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OPTIMIZATION OF FISHING ACTIVITY AND REPOPULATION IN A SIMULATED MODEL

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Abstract

Study of a model representing the growth of a coastal population under an external forcing.

The work first concerned the determination of the state equation governing the problem and the definition of the variables and parameters required to deal the problem.

Then has been done the analysis of the problem using the Lagrange operators method in order to obtain the fundamental equations to write down the model's code.

Once discretized the equations and defined the fundamentals matrices has been possible to implement the code and use it to simulate different dynamic situations of a coastal population growth, with and without an external forcing.

It resulted that the optimization code enables to find the optimal fishing/repopulation vector which guarantee the survival of the species.

Introduction

Today resource management is fundamental in every economic sector; energy, money, food, in a industry as much as in a natural environment.

Recently the worldwide demand for goods resulted in a critical unbalance in different sectors, leading sometimes to an environment and resources abuse which would results in a resources exhaustion.

This is the case of fishing, where the global demand of fish, crustaceans and other edible species has almost become a serious threat to the survival of several species in different places around the world.

A reckless fishing will also destroy natural habitats and will get a strong interference in the natural processes that affect the life cycle of the species, resulting thus in a destructive external forcing that must be adjusted to avoid the irreversible erasing of one or several species.

In order to establish a correct fishing way, it is of primary importance define a maximum fishing rate for each moment in a time period typical for a life cycle of a determined species.

This evaluation must be done after to have determined the natural life cycle of the species without any disturbance, in order to compare the natural evolution with the fishing forcing that can be tolerated by the marine populations.

In this work we tried to do something like this, determining a analytical model which could represent the dynamic of the population growth for some species distributed along a coastline interested by a considerable along-shore current.

The target was to find the optimal forcing vector to maintain a sufficient abundance level in the populations in order to allow the survival of the species.

Methods

First of all we started with the analysis of the growth dynamic for a generic population of species.

The situation requires the division in number j sectors of the analyzed coastline, corresponding to number j sub-populations.

Each of those populations is connected to the other with different weights, so we can represent this connection net by a connectivity matrix which reports the relationship between each site.

This matrix has a Gaussian distribution along its rows but due to the nature of the intense along-shore current the matrix must be asymmetric [1].Figure 1 shows an example.

The population growth is also affected by the natural mortality which can be represented with a diagonal matrix, by a density dependent settlement rate expressed as a matrix and at last by the fishing rate.

The relations between those factors can be written according to the following state equation which governs the problem:

$$\frac{d\underline{n}}{dt} = \left(\underline{\underline{K}}(t)\underline{\underline{S}}(n) - \underline{\underline{M}} - \underline{\underline{F}} \right)\underline{n} \qquad [1]$$

where

$\underline{\boldsymbol{n}}$ is the state vector containing the number of individuals in each population,

$\underline{\underline{\boldsymbol{K}}}(\boldsymbol{t})$ is the time dependent dispersal matrix, which defines the probability of competent larval delivery to each of j local populations per unit time per local adult at time t. Its diagonal therefore represents the level of self-recruitment of the populations. Furthermore it is composed by a constant part, $\underline{\underline{\boldsymbol{Ko}}}$, and by a time variable part, $\gamma\underline{\underline{\boldsymbol{K_t}}}(\boldsymbol{t})$ normally distributed, such that $< \underline{\underline{\boldsymbol{K_t}}}(\boldsymbol{x}, \boldsymbol{y}) > = \boldsymbol{0}$ and

$< \underline{\underline{\boldsymbol{K_t}}}(\boldsymbol{x}, \boldsymbol{y}) >^{\boldsymbol{2}} = \boldsymbol{1}$.

