Seminari DICAT
14 Marzo, 2007

Mechanics of masonry structures: arches, shear walls and vaults

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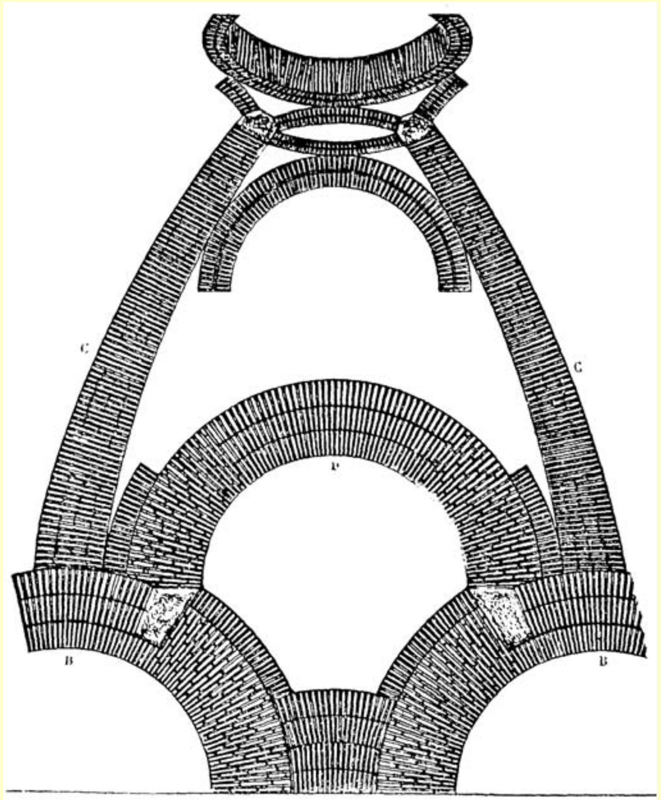
Layout:

- Historic and old masonry buildings
- Modelling: general aspects
- Columns, arches and bridges
- Walls
- Domes
- Conclusions

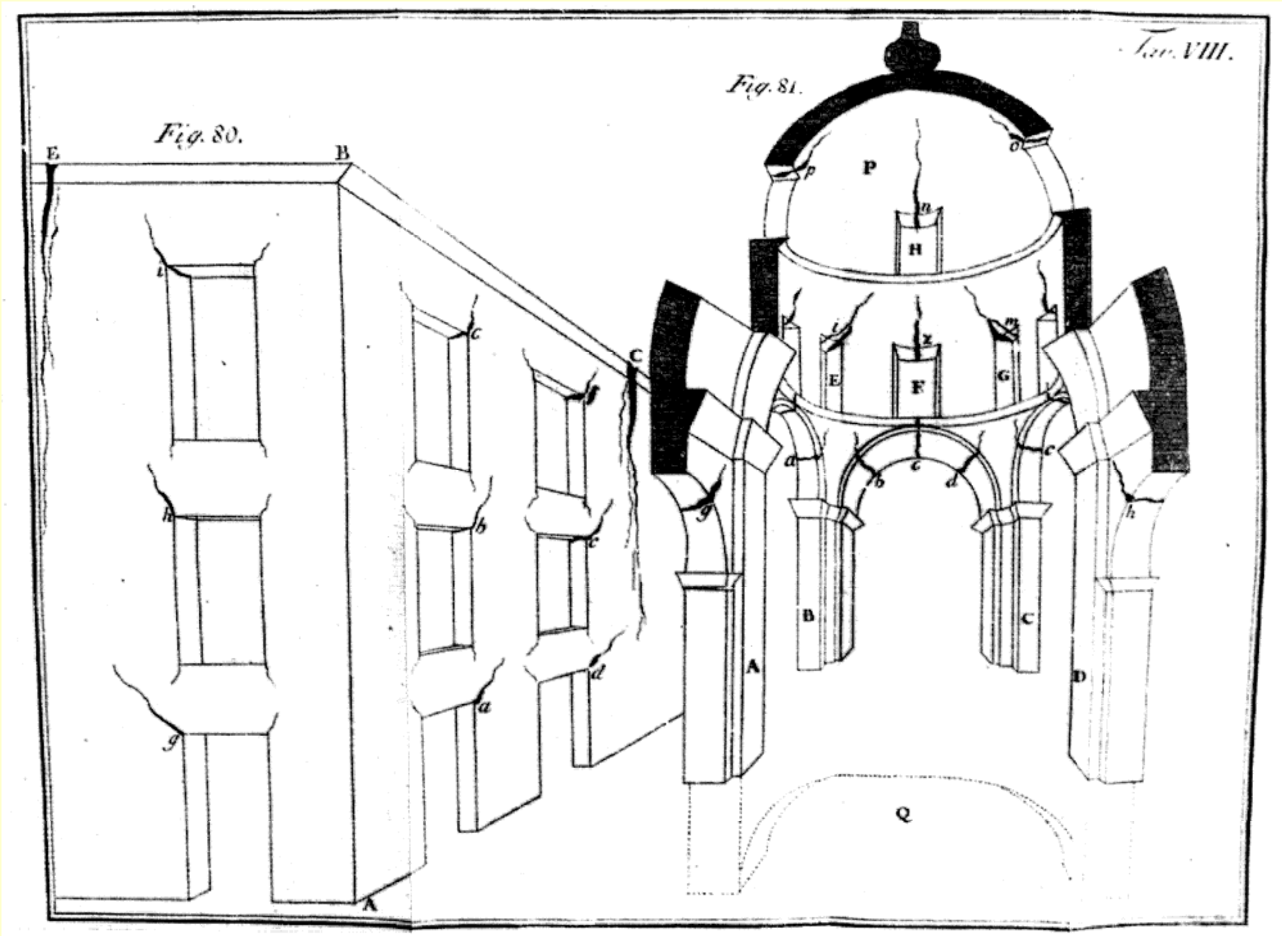


Piranesi: Pantheon (Choisy)

Web site:

prinpontimuratura.diseg.unige.it

1. Historic masonry constructions: from damage to safety



V. Lamberti, Statica degli edifici, Napoli, 1781

1. Knowledge about historic constructions:

- Historical research
- Historic construction techniques and materials
- Inspection-damage

2. Mechanical modeling:

- Interpretation of damages – diagnosis
- Simulation
- Assessment
- Evaluation of strengthening techniques

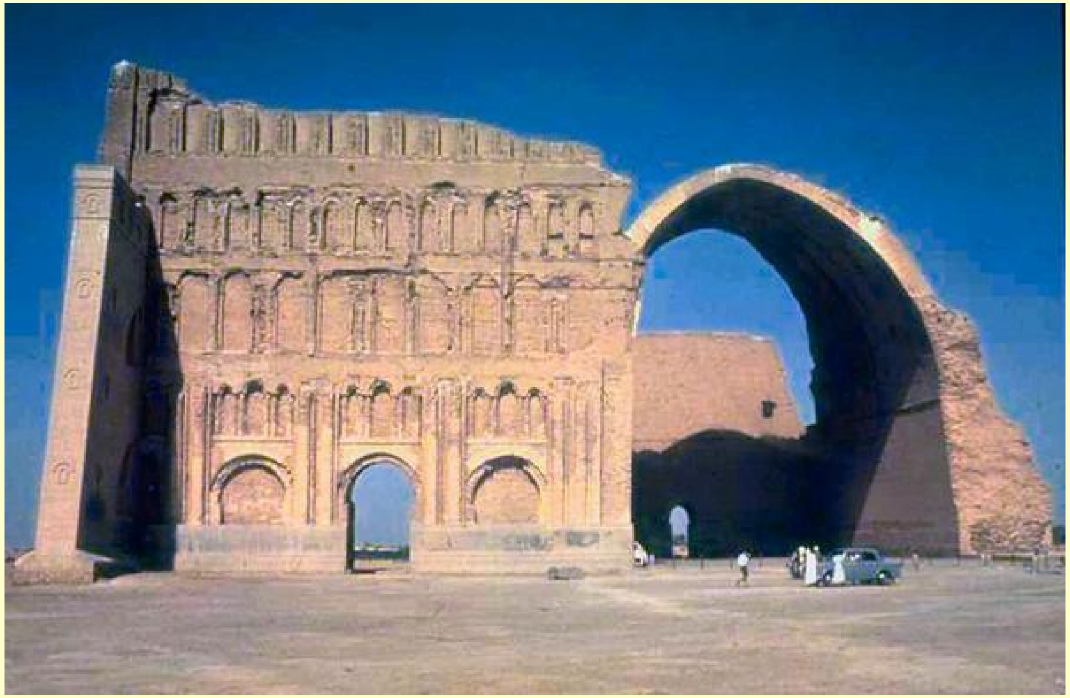
3. Design

- Assessment of structural safety
- Design of repairs (if required)

Arches

Temple of Sethi I and Ramses II, XIX Dinasty

Roma,
Mercati traianei



Palace at Ctesiphon, A.D. 550

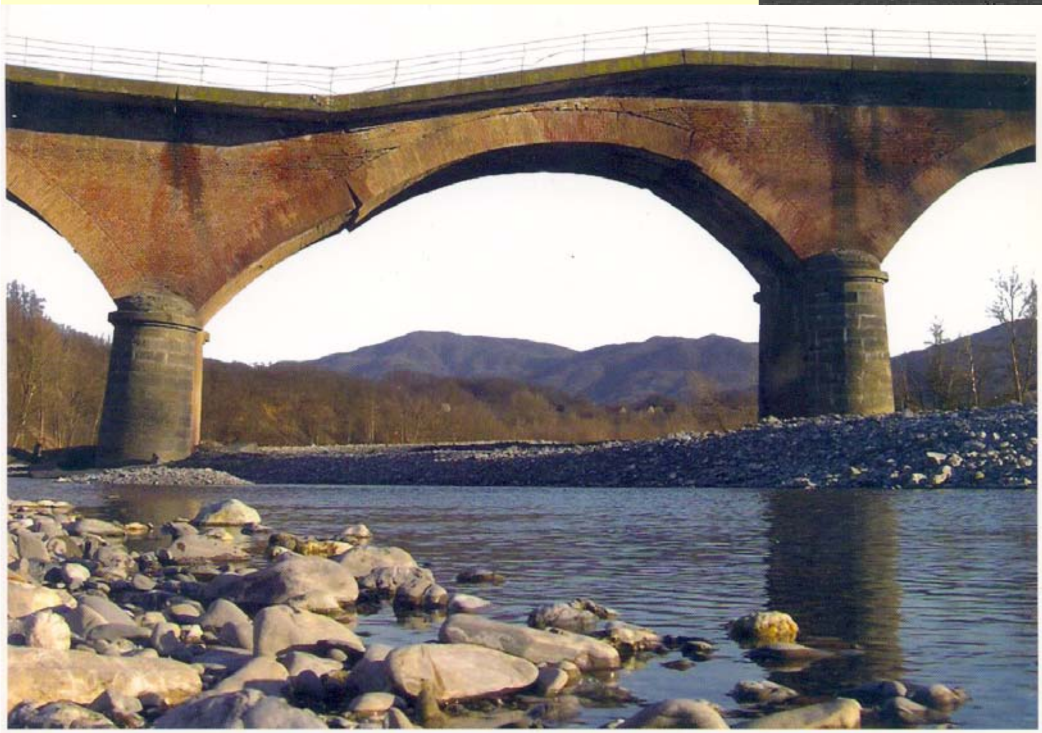
Arches

Umbria-Marche Earthquake, 1997

Masonry bridges

Prestwood
Bridge (Page, 1993)

Road bridge
Arquata S., Alessandria



Railway bridge (Bologna-Piacenza)

Masonry walls

Out-of-plane collapse

Umbria-Marche Earthquake, Colfiorito, 1997

Masonry walls

In-plane collapse

Umbria-Marche Earthquake, Colfiorito, 1997

South Piemonte
Earthquake, 2003

Vaults

Umbria-Marche Earthquake, 1997

Masonry domes

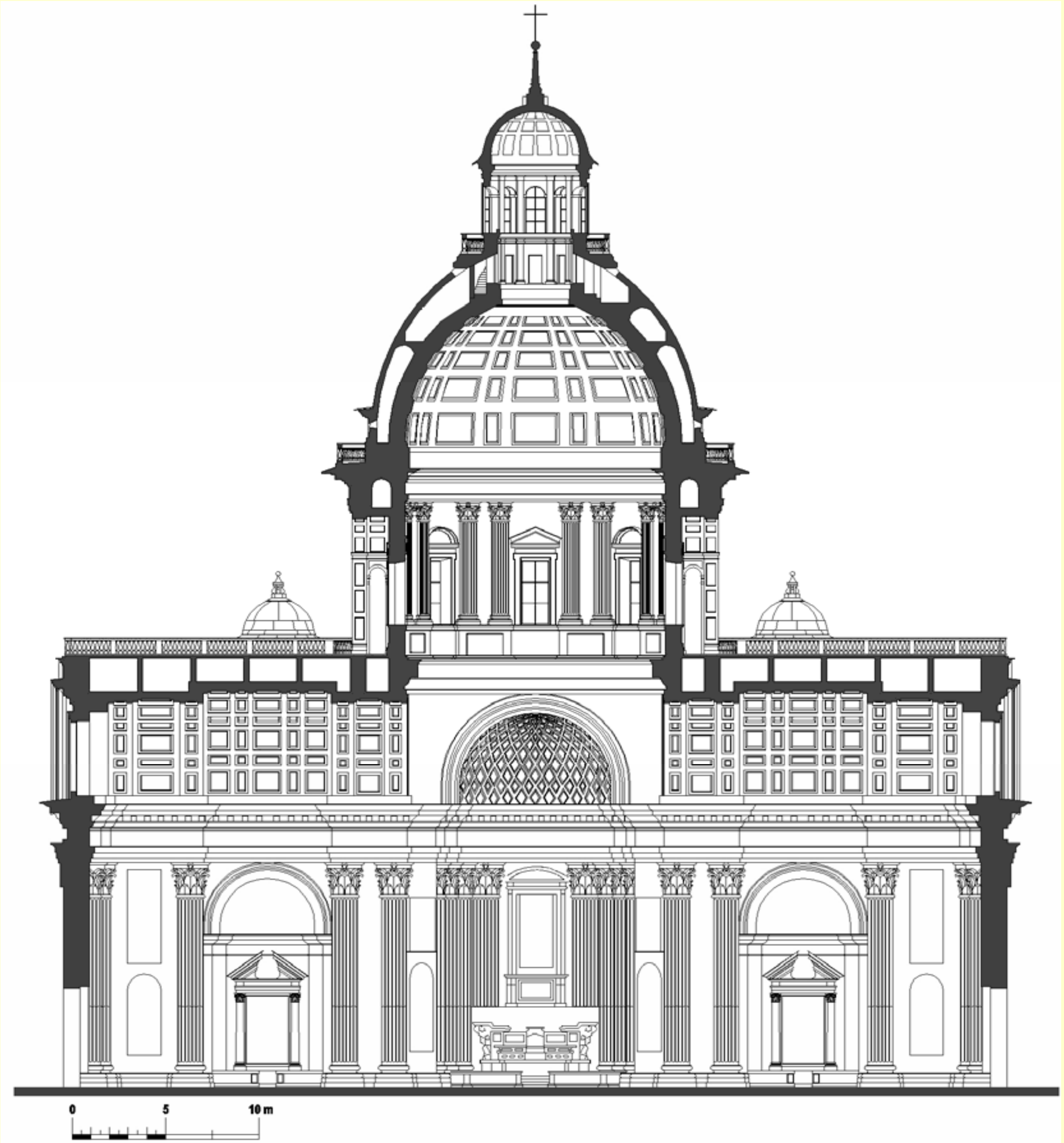
S. Maria del Fiore

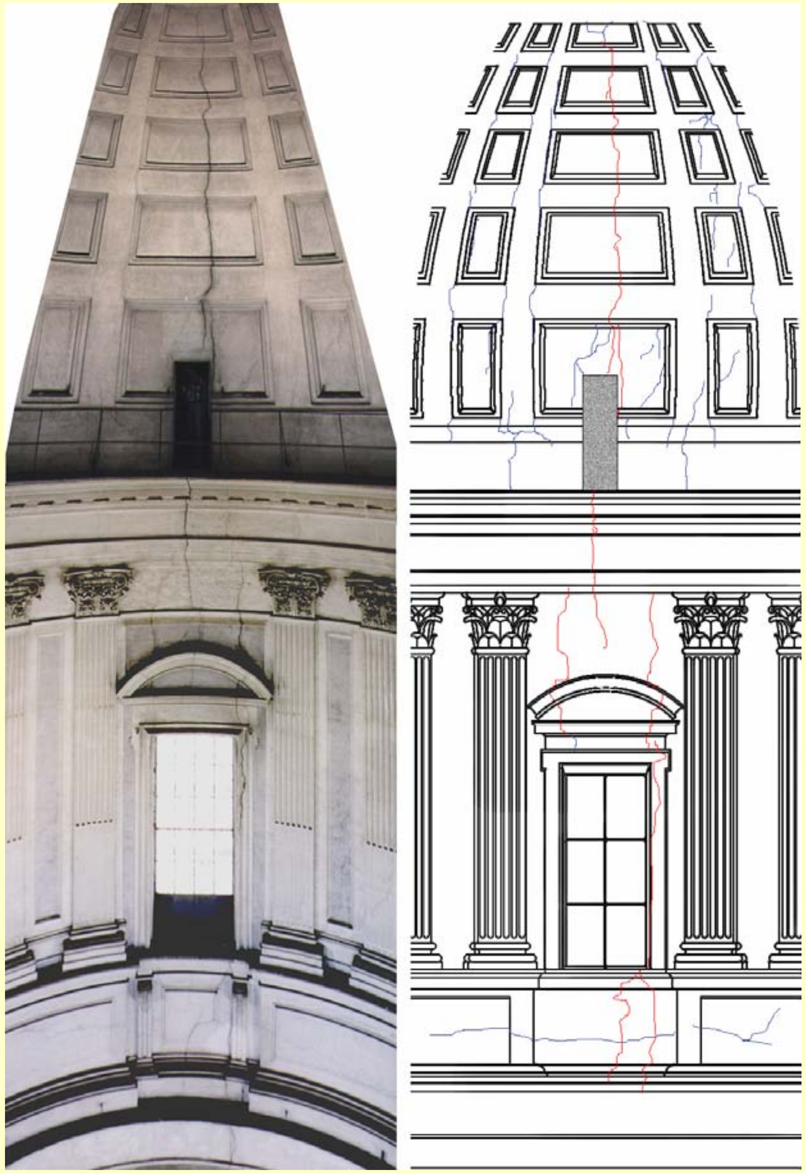
Pantheon

S. Pietro

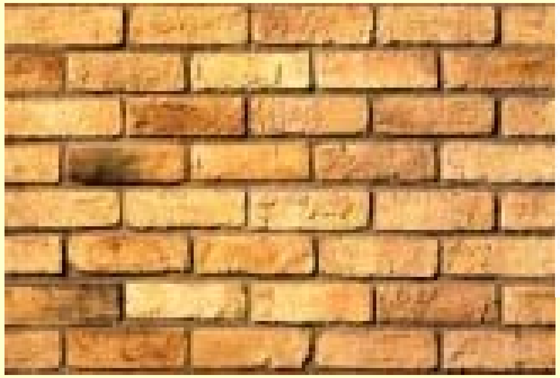
Masonry domes

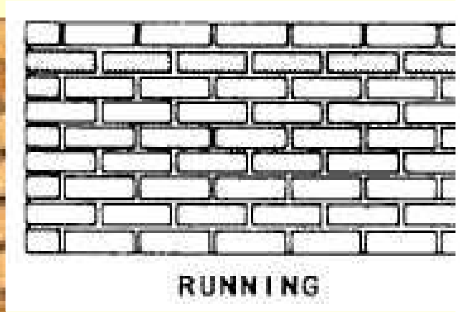
Basilica di S. Maria di Carignano - Genova

