Dynamics of porous filaments in a uniform flow

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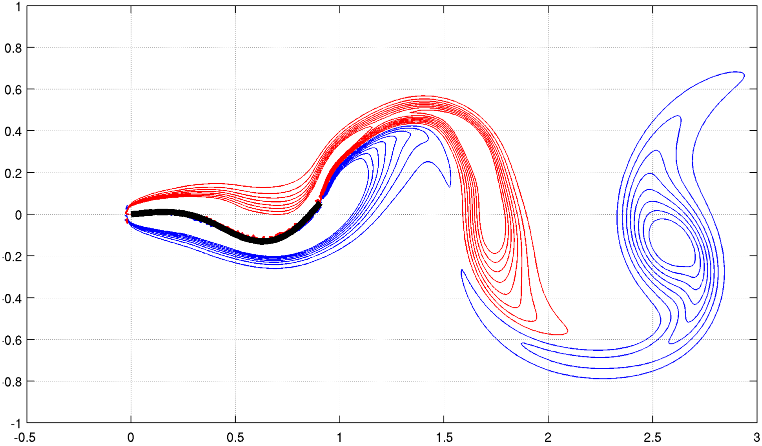
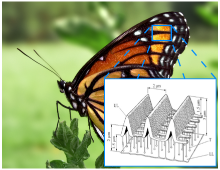
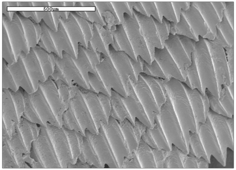


Motivations

The present work places itself into a more general Biomimetics research project, in which we aim at developing new control and drag reduction stategies, based on the introduction of biomimetic irregular surface coatings.

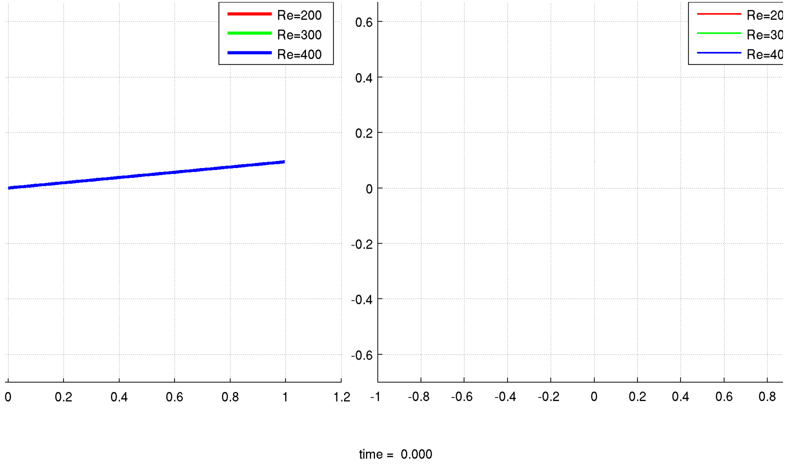




Expected results comprehends

- Delay of transition to turbulence
- Improved fluidodynamic performance both in laminar and in turbulent conditions
- Better understanding of the physics of the phenomenon

Dynamic of filament in a uniform flow



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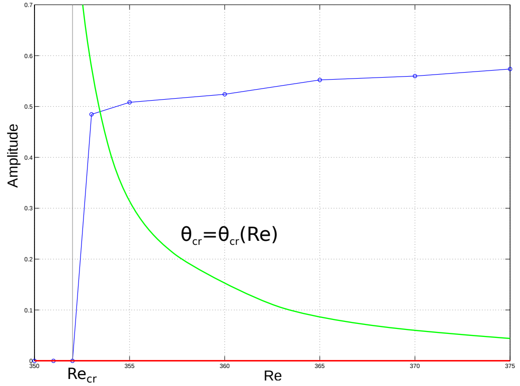

Figure : Bifurcation curve - Bistability.

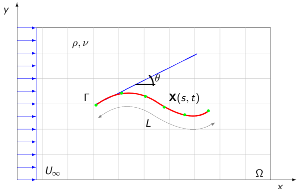

Figure : Model domain and initial condition.