$\underline{\underline{\boldsymbol{S}}}(\boldsymbol{n})$ is the density dependent settlement rate, which is expressed as

$$\underline{\underline{S}}(n) = \underline{\underline{I}} - \underline{\underline{\Sigma}}(n) \qquad [2]$$

where $\underline{\underline{\boldsymbol{\Sigma}}}(\boldsymbol{n})$ = diag(n1/N1 n2/N2 … nj/Nj) , diag(…) denotes a matrix with elements along the diagonal and zeros elsewhere and Nj is the maximum abundance in population j.

$\underline{\underline{\boldsymbol{M}}}$ is the mortality rate matrix expressed as diag(m1 m2 … mj) where mj are the local mortality rates per unit time and

$\underline{\underline{\boldsymbol{F}}}$ is the fishing rate matrix, diag(f1 f2 … fj).

Since the equation [1] shown before is not linear we must linearize it to be able to deal with it.

After linearization we have:

$$\frac{d\underline{\hat{n}}}{dt} = \left(\underline{\underline{K}} - 2\underline{\underline{K}}\,\underline{\underline{\Sigma}}(\underline{n}^*) - \underline{\underline{M}} - \underline{\underline{F}}\right)\underline{\hat{n}} \qquad [3]$$

where $\underline{\hat{\boldsymbol{n}}}$ is the perturbation defined as

$$\underline{\hat{n}} = \underline{n} - \underline{n}^* \qquad [4]$$

with $\underline{\boldsymbol{n}}^*$ solution of equation [1].

The purpose of this work is to obtain an optimal fishing rate which enables the survival of the marine population allowing at the same time the fishing in that place.

So the new state equation [3] has been decomposed as follows in order to define a forcing vector usable in the next analysis steps. What we have now is thus:

$$\frac{d\underline{\hat{n}}}{dt} = \left(\underline{\underline{K}} - 2\underline{\underline{K}}\,\underline{\underline{\Sigma}}(\underline{n}^*) - \underline{\underline{M}}\right)\underline{\hat{n}} - \underline{\underline{F}}\,\underline{\hat{n}} \qquad [5]$$

which can be written so:

$$\frac{d\underline{\hat{n}}}{dt} = \left(\underline{\underline{K}} - 2\underline{\underline{K}}\,\underline{\underline{\Sigma}}(\underline{n}^*) - \underline{\underline{M}}\right)\underline{\hat{n}} - \underline{f} \qquad [6]$$

$$\frac{d\underline{\hat{n}}}{dt} = \underline{\underline{A}}\,\underline{\hat{n}} - \underline{f} \qquad [7]$$

where $\underline{\boldsymbol{f}} = \underline{\underline{F}}\,\underline{\hat{n}}$ is the forcing of the problem.

Now we can start the determination of the required equations for the code writing; using the La Grange operators method we will find the already known state equation, the adjoint equation, the initial conditions and the optimized forcing vector.

We define our output as

$$J = \frac{\gamma_1}{2}\left(\underline{\hat{n}}(T) - \underline{\hat{n}}_T\right)^T\left(\underline{\hat{n}}(T) - \underline{\hat{n}}_T\right) + \frac{\gamma_2}{2}\int_0^T \underline{f}^T\,\underline{f}\,dt \qquad [8]$$

where $\underline{\hat{n}}_T$ is the target, i.e. the value desired as final state vector of the populations.

Rewriting the equation states in that way

$$F = \frac{d\underline{\hat{n}}}{dx} - \underline{\underline{A}}\,\underline{\hat{n}} + \underline{f} = 0 \qquad [9]$$

our LaGrangian is

$$L\left(\underline{\hat{n}}, \underline{f}, \underline{a}, \underline{b}\right) = J - \int_0^T \underline{a}F\,dt - \underline{b}\left(\underline{\hat{n}}(0) - \underline{\hat{n}}_0\right) =$$

$$= \frac{\gamma_1}{2}\left(\underline{\hat{n}}(T) - \underline{\hat{n}}_T\right)^T\left(\underline{\hat{n}}(T) - \underline{\hat{n}}_T\right) + \frac{\gamma_2}{2}\int_0^T \underline{f}^T\,\underline{f}\,dt - \int_0^T \underline{a}\left(\frac{d\underline{\hat{n}}}{dx} - \underline{\underline{A}}\,\underline{\hat{n}} + \underline{f}\right)dt - \underline{b}\left(\underline{\hat{n}}(0) - \underline{\hat{n}}_0\right) \quad [10]$$

