Stability of the interface between two immiscible fluids over a periodically oscillating flat surface

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September 17, 2013

Introduction to Eye Anatomy

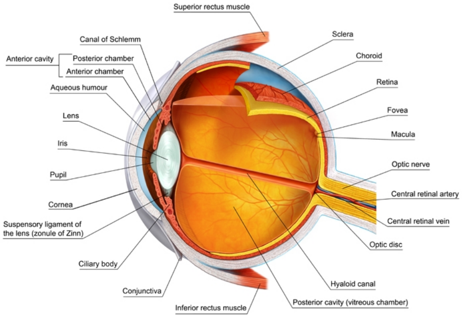


Figure: Eye anatomy

Retinal detachment

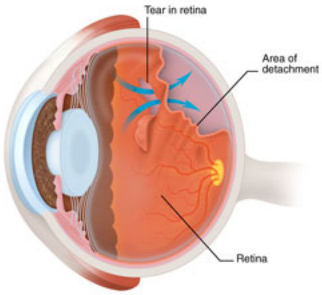


Figure: Retinal detachment

Warning
signs of retinal detachment:

- Flashing lights.
- Sudden appearance of floaters.
- Shadows on the side or periphery of your vision.
- Gray curtain moving across your field of vision.

Motivation

- Vitreous substitutes are often used after vitrectomy to treat retinal detachments.
- Vitreous substitutes cannot be left in the vitreous chamber for too long since they tend to produce emulsifications.
- **How do the physical parameters of the fluids influence the tendency of the system to produce emulsification?**

Introduction

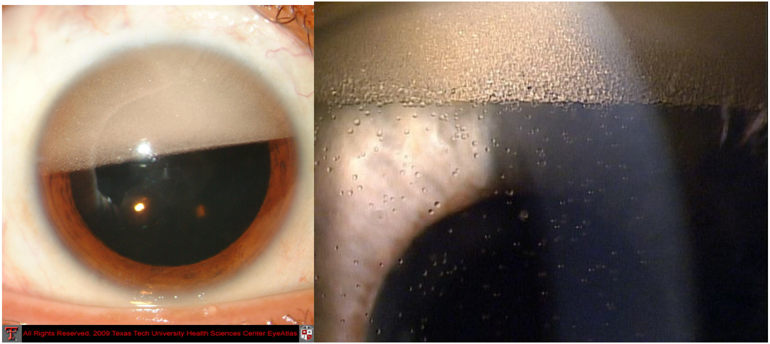
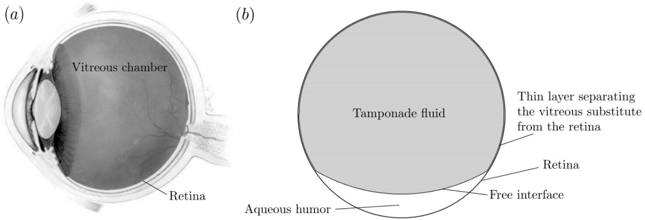


Figure: Emulsification of vitreous substitutes in the vitreous chamber

Fluids commonly used as a vitreous substitutes

- **Silicone oils**;
  - $960 \leq \rho^* \leq 1290$ kg/m^3
  - $10^{-4} \leq \nu^* \leq 5 \times 10^{-3}$ m/s^2
  - $\sigma^* \approx 0.05$ N/m
- **Perfluorocarbon liquids**;
  - $1760 \leq \rho^* \leq 2030$ kg/m^3
  - $8 \times 10^{-7} \leq \nu^* \leq 8 \times 10^{-6}$ m/s^2
  - $\sigma^* \approx 0.05$ N/m
- **Semifluorinated alkane liquids**;
  - $1350 \leq \rho^* \leq 1620$ kg/m^3
  - $4.6 \times 10 \leq \nu^* \leq 10^{-3}$ m/s^2
  - $0.035 \leq \sigma^* \leq 0.05$ N/m

Motivation



The mechanisms leading to emulsification are still unclear

- Shear layer instability of the aqueous-tamponande fluid interface
- Release by the retina of surfactants that decrease the surface tension at the aqueous-tamponande fluid interface

Formulation of the problem. Mathematical model

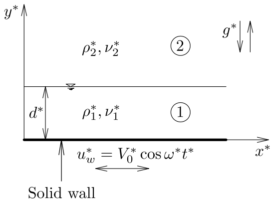


Figure: Geometry of the problem

Assumptions:

- $d^* << R^*$
- 2D-model;
- flat wall oscillating harmonically;
- semi-infinite domain;
- small perturbations;
- quasi-steady approach.