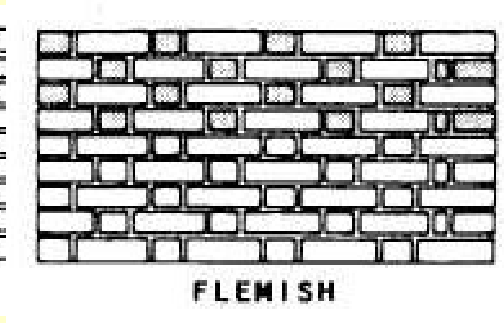


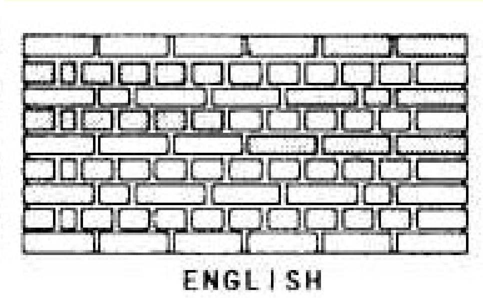


Materials and bond patterns





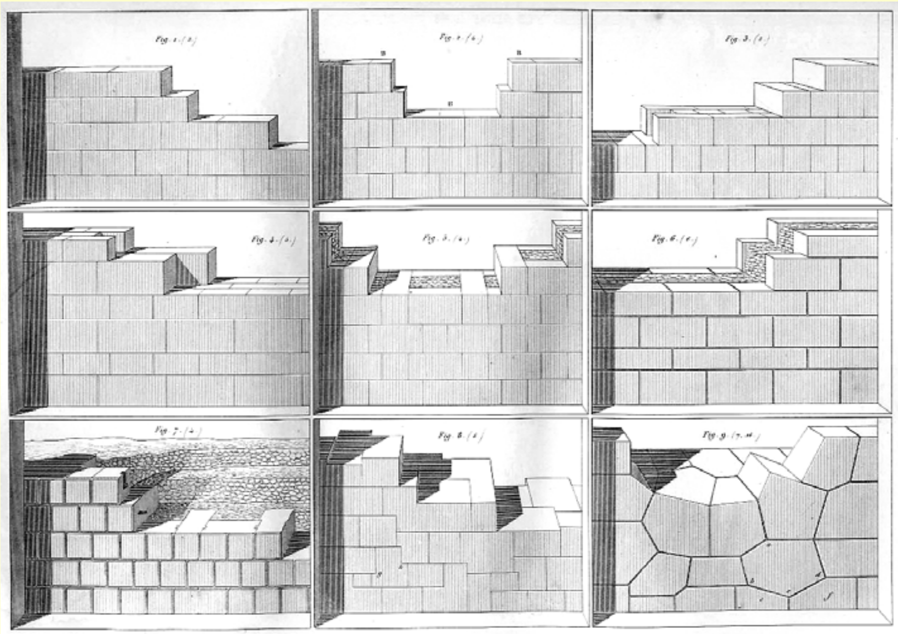




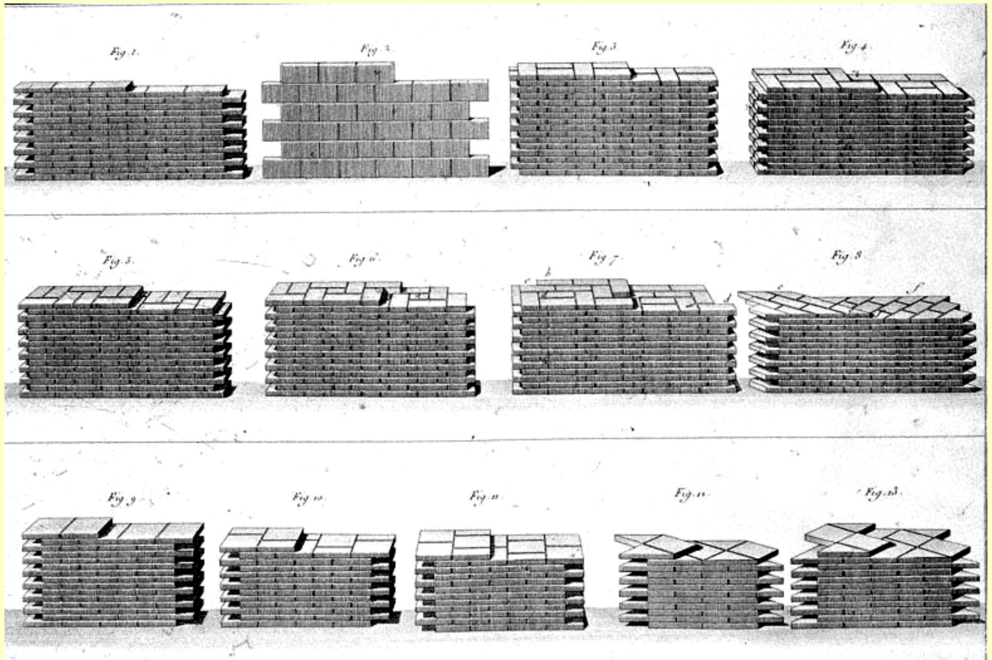
disorder

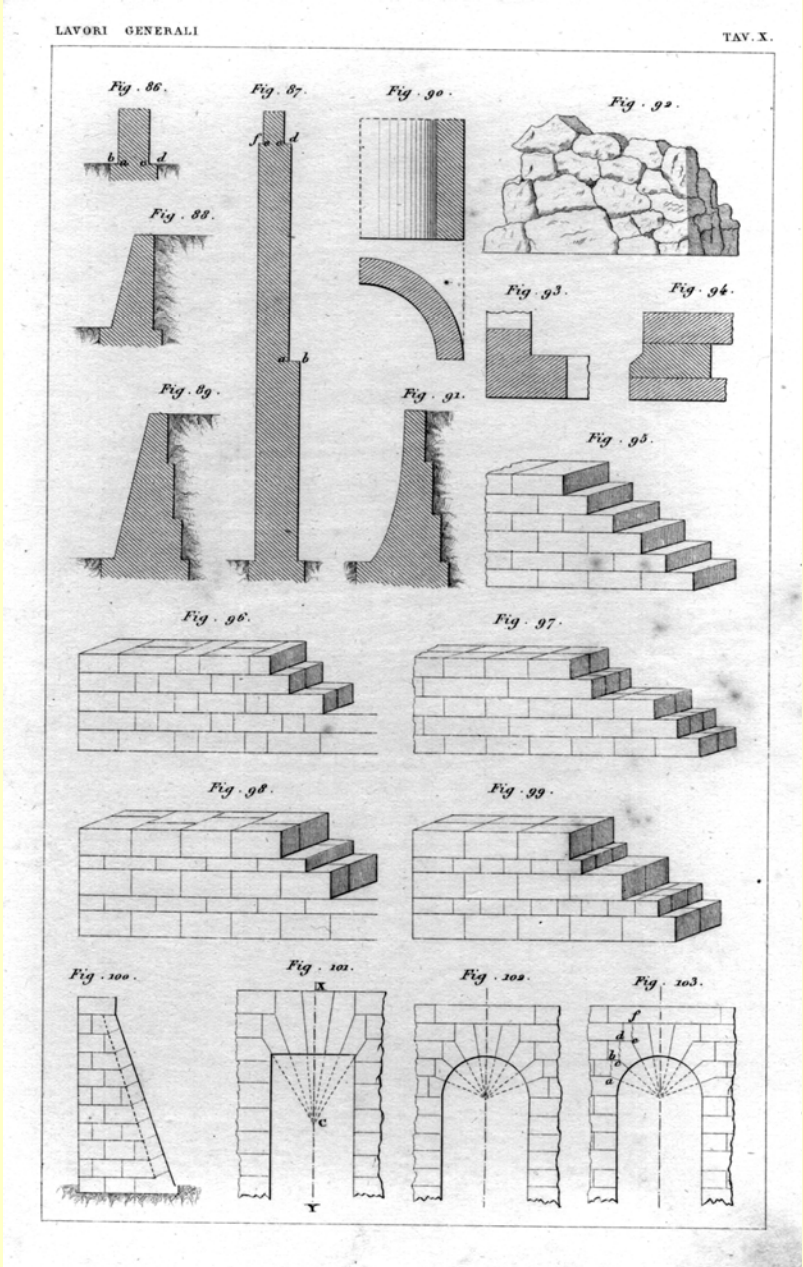
Old building construction techniques

and rules of practice



Rondelet

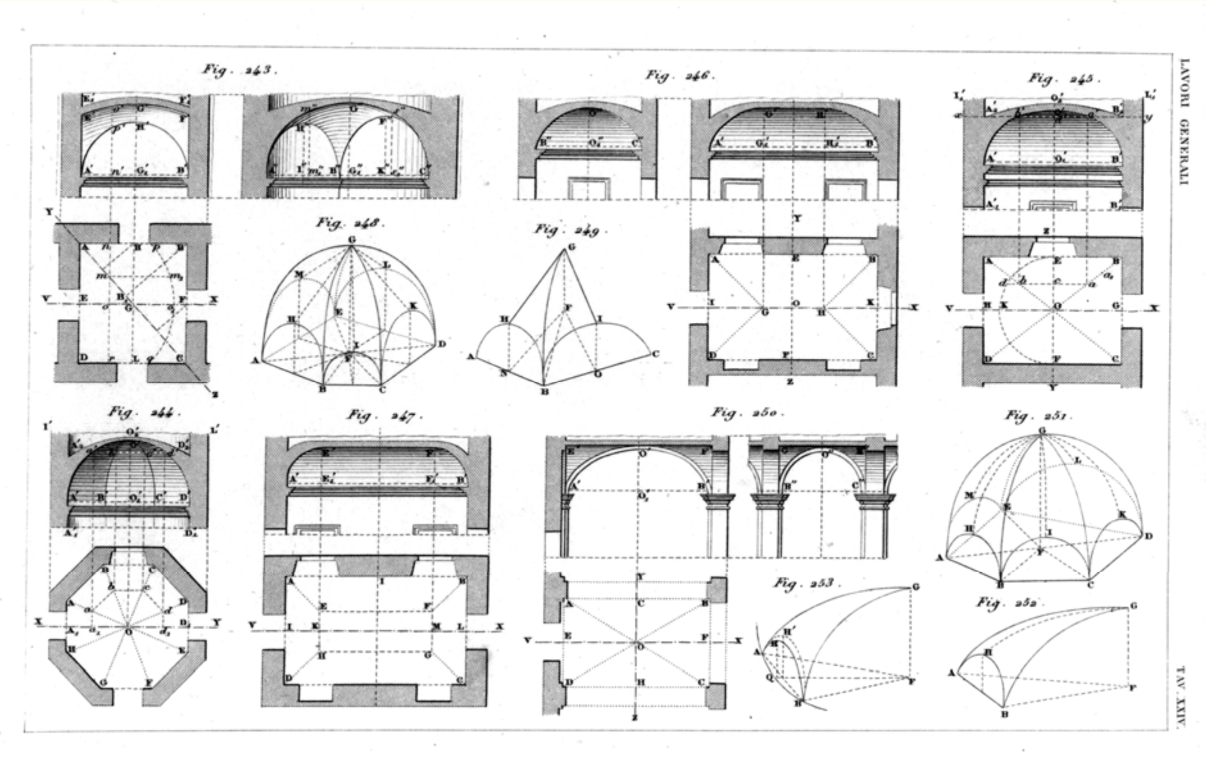


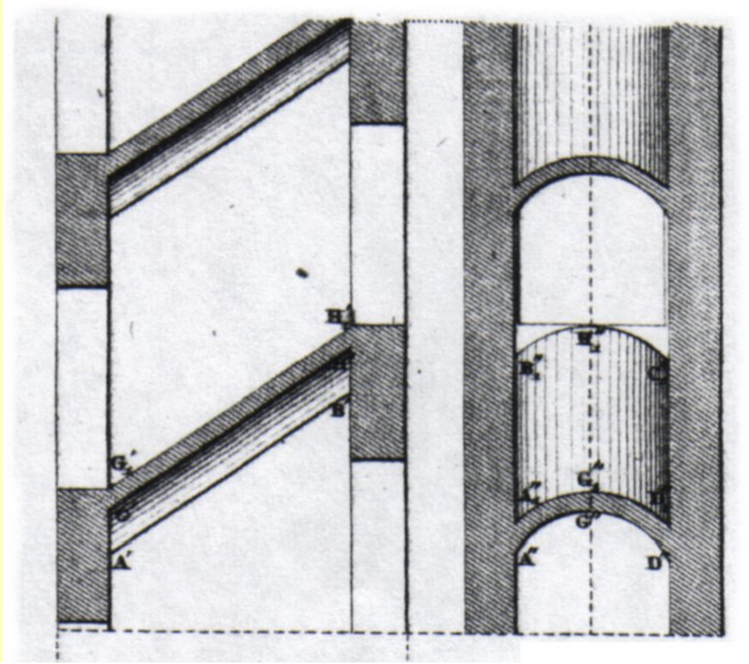


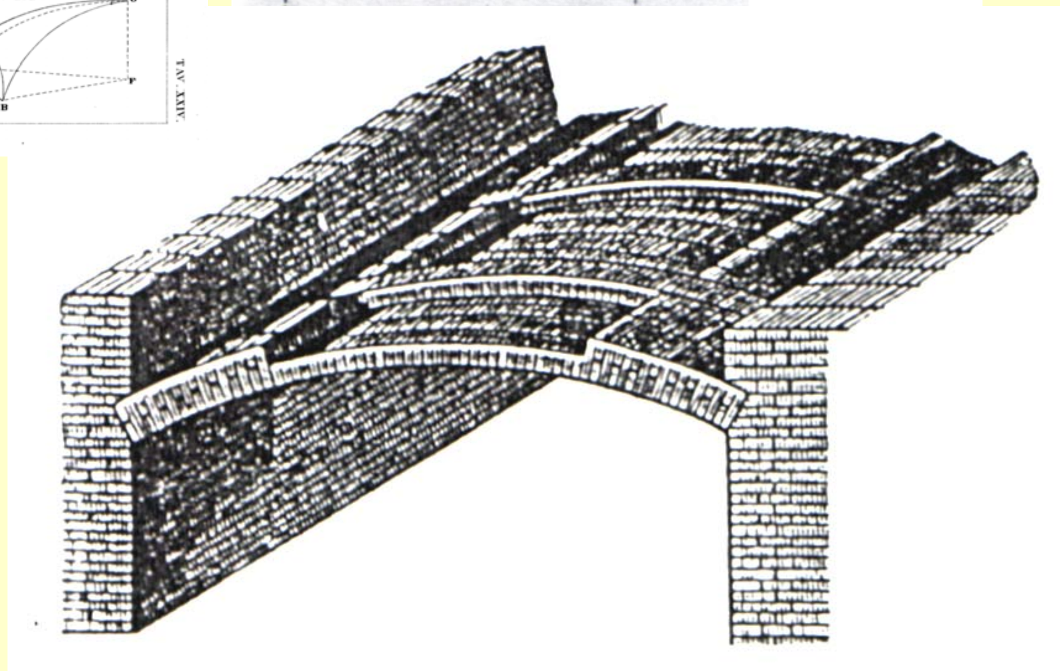
Curioni

Old building construction techniques

and rules of practice

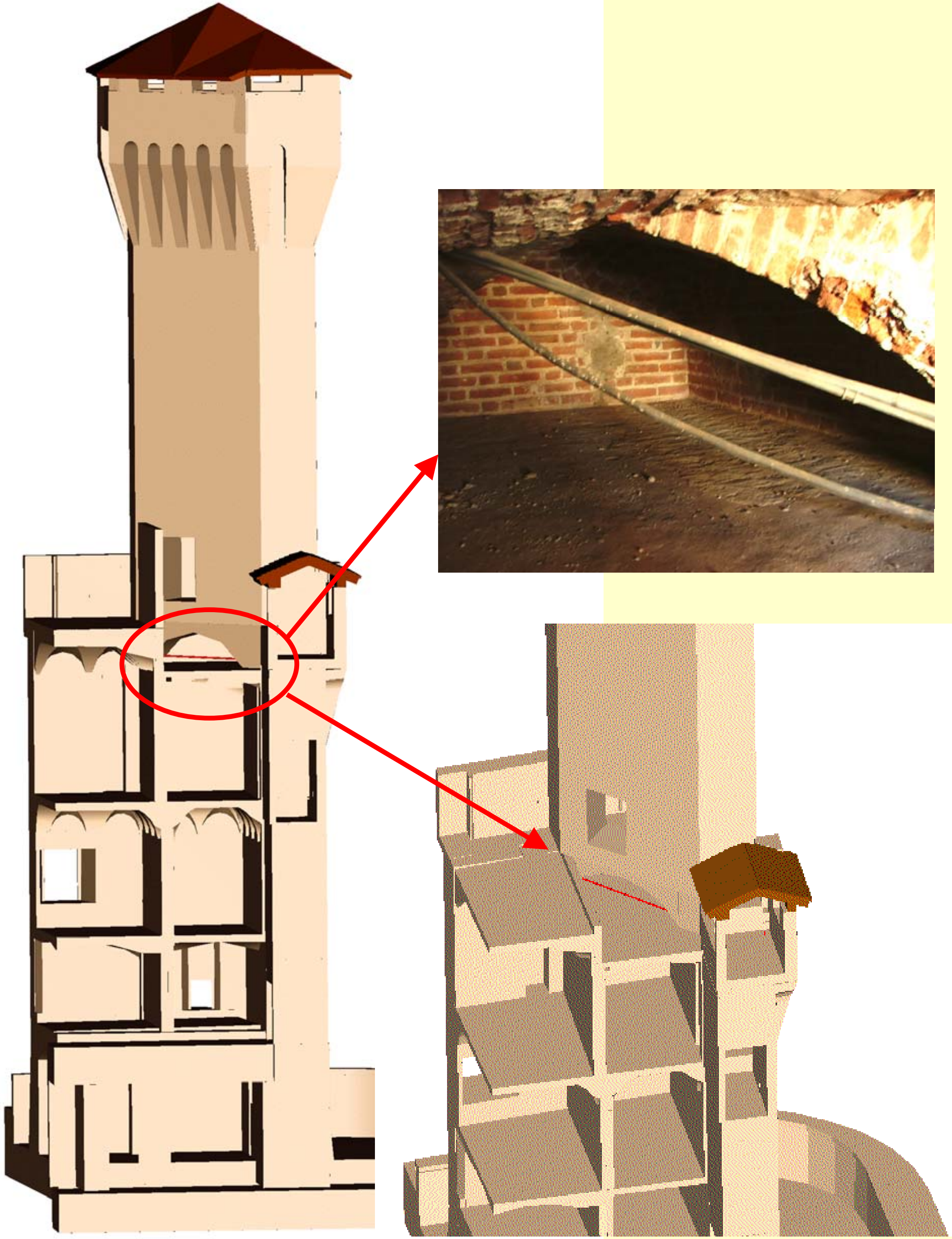






In the absence of rules......

Caste in S. Cristoforo, Genova



2. Modeling: general aspects

The aims of mechanical modeling masonry constructions

- interpretation of the damage and (realistic) assessment of the structural safety;
- selection of the most efficient and less invasive repairs and strengthening techniques (if necessary), compatible with the original design concepts of the construction.

Understanding the relevant mechanical behavior of the construction through proper structural models (avoiding dogmatic conventional assessment procedure)

Masonry

- heterogeneous material (periodic – random bond pattern)
- components: brick unit, stone block, mortar layer
- quasi-brittle behavior
- different types of bond pattern – thick masonry walls
- Randomness of the material parameters
- to be calibrated by *in situ* set up
- constitutive modeling based on the geometry and assembly of the components and their constitutive models

2. Modeling: general aspects (cont'd)

The masonry construction

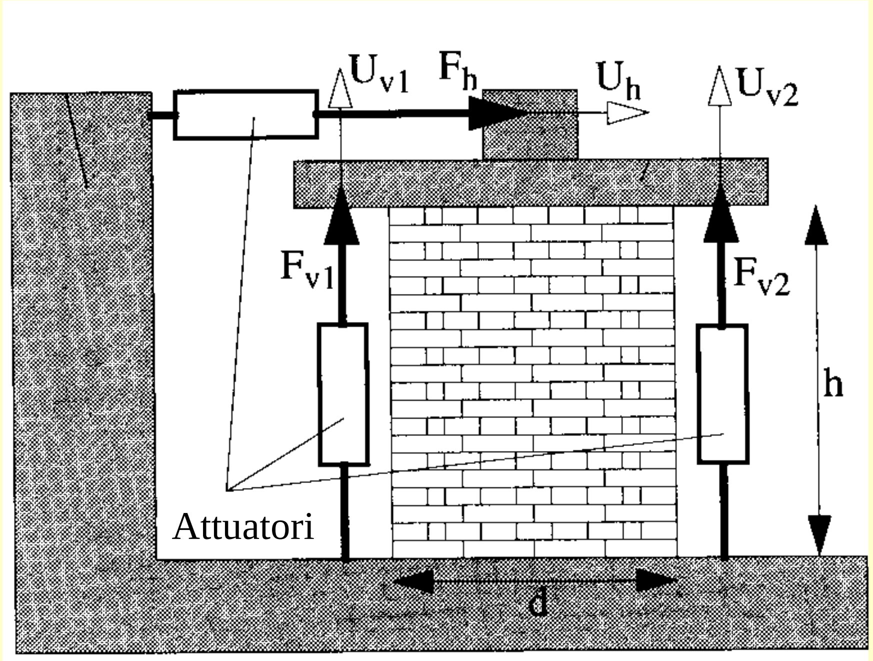
- Construction Versus structure
- Mechanical interaction among the construction elements (vaults, walls, columns, arches, ...)
- Building – foundation interactions
- Modification and extension of the construction (superfetations, growth, etc...)
- Building to building interaction (Historic centres and urban aggregates)

Other aspects

- Sensitivity to the applied loads: static (weigth loads) V/s dynamic (seismic, traffic...) loads.
- Sensitivity to the construction sequence
- Influence of initial stresses and strains and quasi-brittle behavior of masonry: how to approach the safety assessment?
- Chemo-physical degradation and residual life

2. Modeling: general aspects

Imposed horizontal displacement on compressed walls



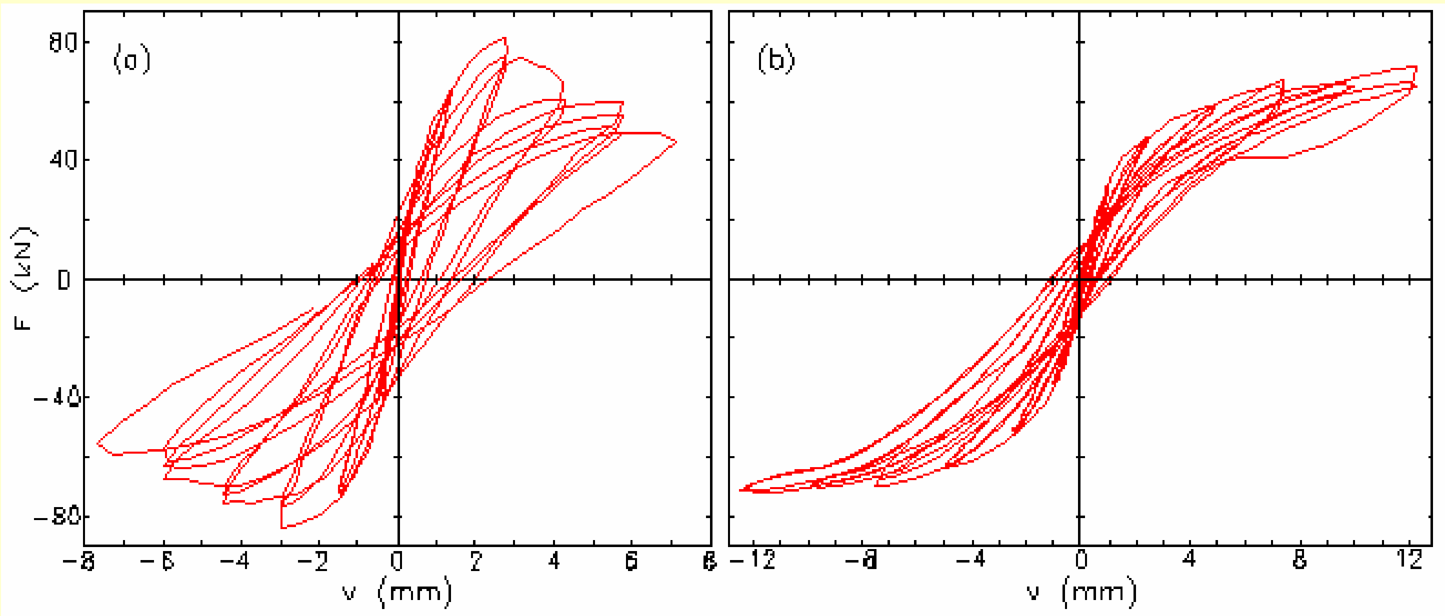
Cyclic shear test set up (Anthoine et al., 1994)