Proceeding, placing the derivatives of L respect to $\underline{\hat{n}}, \underline{f}, \underline{a}$ *and* $\underline{b}$ equal to zero we obtain the following results in the order:

$$\frac{dL}{d\underline{a}} = 0 \Leftrightarrow -\int_0^T \delta\underline{a}\left(\frac{d\underline{\hat{n}}}{dx} - \underline{\underline{A}}\,\underline{\hat{n}} + \underline{f}\right)dt = 0 \Leftrightarrow \frac{d\underline{\hat{n}}}{dt} - \underline{\underline{A}}\,\underline{\hat{n}} + \underline{f} = 0 \qquad \textit{state equation} \qquad [11]$$

$$\frac{dL}{d\underline{b}} = 0 \Leftrightarrow -\delta\underline{b}\left(\underline{\hat{n}}(0) - \underline{\hat{n}}_0\right) = 0 \Leftrightarrow \underline{\hat{n}}(0) = \underline{\hat{n}}_0 \qquad \textit{initial condition} \qquad [12]$$

$$\frac{dL}{d\underline{\hat{n}}} = 0 \Leftrightarrow \gamma_1\left(\underline{\hat{n}}(T) - \underline{\hat{n}}_T\right)\delta\underline{\hat{n}} - \left[\underline{a}(t)\delta\underline{\hat{n}}(t)\right]_0^T - \int_0^T\left(-\frac{d\underline{a}}{dt} - \underline{\underline{A}}^T\underline{a}\right)\delta\underline{\hat{n}}\,dt - \underline{b}\delta\underline{\hat{n}}_0 = 0 \quad \Leftrightarrow$$

$$\Leftrightarrow \quad \left[\gamma_1\left(\underline{\hat{n}}(T) - \underline{\hat{n}}_T\right) - \underline{a}(T)\right]\delta\underline{\hat{n}} - \int_0^T\left(-\frac{d\underline{a}}{dt} - \underline{\underline{A}}^T\underline{a}\right)\delta\underline{\hat{n}}\,dt - \left[\underline{a}(0) - \underline{b}\right]\delta\underline{\hat{n}}_0 = 0 \qquad [13]$$

which gives

$$\underline{a}(T) = \gamma_1\left(\underline{\hat{n}}(T) - \underline{\hat{n}}_T\right) \qquad [14]$$

$$-\frac{d\underline{a}}{dt} = \underline{\underline{A}}^T\underline{a} \qquad \textit{adjoint equation} \qquad [15]$$

$$\underline{b} = \underline{a}(0) \qquad [16]$$

$$\frac{dL}{d\underline{f}} = 0 \Leftrightarrow \gamma_2\int_0^T \underline{f}^T\,\delta\underline{f}\,dt - \int_0^T \underline{a}^T\,\delta\underline{f}\,dt = 0 \Leftrightarrow \int_0^T \left(\gamma_2\underline{f} - \underline{a}\right)\delta\underline{f}\,dt = 0 \Leftrightarrow$$

$$\Leftrightarrow \gamma_2\underline{f} = \underline{a} \Leftrightarrow \underline{f} = \frac{\underline{a}}{\gamma_2} \qquad \textit{optimized forcing} \qquad [17]$$

Next step is the discretization of all equations found above. Since in the code implementation have been used many time steps, resulting in very small time intervals, we used the explicit method sure to keep the system stable. However, the stability check shown in fig 2 shows the perfect stability characterizing the system.

