Transition to turbulence at the bottom of a solitary wave

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Outline

1. Motivation : solitary waves & sediment transport
2. Laminar turbulent transition
3. Boundary layer flow
4. Linear stability analysis
5. Results
6. Conclusions

Background

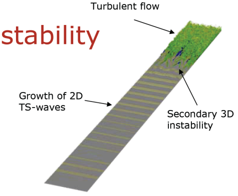
Far from the coast the influence of surface waves on the bottom layer is insignificant. As the waves move closer to the coast the shear stress in the boundary layer increases and destabilizes the upper layers of sediment. Even closer to the coast the boundary layer changes from laminar to turbulent and sediment transport becomes even more intense. This is due to the large vortical structures that "picks" up sediment from the bottom to a level where the local velocity is higher and consequently enhance transport.

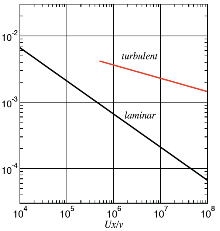
The main differences between laminar and turbulent flow when it comes to sediment transport are

- Laminar flow : forces act locally on the sediment and the grain "diameter" becomes the important length scale
- Turbulent flow : large vortices "picks" up sediment, mixing, transport

It is therefore of importance to understand in what circumstances (parametrically) the flow transitions

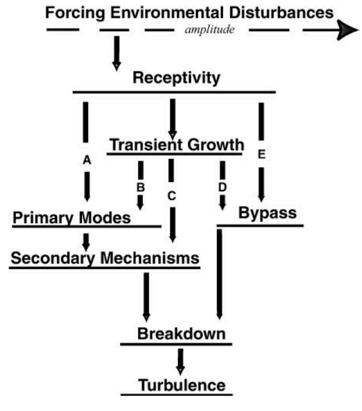
Why study laminar-turbulent transition ?



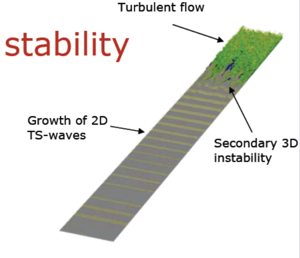


- increase our understanding in a field of study which is still not completely understood
- fluid forces (lift, drag, ...) increase when the flow becomes turbulent
- mixing is enhanced due to turbulence
- sediment transport increases due to coherent structures (vortices)

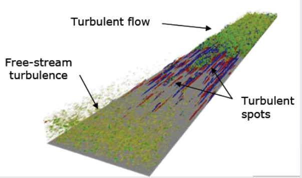
Routes to transition : highly dependent on Tu



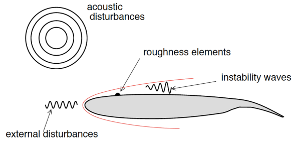
$Tu \sim 0.1\%$



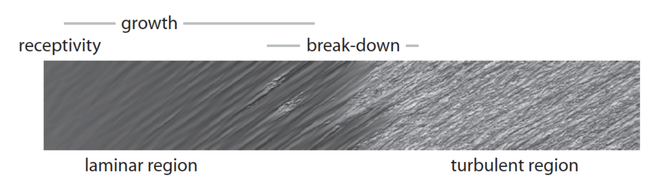
$Tu \sim 10\%$



Classical route to transition : low Tu, Modal analysis



1. Receptivity: Initial amplitudes of unstable waves need to be estimated to capture transition "location"
2. Disturbance growth is initially linear and accurately predicted by Linear Stability Theory (LST)
3. Breakdown of disturbances, nonlinear process, finally leading to turbulence



Transition scenario in solitary wave boundary layer

Few investigations exist concerning the common "route" to transition in solitary wave boundary layers.

We don't know

- The receptivity mechanisms (environmental *Tu*, wall roughness, ...)
- If the dominant linear growth is modal or nonmodal
- ...consequently if the dominant waves are 2D or 3D
- The late stages in the transition process (nonlinear, saturation, ...)

Where do we start ?

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Where do we start ? Classical Modal analysis

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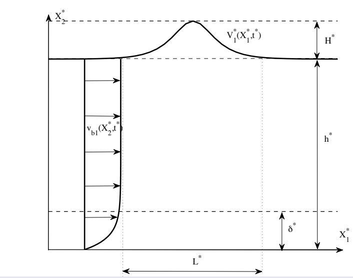
Where do we start ? Classical Modal analysis Compare with DNS & Exp.