Hysteresis & damage

Dominant NL elastic response NTR

Squat wall

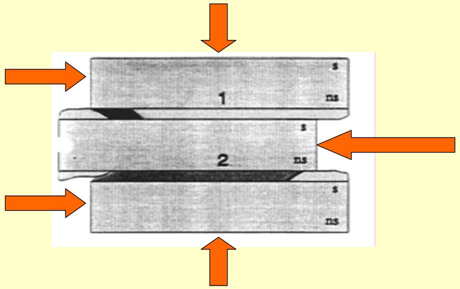
b=100cm
h=135cm

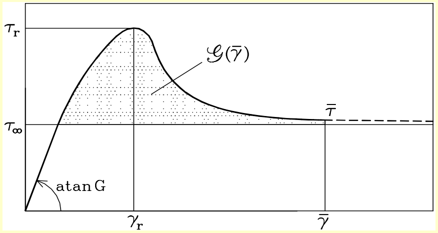


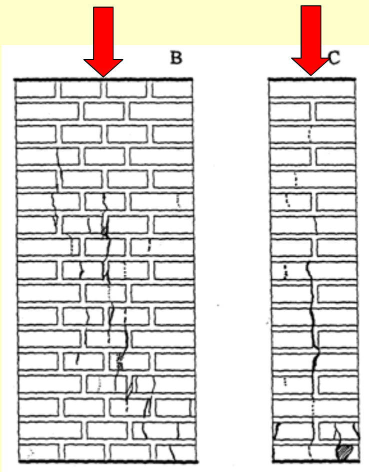
Slender wall

b=100cm
h=200cm

2. Modeling: introductory aspects







The constitutive ingredients

Elasticity

Unilateral contact

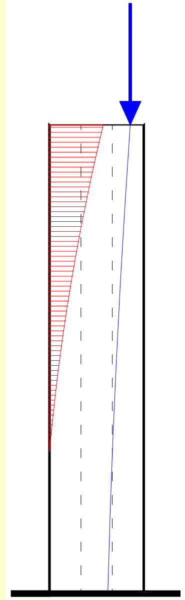
Plasticity

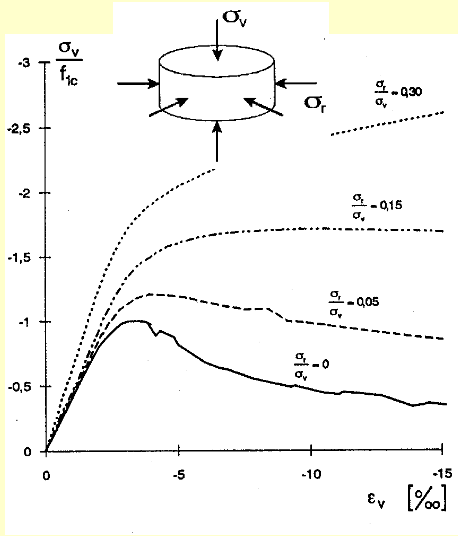
Friction

Damage

Fracture

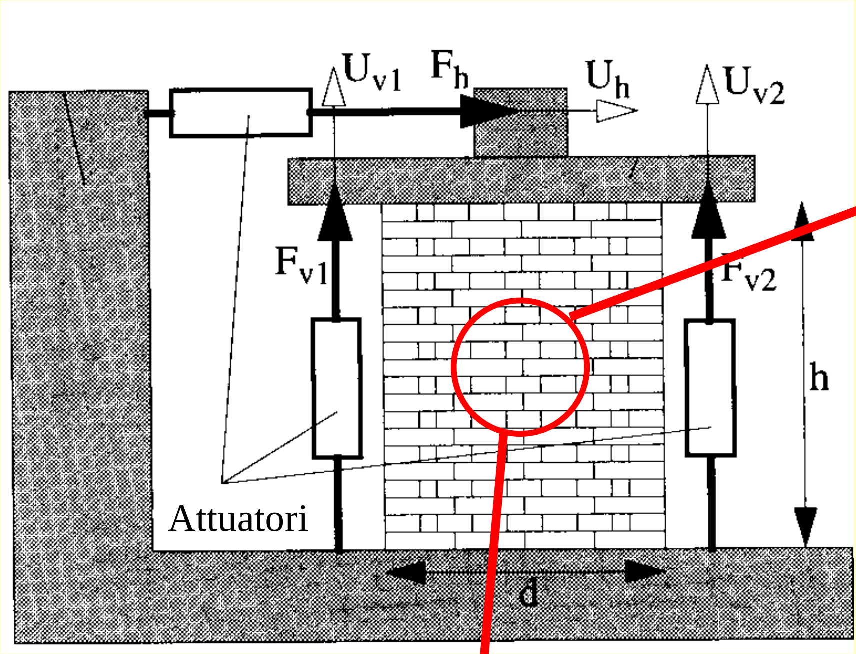
Viscoelasticity

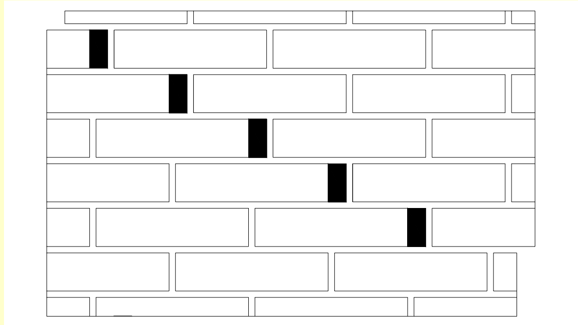




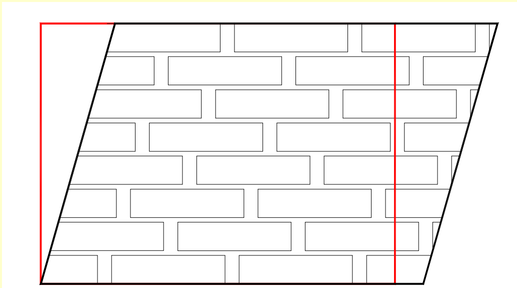
Homogenization

Periodic bond pattern masonry

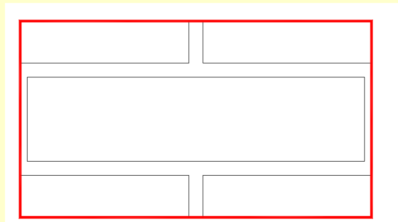




localization



Homogeneous macro-strain

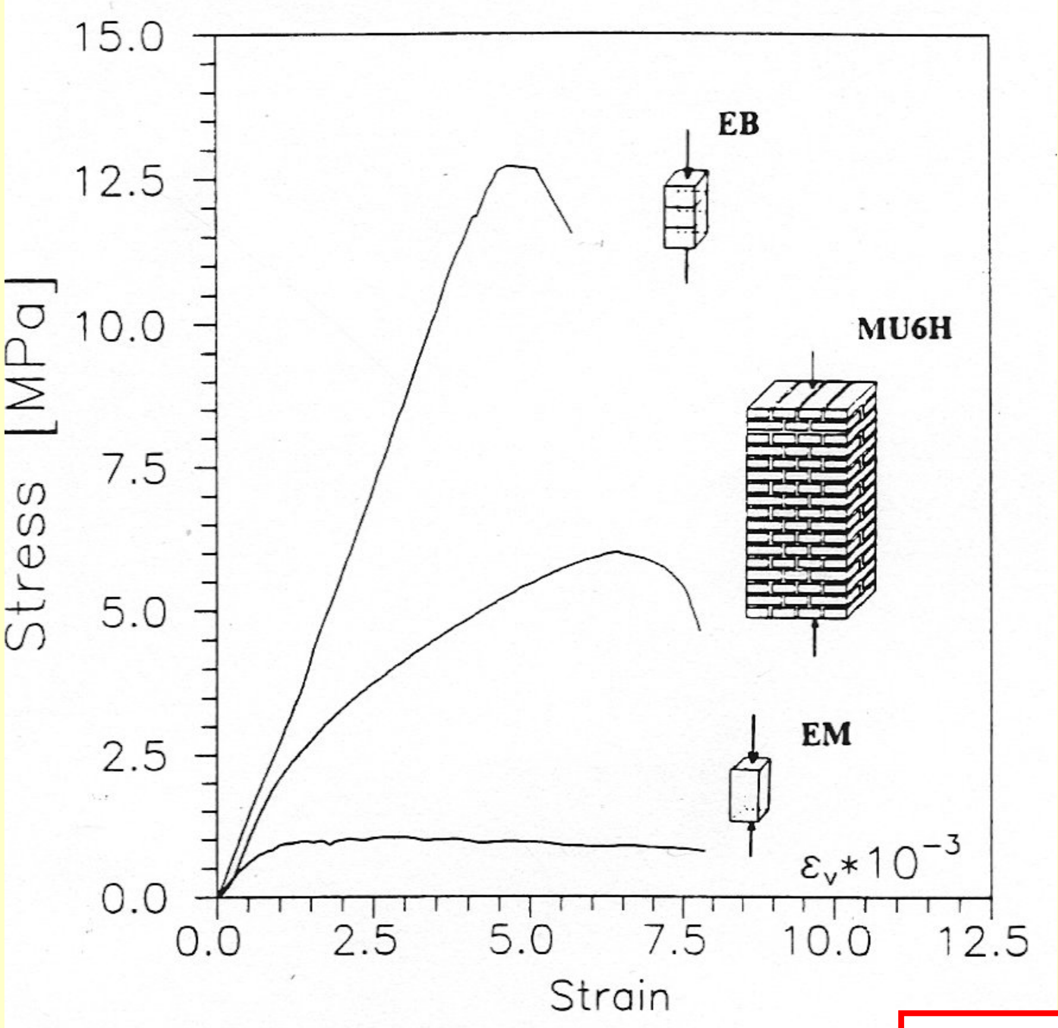


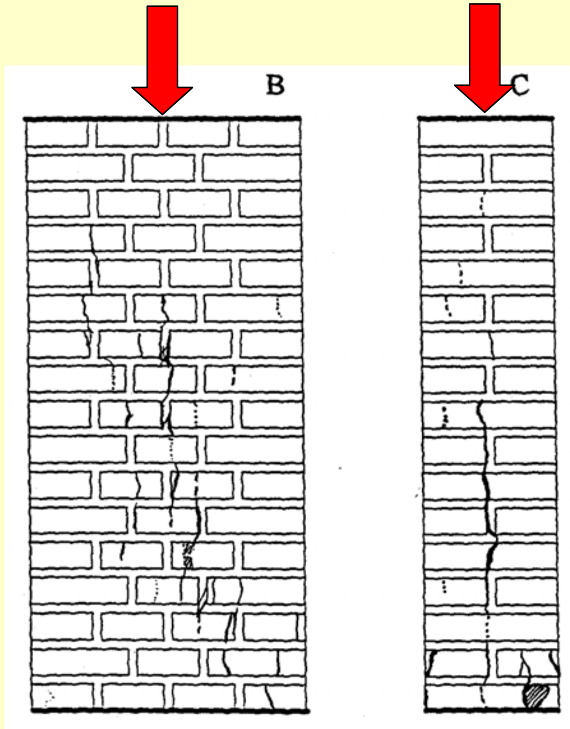
RVE

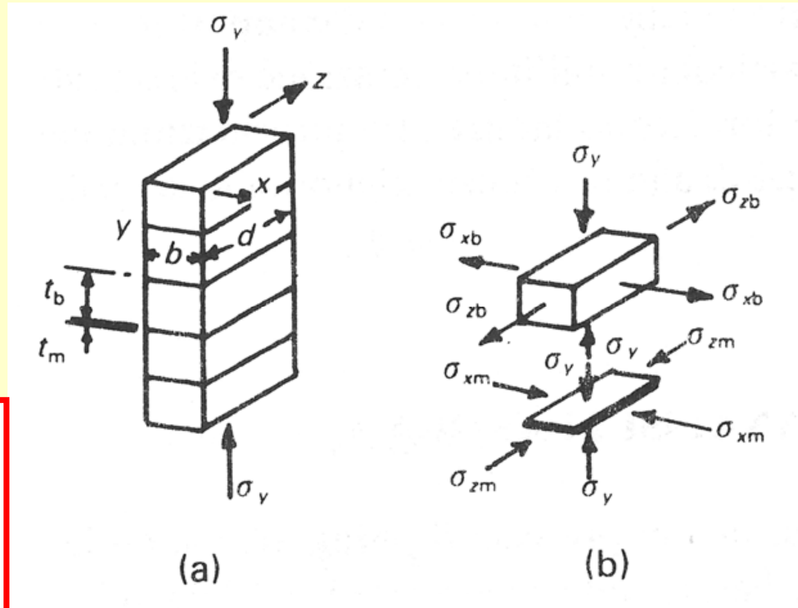
Macro Σ, E

micro σ, ε

Compressive strength







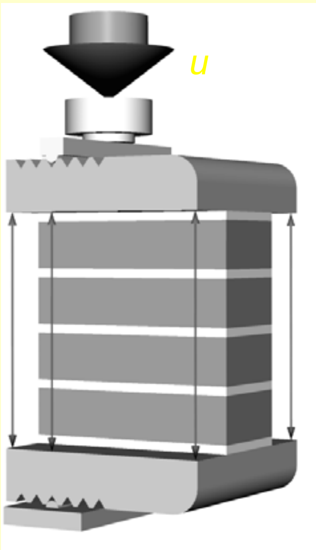
Hilsdorf, 1969

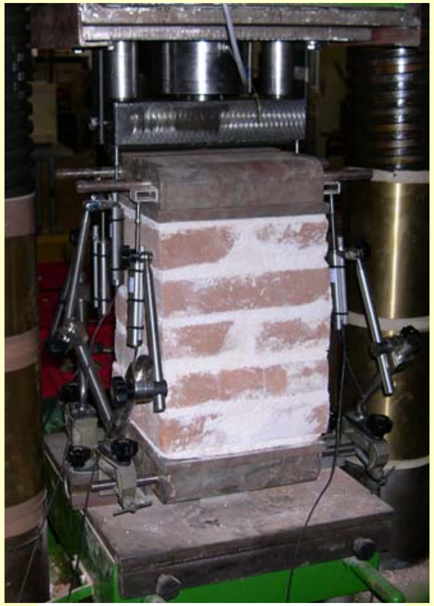
h_b brick unit thickness
h_m mortar layer thickness

$\alpha = h_b/h_m$

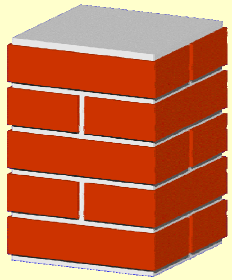
$$f_M = \frac{\alpha\, f_b^t + f_m^t}{\alpha \frac{f_b^t}{f_b^c} + \frac{f_m^t}{f_m^c}}$$

Eccentrically loaded columns & arches

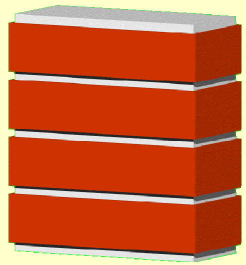




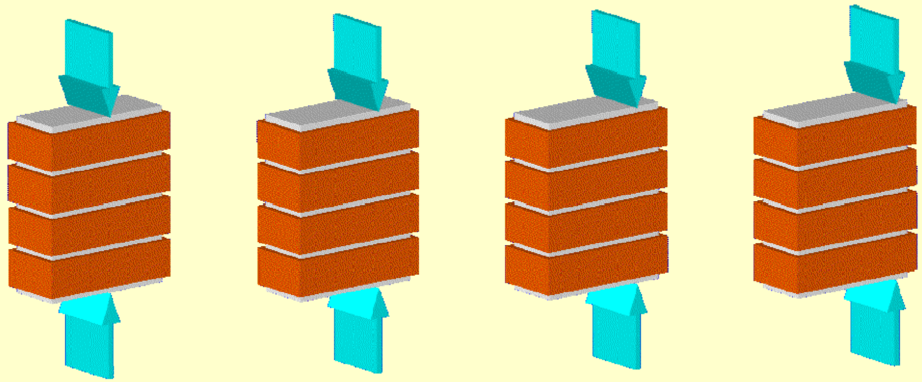
Experimental set up



1 unit stack



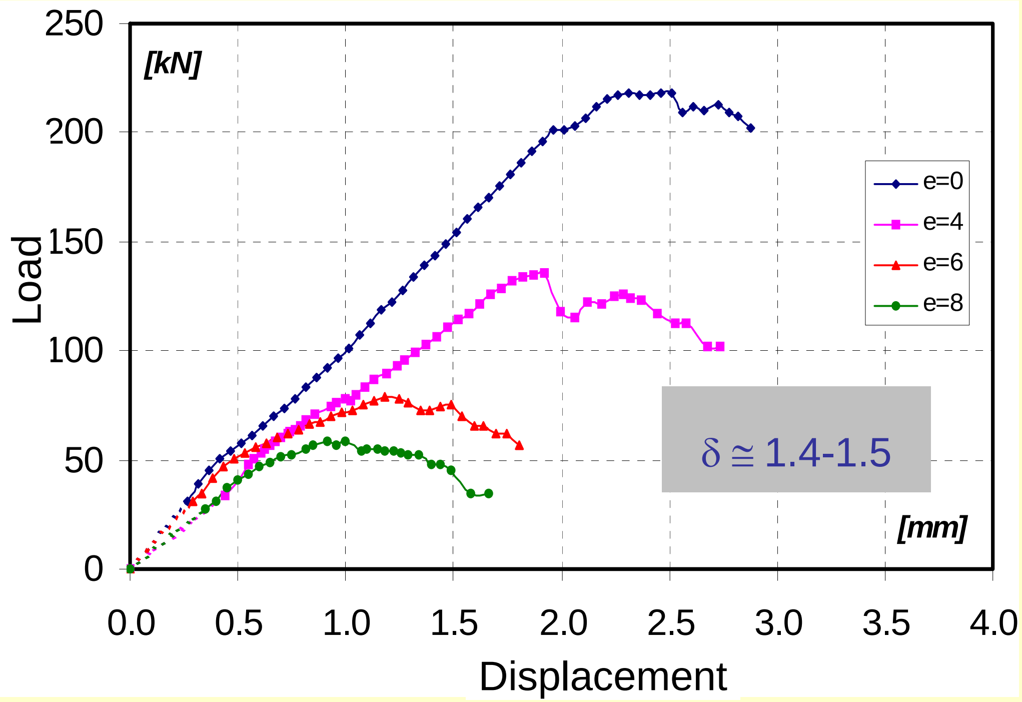
2 unit stack



***e* = 0** **_e_ = 4cm** **_e_ = 6cm** **_e_ = 8cm**

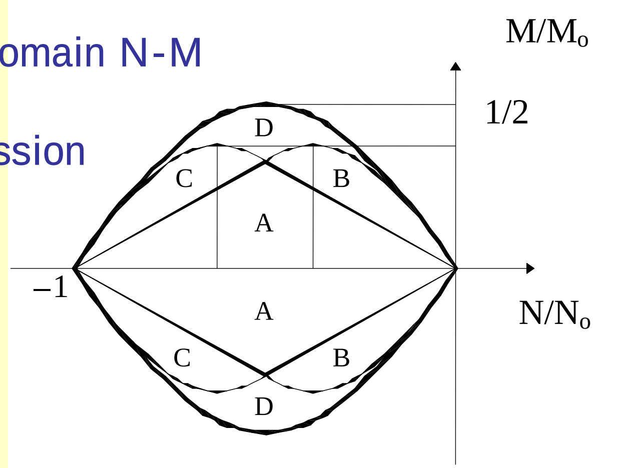
Corradi et al, 2006

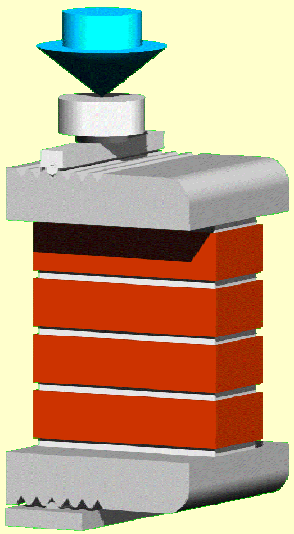
Eccentrically loaded columns & arches



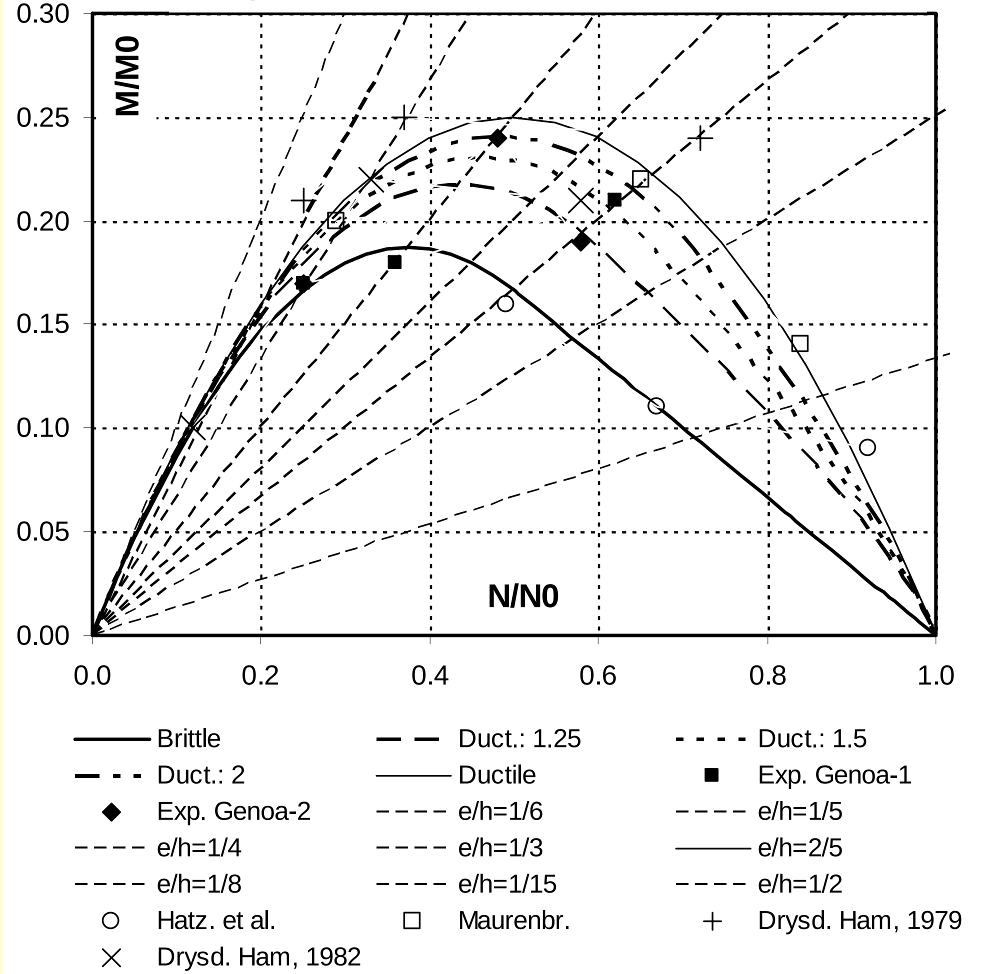
Theoretical limit domain N-M

NTR + EPP compression



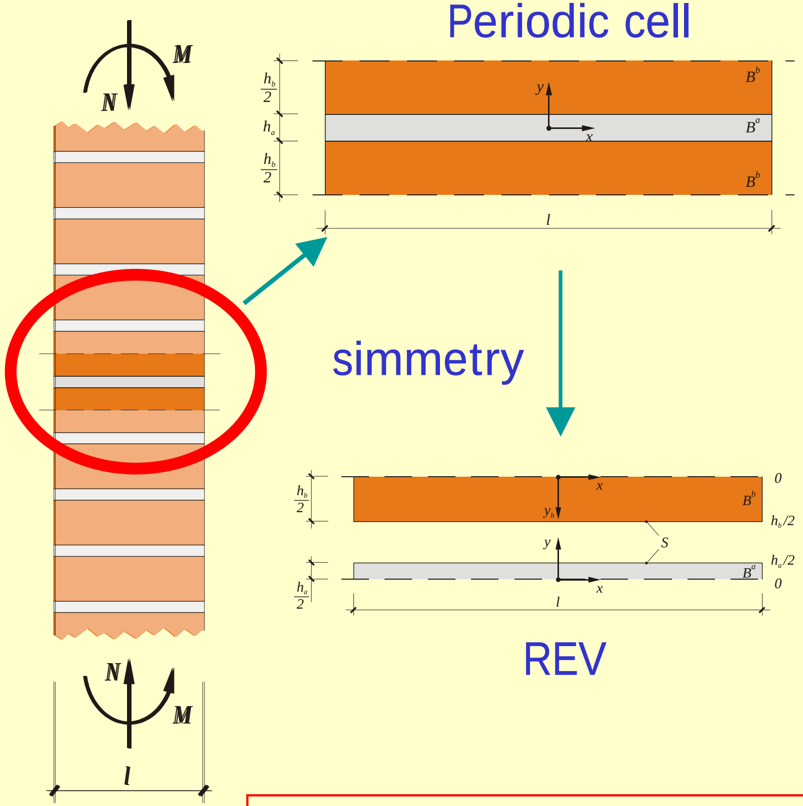


Experimental limit domains N-M



Brencich e Gambarotta, RILEM, 2005

Eccentrically loaded columns & arches



Assumed tension field

$$\Phi^a(x,y) = a_0^p\left[f_0^p(x) + r\, f_0^d(x)\right] + \sum_{n=1}^{N}\sum_{m=1}^{M_a} a_{nm}\, f_n(x)\, g_m^a(y)$$

$$\Phi^b(x,y) = a_0^p\left[f_0^p(x) + r\, f_0^d(x)\right] + \sum_{n=1}^{N}\sum_{m=1}^{M_b} b_{nm}\, f_n(x)\, g_m^b(y)$$

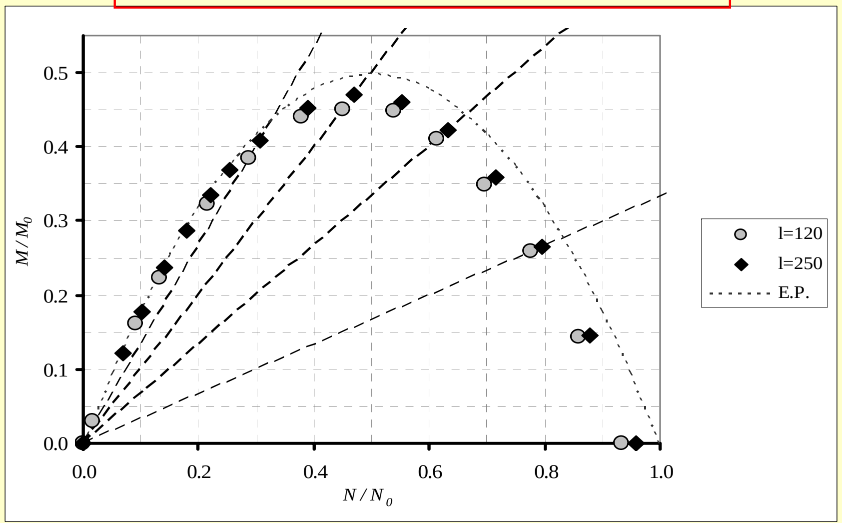
+ B.C. on f() e g()

+ plastic admissibility

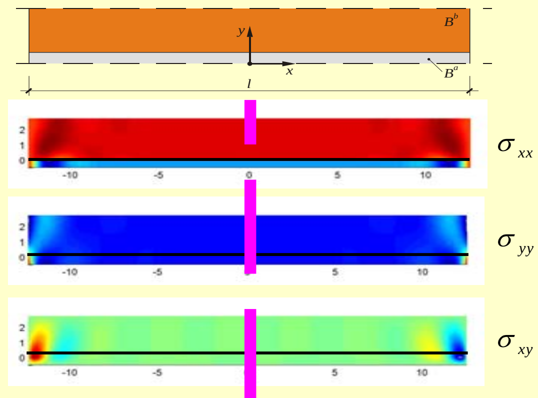
+ unilateral – frictional brick-layer interface

PPLIN
$$\begin{cases} \min N = \mathbf{c}^T \mathbf{a} \\ \text{t.c.} \begin{cases} \mathbf{S}\,\mathbf{a} \le \tilde{\mathbf{d}} \\ \mathbf{A}_{eq}\,\mathbf{a} = \mathbf{0} \\ \mathbf{A}_{att}\,\mathbf{a} \le \mathbf{0} \end{cases} \end{cases}$$

Influence of the unit shape ratio h/b

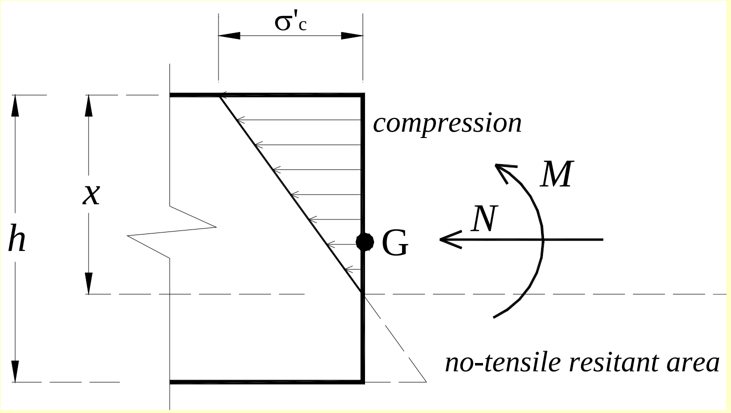


Concentric load Boundary effects

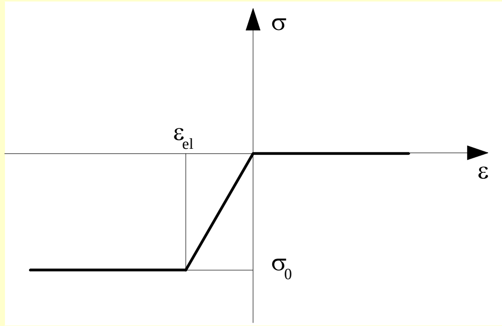


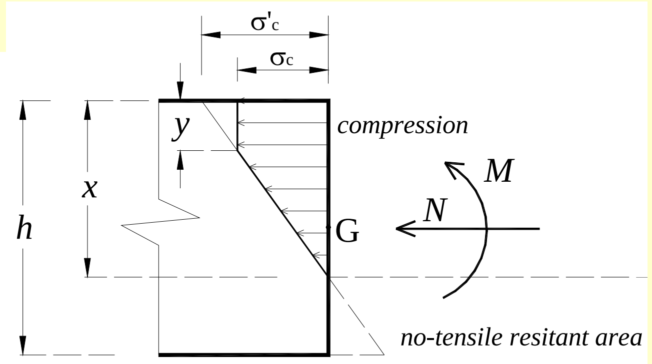
3. Masonry bridges

Incremental analysis – Castigliano
Iterative updating of the compressed section

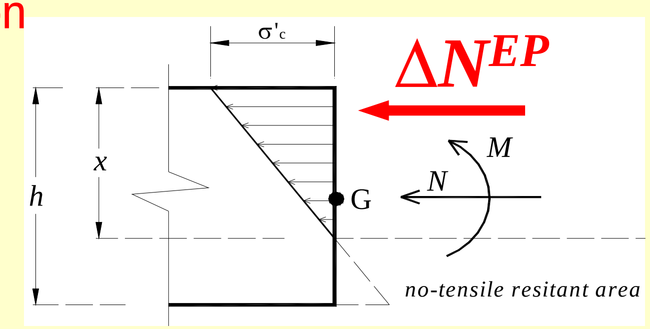


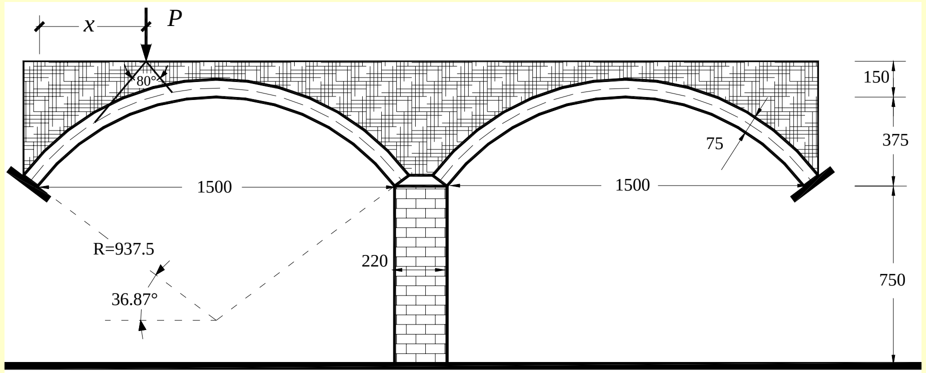
NRT



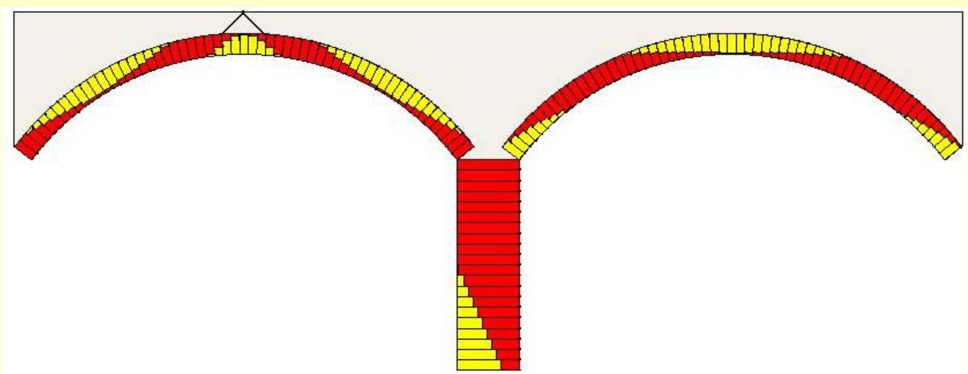


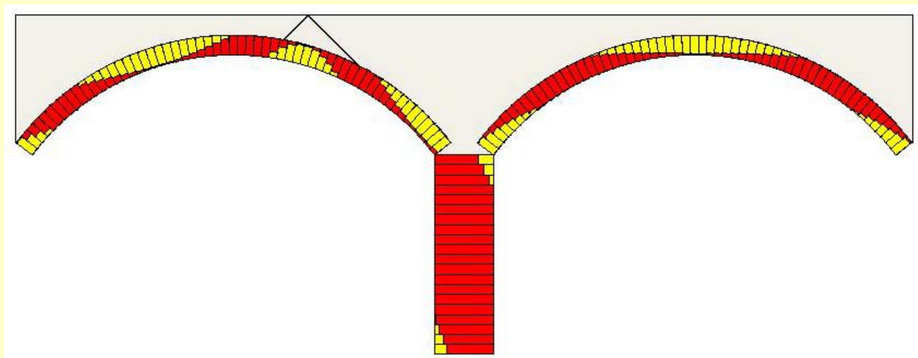
NRT + EPP compression





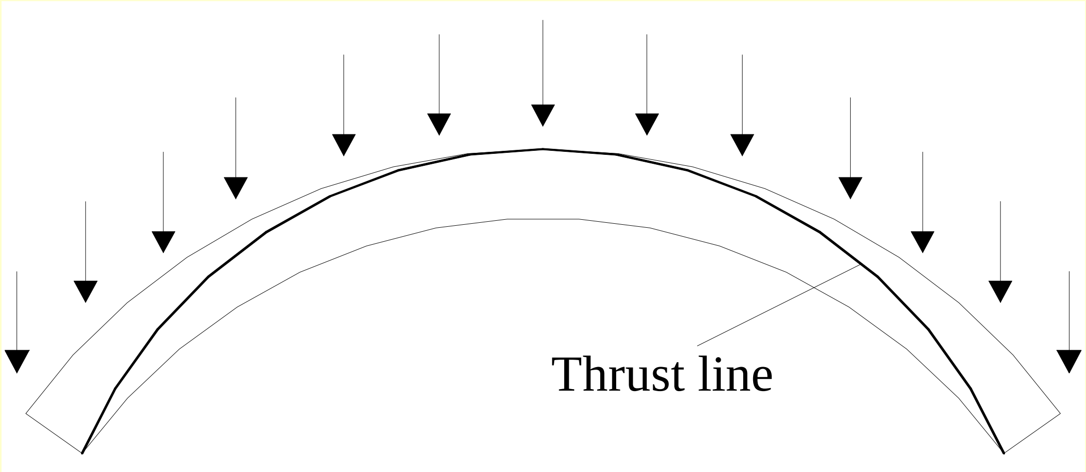
Brencich et al, 2003





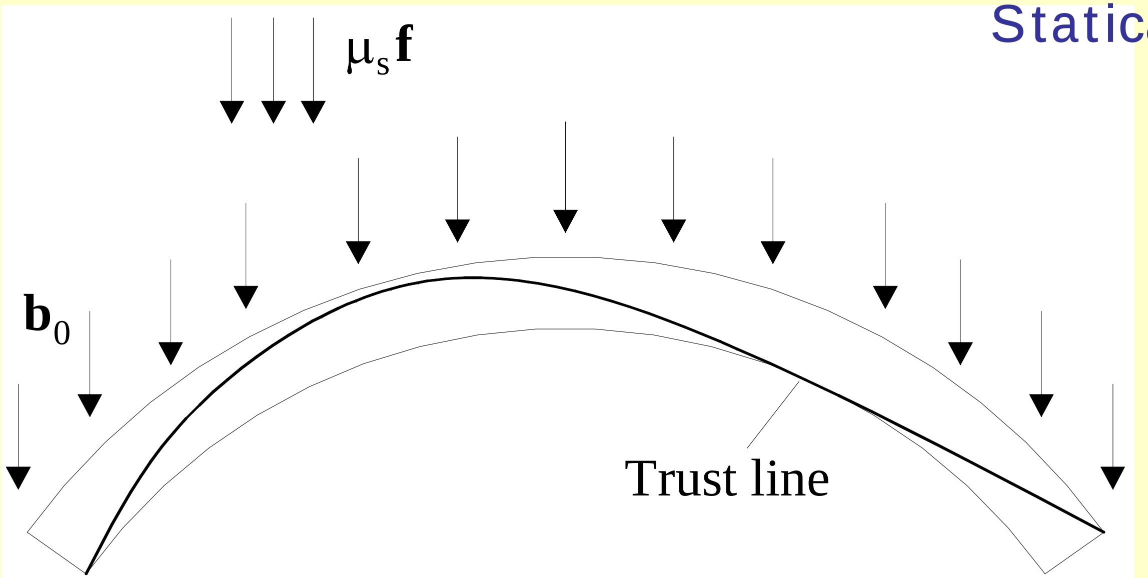
Limit analysis - NTR model

Kooharian, Heyman, ...