Discretizing the state equation we have

$$\frac{\underline{\hat{n}}^{i+1} - \underline{\hat{n}}^{i}}{\Delta t} = \underline{\underline{A}}\,\underline{\hat{n}}^{i} - \underline{f}^{\,i} \qquad [18]$$

which treated becomes

$$\underline{\hat{n}}^{i+1} = \underline{\underline{B}}\,\underline{\hat{n}}^{i} - \underline{f}^{\,i} dt \qquad [19]$$

$$with \qquad \underline{\underline{B}} = \left(\underline{\underline{I}} + dt\underline{\underline{A}}\right) \qquad [20]$$

Equation [19] must be put in the code loop for the calculation of the integral from 0 to T with

$\underline{\hat{n}}^{i=0} = \underline{\hat{n}}_0 = \underline{\hat{n}}(0)$ as initial condition.

For the adjoint instead we have:

$$\underline{a}^{i} = \underline{\underline{B}}^{T}\,\underline{a}^{i+1} \qquad [21]$$

which, inserted in the loop for the calculation of the integral from T to 0 with eq.[14] (which discretized is $\underline{\hat{a}}^{N} = \gamma_1\left(\underline{\hat{n}}^{N} - \underline{\hat{n}}_T\right)$ [22]) as initial condition leads to the $\underline{\hat{a}}(0)$ evaluation, essential to determine the optimized forcing vector, according to equation [17].

This method includes adjoint, whose error can be appreciated, already discretized, as

$$error = \left|\left(\underline{a}^{N}\right)^{T}\underline{\hat{n}}^{N} - \left(\underline{a}^{0}\right)^{T}\underline{\hat{n}}^{0} + \left(\frac{1}{\gamma_2}\right)\sum_{i=0}^{N-1}\left(\left(\underline{a}^{i+1}\right)^{T}\underline{a}^{1}dt\right)\right| \qquad [22]$$

Now is possible to write down the code for the simulation of the growth dynamic of the marine species living in a generic coastline. We try to achieve the species survival acting on the output reported in equation [8] choosing an appropriate value of $\underline{\hat{n}}_T$.

Results and discussions

What it results from the many simulations performed is the survival of all the populations after an established time T, although the growth trend is often decreased by the acting forcing, interpreted as a fishing action.

The particular structure of the output allows us to interpret the external forcing vector $\underline{f}$ not only as a fishing rate, but also as a repopulation forcing when, during the simulation running, a population reaches a critical value or takes a descendent growth trend.

The magnitude and the sign of the forcing vector components also depend on the initial condition $\underline{\hat{n}}_0$ and the target value $\underline{\hat{n}}_T$.

For example if we start from a low value of $\underline{\hat{n}}_0$ and we have relatively high value for $\underline{\hat{n}}_T$, i.e. higher than the final value $\underline{\hat{n}}\,(T)$ of the system without forcing, the resulting forcing will be a repopulation vector whose components will have negative sign and magnitude proportional to the difference between initial condition and target value. Figure y show it.

Instead, for all cases with $\underline{\hat{n}}_T$ sufficiently low compared to the unforced value of $\underline{\hat{n}}\,(T)$, the external forcing will simply result as a fishing vector with the most of its components with a positive sign.

In figures 4.1-4.2 we can observe the trend of a forcing vector and of its components for a simulation run with a output target set at 25% of the maximum abundance.

Therefore the code performed managed to find the optimized fishing condition for a determined environment guarantying the survival of the species which live in it. It also verifies the validity of the optimization method in such a kind of analysis.

