjoint of the direct-adjoint system.

Adjoint of the Direct-Adjoint system

Introduce the adjoint variables $\mathbf{x}^+$ and $\mathbf{p}^+$ and multiply with the direct-adjoint equations, then integrate in time from $t = 0$ to $t = T$. Here we consider that $\mathbf{u}$ has dimension $m = 1$.

$$\int_0^T \mathbf{x}^{+H} \left(\frac{\partial \mathbf{x}}{\partial t} - A\mathbf{x} + \frac{1}{l^2} B R^{-1} B^H \mathbf{p} \right) dt + \int_0^T \mathbf{p}^{+H} \left(\frac{\partial \mathbf{p}}{\partial t} + A^H \mathbf{p} + Q\mathbf{x} \right) dt = 0.$$

Adjoint of the Direct-Adjoint system

Using integration by parts, and considering that both R and Q are symmetric, we obtain

$$-\int_0^T \mathbf{p}^H \left(\frac{\partial \mathbf{p}^+}{\partial t} - A\mathbf{p}^+ - \frac{1}{l^2} B R^{-1} B^H \mathbf{x}^+ \right) dt - \int_0^T \mathbf{x}^H \left(\frac{\partial \mathbf{x}^+}{\partial t} + A^H \mathbf{x}^+ - Q\mathbf{p}^+ \right) dt$$

$$+ \left[\mathbf{p}^H \mathbf{p}^+ \right]_0^T + \left[\mathbf{x}^H \mathbf{x}^+ \right]_0^T = 0.$$

If we now define the new adjoint equations as

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$$+ \left[\mathbf{p}^H \mathbf{p}^+\right]_0^T + \left[\mathbf{x}^H \mathbf{x}^+\right]_0^T = 0.$$

If we now define the new adjoint equations as

$$\frac{\partial \mathbf{p}^+}{\partial t} = A\mathbf{p}^+ + \frac{1}{l^2} BR^{-1}B^H\mathbf{x}^+,$$

$$\frac{\partial \mathbf{x}^+}{\partial t} = -A^H\mathbf{x}^+ + Q\mathbf{p}^+,$$

Adjoint of the Direct-Adjoint system

with $\mathbf{x}^+(t = T) = 0$ and $\mathbf{p}(t = T) = 0$, the remaining terms are

$$\mathbf{x}^{+H}(0)\mathbf{x}(0) + \mathbf{p}^{+H}(0)\mathbf{p}(0) = 0.$$

Recall that the original linear relation was

$$K\mathbf{x}_0 = -\frac{1}{l^2}R^{-1}B^H\mathbf{p}_0$$

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$$\mathbf{x}^{+H}(0)\mathbf{x}(0)+\mathbf{p}^{+H}(0)\mathbf{p}(0)=0.$$

Recall that the original linear relation was

$$K\mathbf{x}_0=-\frac{1}{l^2}R^{-1}B^H\mathbf{p}_0$$

- Choosing $\mathbf{p}^{+H}(t=0)$ as one row of $-\frac{1}{l^2}R^{-1}B^H$ $(m=1)$
- we can identify one row of K as $\mathbf{x}^{+H}(0)$. $(m=1)$

Solution procedure

If we let $\mathbf{x}^+ \rightarrow -\mathbf{p}$ and $\mathbf{p}^+ \rightarrow \mathbf{x}$ we easily obtain the original (Direct-Adjoint) system. (self-adjoint)

Finally: solve the original linear system with new b.c.

$$\frac{\partial \mathbf{x}}{\partial t} = A\mathbf{x} - \frac{1}{l^2} B R^{-1} B^H \mathbf{p} \quad \text{on} \quad 0 < t < T, \quad \mathbf{x}^H(0) \quad \text{is one row of} \quad \frac{1}{l^2} R^{-1} B^H,$$
$$\frac{\partial \mathbf{p}}{\partial t} = -A^H \mathbf{p} - Q\mathbf{x} \quad \text{on} \quad 0 < t < T, \quad \text{with} \quad \mathbf{p}(T) = 0.$$

One row of K is then given by $-\mathbf{p}^H(0)$ (since $\mathbf{x}^+ = -\mathbf{p}$).