Definition of the basic flow : surface



Assume
$H = H^*/h^* << 1$
$\mu = h^*/L^* << 1$ (Boussinesq)

with $H \sim \mu^2$, neglecting the wave damping and H^2 terms one obtains (Grimshaw, 1971) the free surface elevation and wave propagation velocity as

$$\eta^*(X_1^*, t^*) = H^* sech^2 \left(\sqrt{\frac{3H}{4}} \zeta \right)$$

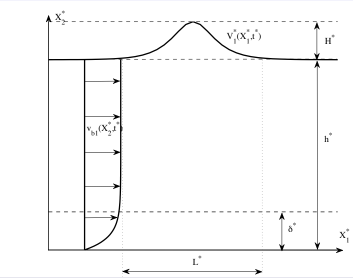
$$V_1^*(X_1^*, t^*) = H\sqrt{g^* h^*} sech^2 \left(\sqrt{\frac{3H}{4}} \zeta \right)$$

where

$$\zeta = (X_1^* - \sqrt{g^* h^*} t^*)/h^* = X_1 - t$$

Note that:
$U_{ref}^* = H\sqrt{g^* h^*}$ $\quad L_{ref}^* = H^*$ and $Re = U_{ref}^* L_{ref}^* / \nu^* = H\sqrt{g^* h^*} H^* / \nu^* \sim (H/\delta)^2$

Definition of the basic flow : bottom boundary layer



$$Re_\delta = H\sqrt{g^* h^*}\delta^*/\nu^* = \sqrt{Re}$$

The upper (air) boundary layer is neglected (τ_{xy} small)

In the bottom boundary layer viscous and inertial effects should balance

$$\frac{\partial}{\partial t^*} \sim \sqrt{g^* h^*}/h^*, \quad \nu^* \frac{\partial^2}{\partial X_2^{*2}} \sim \nu^*/\delta^{*2}$$

$$\rightarrow \delta^* \sim \sqrt{\nu^* h^*/\sqrt{g^* h^*}}$$

Here : $\delta^*/h^* << 1$, consequently we can use

Boundary Layer Approximation

v_{b2}^* is negligible (continuity equation)

$\partial p^*/\partial X_2^* = 0$ (y momentum equation)

v_{b1}^* is then obtain by solving

$$\frac{\partial v_{b1}^*}{\partial t^*} = \left.\frac{\partial V_1^*}{\partial t^*}\right|_{X_2=0} + \nu^* \frac{\partial^2 v_{b1}^*}{\partial X_2^{*2}}$$

b.c : $v_{b1}^* = 0$ at $X_2^* = 0$ and $\frac{\partial v_{b1}^*}{\partial X_2^*} \rightarrow 0$ as $\rightarrow \infty$

Definition of the basic flow : solution

Following Mei, "The applied dynamics of ocean surface waves" (1989), the solution can be written as

$$v_{b1}(X_2,\zeta) = sech^2\left(\sqrt{\frac{3H}{4}}\zeta\right) - \frac{2}{\sqrt{\pi}}\int_0^{\infty} sech^2\left[\sqrt{\frac{3H}{4}}\left(\frac{1}{2}\frac{X_2^2}{\xi^2}+\zeta\right)\right] e^{-\xi^2}\,d\xi, \quad \text{with} \quad \zeta = X_1$$

Case : Sumer et al. (2010), $H = 0.12$, $\delta = 0.0005$

Movie 1

Definition of the basic flow : solution contd.

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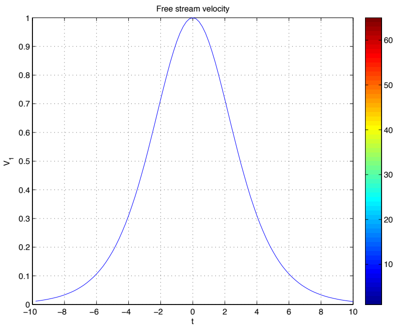

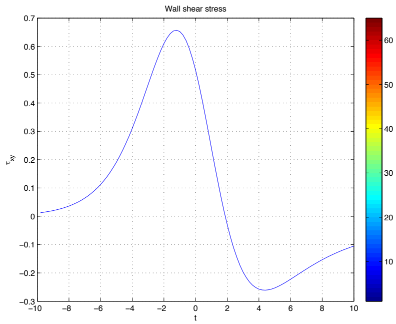

Definition of the basic flow : solution contd.

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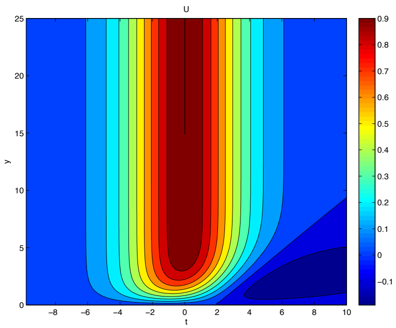