Scaling and Dimensionless Parameters

$$\mathbf{x} = \frac{\mathbf{x}^*}{d^*}, \quad \mathbf{u_i} = \frac{\mathbf{u_i}^*}{V_0^*}, \quad p_i = \frac{p_i^*}{\rho_1^* V_0^{*2}}, \quad t = \frac{V_0^*}{d^*} t, \quad \omega = \frac{d^*}{V_0^*} \omega^*$$

$$m = \frac{\mu_2^*}{\mu_1^*} \qquad \gamma = \frac{\rho_2^*}{\rho_1^*}$$

$$R = \frac{V_0^* d^*}{\nu_1^*} \qquad Fr = \frac{V_0^*}{\sqrt{g^* d^*}}$$

$$S = \frac{\sigma^*}{\rho_1^* d^* V_0^{*2}}$$

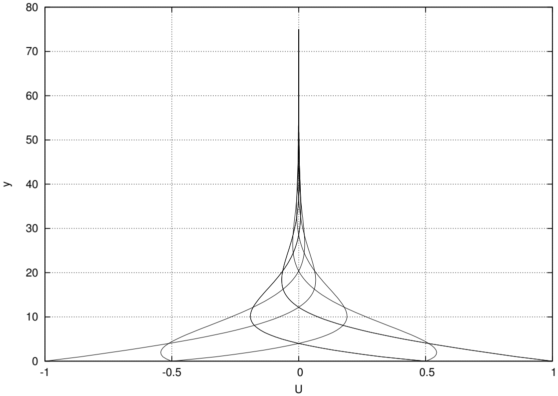
Basic flow

$$U_1 = (c_1 e^{-ay} + c_2 e^{ey}) e^{i\omega t} + c.c., \qquad P_1 = -Fr^{-2} y + const,$$
$$U_2 = c_3 e^{-by} e^{i\omega t} + c.c., \qquad P_2 = -\gamma Fr^{-2} y + const$$

where

$$a = \sqrt{i\omega R} \qquad b = \sqrt{\frac{i\gamma\omega R}{m}}$$

Basic flow



Range of variability of the dimensionless parameters

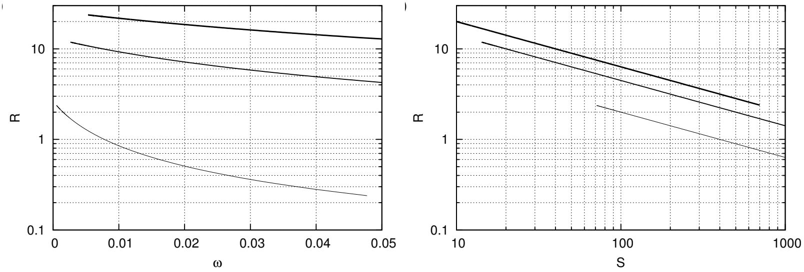


Figure: Relationship between R and ω and S and ω obtained adopting feasible values of eye movement. From thin to thick curves: $d = 1 \times 10^{-5}$m, $d = 5 \times 10^{-5}$m, $d = 1 \times 10^{-4}$m

Outline of the Solution

Flow is decomposed:

$$\mathbf{u_i} = \mathbf{U_i} + \mathbf{u_i}', \quad p_i = P_i + p_i'$$

Stream function:

$$\bar{u}_i = \frac{\partial \psi_i}{\partial y}, \ \bar{v}_i = -\frac{\partial \psi_i}{\partial x}$$

which is expanded in Fourier modes in such a way:

$$\psi_i = e^{i\alpha(x-\Omega t)}\hat{\psi}_i(y,\tau) + c.c$$

where

$$0 \leq \tau \leq 2\pi/\omega$$

The system governing the stability is consist of **two Orr-Sommerfeld equations** and boundary conditions.