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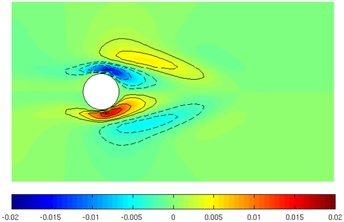
ADVANTAGE

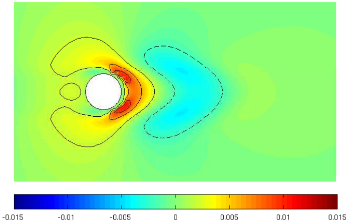
Avoid solving for $X_{n\times n}$; solve original system $\mathbf{x}_{n\times 1}$ m times

Results: K for $Re = 55$

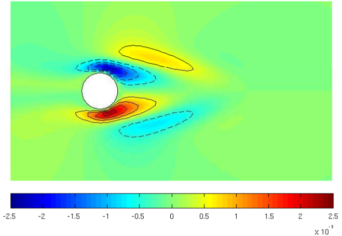
$K_u,\ l^2 = 1$



$K_v,\ l^2 = 1$



$K_u,\ l^2 \rightarrow \infty$



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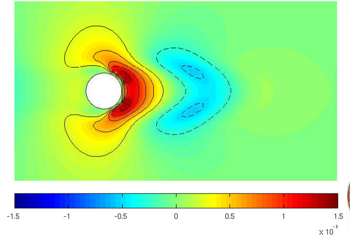
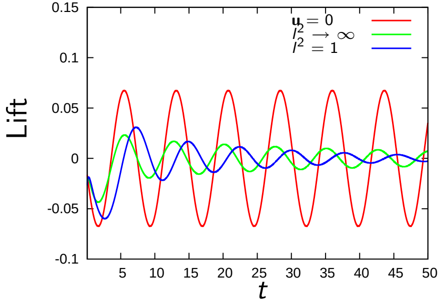


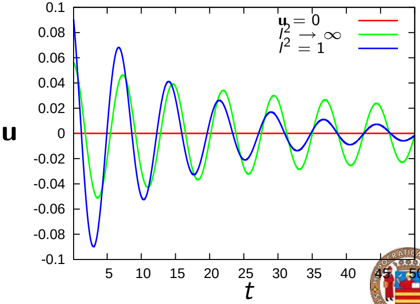
Control of vortex shedding

In the temporal evolution of the lift (C_L) and control $\mathbf{u}$:

- C_L and $\mathbf{u}$ tend to zero as the control is applied
- Control $\mathbf{u}$ strengthens as l^2 decreases

Test case: $Re = 55$, control is turned on at $t = 0$



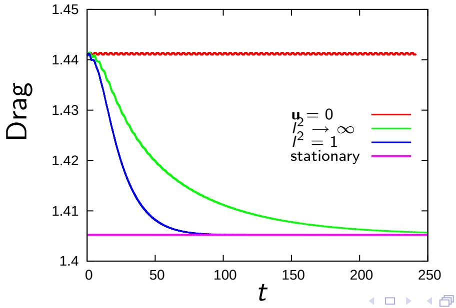


Control of vortex shedding

In the temporal evolution of drag (C_D) coefficient:

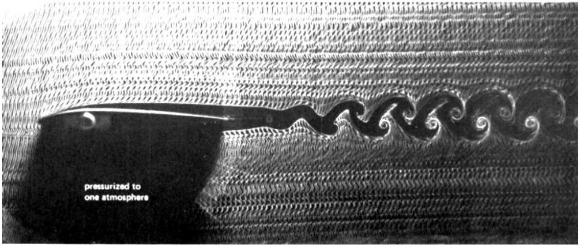
- As the control is applied C_D tends to the constant value corresponding to the steady state solution
- The control acts more quickly as l^2 is decreased