Asymmetric Connectivity Matrix									
0,606531	1	0,606531	0,135335	0,011109	0,000335	3,73E-06	1,52E-08	2,29E-11	1,27E-14
0,135335	0,606531	1	0,606531	0,135335	0,011109	0,000335	3,73E-06	1,52E-08	2,29E-11
0,011109	0,135335	0,606531	1	0,606531	0,135335	0,011109	0,000335	3,73E-06	1,52E-08
0,000335	0,011109	0,135335	0,606531	1	0,606531	0,135335	0,011109	0,000335	3,73E-06
3,73E-06	0,000335	0,011109	0,135335	0,606531	1	0,606531	0,135335	0,011109	0,000335
1,52E-08	3,73E-06	0,000335	0,011109	0,135335	0,606531	1	0,606531	0,135335	0,011109
2,29E-11	1,52E-08	3,73E-06	0,000335	0,011109	0,135335	0,606531	1	0,606531	0,135335
1,27E-14	2,29E-11	1,52E-08	3,73E-06	0,000335	0,011109	0,135335	0,606531	1	0,606531
2,58E-18	1,27E-14	2,29E-11	1,52E-08	3,73E-06	0,000335	0,011109	0,135335	0,606531	1
1,93E-22	2,58E-18	1,27E-14	2,29E-11	1,52E-08	3,73E-06	0,000335	0,011109	0,135335	0,606531

Figure 1: Asymmetric connectivity matrix. the values inserted in each cell report the connection level between the two corresponding sectors, for example in the cell identified by row 2 and column 5 is reported the connection between population 2 and 5.

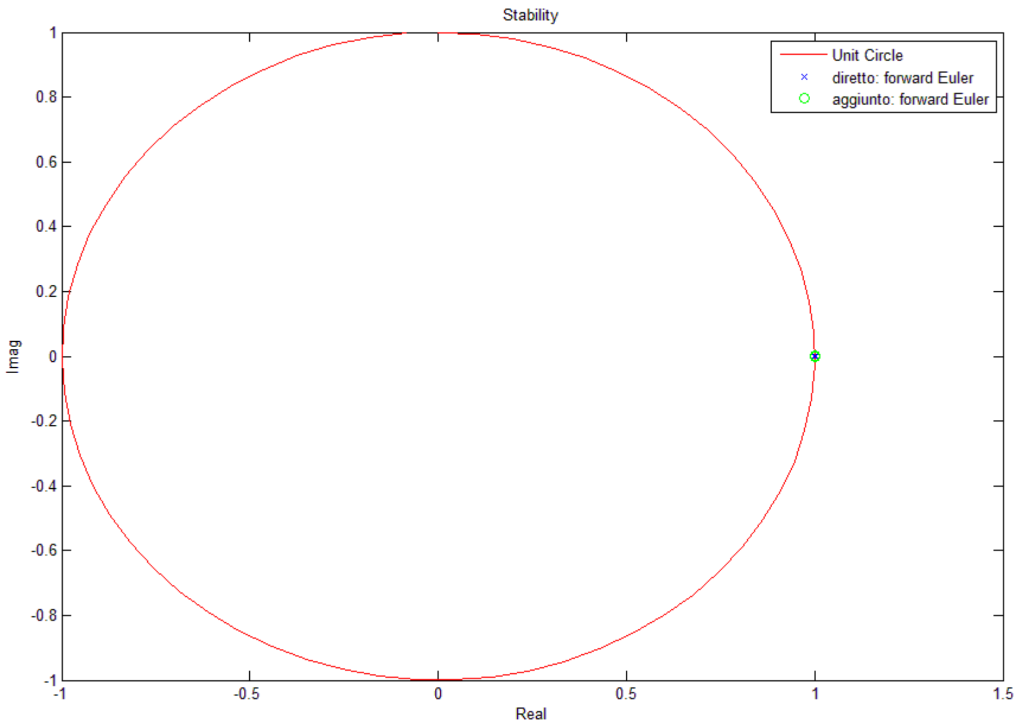

Figure 2: stability plot. It is shown a perfect system stability.