Hypotheses:

1. No tensile strength masonry NTR
2. Infinite compressive strength
3. No sliding failure
4. Small displacement and rotations



Statically admissible stress field

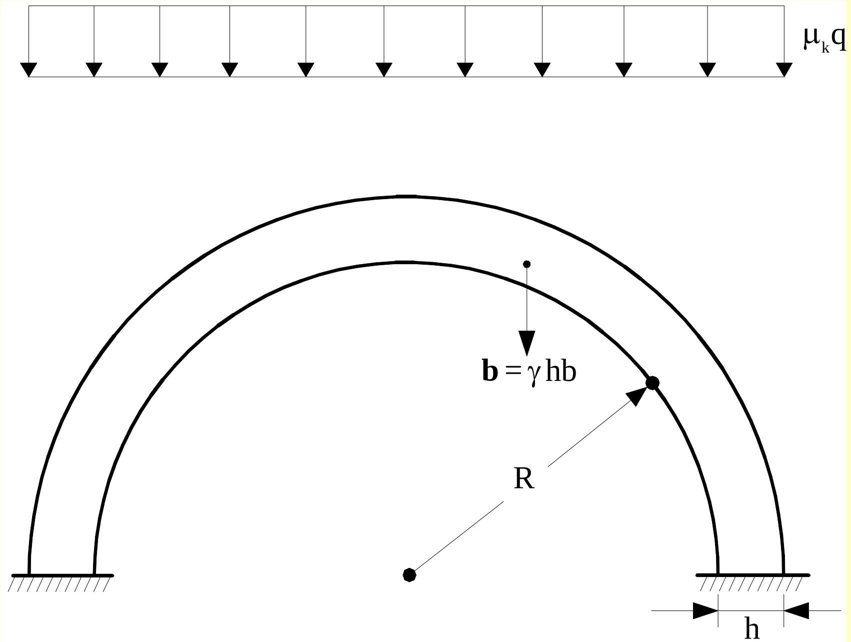
Safe theorem

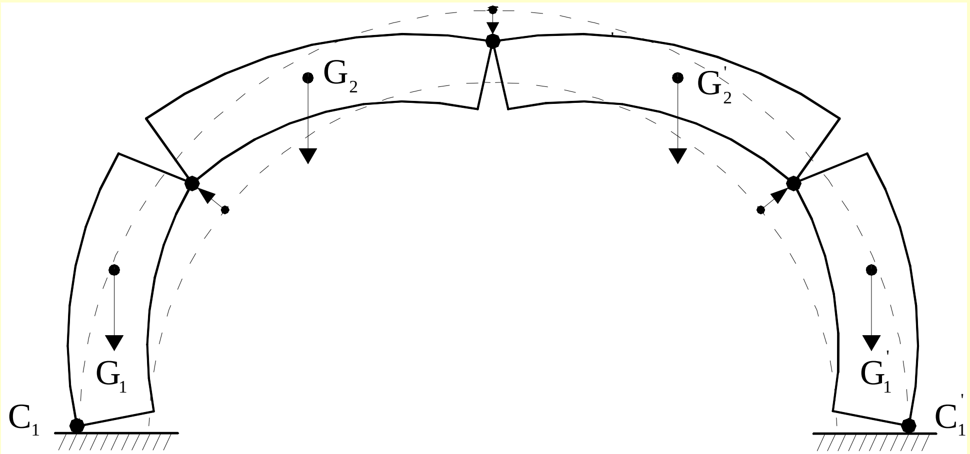
$$\mu_c = \max \mu_s .$$

Limit analysis – NRT model

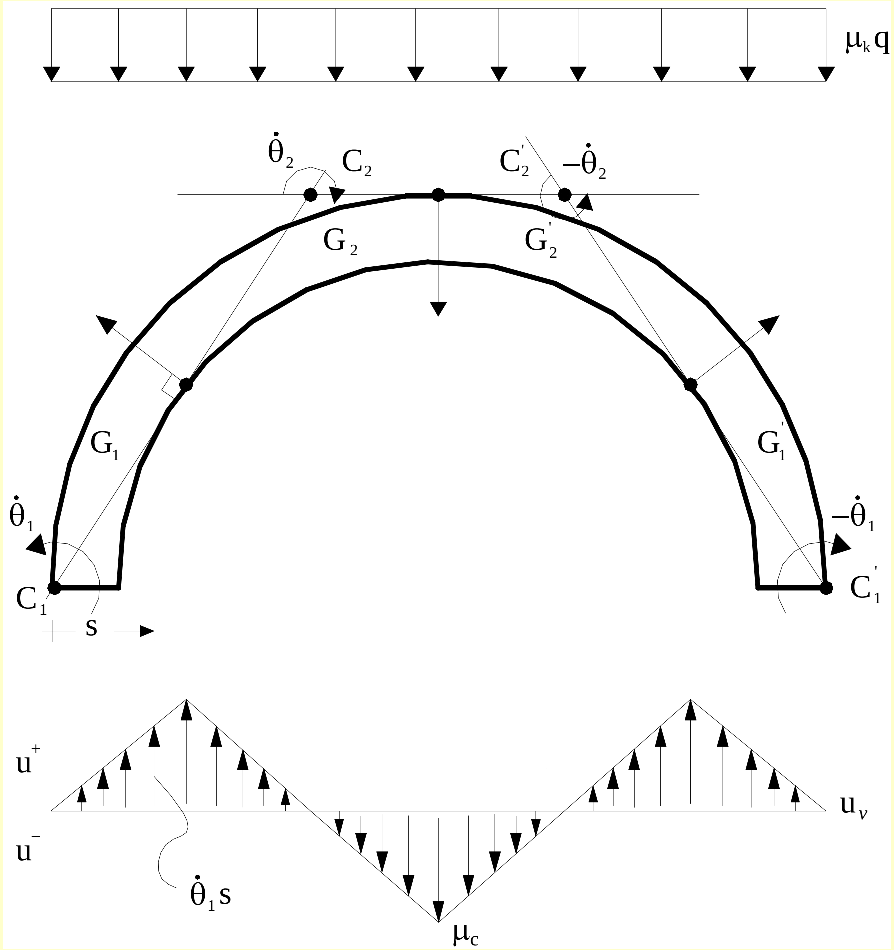
Kinematically admissible mechanisms

$$\mu_k = -\int_{\mathscr{S}} \gamma \, hb \, \dot{u}_v(s) \, ds \Big/ \int_{\mathscr{S}} q \, \dot{u}_v(s) \, ds$$





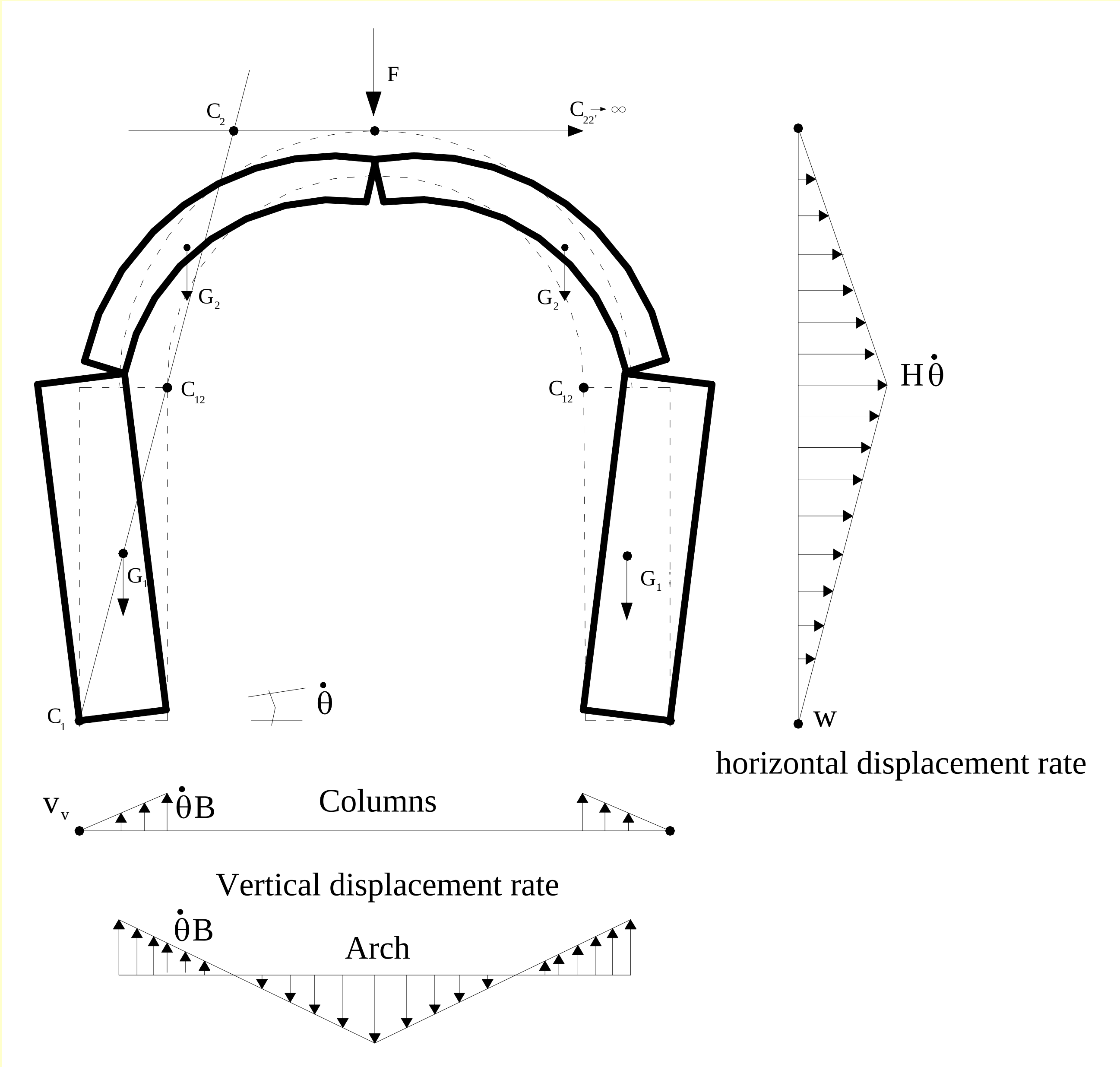
Potential failure mechanism.

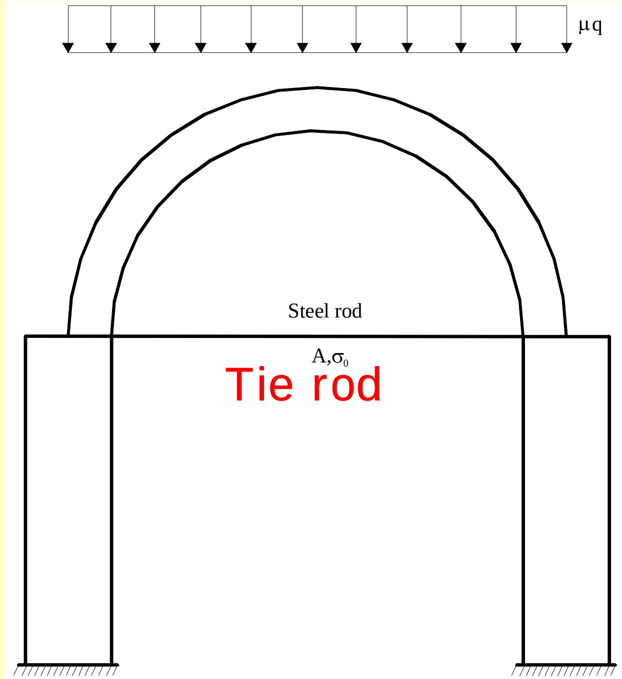


Kinematic theorem

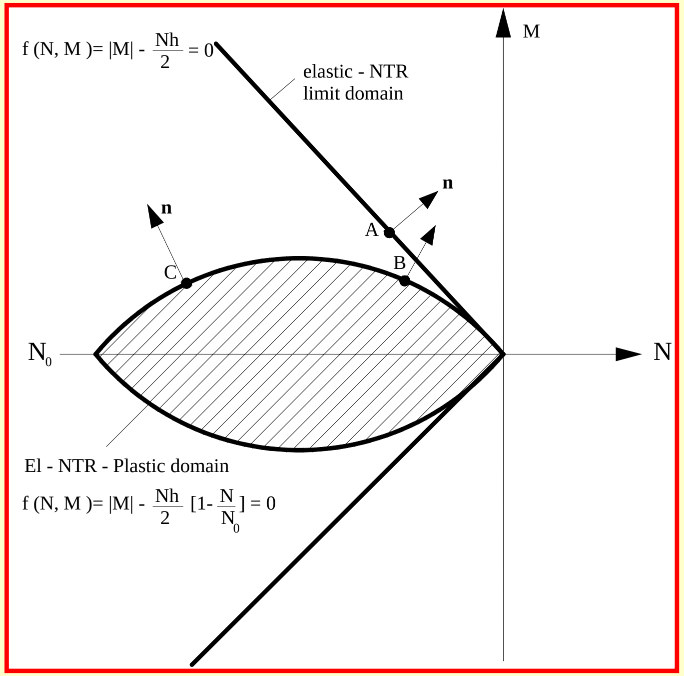
$$\mu_c = \min \mu_k$$

Limit analysis: applications





Effects of the limited compressive strength (2° hypot.)



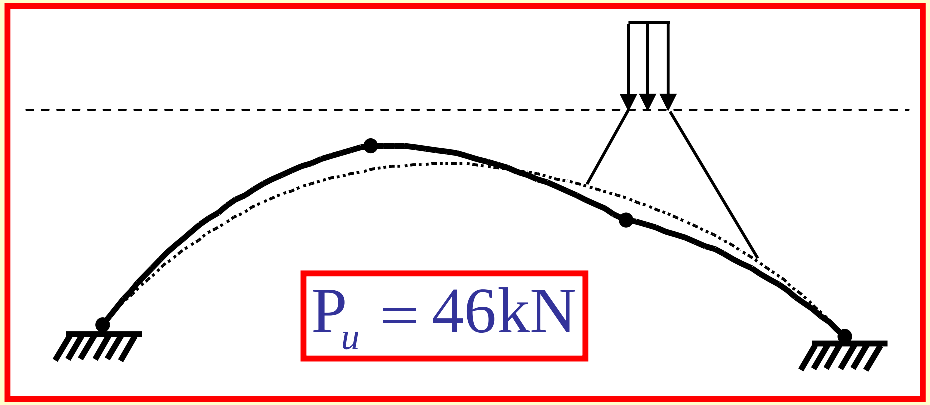
Masonry bridges:
Vault – fill interaction

Tests on full scale masonry bridges: Prestwood Bridge

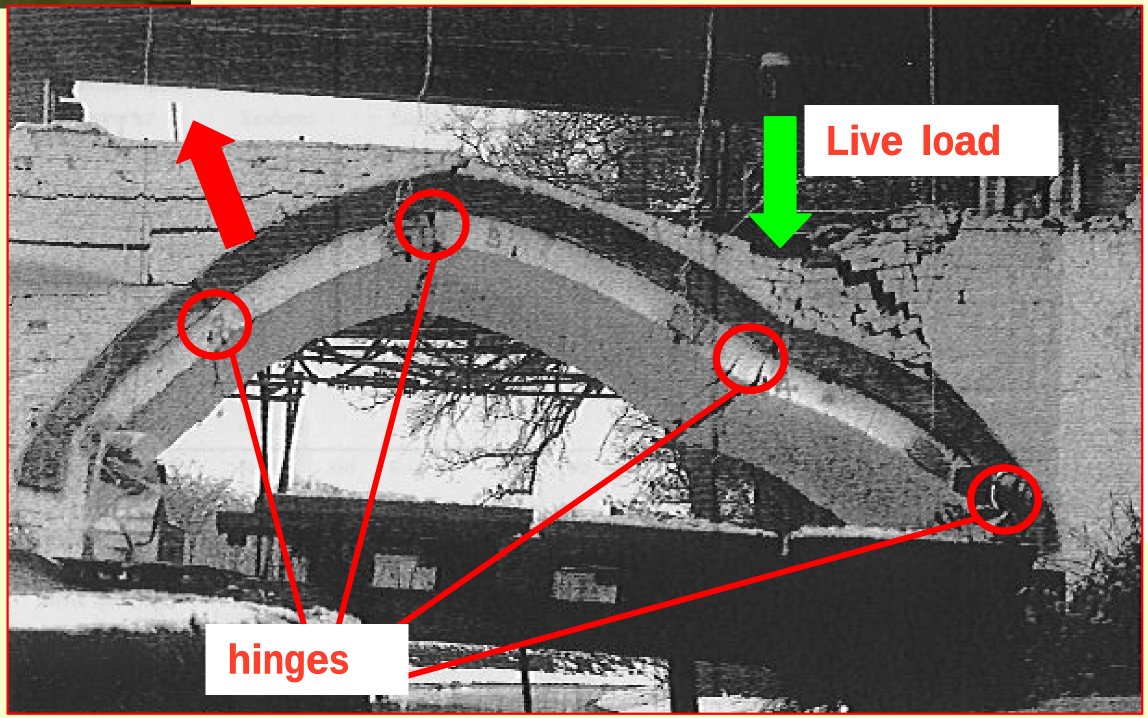
Page, 1993

$P_{exp} = 228kN$

Heavy not resisting fill

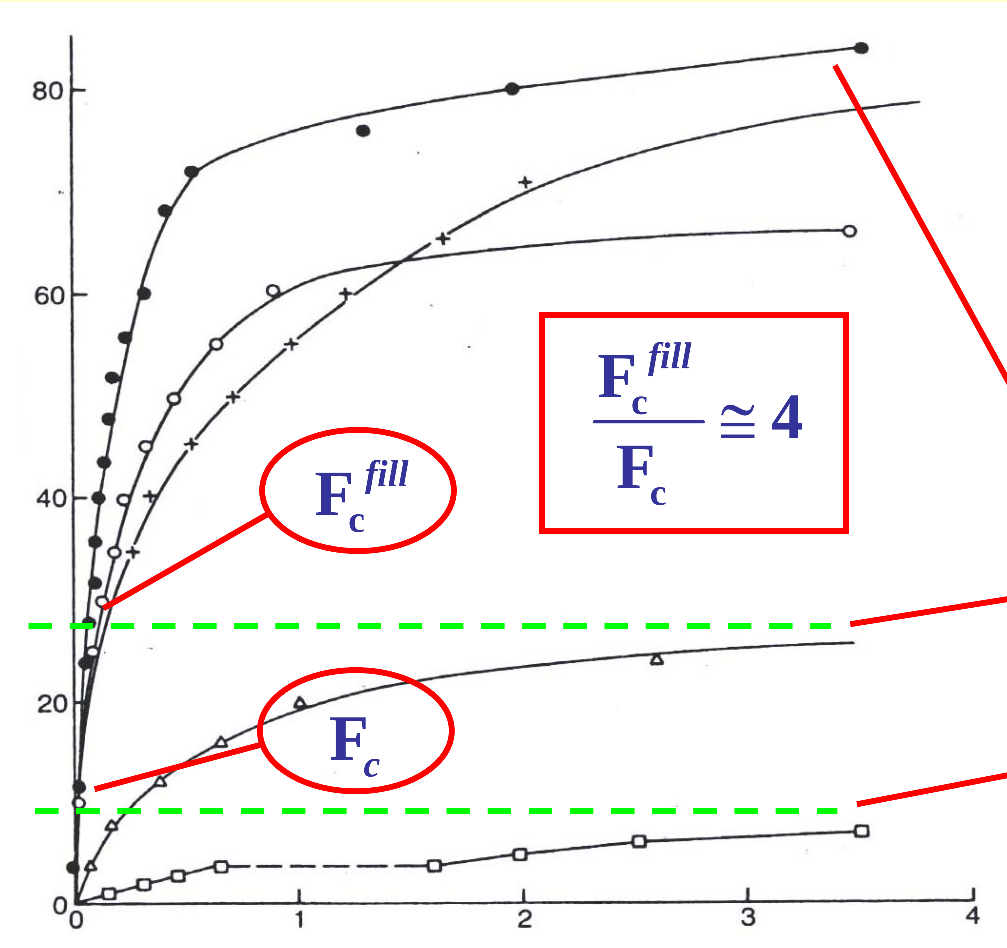


$P_u = 46kN$



Tests on model scale bridges

(Royles & Hendry, 1991)



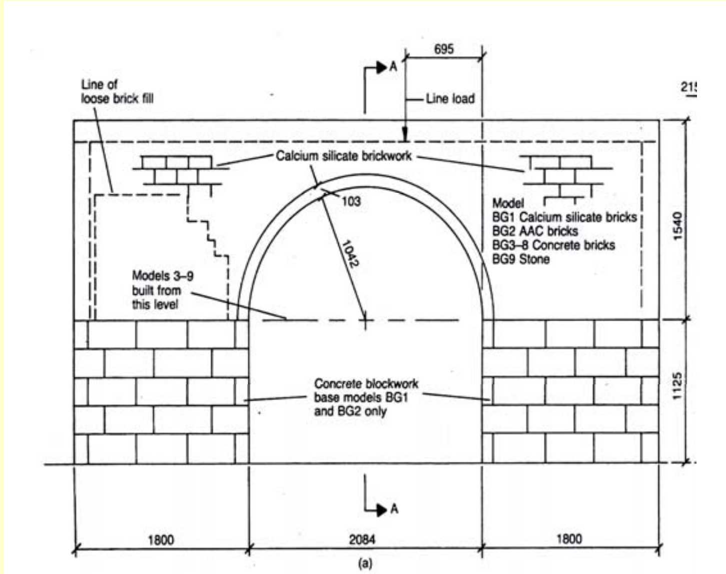
Total load applied F (kN)

v (mm)

F_c^{fill}

F_c

$$\frac{F_c^{fill}}{F_c} \cong 4$$



Complete bridge

Vault and fill

Vault

Crisfield (1985)

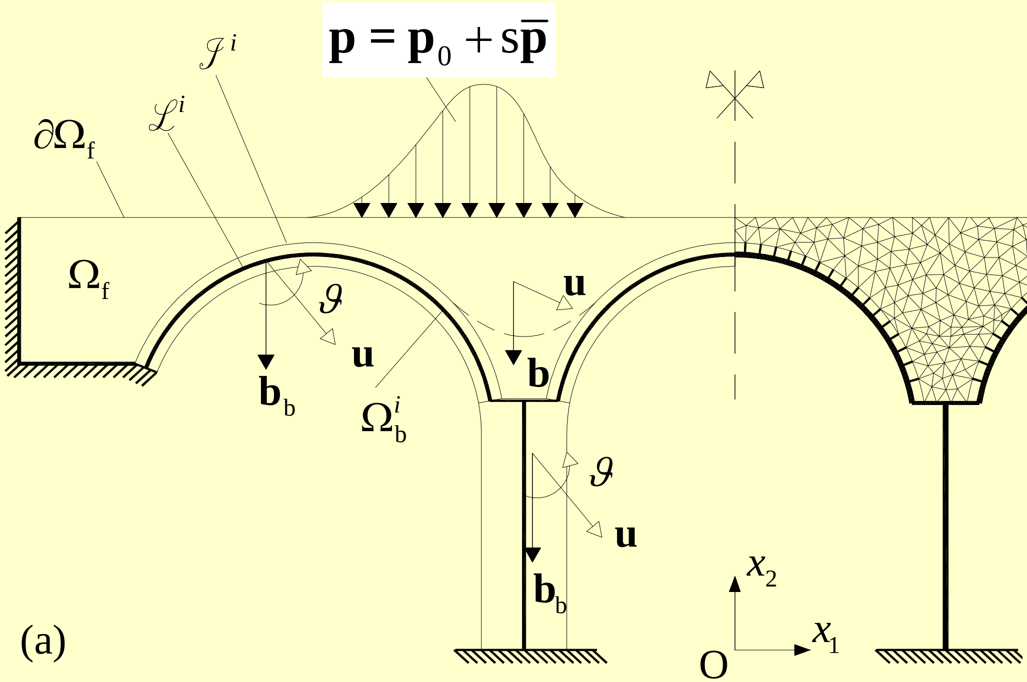
Choo et al. (1991)

Owen et al. (1998)

Bicanic et al. (2003)