Test case: $Re = 55$, control is turned on at $t = 0$



Vortex shedding past a low-Reynolds-number airfoil

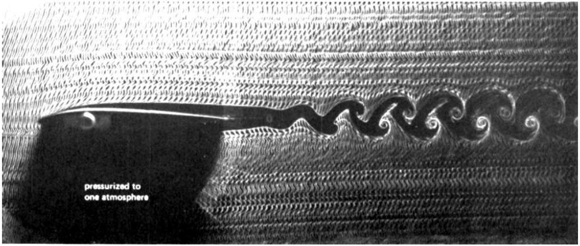
At sufficiently low *Re* the flow around an airfoil is 2D, laminar and without separation. In these conditions and above a critical value of *Re* the wake oscillates at a recognizable frequency: *e.g.*, *McAlister & Carr (1978)*, $Re_\delta = 145$ ($Re_c = 21000$), $St \approx 0.43$.



McAlister & Carr (1978)

Schematization

In such flow conditions it is reasonable to assume that the flow at the trailing edge is approximately given by a double Falkner-Skan profile[1]. Depending on the F-S profile and the *Re* number the flow on the entire airfoil might also be sub-critical with respect to convective disturbances.



McAlister & Carr (1978)

[1] Woodley & Peake, *J. Fluid Mech. (1997)*

Related investigations

Flow condition: 2D, laminar flow, no separation

Experiments

- **Airfoil** $Re_c = 21000$: McAlister & Carr (1978)
- **Flat plate** Taneda (1958)

Numerical (trailing edge profile given by double Falkner-Skan profile)

- **Linear local** Woodley & Peake (1997), Taylor & Peake (1999).
- **Nonlinear** Pier & Peake (2008, 2009).

Problem formulation

- 2D, incompressible flow
- Semi-infinite flow domain downstream of the trailing edge
- At trailing edge: double Falkner-Skan profile with pressure gradient[2] $-0.09 \leq m \leq 0$
- $Re_\delta = U_\infty \delta / \nu$
- Length scale $\delta = (\nu x / U_\infty)^{0.5}$

[2] $U_\infty = C\, x^m$

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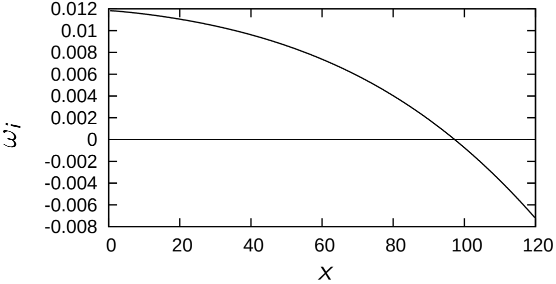
Is it sufficient to use the trailing edge as inlet of the computational domain ?

This was the assumption in Woodley & Peake (1997), Taylor & Peake (1999), Pier & Peake (2008, 2009), and many similar examples...

[2] $U_\infty = C\, x^m$

Trailing edge

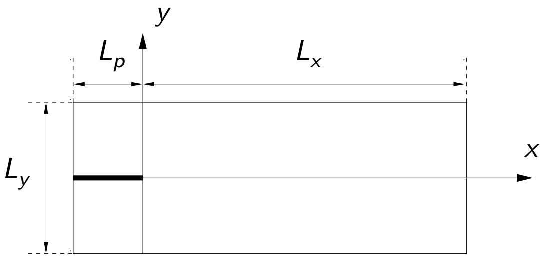
- Local stability analysis show that the Falkner-Skan profile at the trailing edge is already absolutely unstable.