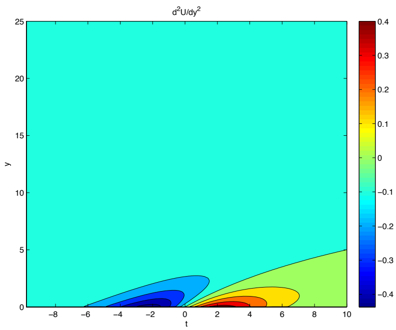

Governing equations

We consider analyzing 2D perturbations and assume a decomposition as

$$(v_1, v_2, p) = (v_{b1}, 0, p_b) + \epsilon(v_{p1}, v_{p2}, p_p) \quad \text{where} \quad \epsilon << 1,$$

and $U^*_{ref} = H\sqrt{g^* h^*}$, $L^*_{ref} = \delta^*$, $t^*_{ref} = L^*_{ref}/U^*_{ref}$, $p^*_{ref} = \rho^* H g^* \delta^*$. It is further imposed a non-slip condition at the bottom wall and the perturbations are assumed to decay far from the bottom (free stream)

The system is further reduced using the stream function such that $v_{p1} = \partial\psi/\partial x_2$ and $v_{p2} = -\partial\psi/\partial x_1$ which, by definition, satisfies continuity. Introducing the

decomposition, stream function, and dropping ϵ^2 terms (linearising) gives the following equation

$$\frac{\partial}{\partial t}(\Delta\psi) + \frac{H}{\delta}\left[v_{b1}\frac{\partial}{\partial x_1}(\Delta\psi) - \frac{\partial^2 v_{b1}}{\partial x_2^2}\frac{\partial\psi}{\partial x_1}\right] = \frac{1}{2}\Delta^2\psi,$$

where $\Delta = \partial/\partial x_1^2 + \partial/\partial x_2^2$

$Re_\delta \sim H/\delta$.

Governing equations contd.

Consider that $H^* >> \delta^*$ which means that $H/\delta >> 1$ Remember $Re_\delta \sim H/\delta$.

The amplitude of the perturbation is assumed to grow on a time scale much faster that the basic flow.

The modal form of the stream function can therefore be written

$$\psi(x_1, x_2, t) = f(x_2, t) \exp\left[i\alpha \left(x_1 - \frac{H}{\delta} \int c(\tau) d\tau \right) \right],$$

which means that we seek an asymptotic solution at each instant in time for the basic flow. The governing equation becomes

$$[v_{b1}(x_2, t) - c(t)]\hat{\Delta} f(x_2, t) - \frac{\partial^2 v_{b1}}{\partial x_2^2} f(x_2, t) = \frac{1}{2i\alpha(H/\delta)} \hat{\Delta}^2 f(x_2, t),$$

Note that the viscous effect is accounted for in the term $1/(H/\delta)$. Further, the variables t and X_1 appear only in the combination $\zeta = X_1 - t$.

This is an eigenvalue problem for the complex valued variable $c(t)$. The solution $c(t)$ is the so called dispersion relation and we can note that $c = c(H, \delta, \alpha, \zeta)$.

The real part c_r is the phase speed and the imaginary part c_i is the growth rate. $c_i > 0$ means un unstable solution, i.e. a solution which grows in time.

Example result from LST (Absolut value)

Case : Sumer et al. (2010), $H = 0.12$, $\delta = 0.0005$, $\alpha = 0.2$

Movie 1

Example result from LST (Real part)

Case : Sumer et al. (2010), $H = 0.12$, $\delta = 0.0005$, $\alpha = 0.2$

Movie 1

Outline

The results are presented in the following way

- Experiments by Sumer et al. (2010)*
    - U-shaped water tunnel excited by piston mechanism
    - $L \times H \times B = 10 \times 0.29 \times 0.39 m^3$
    - Flow visualization with color CCD camera (25 frames/second)
    - shear stress (hot film probe) and
      free stream velocity (Laser doppler anemometer, LDA) measurements
- Linear Stability Analysis : critical conditions (ζ, *alpha*)
- Comparison with Direct Numerical Simulation

*Sumer et al. (2010), "Coherent structures in wave boundary layers. Part 2. Solitary motion", Journal of Fluid Mechanics, **646**, 207-231

Video (plan view) from Sumer et al. (2010)

Video from experiments by sumer et al. (2010) where $H = 0.12$, $\delta = 0.0005$, flow from left to right. The video shows the vortex tubes in plan view.

Movie 1

Video (side view) from Sumer et al. (2010)

Video from experiments by sumer et al. (2010) where $H = 0.11$, $\delta = 0.00054$, flow from left to right. The video shows the vortex tubes in side view.

Movie 2

Video (plan view) from Sumer et al. (2010)

Video from experiments by sumer et al. (2010) where $H = 0.199$, $\delta = 0.00043$, flow from left to right. The video shows the vortex tubes in side view.

Movie 3

Linear stability results

From the linear stability results we obtain the phase velocity c_r (or frequency $\omega = \alpha c_r$) and growth rate for a given set of parameters $c = c(H, \delta, \alpha, \zeta)$. It does not give the whole transition scenario but at least the initial part (critical values) indicating the physical mechanisms which later on lead to transition and turbulent flow. Parameters in the figure taken from Sumer et al. (2010).