Limit analysis of the bridge

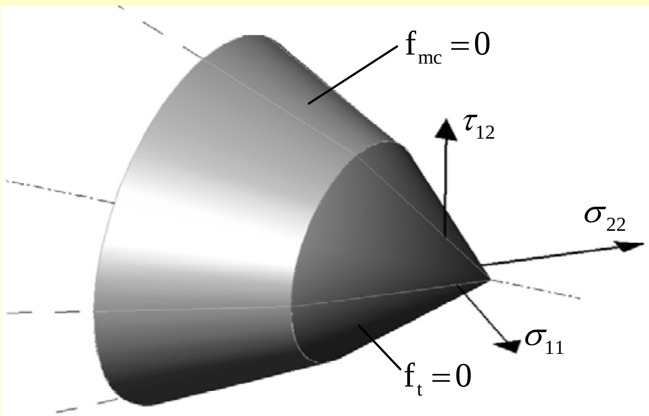
Cavicchi & Gambarotta, 2005, 06, 07



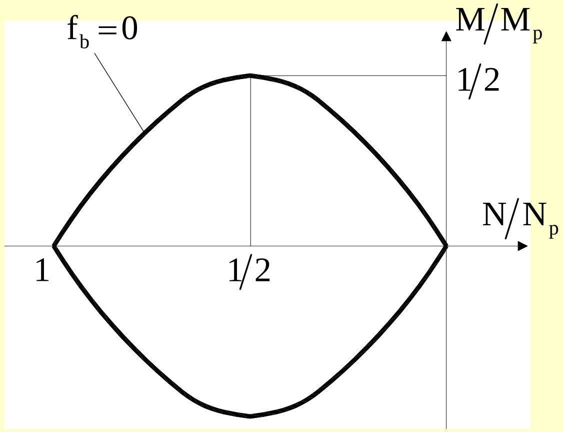
Hypotheses:

1. Arches & piers : NTR – EPP in compression
2. Fill: Mohr-Coulomb + Cut-off
3. FE discretization
4. Plane strain/plane stress
5. Piecewise linearization of the limit domains

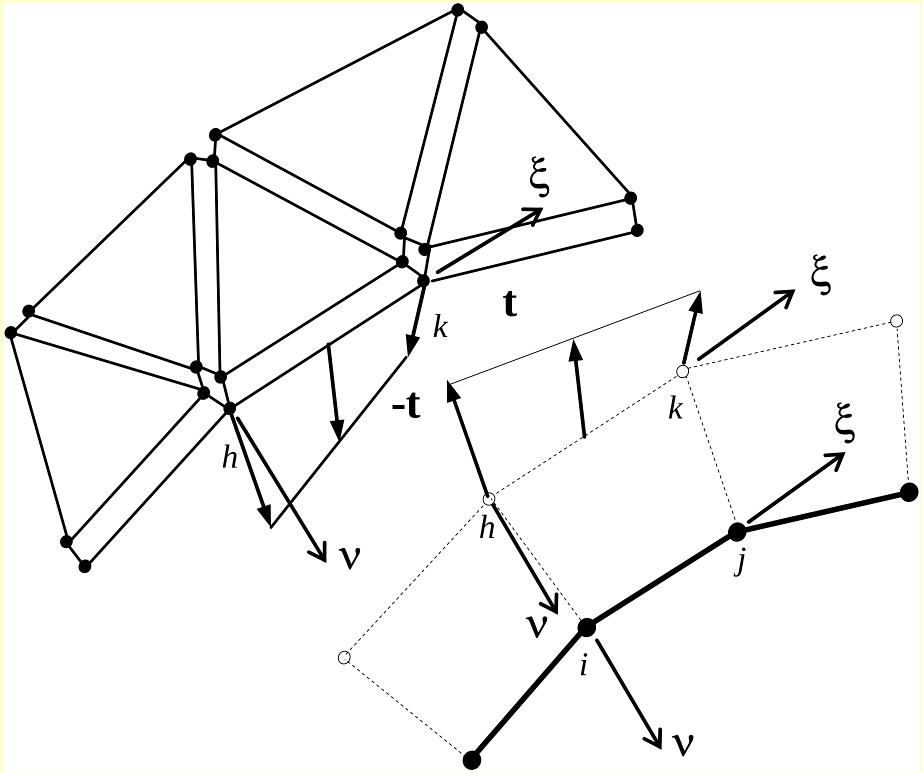
Limit domains



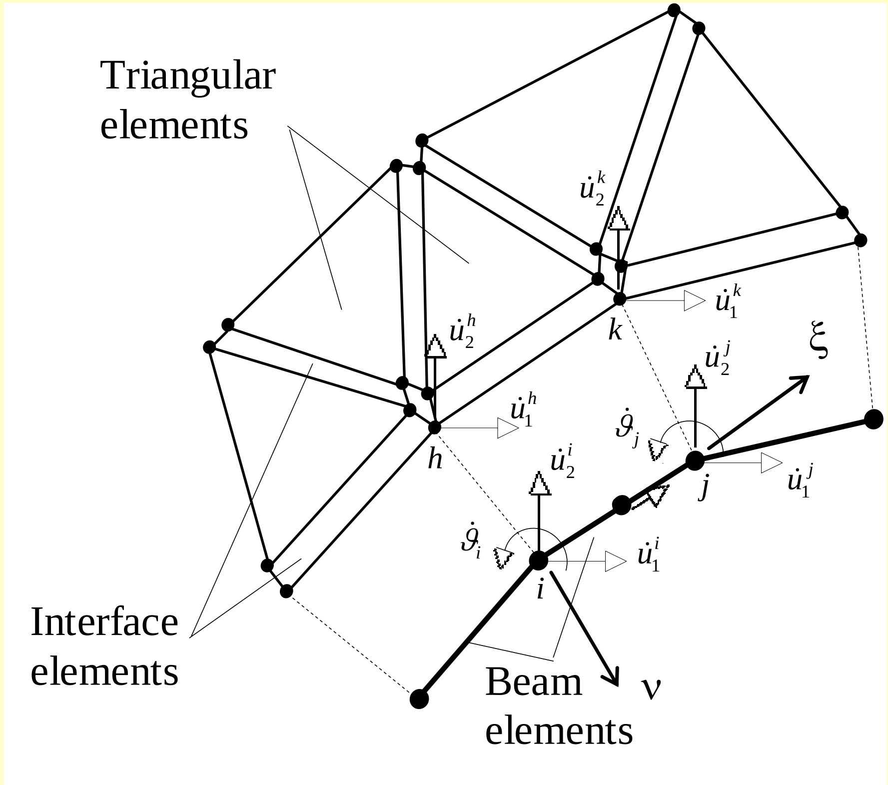
Fill: Mohr Coulamb + Cut off



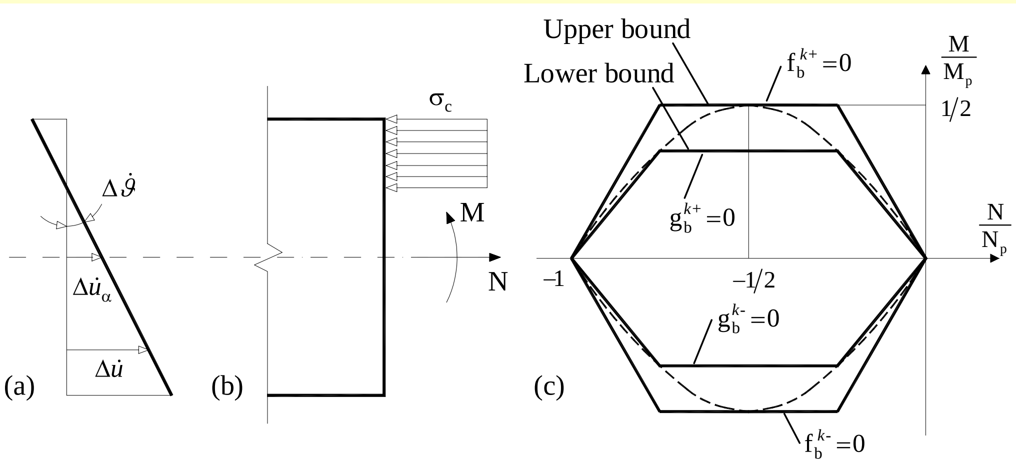
Arch: NTR - EEP in compression

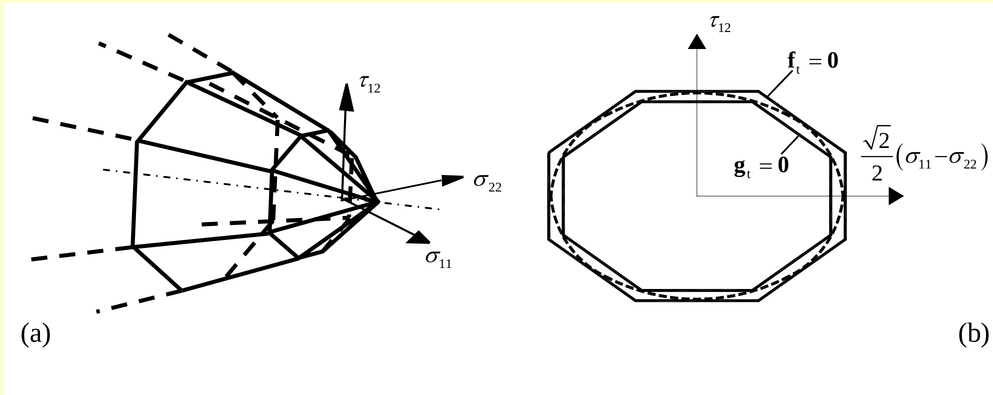


Equilibrium FE model
arch – fill interaction



Compatible FE model
Arch – fill interaction

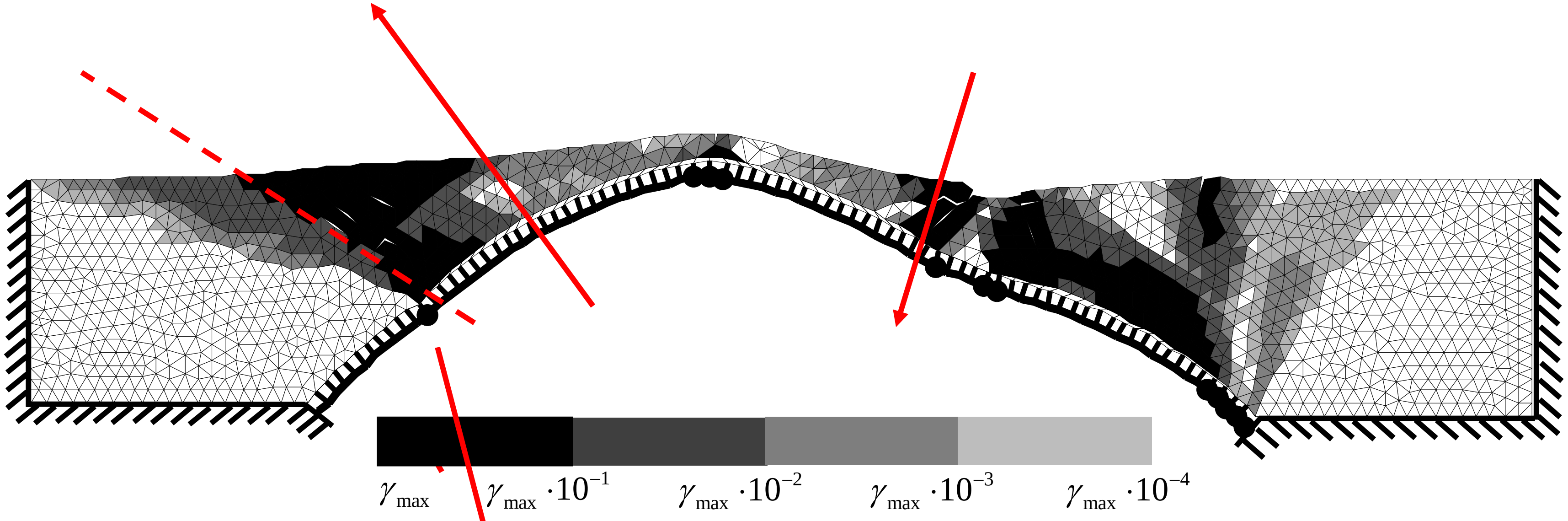




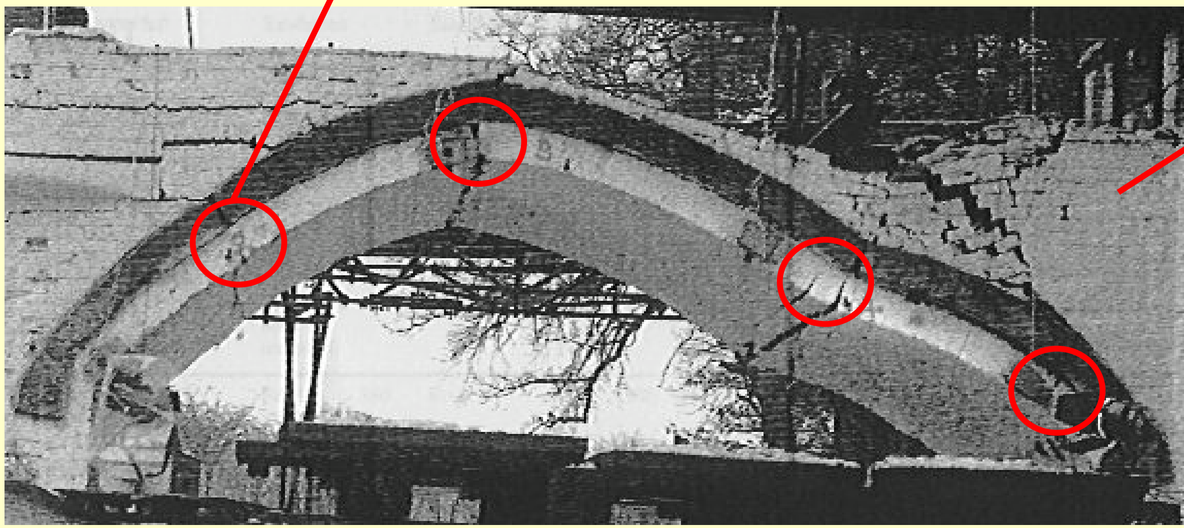
Piecewise linearization of the limit domains

Prestwood Bridge collapse: numerical simulation

U. B. - collapse mechanism (plane strain)



Hinge at haunch



$\sigma_c = 4.5\,\text{MPa}$

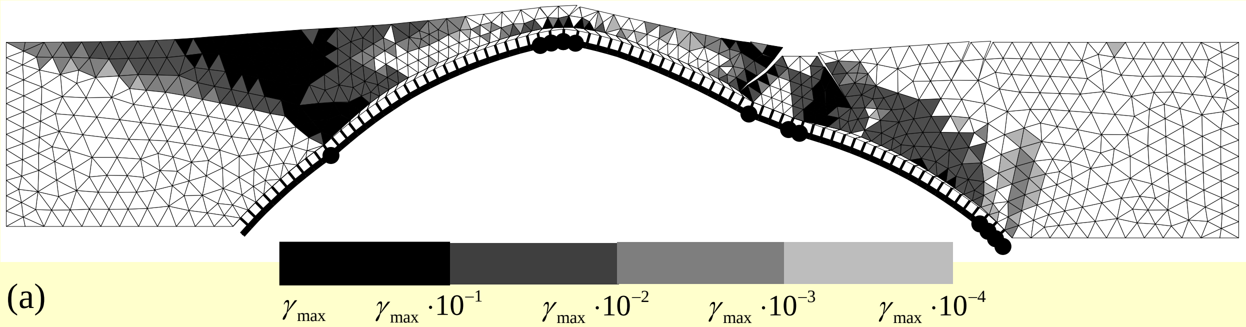
$\varphi = 37^{o}$

$c = 10\,\text{kPa}$

(Page, 1993)

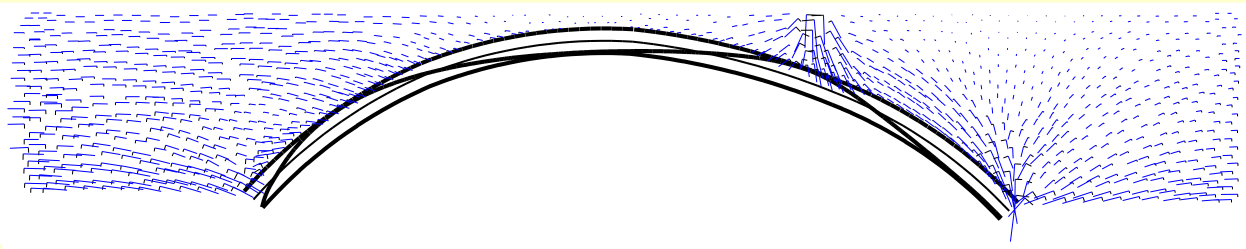
$P_{exp} = 228\,\text{kN}$

Plane strain



U. B. -Collapse mechanism

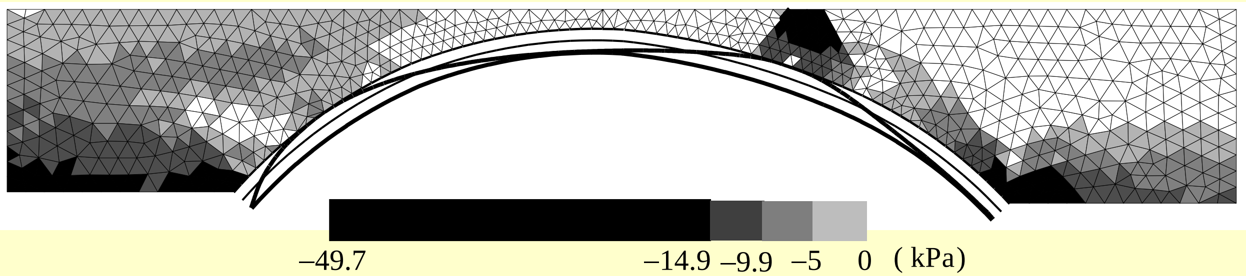
$$P_U = 228\text{kN}$$



(b)

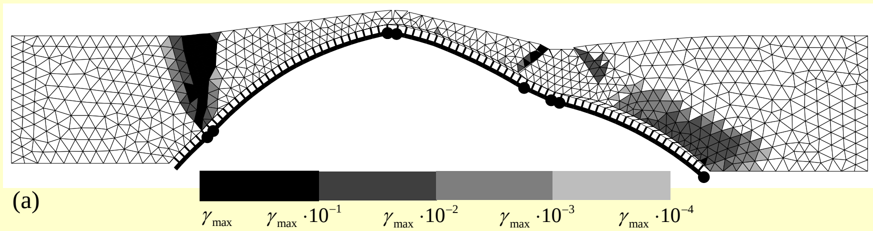
L.B. – Principal stress field

$$P_L = 184\text{kN}$$

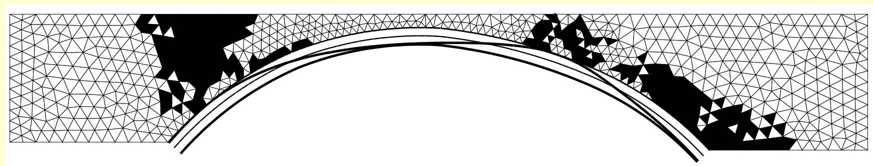


Lateral pressure

Plane stess



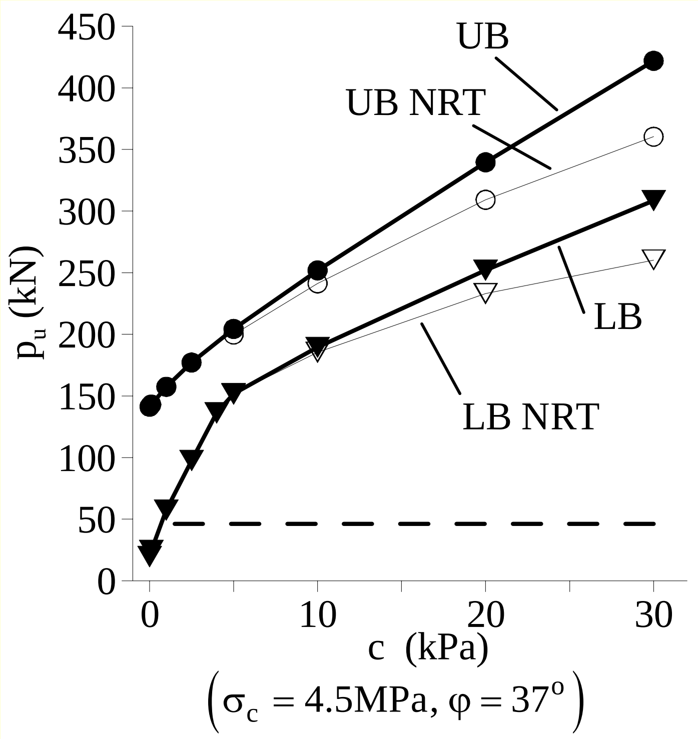
$$P_U = 184\text{kN}$$

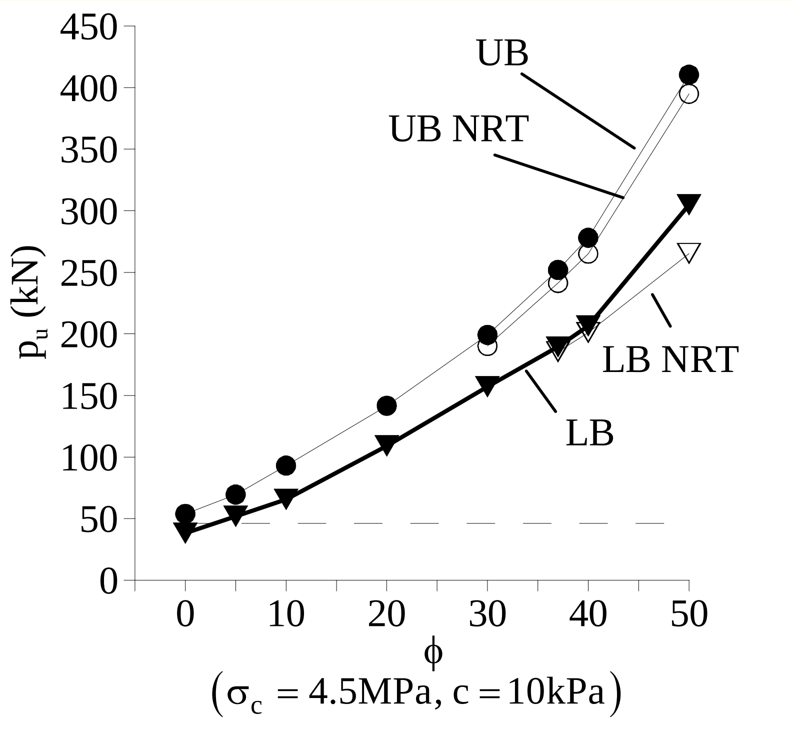


(b)

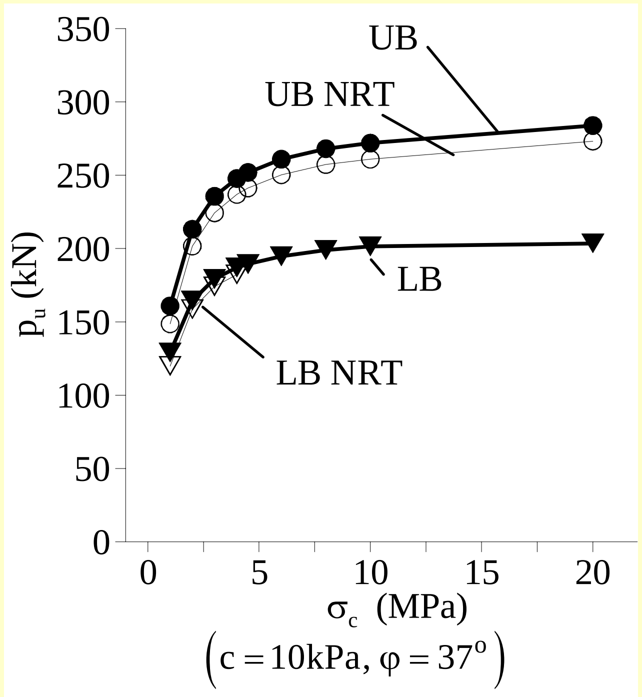
$$P_L = 160\text{kN}$$

Inluence of the cohesion and the angle of internal friction on the collapse load

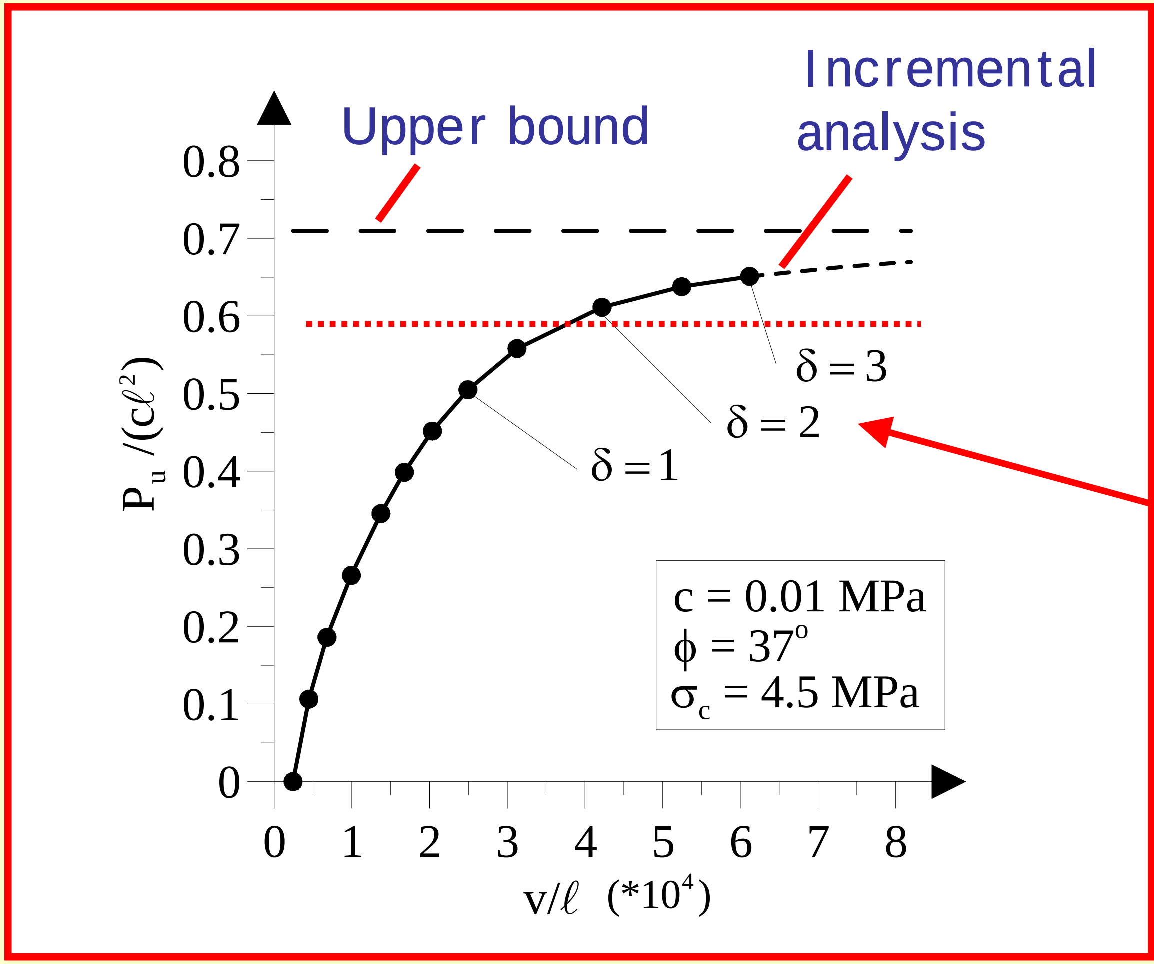




Influence of the masonry compressive strength on the collapse load



Load\deflection curve and ductility demand



Masonry ductility:

$$\delta = \frac{\varepsilon}{\varepsilon_c}$$

Vertical displacement v

Multi span bridge: Fill – arches – piers interaction

Fill model properties:

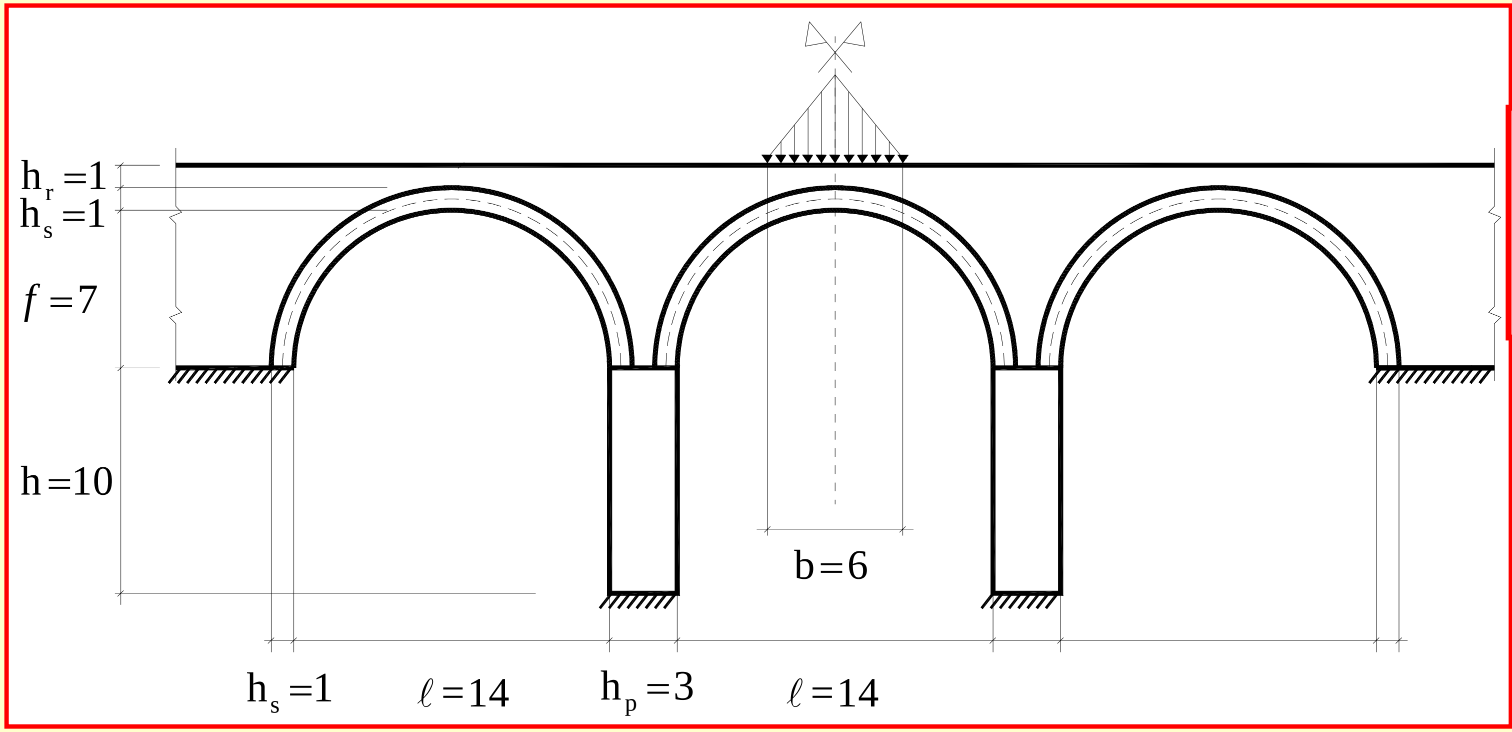
Fill density $\rho = 18\ kN/m^3$

Discrete domain planes $p = 36$

Arch model properties:

Masonry density $\rho = 18\ kN/m^3$

Discrete domain planes $p = 48$

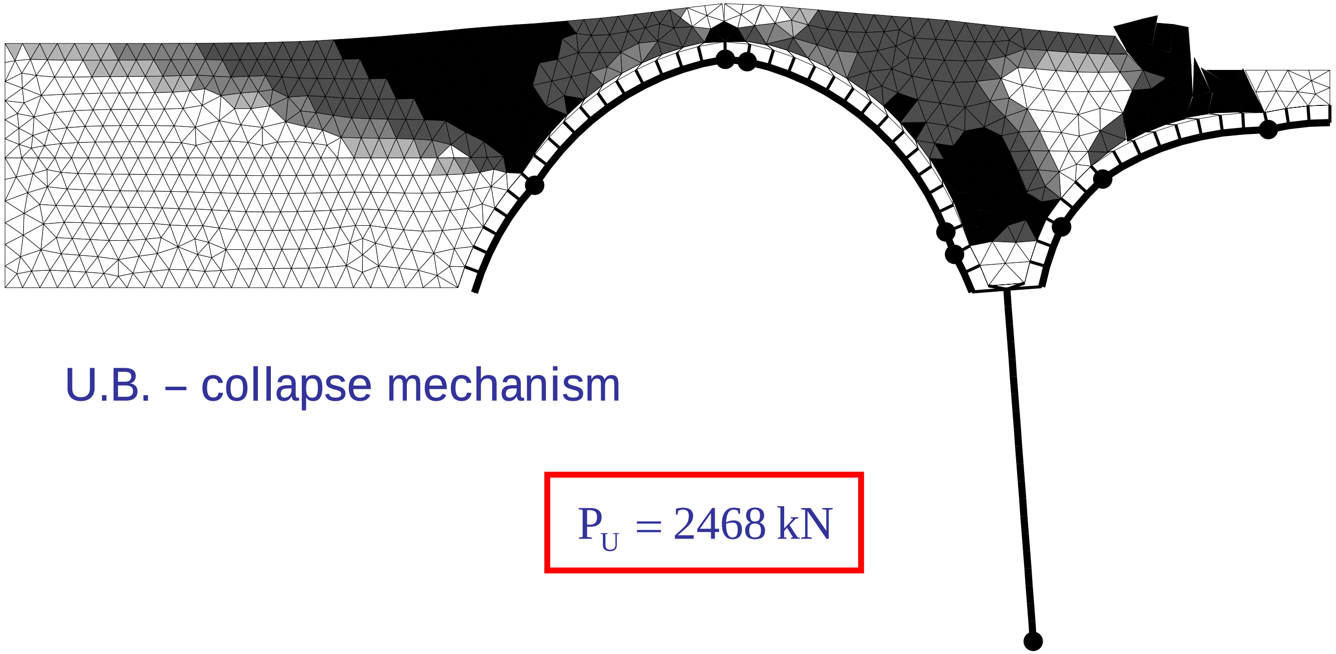


$\sigma_c = 12\,MPa$

$\varphi = 30^o$

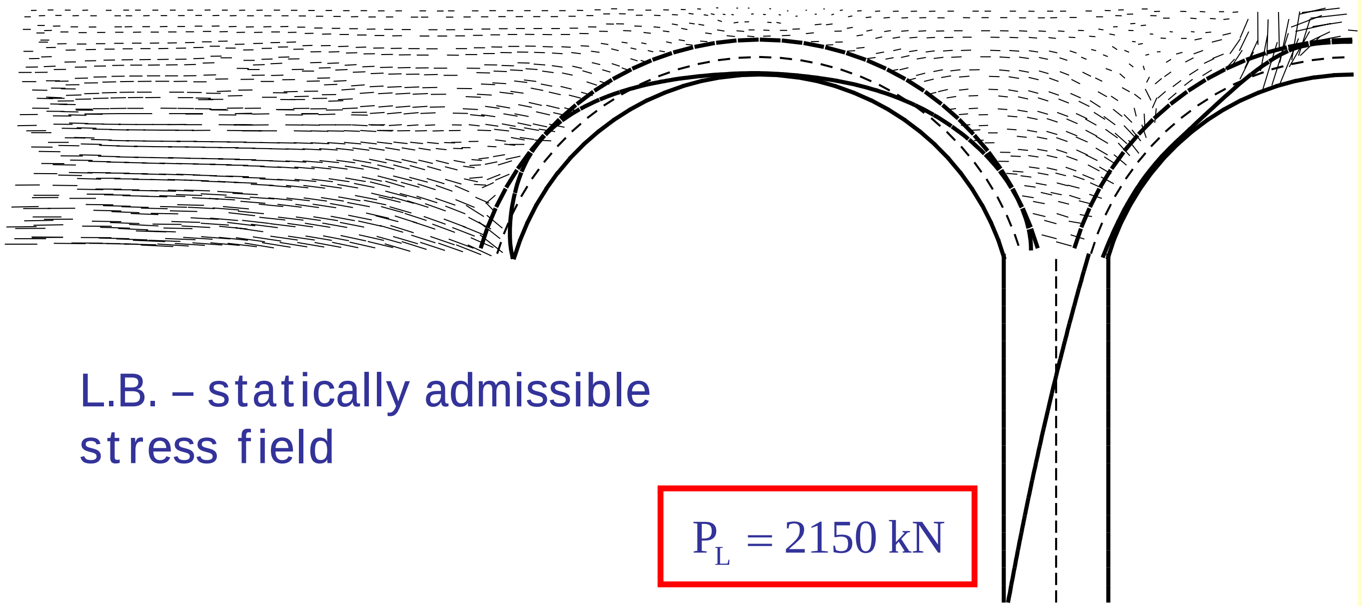
$c = 20kPa$

Multi span bridge



U.B. – collapse mechanism

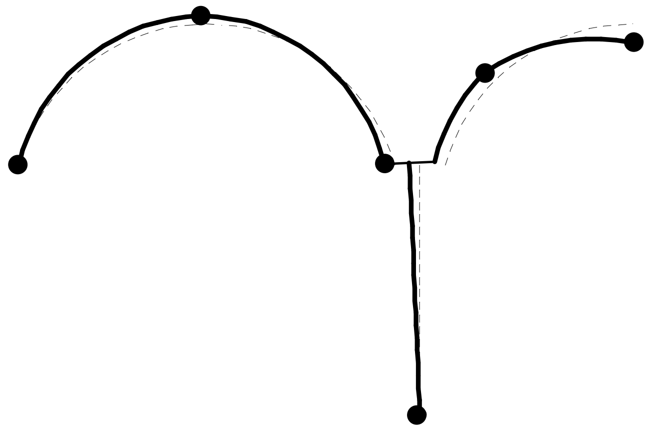
$P_U = 2468 \text{ kN}$



L.B. – statically admissible stress field

$P_L = 2150 \text{ kN}$

Non resistant fill



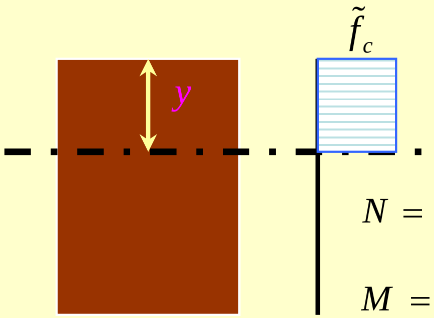
$P_u = 625 \text{ kN}$

$(P_u = 923 \text{ kN})$

Sterpi et al., 2006

Masonry bridges: probabilistic analysis

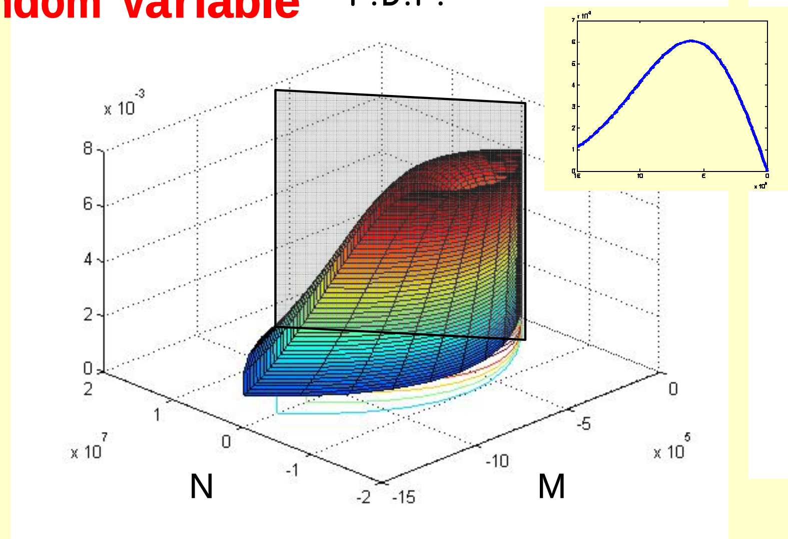
Compressive strength: a random variable

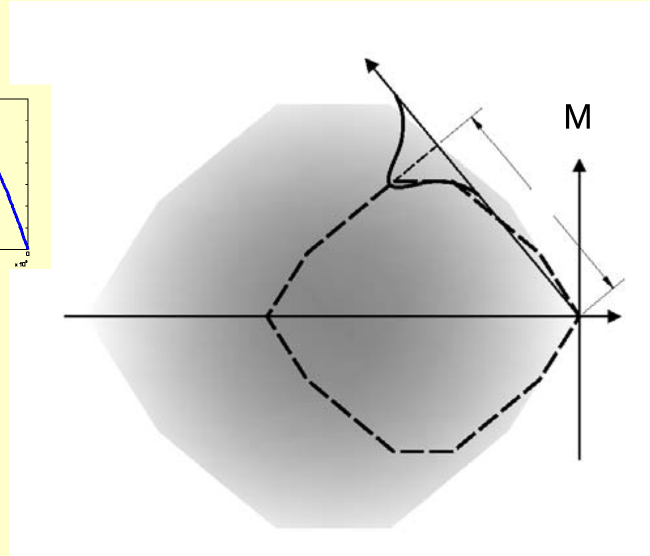


$$N = \tilde{f}_c b y$$

$$M = \frac{\tilde{f}_c b y}{2}(h-y)$$

P.D.F.

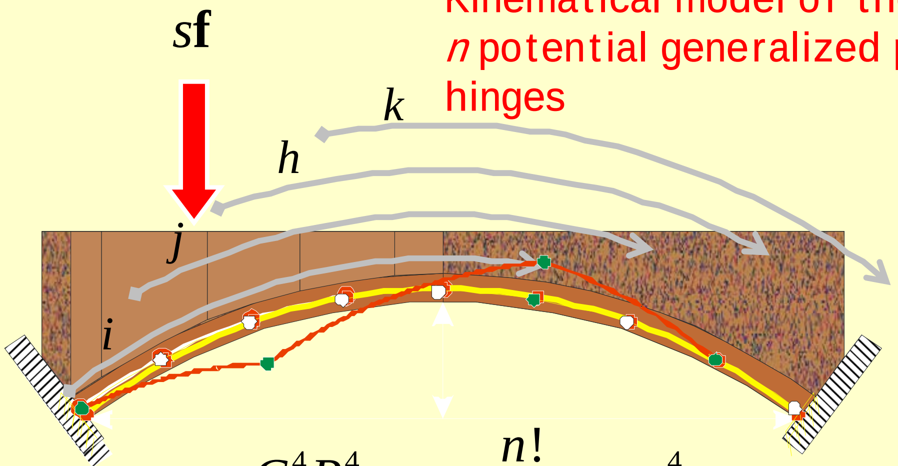




$2m$ piecewise linearization

Kinematical model of the arch

n potential generalized plastic hinges



$$n_t = C_n^4 R_m^4 = \frac{n!}{4!(n-4)!} m^4$$

Mechanism enumeration

$$P_r(s) = \mathrm{Prob}\left[\exists \mathbf{u} \in \mathcal{U} : \mathbf{f}_0^T \mathbf{u} + s\mathbf{f}^T \mathbf{u} > \tilde{\mathbf{r}}^T \boldsymbol{\lambda}_p \right]$$

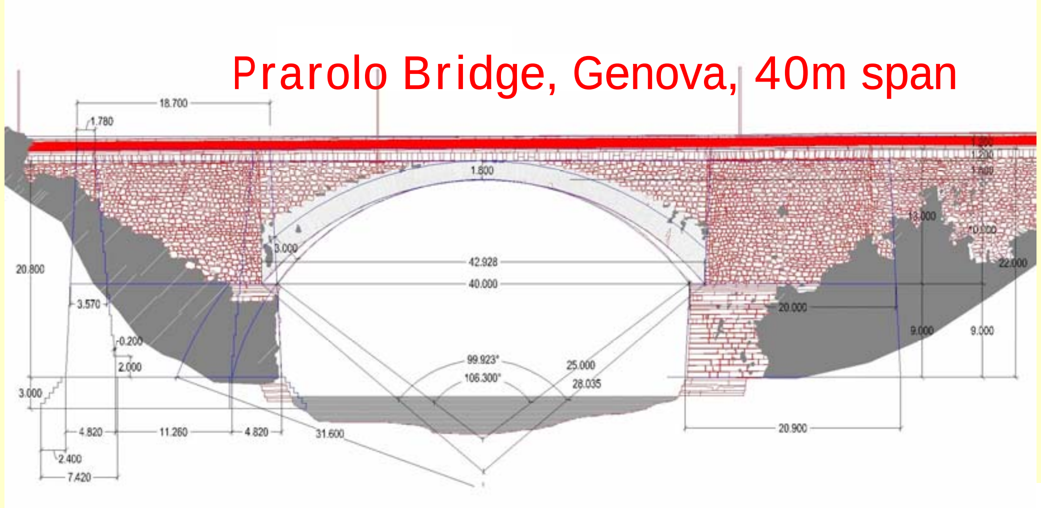
Discrete model – failure - i-th mechanism

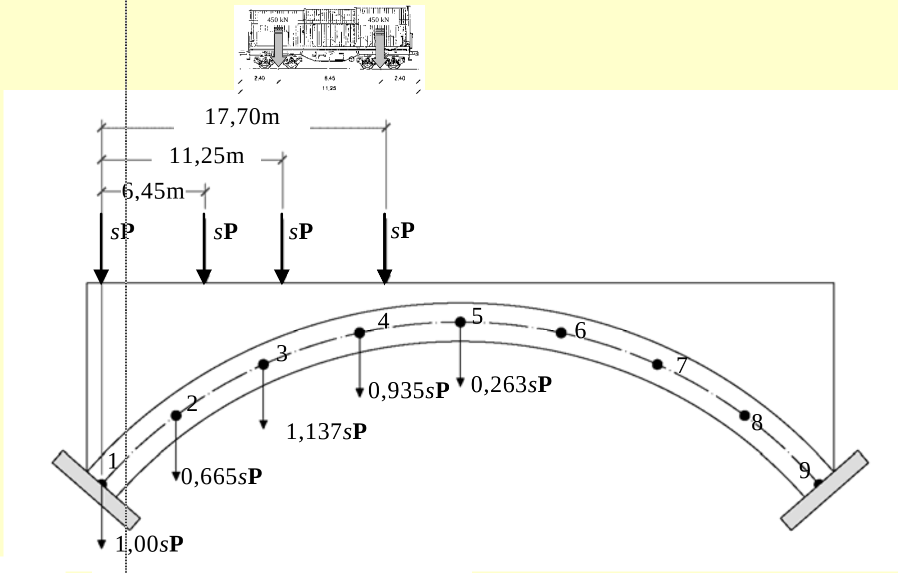
$$[E_i] = \left[\mathbf{f}_0^T \mathbf{u}_i + s\mathbf{f}^T \mathbf{u}_i > \tilde{\mathbf{r}}^T \boldsymbol{\lambda}_{pi}\right] \qquad \tilde{s}_i = \frac{\tilde{\mathbf{r}}^T \boldsymbol{\lambda}_{pi} - \mathbf{f}_0^T \mathbf{u}_i}{\mathbf{f}^T \mathbf{u}_i}$$

$$P_r[E_i] = P_r[\tilde{s}_i < s_0]$$

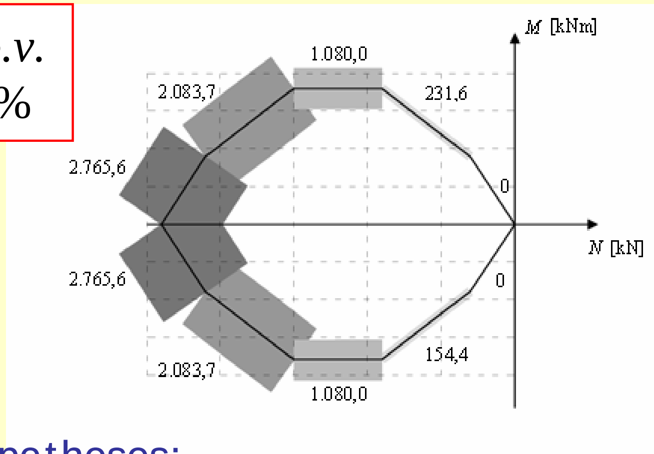
Approximations: bounds on the C.D.F. $\Longrightarrow$ $\max P_r(E_i) \le P_r\left(\bigcup_i E_i\right) \le \sum_i P_r(E_i)$

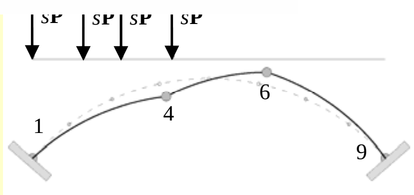
Masonry bridges: probabilistic analysis



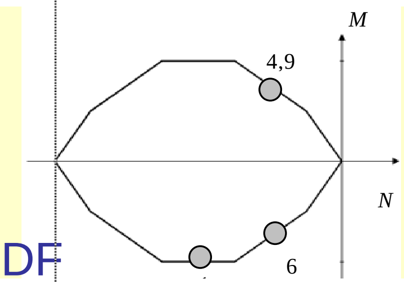


c.o.v.
15%





First mechanism



Hypotheses:

Statistically independent random variables

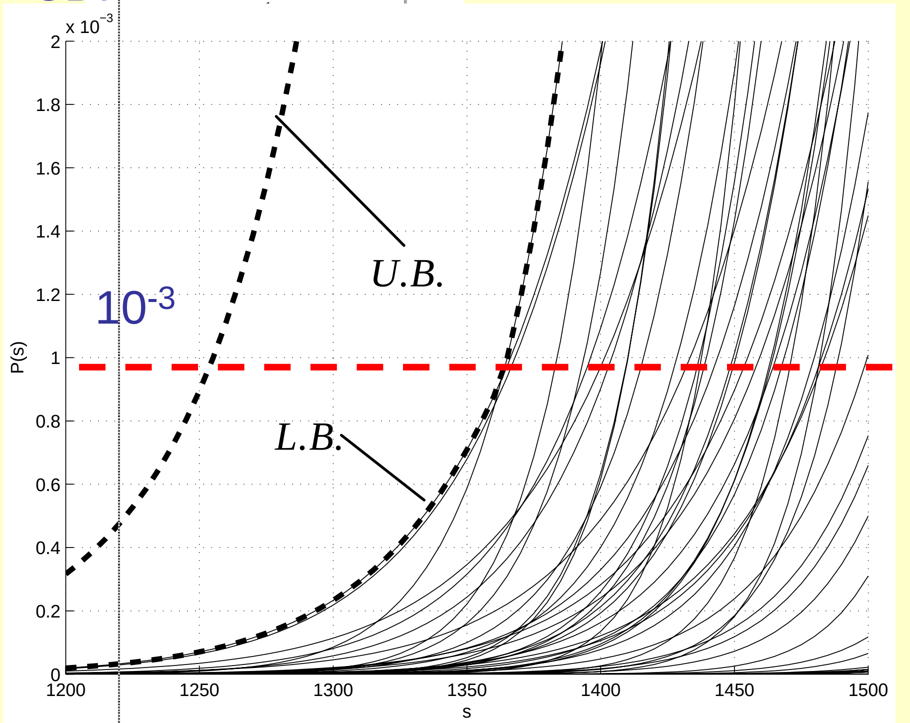
The compressive masonry strength is gaussian

The structural strength (upper bound theorem) is gaussian

$$\bar{s} = \frac{\bar{D}_{\text{int}} - W_0}{W_a}$$

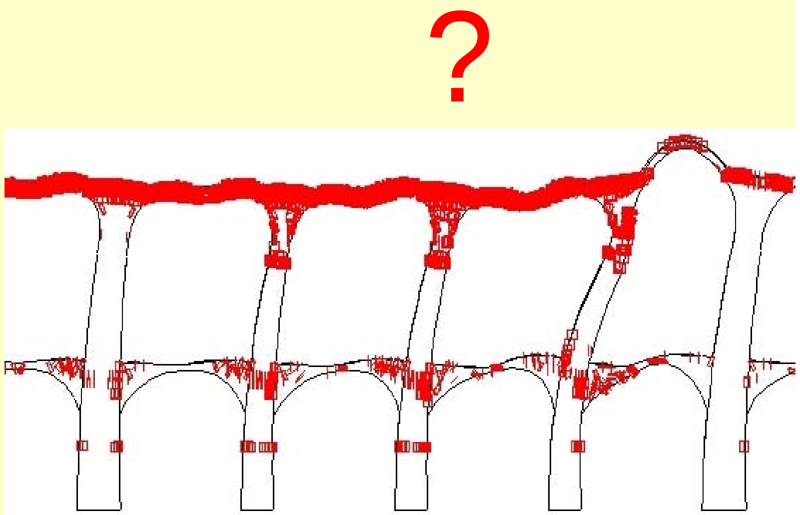
$$c.o.v. = \frac{\sqrt{\boldsymbol{\lambda}^T \mathbf{C}(\tilde{\mathbf{r}})\boldsymbol{\lambda}}}{\bar{D}_{\text{int}} - W_0}$$