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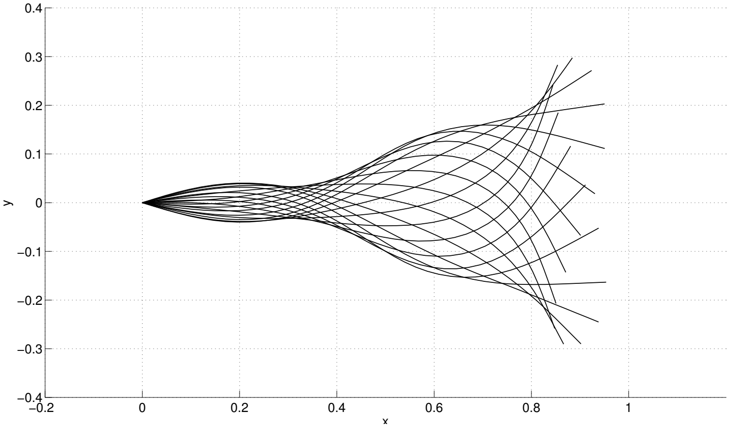


Figure : Filament snapshots at different times over a single period.

State of the art

- Zhu and Peskin, "Simulation of a Flapping Flexible Filament in a Flowing Soap Film by the Immersed Boundary Method" (2002), stretching and bending rigidity
- Kim and Peskin, "2-D Parachute Simulation by the Immersed Boundary Method" (2006), stretching rigidity and porosity
- Kim and Peskin, "Penalty Immersed Boundary Method for an Elastic Boundary with Mass" (2007), stretching and bending rigidity and mass

In our DNS we tried to simulate a filament with both stretching and bending rigidity, porosity and mass.

The Immersed Boundary Method

- Proposed by Peskin (2002) in order to simulate blood flow around cardiac valves.
- Does not require body-fitted or unstructured meshes, the Navier-Stokes equations are solved on a background Eulerian mesh, thus preserving the accuracy and efficiency of the solution.
- The IB is described by a set of Lagrangian points interconnected by springs.
- Lagrangian points does not require to conform to the Eulerian mesh as the informations between meshes are effectively passed by mean of a smoothed approximation of the Dirac Delta function.
- Handles efficiently moving boundaries.

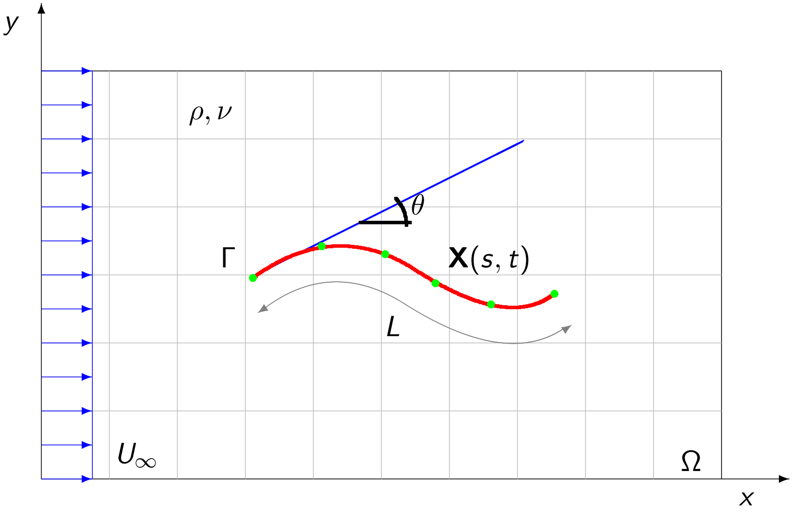
The Immersed Boundary Method

The main drawback of this IBM formulation is that it leads to stiff problems since the stretching rigidity of the boundary has to be high enough to enforce incompressibility. Given the oscillation frequency of a simple oscillator

$$\omega = \sqrt{\frac{k}{m}}$$

as k increases the characteristic period decreases, so the integration time step needed to fully describe the dynamics must decrease as well.

The Immersed Boundary Method



The Immersed Boundary Method

The ruling equations can be non-dimensionalized as in Bagheri et al.[1] (2012).

$$x^* = \frac{x}{L}, \mathbf{u}^* = \frac{\mathbf{u}}{U_\infty}, \mathbf{f}^* = \frac{\mathbf{f}}{\rho_0 U_\infty^2 / L}, \mathbf{F}^* = \frac{\mathbf{F}}{\rho_0 U_\infty^2}$$

$$K_s^* = \frac{K_s}{\rho_0 U_\infty^2 L}, K_b^* = \frac{K_b}{\rho_0 U_\infty^2 L^3}$$

Doing so, several dimensionless parameters arises:

$$Re = \frac{U_\infty L}{\nu}, \quad Fr = \frac{\sqrt{gL}}{U_\infty}, \quad M = \frac{\rho}{\rho_0 L}$$

$$E = \frac{K_s}{\rho_0 U_\infty^2 L}, \quad B = \frac{K_b}{\rho_0 U_\infty^2 L^3}, \quad L = \lambda \rho_0 U_\infty$$

[1] Bagheri, S., Mazzino, A., and Bottaro, A., in *PRL*, **109**, 154502 (2012).