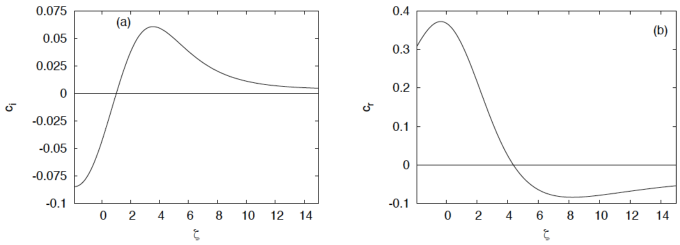

FIGURE 1. Growth rate c_i (a) and phase velocity c_r (b) plotted versus ζ for $\alpha = 0.20, H = 0.12$ and $\delta = 5.0 \times 10^{-4}$.

Linear stability results

A worst case scenario can be assumed which requires to "scan" the whole parameter space, $c = c(H, \delta, \alpha, \zeta)$. In such a way critical conditions can be established as shown in the figure. Here it is shown that the instability occurs for $\zeta > 0$ which means the deceleration phase.

In this case $\zeta_c = 1.0$ and the corresponding wave number $\alpha_c = 0.2$.

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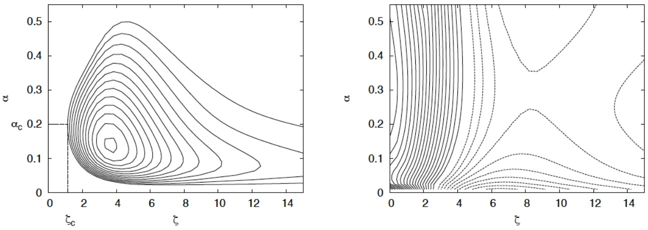

FIGURE 2. (a) Growth rate c_i ($\Delta c_i = 0.005$, only positive values are plotted); and (b) phase velocity c_r ($\Delta c_r = 0.02$, continuous lines, $c_r \geqslant 0$); broken lines $c_r < 0$). plotted versus ζ and α for $H = 0.12$ and $\delta = 5.0 \times 10^{-4}$.

Comparison with DNS

In Direct Numerical Simulations (DNS) the flow is computed without any approximations. It is therefore a "numerical experiment" to compare the Linear Stability (LST) results with. Two different DNS computations have been performed.

- Given initial condition of the perturbations
- Model of distributed wall roughness during the whole wave cycle

This gives different Receptivity scenarios and it is shown that the latter agrees better with LST (worst case scenario).

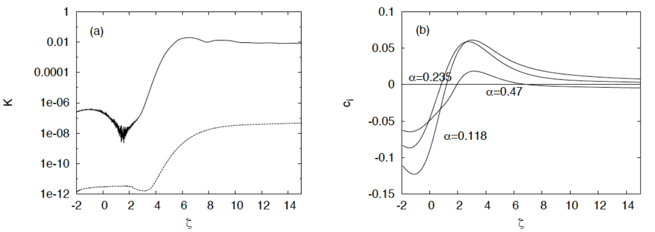

FIGURE 3. (a) Dimensionless kinetic energy per unit area K of the perturbations of the laminar boundary layer under a solitary wave, computed using the numerical approach of Vittori & Blondeaux (2008) for $H = 0.20$ and $\delta = 8 \times 10^{-4}$. The broken line is the value of K obtained introducing a perturbation of the laminar flow at the beginning of the numerical simulation and considering a perfectly plane wall, the continuous line is the value obtained introducing wall imperfections and considering vanishing initial condition. (b) Growth rate c_i plotted versus ζ for $H = 0.20$, $\delta = 8 \times 10^{-4}$ and three different values of α.

Summary of results : comparison between LST and experiments

Summary of experiments by Sumer et al. (2010) in comparison with Linear stability results. A reasonable agreement is found regarding the critical wave number α_c, while the critical time (LST) is under estimated.

Exp. no :	H	δ	α_c LST	α_c exp	ζ_c LST	ζ_c exp
1	0.12	0.0005	0.2	0.21-0.3	1.01	3.18
2	0.108	0.00054	0.2	0.23-0.3	1.16	4.77
3	0.199	0.00043	0.21	0.23-0.27	0.53	2.23
4	0.096	0.0006	0.205	0.19-0.26	1.39	4.81

Conclusions

- The solitary wave boundary layer is unstable if the height H exceeds a certain threshold, for a given boundary layer thickness δ.
- The instability sets in during the deceleration phase (for the parameters investigated).
- The critical wave length found by LST is similar to the distance between the vortex tubes found in the experiments by Sumer et al. (2010)
- The threshold wave height is under estimated by LST
- The discrepancy between DNS and LST might be explained considering different receptivity scenarios.