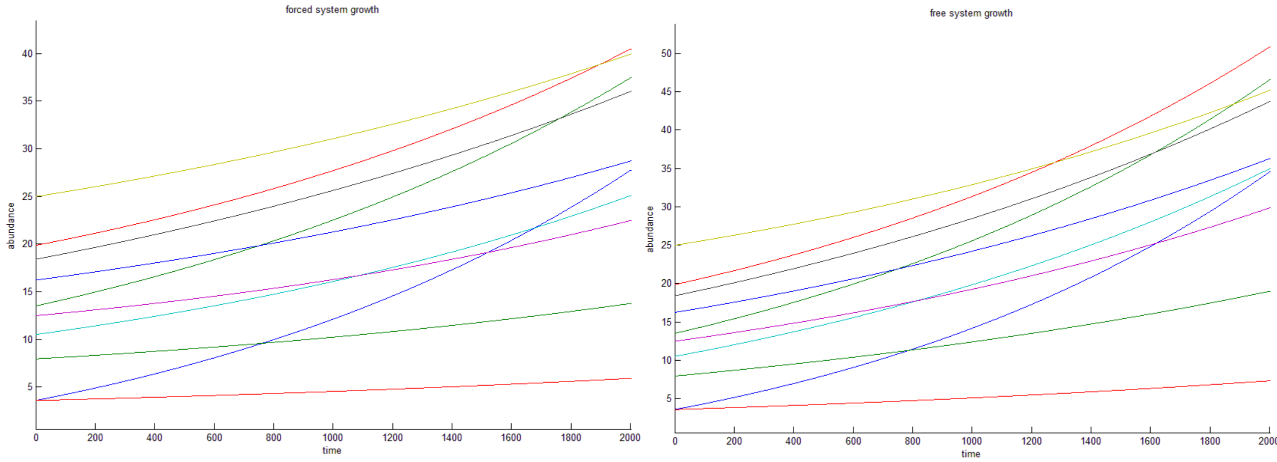

Figure 3.1: forced system growth compared with free system growth. Are plotted the different components of the state vector in   function with time. It’s visible an attenuation of the growth in the forced system.

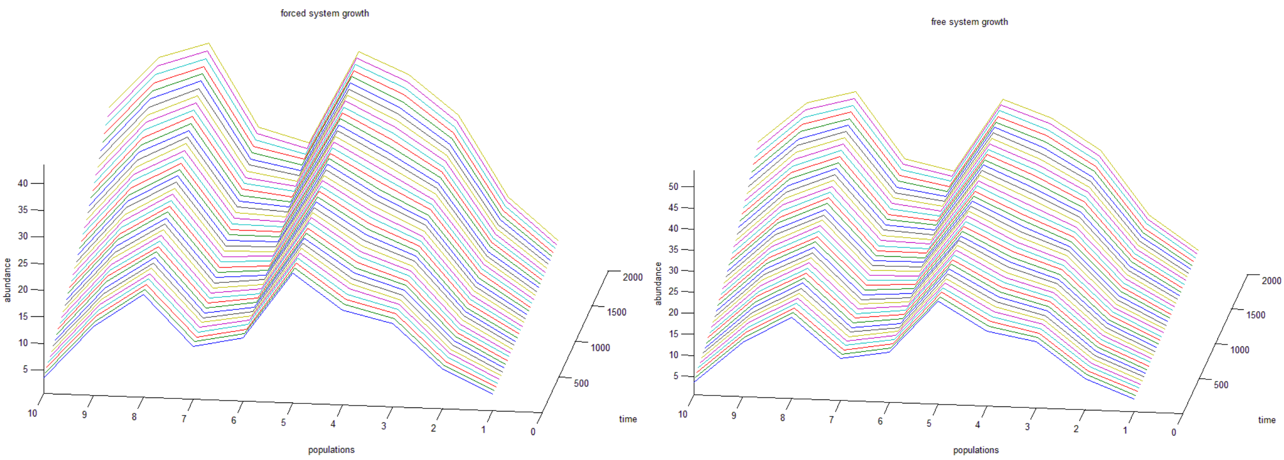

Figure 3.2: vector evolution in time for the forced system and for the free system.

