STABILITY ANALYSIS OF THE INTERFACE BETWEEN AQUEOUS HUMOUR AND VITREOUS SUBSTITUTES IN VITRECTOMISED EYES

Krystyna Isakova[1], Jan Pralits[1], Rodolfo Repetto[1] and Mario Romano[2]

[1] Department of Civil, Chemical and Environmental Engineering, University of Genoa, Italy
E-mail: *krystyna.isakova@edu.unige.it, jan.pralits@unige.it, rodolfo.repetto@unige.it*
[2] Istituto Clinico Humanitas, Milan, Italy
E-mail: *romanomario@email.it*

Abstract: We study the stability of the interface between to immiscible fluids, the motion of which is induced by oscillations of a solid flat wall. The work represents a first step towards a mechanical understanding of the conditions leading to emulsification of vitreous substitute in the vitreous chamber.

Keywords: *Vitreous substitutes, emulsification, hydrodynamic stability.*

1 Introduction

Retinal detachment (RD) occurs when the sensory layer of the retina detaches from the retinal pigment epithelium. This is a serious condition that might lead to permanent loss of sight and needs immediate treatment. A commonly adopted surgical technique to tread RD is vitrectomy: the vitreous body is surgically removed from the vitreous chamber and it is replaced by “vitreous substitutes”, typically silicon oils. Vitreous substitutes are immiscible with water and their role is to push back the retinal in contact with the pigment epithelium. Owing to their hydrophobic properties tamponade fluids coexist in the vitreous chamber with a certain amount of aqueous humour (the fluid produced in the anterior part of the eye) and, typically, a thin layer of aqueous separates the tamponade fluid from the vitreous chamber wall, see figure $1(b)$. It is known that vitreous substitutes cannot be left for too long in the vitreous chamber since they tend to produce an emulsion. Several clinical studies have tried to investigate the conditions leading to the formation of an emulsion, but the physics of the problem remains poorly understood. Many authors, however, believe that shear stresses at the oil-aqueous interface generated during eye rotations might be responsible to the generation of emulsion (e.g. Toklu et al., 2012).

2 Mathematical model

Owing to the lack of understanding of this instability process we consider a highly idealised problem that, in our view, represents the suitable starting point to understand the basic mechanisms of this instability. We consider the geometry represented in figure 1 a flat solid surface representing the vitreous chamber wall (located at $y^* = 0$), performing sinusoidal oscillations of frequency ω^* and amplitude V^*/ω^* along the x^* direction. We assume that two immiscible fluids occupy the region of space . The interface between the two fluids is at $y = d^*$; fluid 1 (representing the aqueous) occupies the region $0 \leq y^* \leq d^*$, and fluid 2 (representing the tamponade fluid) extends from d to infinity. This geometrical configuration well represents the real case when the thickness of the aqueous layer is much smaller than the radius of the eye, so that the retinal surface can be thought of as a flat surface. We study the linear stability of the interface using the normal mode analysis and assuming quasi-steady flow conditions, e.g. assuming that perturbations evolve on a time scale smaller than the period of oscillation of the basic flow. The stability problem leads to two Orr-Sommerfeld equations for the streamfunctions in fluids 1 and 2, coupled with suitable boundary conditions. This is solved numerically employing a second-order finite-difference scheme with uniform discretisation.

3 Results and discussion

In Figure 2 we show neutral stability curves, i.e. curves on which the growth rate of perturbations is zero, on the plane $L - (\omega^* t^*/\pi)$, where L represents the perturbation wavelength (scaled with d^*, and t^* is time. Each curve corresponds to a different value of the ratio between fluid kinematic viscosities $m = \nu_2^*/\nu_1^*$. The figure shows that sufficiently long waves are linearly unstable during certain phases of the basic flow cycle. Whether instability will actually occur over long time scales clearly depends on the value of the growth rate and on the initial magnitude of perturbations.

Results show that the shortest dimensionless unstable wavelength L_{min} i) grows if the ratio m between the viscosities of the two fluids increases, ii) decreases if the surface tension decreases and iii) decreases if the Reynolds number $R = V^* d^*/\nu_1^*$ increases.