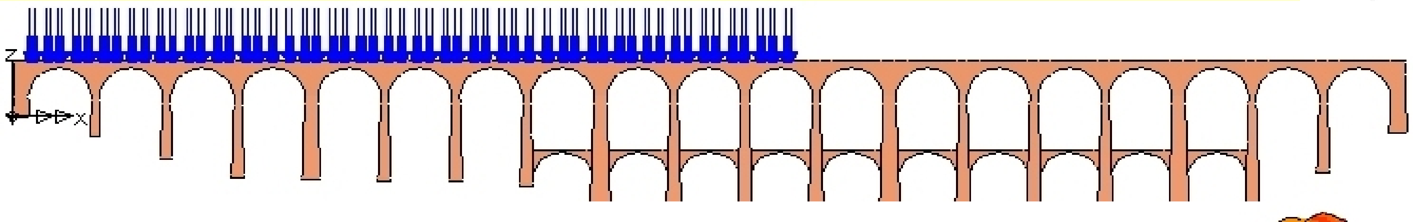
CDF



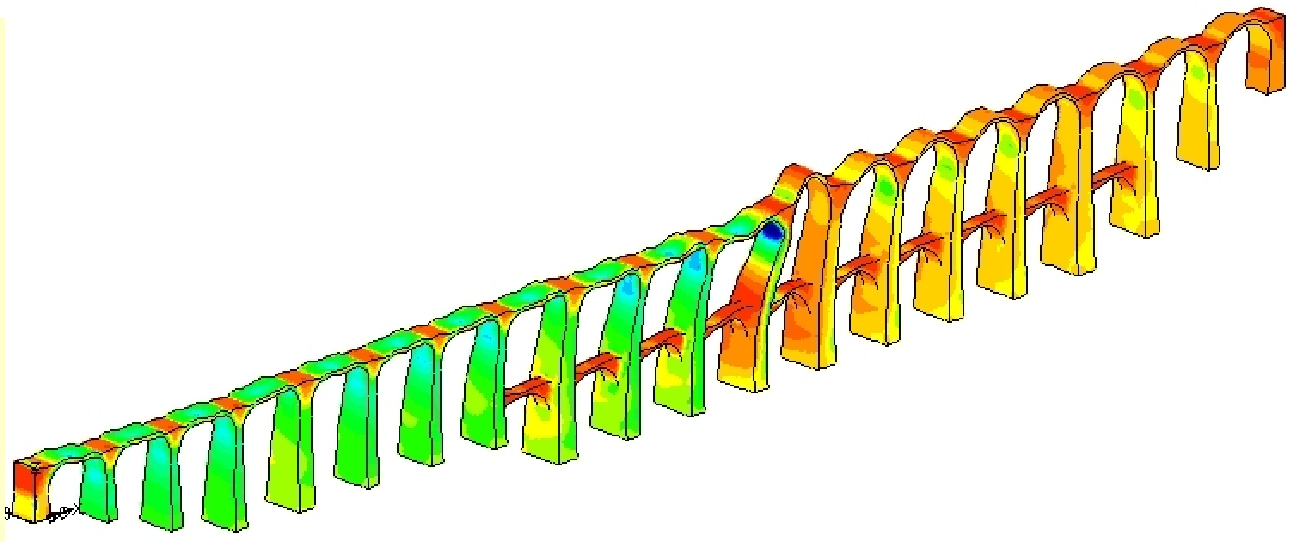
Masonry railway bridges

Open problems



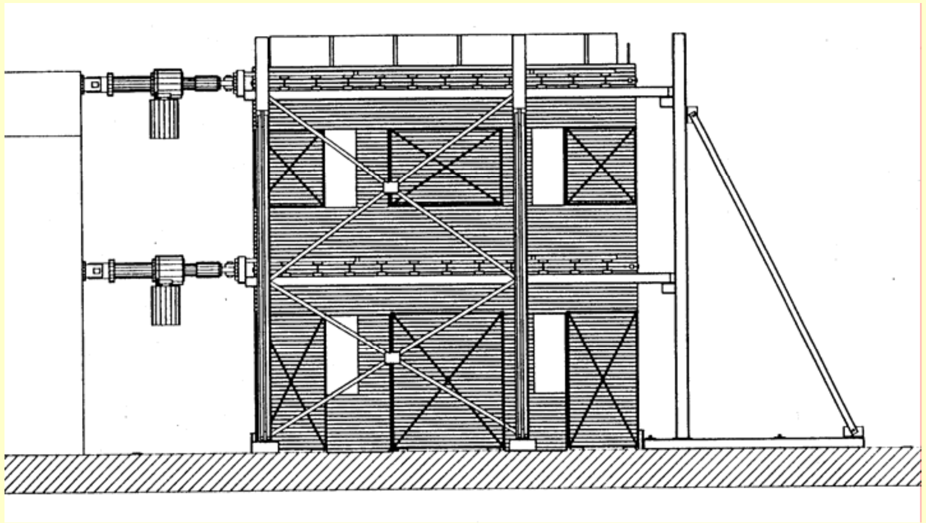


Non linear analysis including damage and cracking

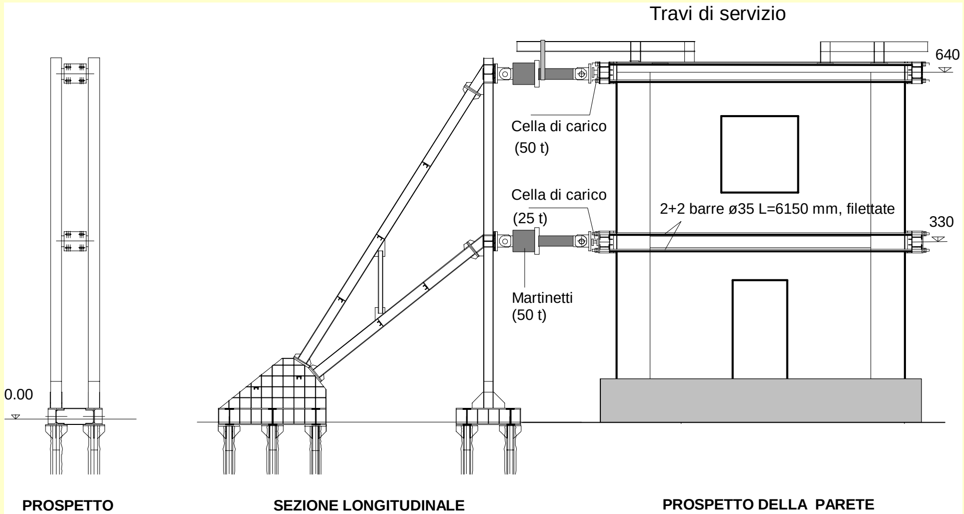


4. Masonry walls – Simulation of in-plane response to seismic actions

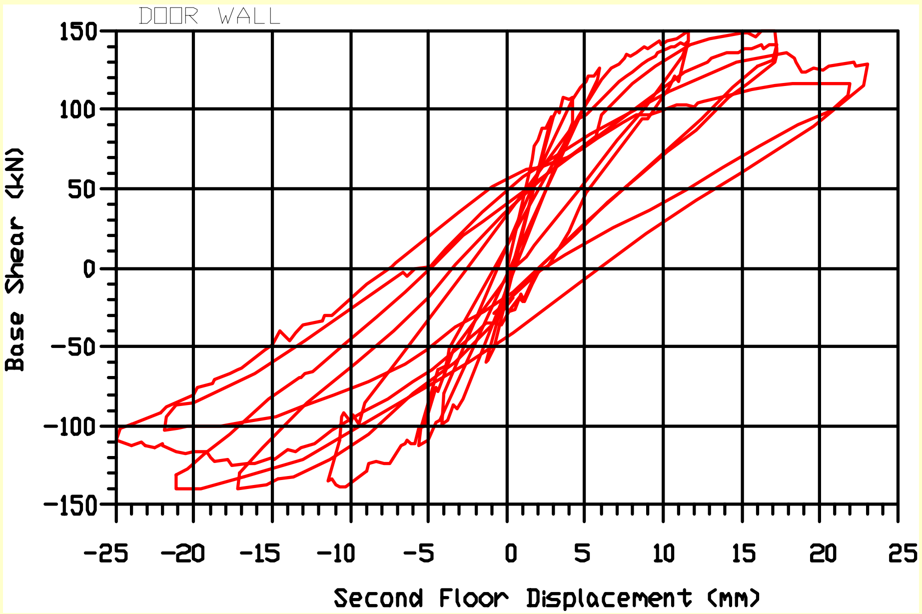
Cyclic horizontal forces, anisotropic damage, damage localization, hysteretic dissipation, inertial vertical forces …………

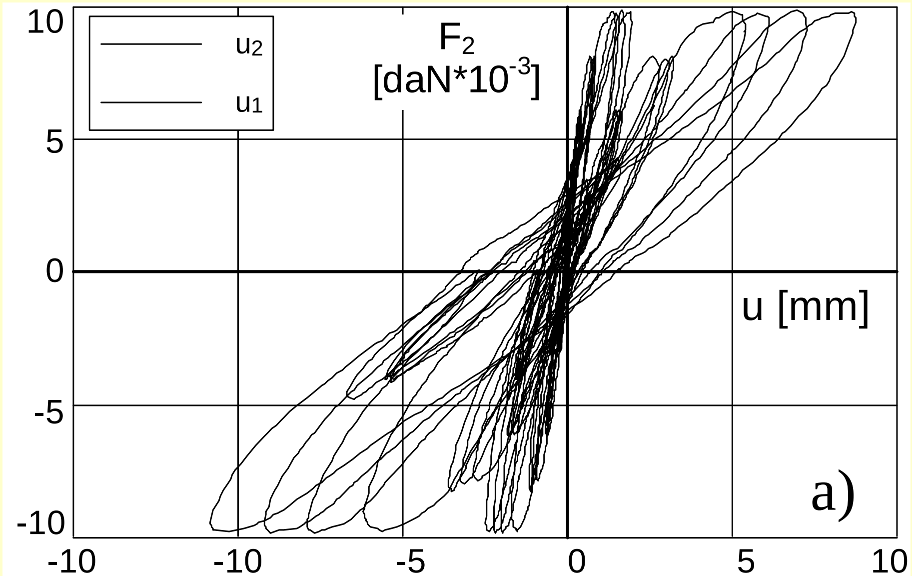


Brick masonry wall tested in Pavia, Magenes et al. (1994).



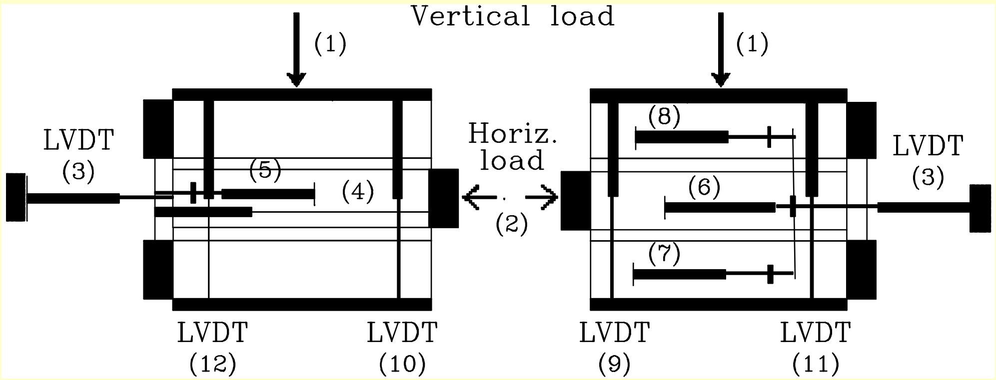
Block masonry wall in S.Sisto (Beolchini et al., 1997).





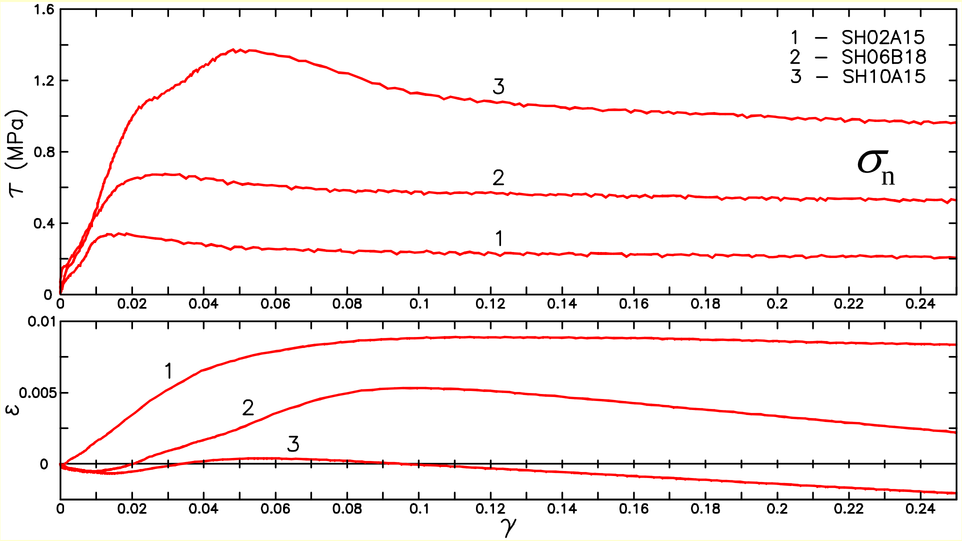
4. Shear wall – in-plane response

Shear testing on brick-mortar assemblages



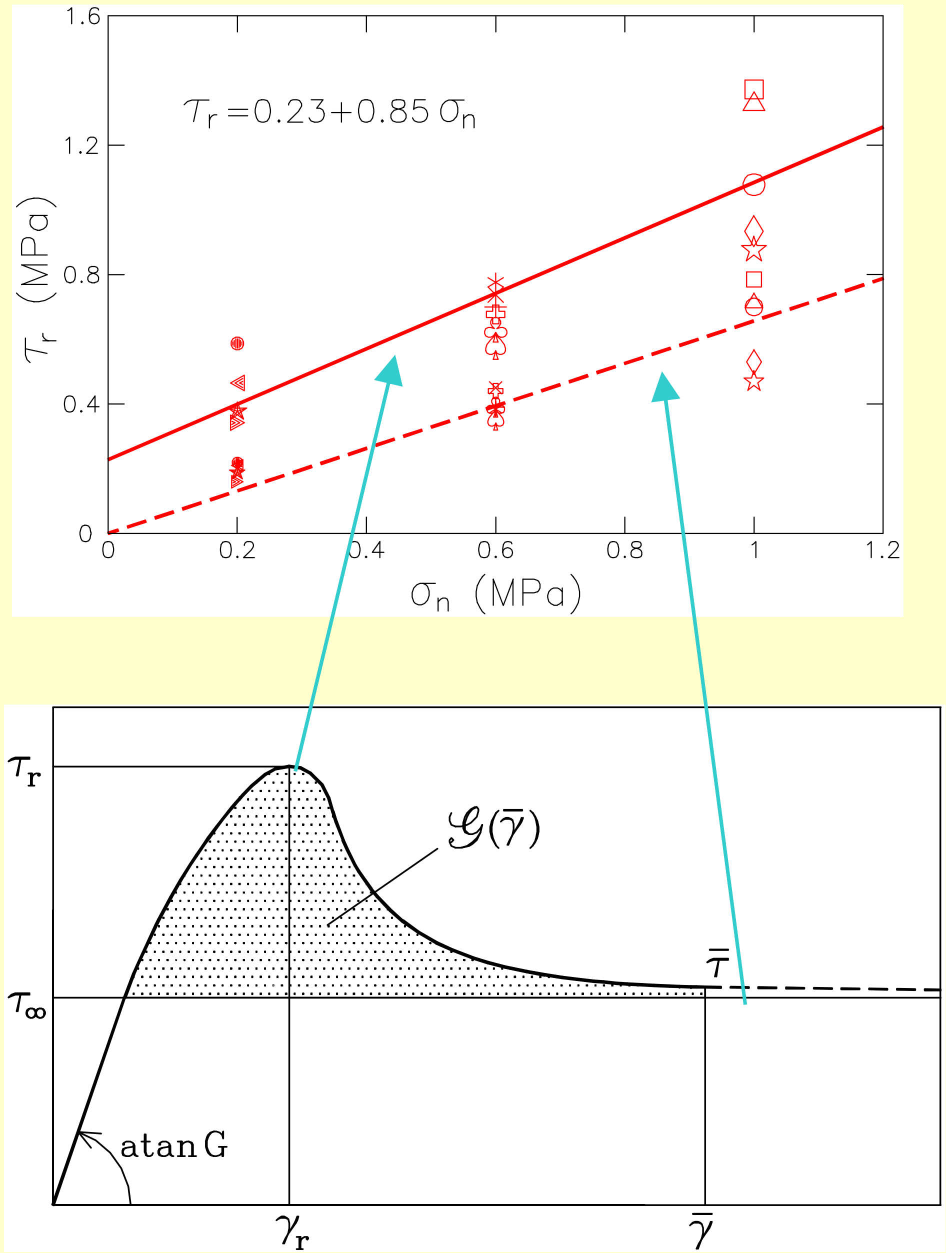
Shear test apparatus - Triplet
(Binda et al., 1995).

τ mean shear stress - γ mean shear strain



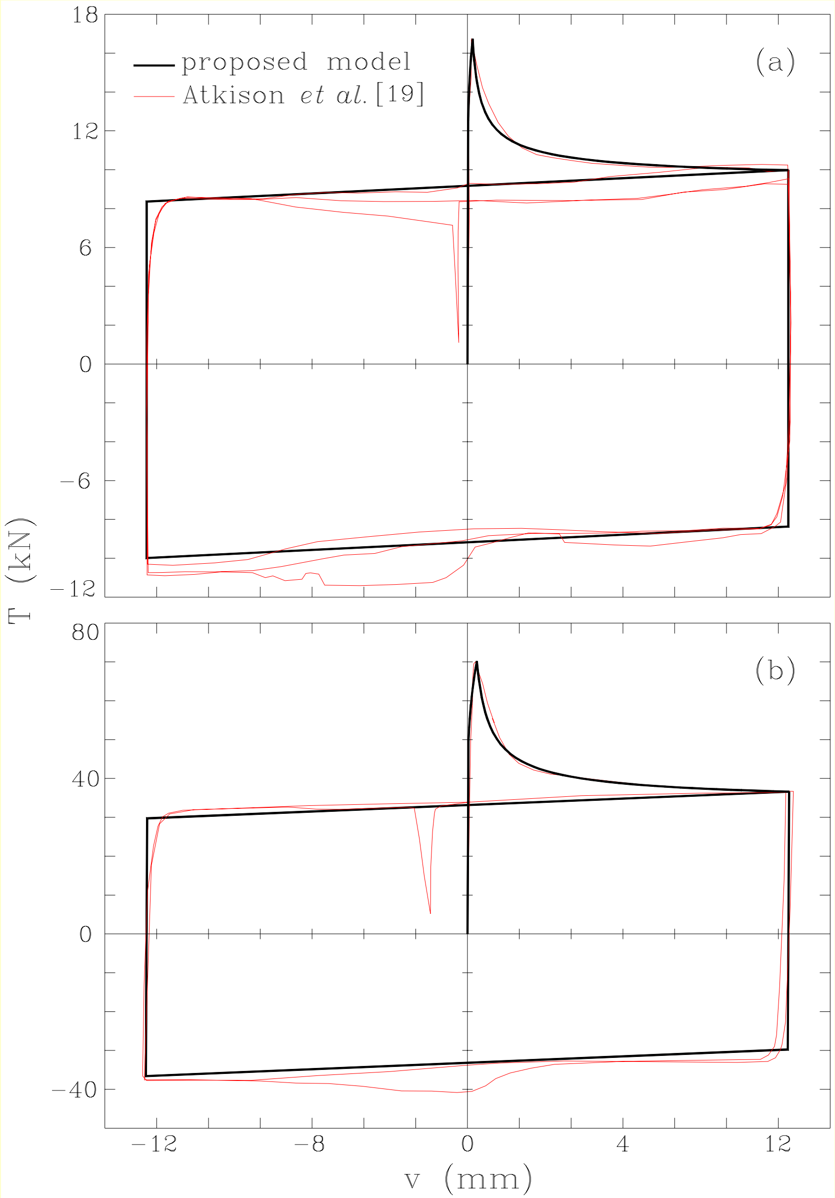
ε - mean normal extension

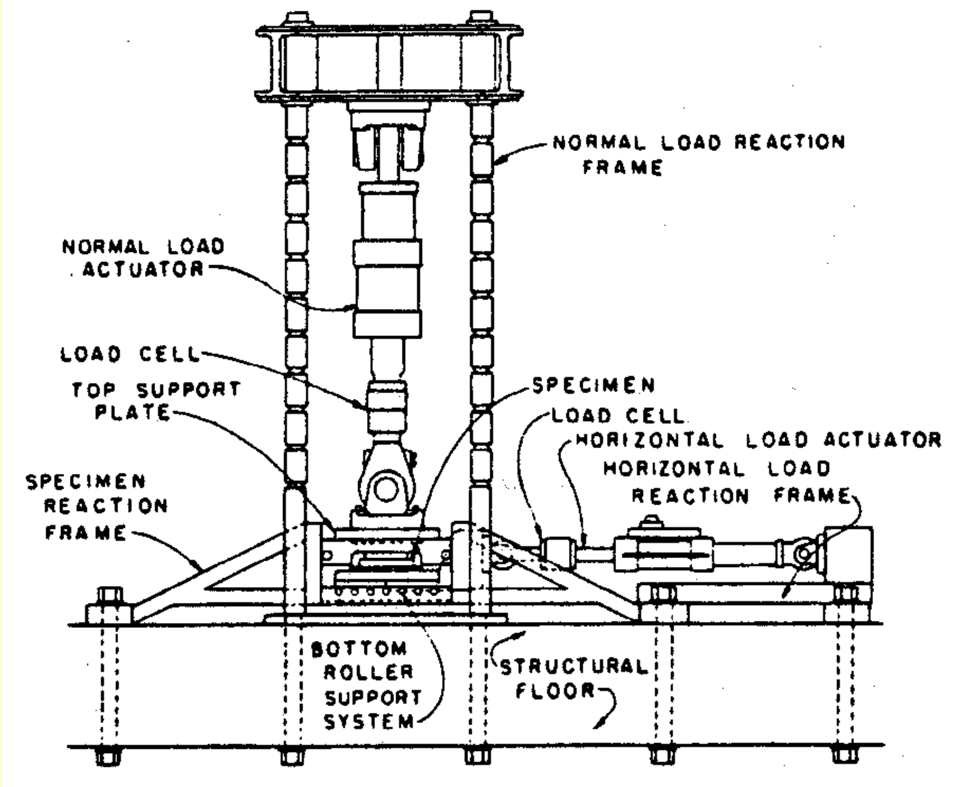
Experimental results

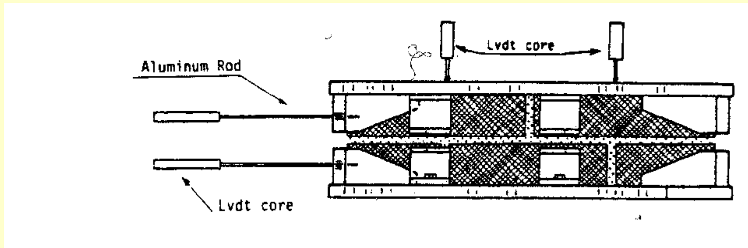


Phenomenological description

4. Shear wall – in-plane response



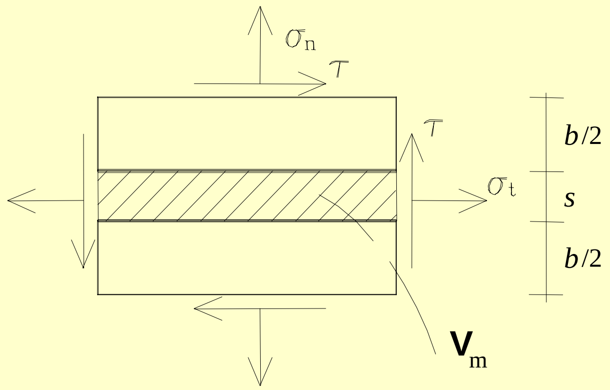




Direct cyclic shear test by Atkinson et al., 1989.