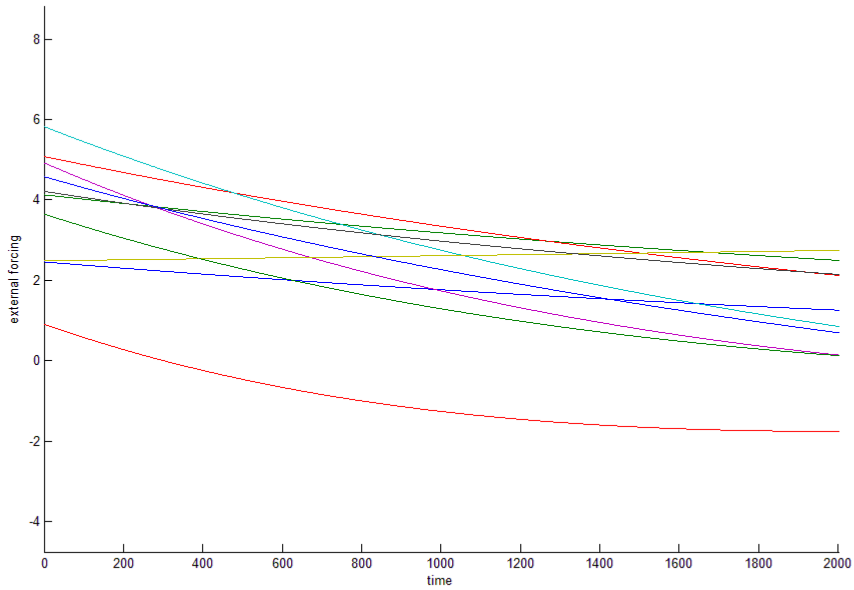

Figure 4.1: external forcing in function with time. It's evident a component which becomes negative, therefore interpretable as a repopulation component. All the other positive components are thus fishing rates.

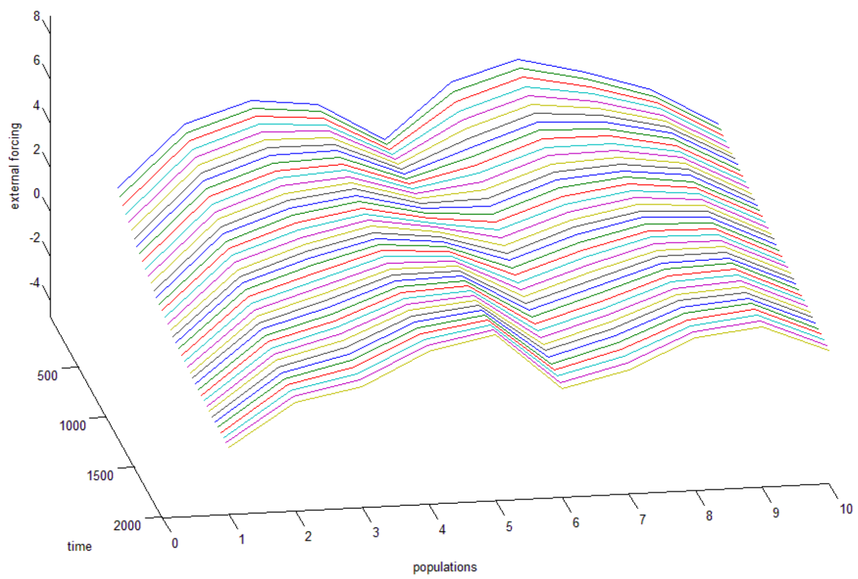

Figure 4.2: external forcing vector evolution in time.

References

[1] C. Aiken, S.A. Navarrete, ”Stability of a simple meta-population model subject to environmental variability”,2008

[2] J. Pralits, “Advanced fluid dynamics ”, Internal notes, University of Genoa, 2012

[3] D. Kaplan, “Along-shore advection and marine reserves: consequences for modeling and management”, 2006

[4] Mat lab users license, 2012

[5] A. M. Bradley, “PDE-constrained optimization and the adjoint method”, 2010