Neutral Curves

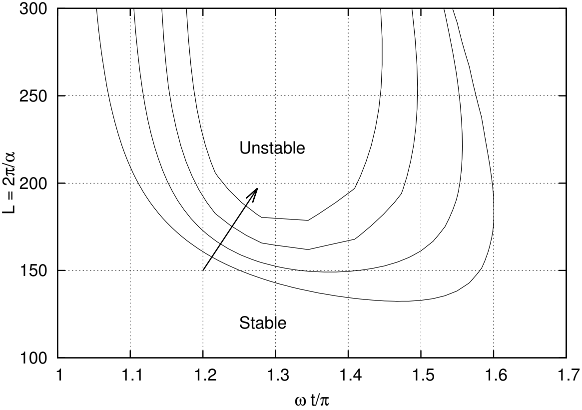


Figure: $S = 14$, $\gamma = 1.0$, $R = 12$, $\omega = 0.003$

Dependence on m

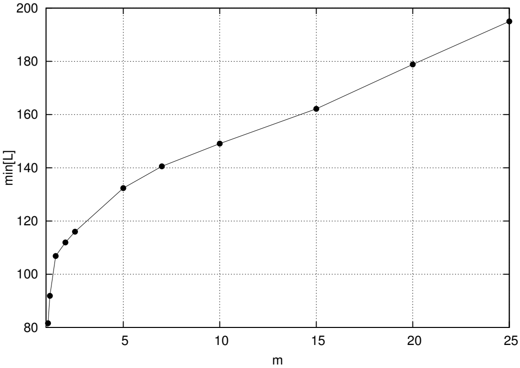


Figure: $S = 14,\ \gamma = 1.0,\ R = 12,\ \omega = 0.003$

Dependence on S

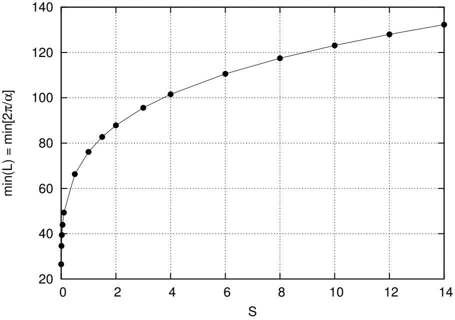


Figure: $R = 12$, $m = 5.0$, $\gamma = 1.0$, $\omega = 0.003$

Dependence of R

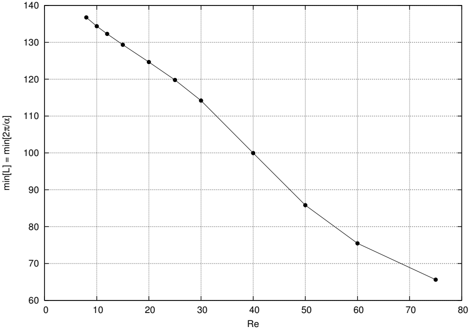


Figure: $S = 14$, $m = 5.0$, $\gamma = 1.0$, $\omega = 0.003$

Dependence on γ

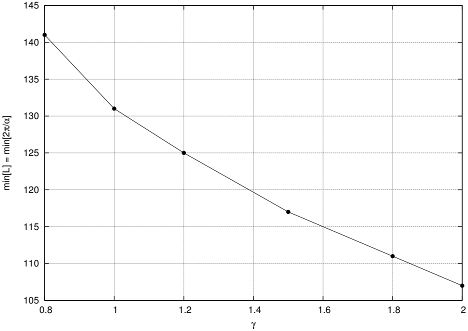


Figure: $S = 14$, $m = 5.0$, $R = 12$, $\omega = 0.003$

Conclusions

- Linear stability analysis of two fluids having the same densities and different viscosities shows that waves long enough are linearly unstable during certain phases of the cycle.
- The length of unstable waves becomes longer with viscosity ratio.
- The system can be destabilized either by decreasing the surface tension or by increasing the Reynolds number.
- Heavier fluid on top together with the gravity effect bring system to unstable region.
- The shortest unstable perturbation has a dimensional wavelength $L^* = 6\text{mm}$. This value is twice as small as the eye radius.

Future developments

- Extension of the present work:
  - Energy analysis;
  - Floquet analysis;
  - Non-modal analysis;
- Changing geometry:
  - Including the roughness of the surface;
  - Including the curvature of the surface;
  - Building 2D and 3D model