The Immersed Boundary Method

This results in the introduction of a body-force field $\mathbf{f}$ that locally mimics the no-slip condition, i.e. such that a desired velocity $\mathbf{V}$ can be assigned over a boundary Γ.

$$\begin{cases} \frac{\partial \mathbf{u}}{\partial t}(\mathbf{x},t) + \mathbf{u}(\mathbf{x},t) \cdot \nabla \mathbf{u}(\mathbf{x},t) = -\nabla p(\mathbf{x},t) + \frac{1}{Re}\nabla^2 \mathbf{u}(\mathbf{x},t) + \mathbf{f}(\mathbf{x},t) \\ \nabla \cdot \mathbf{u}(\mathbf{x},t) = 0 \end{cases}$$

where

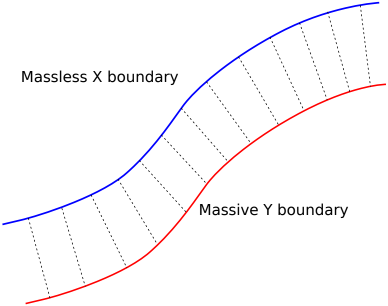
$$\mathbf{f}(\mathbf{x},t) = \int_\Gamma \mathbf{F}(s,t)\delta(\mathbf{x} - \mathbf{X}(s,t))ds$$

in order to fulfill the momentum conservation between solid and fluid, where $\mathbf{F}$ represents the solid stresses.

Massive boundaries

Following the approach showed in Kim and Peskin[2] (2007), we split the IB in two different Lagrangian sets.

In the numerical model we have to add to the familiar equations the dynamical equation of the massive boundary $\mathbf{Y}$ and the elastic forces $\mathbf{F}_K$.



[2] Kim, Y., and Peskin, C. S., in *PoF*, **19**, 053103 (2007).

Porous boundaries

Following the derivation by Kim and Peskin[3] (2006), porosity can be simulated by allowing a relative slip in the normal direction between the IB and the surrounding flow, given by:

$$\mathbf{V}(s,t) = \mathbf{u}_{\mathbf{int}}(s,t) + \lambda(\mathbf{F}(s,t)\cdot\mathbf{n})\mathbf{n}$$

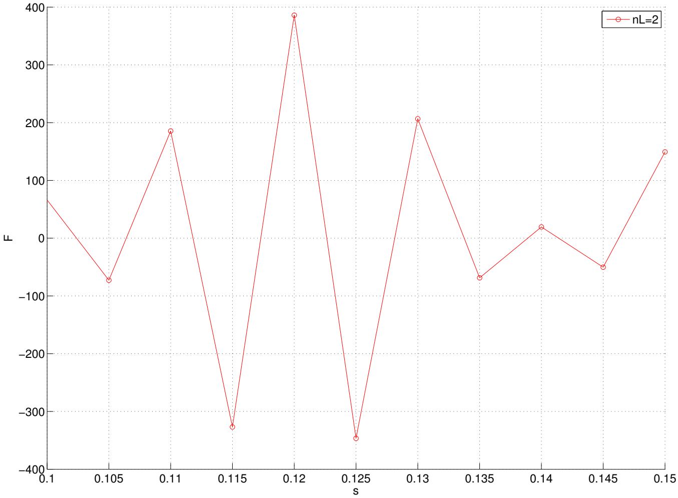
Unlike the classic IBM where **F** comes into play only after integration, here its punctual value is required.

This could cause numerical instabilities when we deal with boundaries having bending stiffness if $\mathbf{F}(s)$ is not a continous function

$$\mathbf{F}_b(s,t) = \gamma \frac{\partial^4 \mathbf{X}(s,t)}{\partial s^4}$$

[3] Kim, Y., and Peskin, C. S., in *SIAM JSC*, **28** (6), 2294-2312 (2006).

Lagrangian forces



Possible strategies

We tried several ways in order to address this issue:

- lower the integration time step,
- increase the Lagrangian set.

