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Stability of the interface between two immiscible fluids over a periodically oscillating flat surface

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Introduction to Eye Anatomy

Figure: Eye anatomy
Warning signs of retinal detachment:

- Flashing lights.
- Sudden appearance of floaters.
- Shadows on the side or periphery of your vision.
- Gray curtain moving across your field of vision.

**Figure:** Retinal detachment
Vitreous substitutes are often used after vitrectomy to treat retinal detachments.

Vitreous substitutes cannot be left in the vitreous chamber for too long since they tend to produce emulsifications.

How do the physical parameters of the fluids influence the tendency of the system to produce emulsification?
Introduction

Figure: Emulsification of vitreous substitutes in the vitreous chamber
Fluids commonly used as a vitreous substitutes

- Silicone oils;
  - $960 \leq \rho^* \leq 1290 \text{ kg/m}^3$
  - $10^{-4} \leq \nu^* \leq 5 \times 10^{-3} \text{ m/s}^2$
  - $\sigma^* \approx 0.05 \text{ N/m}$

- Perfluorocarbon liquids;
  - $1760 \leq \rho^* \leq 2030 \text{ kg/m}^3$
  - $8 \times 10^{-7} \leq \nu^* \leq 8 \times 10^{-6} \text{ m/s}^2$
  - $\sigma^* \approx 0.05 \text{ N/m}$

- Semifuorinated alkane liquids;
  - $1350 \leq \rho^* \leq 1620 \text{ kg/m}^3$
  - $4.6 \times 10 \leq \nu^* \leq 10^{-3} \text{ m/s}^2$
  - $0.035 \leq \sigma^* \leq 0.05 \text{ N/m}$
The mechanisms leading to emulsification are still unclear

- Shear layer instability of the aqueous-tamponade fluid interface
- Release by the retina of surfactants that decrease the surface tension at the aqueous-tamponade fluid interface
Formulation of the problem. Mathematical model

Assumptions:

- $d^* \ll R^*$
- 2D-model;
- flat wall oscillating harmonically;
- semi-infinite domain;
- small perturbations;
- quasi-steady approach.

Figure: Geometry of the problem
Scaling and Dimensionless Parameters

\[ x = \frac{x^*}{d^*}, \quad u_i = \frac{u_i^*}{V_0^*}, \quad p_i = \frac{p_i^*}{\rho_1 V_0^{*2}}, \quad t = \frac{V_0^*}{d^*} t, \quad \omega = \frac{d^*}{V_0^*} \omega^* \]

\[ m = \frac{\mu_2^*}{\mu_1^*}, \quad \gamma = \frac{\rho_2^*}{\rho_1^*} \]

\[ R = \frac{V_0^* d^*}{\nu_1^*}, \quad Fr = \frac{V_0^*}{\sqrt{g^* d^*}} \]

\[ S = \frac{\sigma^*}{\rho_1^* d^* V_0^{*2}} \]
Basic flow

\[ U_1 = (c_1 e^{-ay} + c_2 e^{ey}) e^{i\omega t} + c.c., \]
\[ U_2 = c_3 e^{-by} e^{i\omega t} + c.c., \]
\[ P_1 = -Fr^{-2}y + \text{const}, \]
\[ P_2 = -\gamma Fr^{-2}y + \text{const} \]

where

\[ a = \sqrt{i\omega R} \]
\[ b = \sqrt{\frac{i\gamma \omega R}{m}} \]
Basic flow
Range of variability of the dimensionless parameters

Figure: Relationship between $R$ and $\omega$ and $S$ and $\omega$ obtained adopting feasible values of eye movement. From thin to thick curves:

$d = 1 \times 10^{-5} \text{m}$, $d = 5 \times 10^{-5} \text{m}$, $d = 1 \times 10^{-4} \text{m}$
Outline of the Solution

Flow is decomposed:

\[ u_i = U_i + u_i', \quad p_i = P_i + p_i' \]

Stream function:

\[ \bar{u}_i = \frac{\partial \psi_i}{\partial y}, \quad \bar{v}_i = -\frac{\partial \psi_i}{\partial x} \]

which is expanded in Fourier modes in such a way:

\[ \psi_i = e^{i\alpha(x-\Omega t)}\hat{\psi}_i(y, \tau) + c.c \]

where

\[ 0 \leq \tau \leq 2\pi/\omega \]

The system governing the stability is consist of two Orr-Sommerfeld equations and boundary conditions.
Neutral Curves

Figure: $S = 14$, $\gamma = 1.0$, $R = 12$, $\omega = 0.003$
Dependence on $m$

Figure: $S = 14, \gamma = 1.0, R = 12, \omega = 0.003$
Dependence on $S$

**Figure:** $R = 12$, $m = 5.0$, $\gamma = 1.0$, $\omega = 0.003$
Dependence of $R$

**Figure:** $S = 14$, $m = 5.0$, $\gamma = 1.0$, $\omega = 0.003$
Dependence on $\gamma$

Figure: $S = 14$, $m = 5.0$, $R = 12$, $\omega = 0.003$
Conclusions

- Linear stability analysis of two fluids having the same densities and different viscosities shows that waves long enough are linearly unstable during certain phases of the cycle.
- The length of unstable waves becomes longer with viscosity ratio.
- The system can be destabilized either by decreasing the surface tension or by increasing the Reynolds number.
- Heavier fluid on top together with the gravity effect bring system to unstable region.
- The shortest unstable perturbation has a dimensional wavelength $L^* = 6\text{mm}$. This value is twice as small as the eye radius.
Future developments

- Extension of the present work:
  - Energy analysis;
  - Floquet analysis;
  - Non-modal analysis;

- Changing geometry:
  - Including the roughness of the surface;
  - Including the curvature of the surface;
  - Building 2D and 3D model