Here: Maximum value at trailing edge for $m = -0.09$, $Re = 2000$

Trailing edge

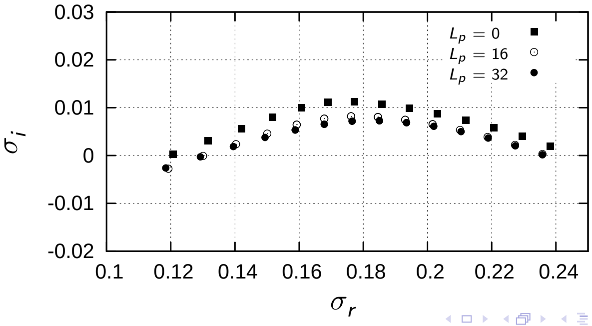
- Local stability analysis show that the Falkner-Skan profile at the trailing edge is already absolutely unstable.
- Solution here: add small plate (L_p) of infinitesimal thickness upstream of trailing edge.



Domain size

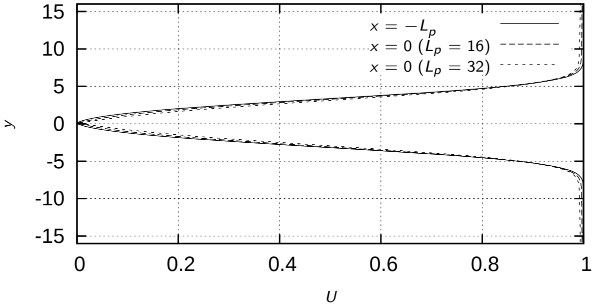
Grid and domain convergence tests have been made comparing results of the unstable global modes. Here computations are performed using $L_x \geq 400$ and $L_p \geq 30$.
An example of the dependence on L_p is given below for $Re_\delta = 200$.

$$Re_\delta = 200,\ m = -0.09,\ L_x = 432,\ L_p = 32,\ L_y = 50$$



Mean flow on upstream plate

The assumption is to have a similarity solution at $x = 0$. However, here the similarity solution is given at $x = -L_p$. How does the mean flow change on the added flat plate ? An example is given for the mean flow used in the previously shown eigenvalue calculation.

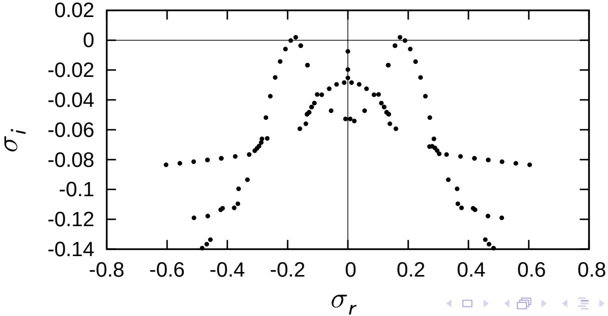


Eigenvalue spectrum

The eigenvalues appear as complex conjugate pairs for a given *Re* and *m*.

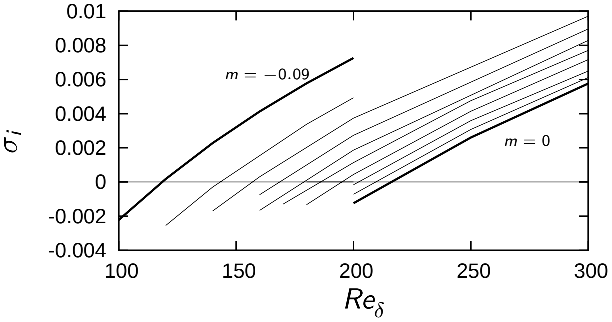
The number of unstable modes increases as one goes above the critical Reynolds number.

$$Re_\delta = 200, \; m = -0.09$$



Critical Reynolds number

The critical Reynolds number as a function of the pressure gradient is found by plotting the growth rate of the least stable mode as a function of *Re* and *m*.