Lower the integration time step

We have to set a very low CFL number ($\sim 10^{-4}$) in order to simulate a filament with an acceptable[4] level of porosity ($\lambda \sim 10^{-2}$).

[4]Kim, Y., and Peskin, C. S., in *SIAM JSC*, **28** (6), 2294-2312 (2006)

Increase the Lagrangian set

We define $nL = \Delta x / \Delta s$ (with Δx and Δs being respectively the Eulerian and Lagrangian meshwidth).

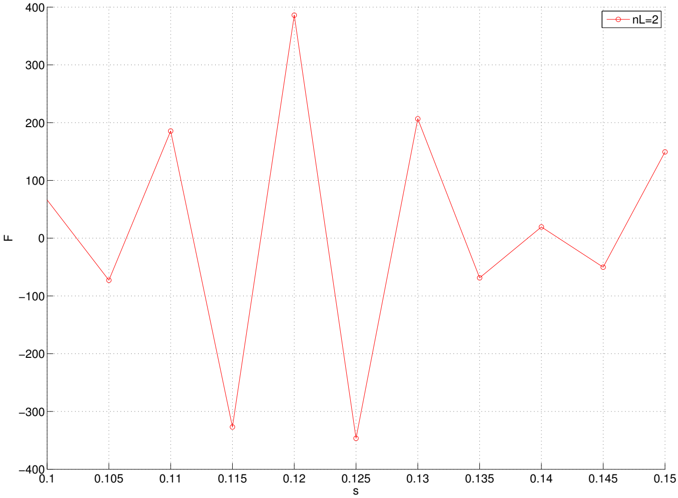
In [5] it is said that

> *"To avoid leaks, the target points should be spaced about half a mesh-width apart (or closer)"*

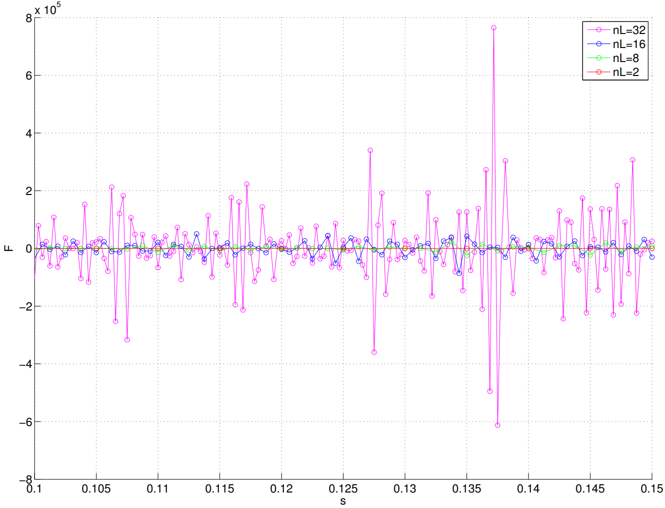
i.e. $nL = 2$. Even though this indication was prescribed not to add unphysical porosity to the IB, we used it as a starting point for our investigation.

[5] Kim, Y., and Peskin, C. S., in *PoF*, **19**, 053103 (2007).