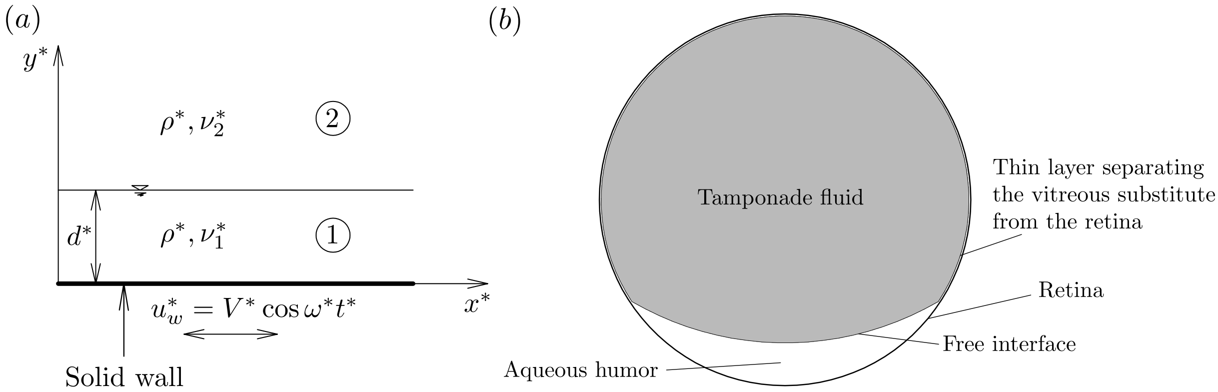

Figure 1: Sketch of the considered geometry and notation (a) and of the vitreous chamber filled with a vitreous substitute (b).

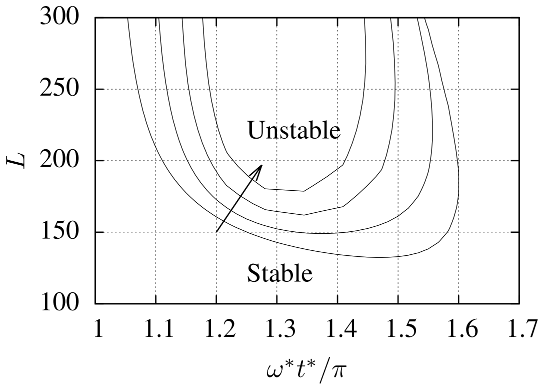

Figure 2: Neutral stability curves in the $L - (\omega^* t^* / \pi)$ plane for different values of the parameter m ($m = 5, 10, 15, 20$). The arrow indicates the direction of increase of $m = \nu_2^* / \nu_1^*$. $R = V^* d^* / \nu_1^* = 12, \omega = d^* / V^* \omega^* = 0.003$, $S = \frac{\sigma^*}{\rho_1^* d^* V^{*2}} = 14$, where σ^* denotes the dimensional surface tension and ρ_1^* the density of fluid 1.

Our results are in qualitative agreement with in-vivo and in-vitro observations of the tendency of vitreous substitutes to produce an emulsion. Indeed, such observations show that:

i) highly viscous vitreous substitutes are more resistant to emulsification than less viscous ones (Crisp et al., 1987; Heidenkummer et al., 1992);
ii) the presence of surfactants in vitreous substitutes, which decrease the surface tension, promotes instability (Yilmaz and Güler, 2008);
iii) patients with increased eye mobility (higher values of R) are more prone to emulsification development.

The model allows us to quantify these effects.

References

A. Crisp, J. de Juan, E, and J. Tiedeman. Effect of silicone oil viscosity on emulsification. *Archives of ophthalmology*, 105(4):546–550, Apr. 1987. ISSN 0003-9950. PMID: 3566611.

H. P. Heidenkummer, A. Kampik, and S. Thierfelder. Experimental evaluation of in vitro stability of purified polydimethylsiloxanes (silicone oil) in viscosity ranges from 1000 to 5000 centistokes. *Retina (Philadelphia, Pa.)*, 12(3 Suppl):S28–32, 1992. ISSN 0275-004X. PMID: 1455079.

Y. Toklu, H. B. Cakmak, S. B. Ergun, M. A. Yorgun, and S. Simsek. Time course of silicone oil emulsification. *Retina (Philadelphia, Pa.)*, 32(10):2039–2044, Dec. 2012. ISSN 1539-2864. doi: 10.1097/IAE.0b013e3182561f98. PMID: 22653542.

T. Yilmaz and M. Güler. The role of nystagmus in silicone oil emulsification after pars plana vitrectomy and silicone oil injection for complex retinal detachment. *European journal of ophthalmology*, 18(1):150–154, Feb. 2008. ISSN 1120-6721. PMID: 18203104.