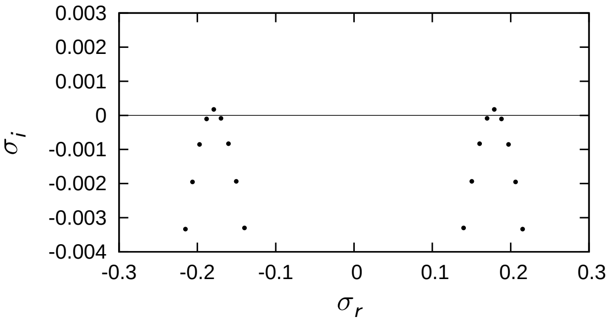


Airfoil wake vortex shedding

AirFoil

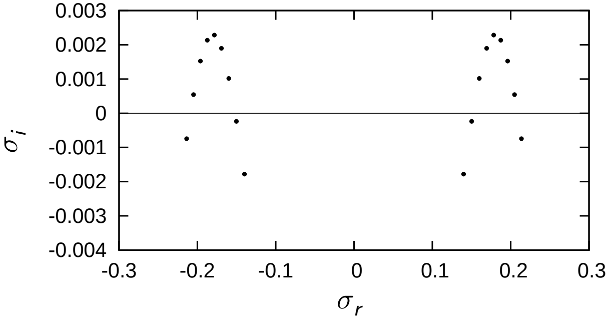
Eigenvalue spectrum, $Re = 120 \;\; m = -0.09$

1 unstable mode



Eigenvalue spectrum, $Re = 140 \ \ m = -0.09$

6 unstable modes



Minimal-control-energy stabilization

The linear feedback matrix K which suppresses vortex shedding modes in an airfoil's wake has been computed using:
Full state information, $Re = U\delta/\nu$

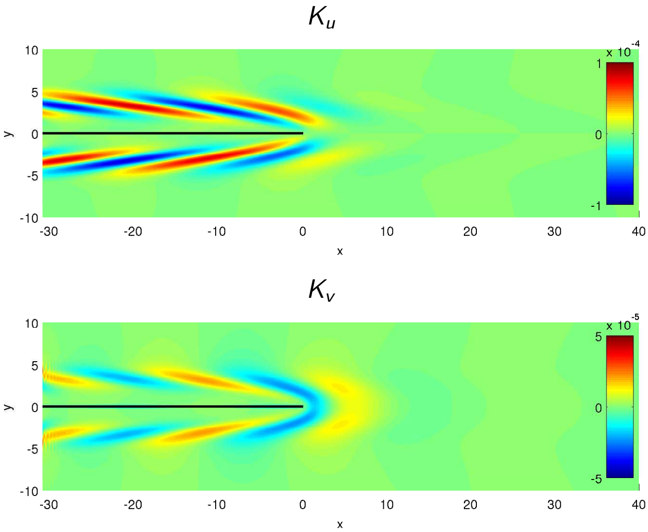
Actuator: unsteady circulation (velocity difference)

An angular oscillation of the whole airfoil, or of a flap, produces an instantaneous change of circulation in the potential flow. In the boundary layer and wake, this appears as a difference between the upper and lower streamwise outer velocities. This difference is used as the control parameter in our simulation.

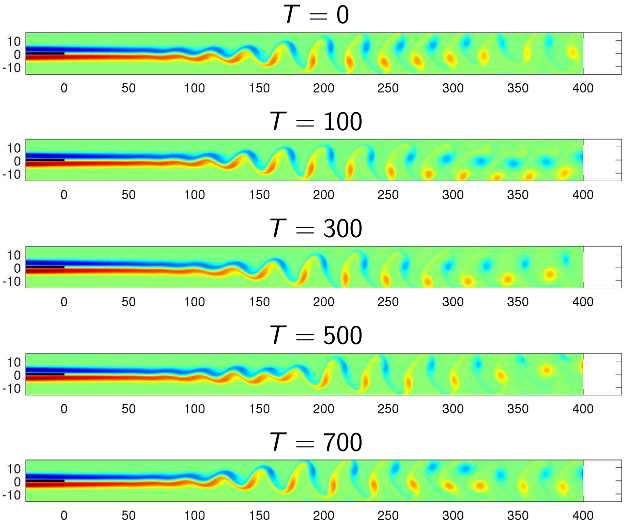
Dimension of control $\mathbf{u}$ is $m = 1$