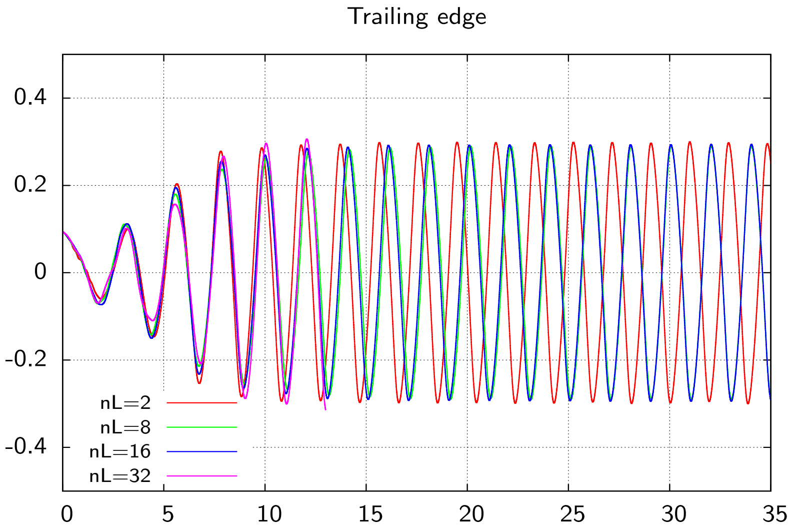
Lagrangian forces



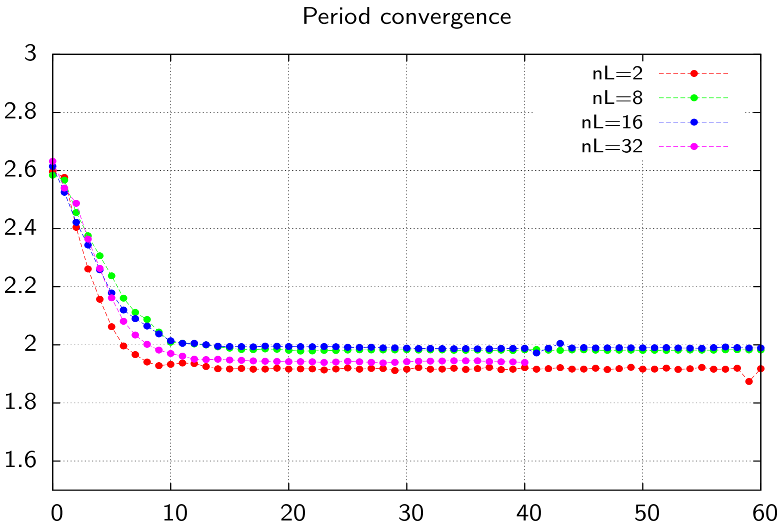
Lagrangian forces



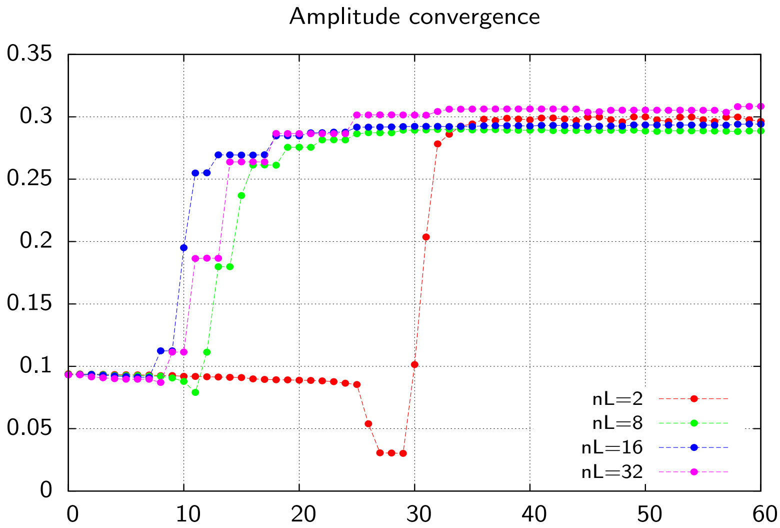
Impermeable simulations



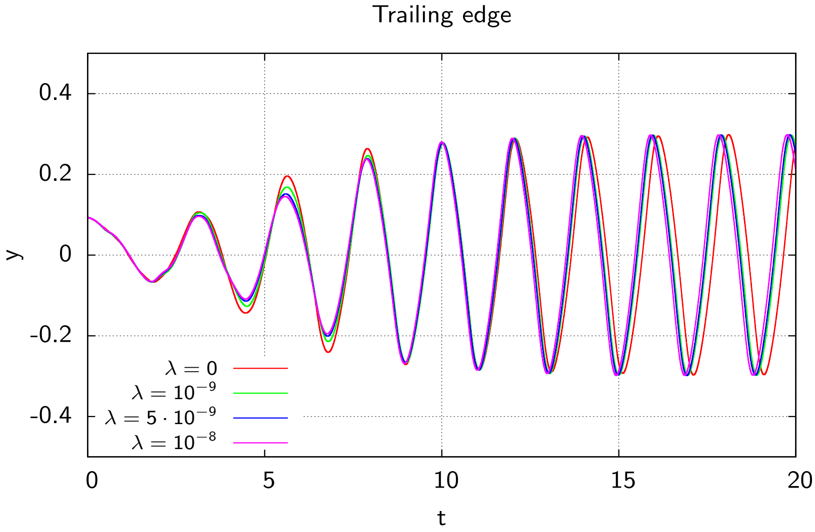
Impermeable simulations



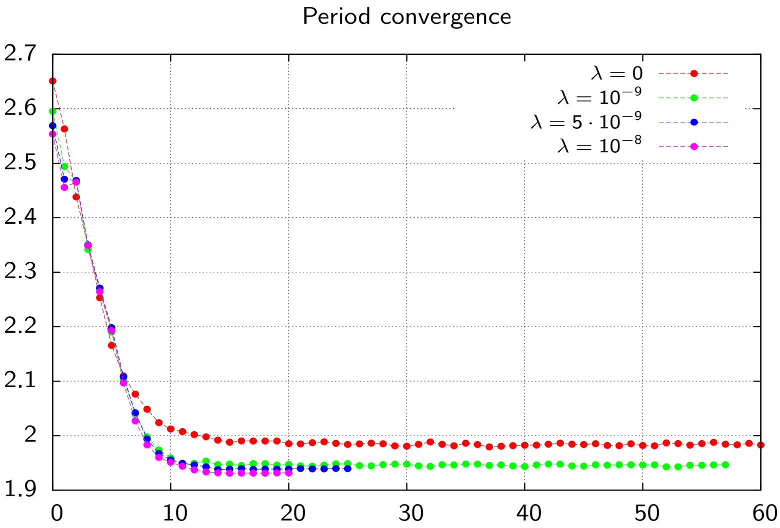
Impermeable simulations



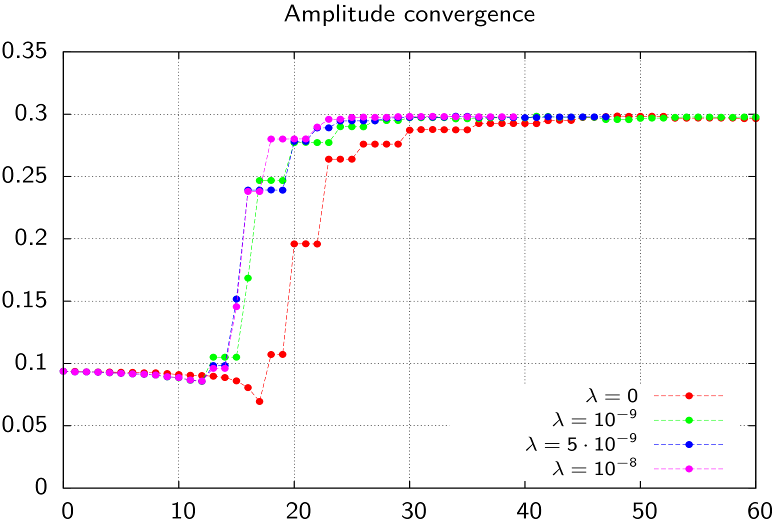
Porous simulations, nL=2



Porous simulations, nL=2



Porous simulations, nL=2



Conclusions and future developments

- A 2D incompressible Navier-Stokes solver (based on the Fractional Step Method) has been written;
- A fluid-structure interaction code taking into account stretching and bending stiffness, mass and porosity has been produced;
- Two different ways of addressing porosity have been explored, neither of them leading to satisfactory results;
- Simulations with $\lambda \simeq 10^{-9}$ have been performed, where porosity affects the oscillation period but not the amplitude;
- We plan to implement of a low-pass filter based on the Fourier transform in order to regularize $\mathbf{F}(s)$ at least at the scale of the Eulerian field, i.e. of the Navier-Stokes equations.

Thanks for the attention!

[1] Zhu, L., and Peskin, C. S., “Simulation of a Flapping Flexible Filament in a Flowing Soap Film by the Immersed Boundary Method”, in *Journal of Computational Physics*, **179**, 452-468 (2002).

[2] Huang, W.-X., Shin, S. J., and Sung, H. J., “Simulation of Flexible Filaments in a Uniform Flow by the Immersed Boundary Method”, in *Journal of Computational Physics*, **226**, 2206-2228 (2007).

[3] Kim, Y., and Peskin, C. S., “Penalty Immersed Boundary Method for an Elastic Boundary with Mass”, in *Physics of Fluids*, **19**, 053103 (2007).

[4] Kim, Y., and Peskin, C. S., “2-D Parachute Simulation by the Immersed Boundary Method”, in *SIAM Journal on Scientific Computing*, **28** (6), 2294-2312 (2006).

[5] Bagheri, S., Mazzino, A., and Bottaro, A., “Spontaneous Symmetry Breaking of a Hinged Flapping Filament Generates Lift”, in *Physical Review Letters*, **109**, 154502 (2012).