NoControl

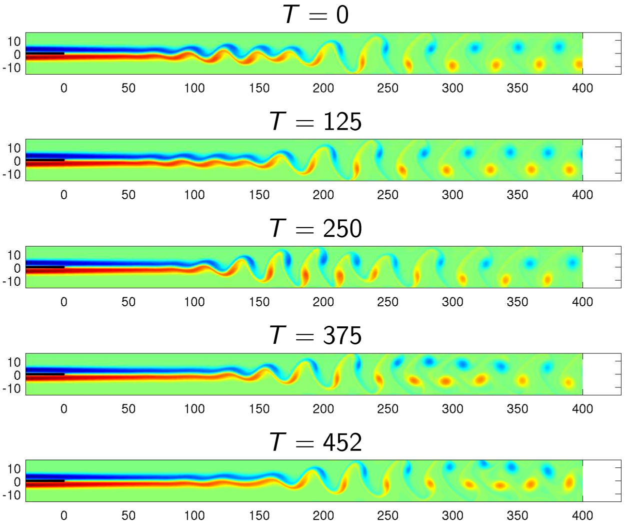
Control kernel K: $Re = 120$, $m = -0.09$



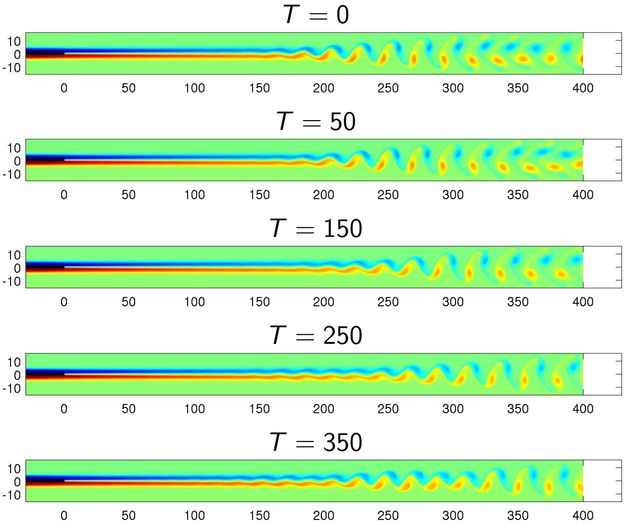
$Re = 120$, $m = -0.09$, 1 unstable mode pair



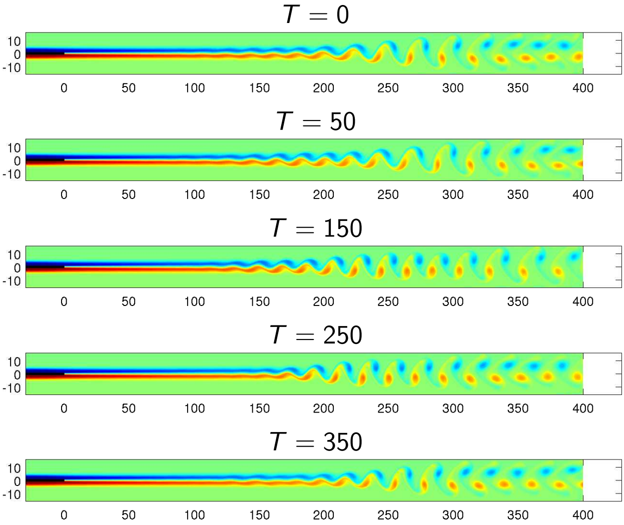
$Re = 140$, $m = -0.09$, 6 unstable mode pairs



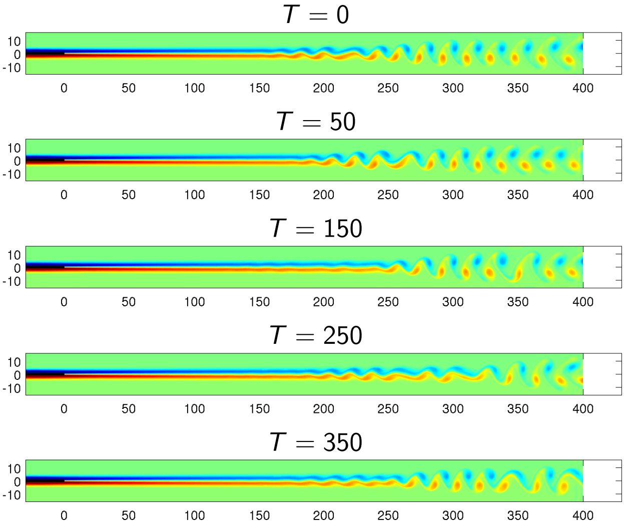
$Re = 178$, $m = -0.05$, 1 unstable mode pair



$Re = 190$, $m = -0.05$, 7 unstable mode pairs



$Re = 220$, $m = 0$, 5 unstable mode pairs



Conclusions: Riccati-less control

- Two exact methods have been developed to solve large-dimensional optimal-control problems:
- MCE: minimal-control-energy stabilization: In the limit $l^2 \to \infty$, K can be determined from the unstable eigenvalues and corresponding left eigevectors only
- ADA: adjoint of the direct-adjoint: The feedback matrix K for the general problem (any value of l^2) can be obtained from the iterative solution of the Adjoint of the Direct-Adjoint system. This is equivalent to solving the original system with appropriate initial condition.
- Both methods have been successfully tested to control vortex shedding behind a cylinder.

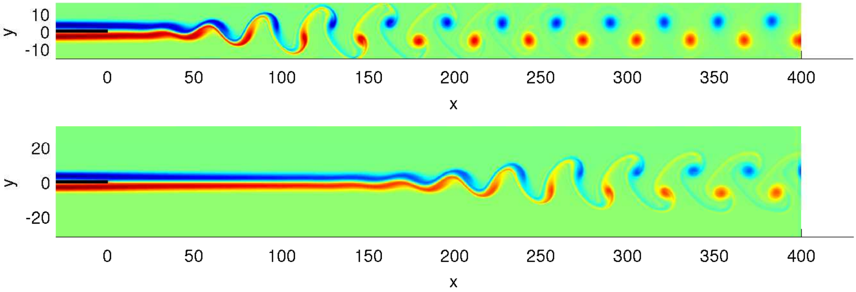
Conclusions: low-Reynolds-number airfoil wake

- The phenomenon looks qualitatively similar to vortex shedding behind a cylinder. However,
  - a local absolute instability already exists at the trailing edge: inclusion of a tract of the upstream flow region is essential;
  - the system easily presents more than one unstable mode.
- Work is still required to arrive at a conclusion regarding the effectiveness of control.
- Of the present tests, one with 6 unstable modes offers the most promising results.

EXTRA SLIDES

Influence of domain size

$Re = 250,\ m = -0.09$



References: cylinder control using rotational oscillation

Aim: reduce C_D

Exp. Tokumaru & Dimotakis (1991), -20%, $Re = 15000$

Feedback control:

Exp. Fujisawa & Nakabayashi (2002) -16% (-70% C_L), $Re = 20000$

Exp. Fujisawa et al.(2001) "reduction", $Re = 6700$

Optimal control (using adjoints):

Num. He et al.(2000) -30 to -60% for $Re = 200 - 1000$

Num. Protas & Styczek (2002) -7% at $Re = 75$, -15% at $Re = 150$

Bergmann et al.(2005) -25% at $Re = 200$ (POD)

Aim: reduce vortex shedding

Feedback control:

Num. Protas (2004) reduction, "point vortex model", $Re = 75$

Optimal control (using adjoints):

Num. Homescu et al.(2002) reduction, $Re = 60 - 1000$