Brick-mortar interface model: coupled damage-frictional interface

Gambarotta e Lagomarsino, 1997



Macro fieds

$$\boldsymbol{\sigma}_m = \{\sigma_t \, \sigma_n \, \tau\}^t$$

$$\boldsymbol{\varepsilon}^*_m = \{0 \; \varepsilon^*_m \, \gamma^*_m\}^t \qquad \text{Inelastic strain}$$

$$\boldsymbol{\varepsilon}_m = \{0 \quad \varepsilon_m \quad \gamma_m\}^t \qquad \text{Total strain}$$

$$\boldsymbol{\varepsilon}_m = \mathbf{K}_m \boldsymbol{\sigma}_m + \boldsymbol{\varepsilon}^*_m$$

$$\begin{cases} \varepsilon^*_m = h(\alpha_m)\; H(\sigma_n)\, \sigma_n \\ \gamma^*_m = k(\alpha_m)\,(\tau - f) \end{cases}$$

α_m damage variable

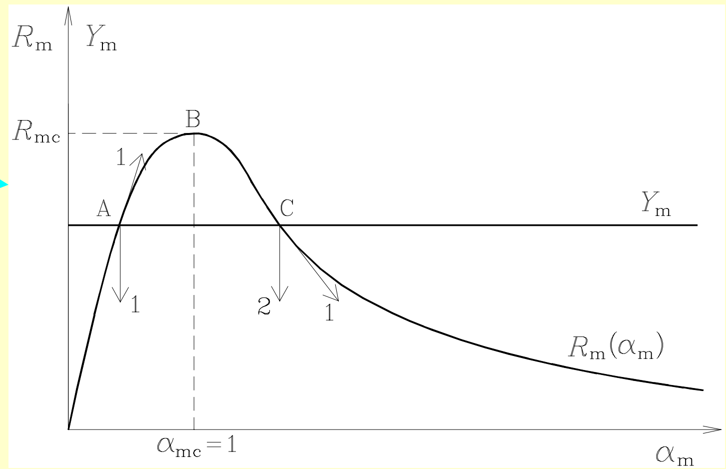
f friction

Conjugate variables

$$Y_m = \tfrac{1}{2} h'(\alpha_m)\; H(\sigma_n)\, \sigma_n^2 + \tfrac{1}{2} k'(\alpha_m)\,(\tau - f)^2 \quad , \; \gamma^*_m$$

Damage evolution

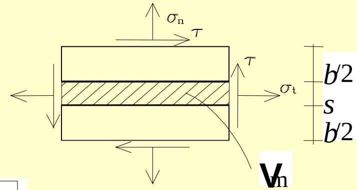
$$\begin{cases} \phi_{dm} = Y_m - R_m \le 0 \\ \phi_{dm} = 0,\; \dot{\phi}_{dm} \le 0,\; \dot{\alpha}_m \ge 0,\; \dot{\phi}_{dm}\dot{\alpha}_m = 0 \end{cases}$$



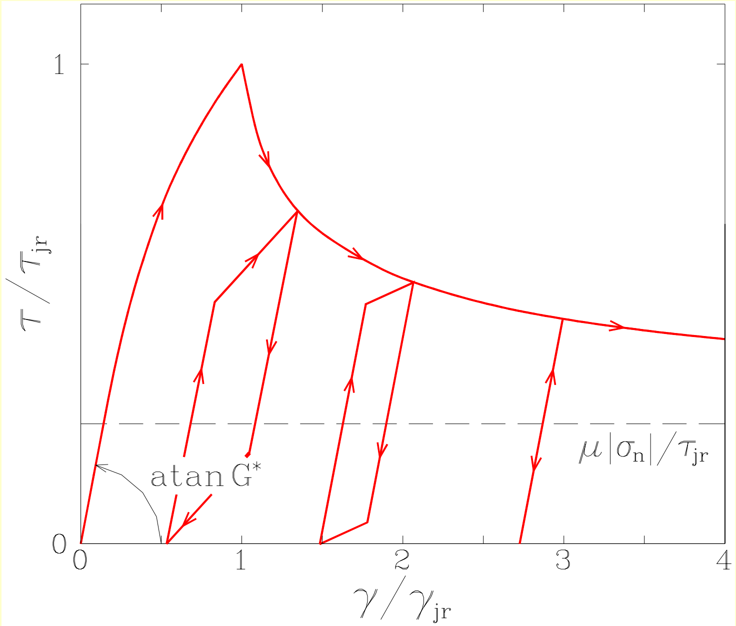
Sliding

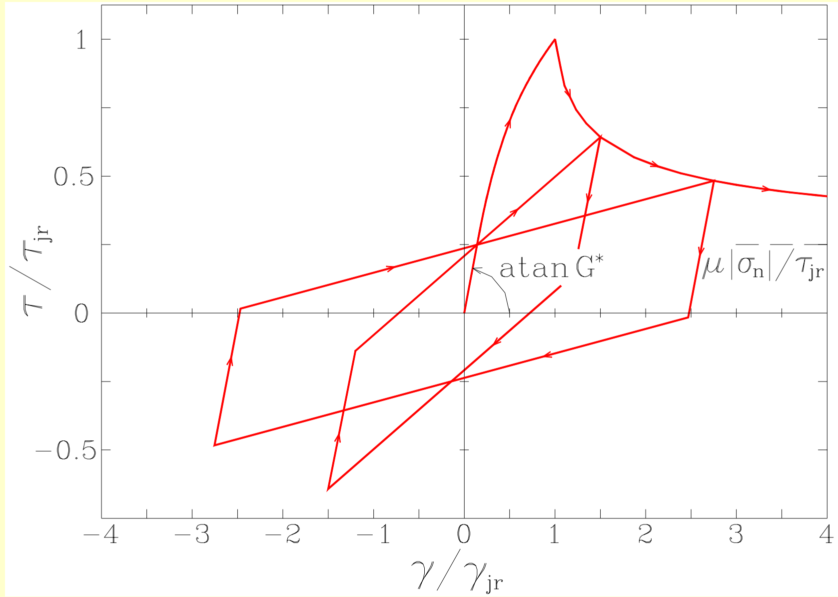
$$\begin{cases} \phi_s = |f| + \mu\sigma_n \le 0 \\ \dot{\gamma}^*_m = \mathbf{v}\dot{\lambda} \quad , \quad \dot{\lambda} \ge 0 \qquad \mathbf{v} = \dfrac{f}{|f|} \end{cases}$$

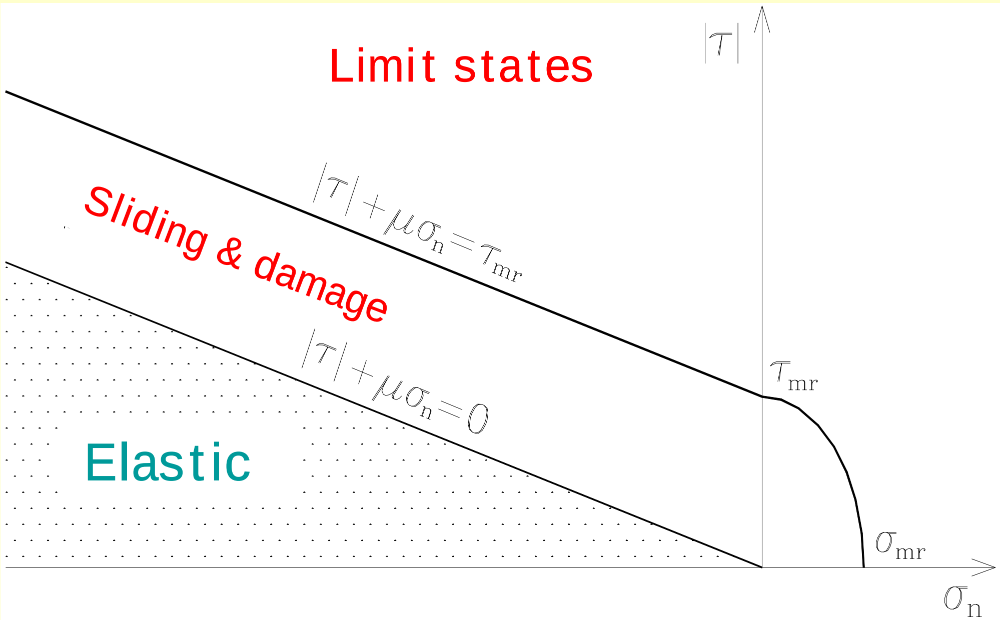
Brick-mortar interface model: coupled damage-frictional interface



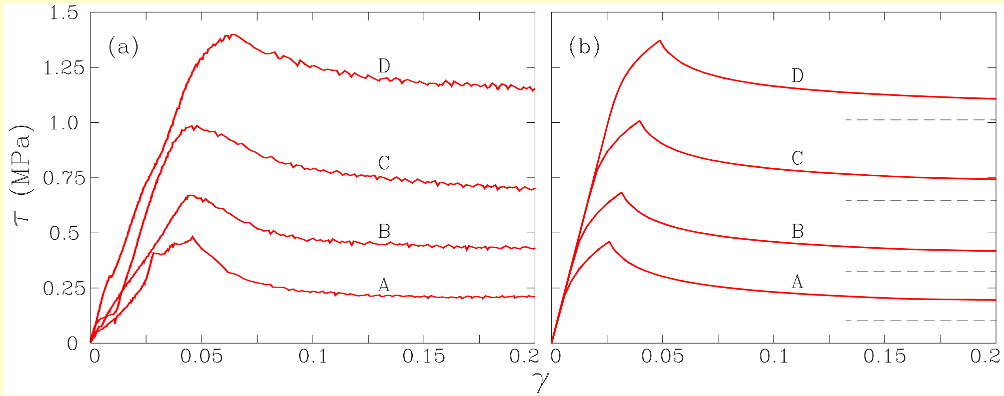
Hysteretic damage



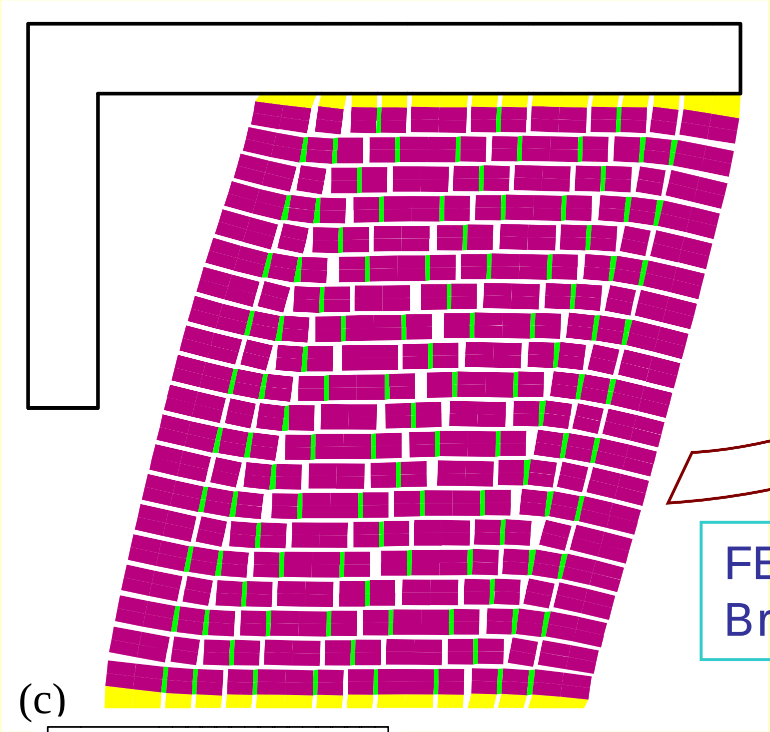


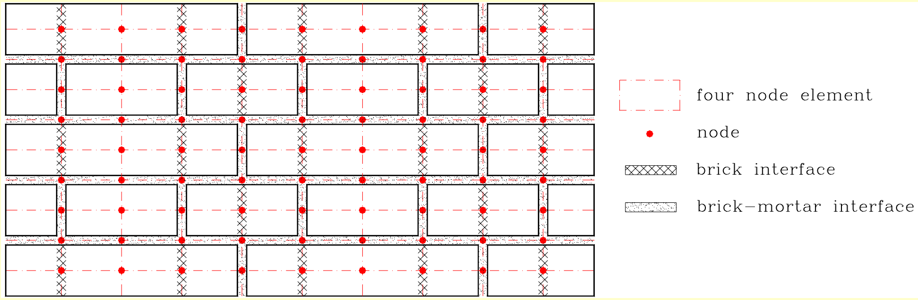


Simulation of experimental results (Binda et al)

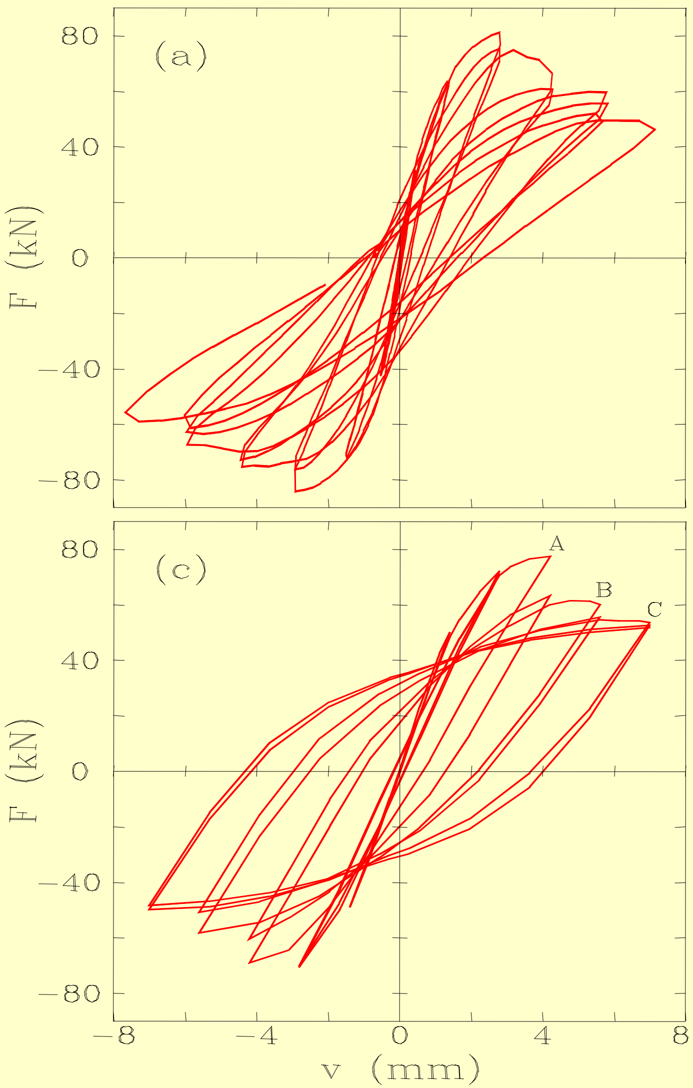


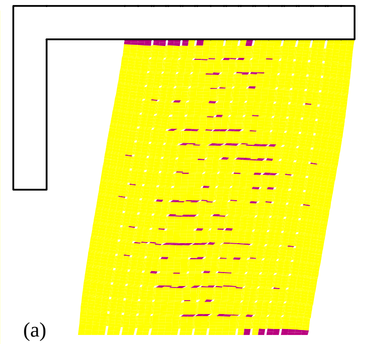
4. Masonry walls – simulation of the in-plane response



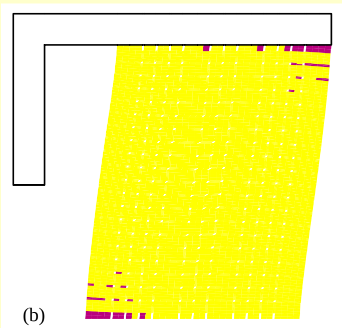


FE model
Brick units + Interface



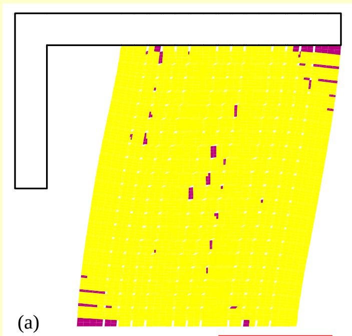


SR



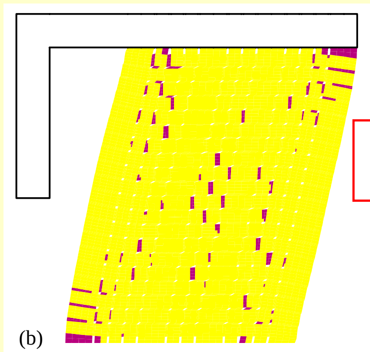
OR

A

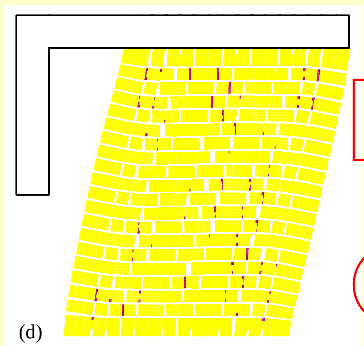


OR

B



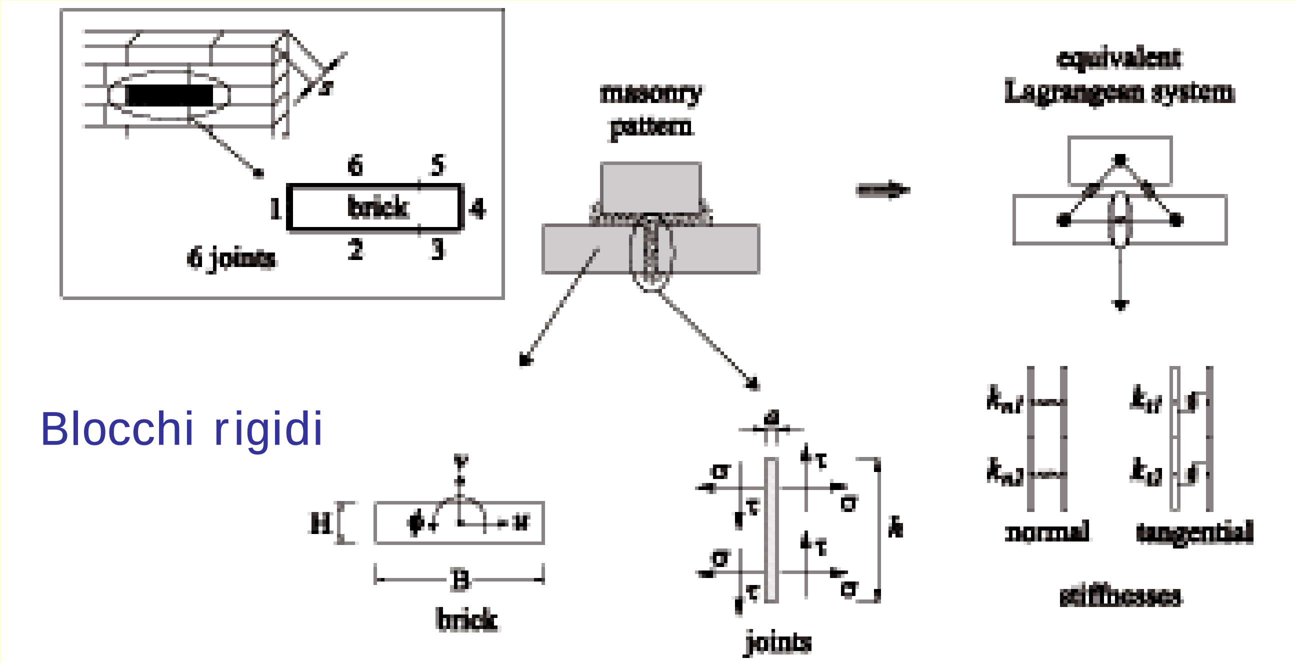
OR



BR

C

4. Masonry walls – Discrete models

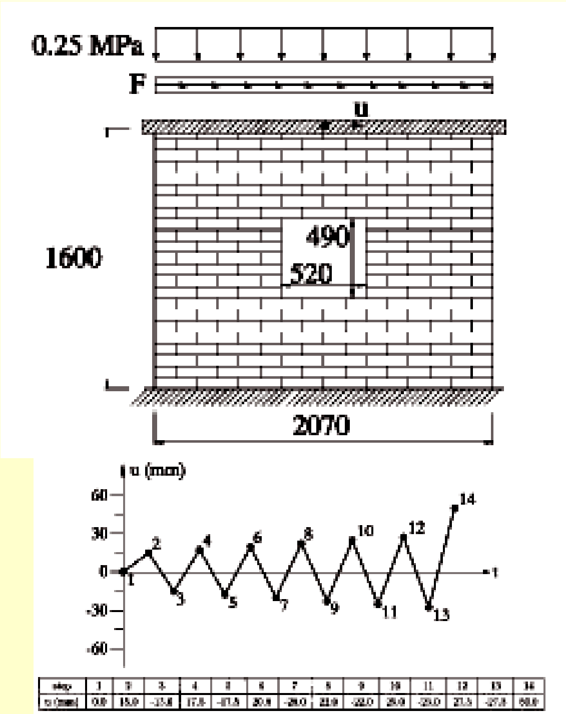


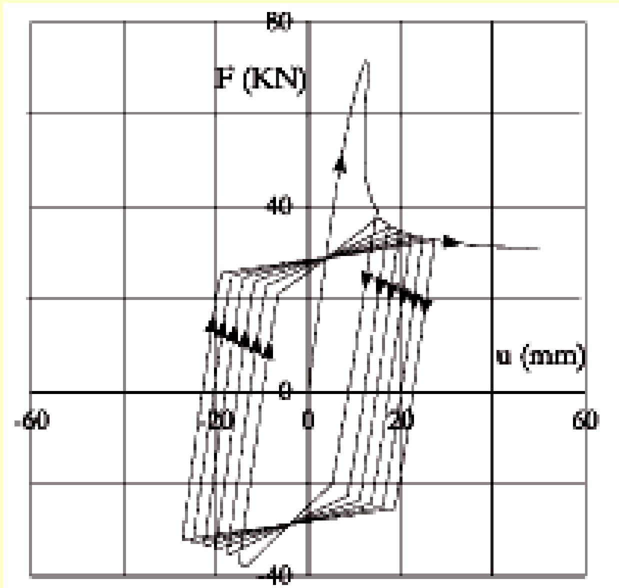
Blocchi rigidi

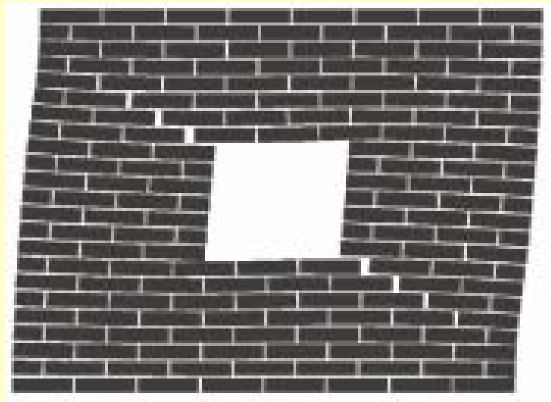
Casciaro et al, 2002
Salerno, Uva, 2006

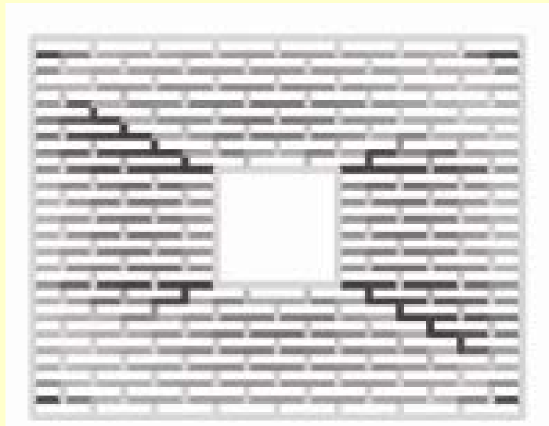
Coupled damage-frictional interface
(Gambarotta e Lagomarsino, 1997)

Mixed FE formulation
Arch-length iterative analysis

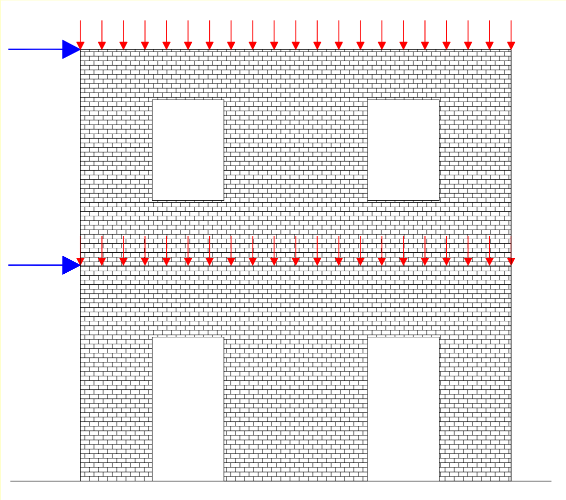


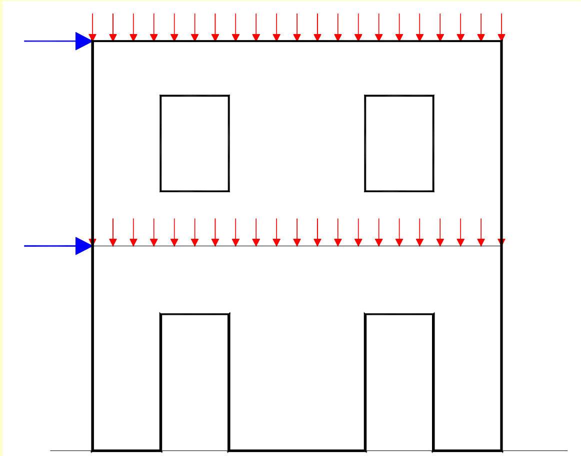






4. Large masonry shear walls – seismic actions





Micro fields $\sigma, \mathbf{u}, \varepsilon, \zeta$

$$\mathbf{u}(\mathbf{x}) = \mathbf{E}\mathbf{x} + \mathbf{u}_{per}$$

$$div\boldsymbol{\sigma} = \mathbf{0} \quad \text{in} \quad \mathcal{E}$$

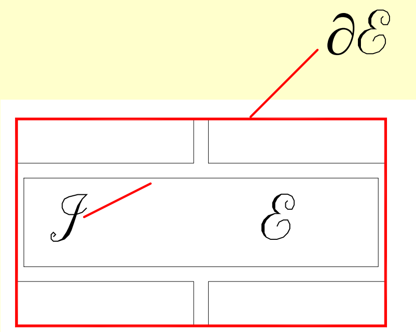
$$\boldsymbol{\sigma}\mathbf{n} \quad \text{antiperiodic on } \partial\mathcal{E}$$

$$\|\boldsymbol{\sigma}\|\mathbf{n} = \mathbf{0} \quad \text{su } \mathcal{I}$$

Macro fields Σ, E, Z

$$\Sigma = \frac{1}{A}\int_{\partial\mathcal{E}} \mathbf{x}\otimes\mathbf{t}ds$$

$$\mathbf{E} = \frac{1}{A}\int_{\partial\mathcal{E}} sym(\mathbf{u}\otimes\mathbf{n})ds$$



Periodic RVE

Micro – costitutive equations

Brick units $\sigma_b \leftrightarrow \varepsilon_b, \zeta_b$

Mortar $\sigma_m \leftrightarrow \varepsilon_m, \zeta_m$

Interface $\sigma_i \leftrightarrow \varepsilon_i, \zeta_i$

ζ internal variables

Macro – costitutive equations

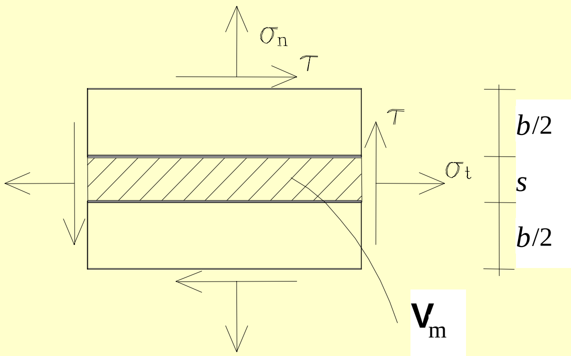
$\Sigma \leftrightarrow E, Z$

Z internal variables

4. Continuum damage-friction model

Layered micro-model

(Gambarotta e Lagomarsino, 1997)



RVE

$$\varepsilon = \mathbf{K}_M \sigma + \eta_m \varepsilon^*_m + \eta_b \varepsilon^*_b$$

$$\varepsilon = \{\varepsilon_1\ \varepsilon_2\ \gamma\}^t$$

$$\sigma = \{\sigma_1\ \sigma_2\ \tau\}^t$$

Mean stress

$$\varepsilon^*_b = \{0\ \varepsilon_b\ \gamma_b\}^t$$

Brick unit

$$\varepsilon^*_m = \{0\ \varepsilon_m\ \gamma_m\}^t$$

interface

Interface

$$\begin{cases} \varepsilon_m = c_{mn}\,\alpha_m\ H(\sigma_2)\,\sigma_2 \\ \gamma_m = c_{mt}\,\alpha_m\,(\tau - f) \end{cases}$$

Internal variables: α_m damage & f interface friction

Conjugate variables ⇨ $Y_m = \frac{1}{2} c_{mn} H(\sigma_2)\,\sigma_2^2 + \frac{1}{2} c_{mt}\ (\tau - f)^2 \quad , \quad \gamma_m$

Brick unit

$$\begin{cases} \varepsilon_b = c_{bn}\,\alpha_b\,H(-\sigma_2)\,\sigma_2 \\ \gamma_b = c_{bt}\,\alpha_b\,\tau \end{cases}$$

Internal variavle: α_b danno nel mattone

Conjugate variable ⇨ $Y_b = \frac{1}{2} c_{bn}\ H(-\sigma_2)\,\sigma_2^2 + \frac{1}{2} c_{bt}\ \tau^2$

Limit conditions:

- **Damage**

$$\phi_{dm} = Y_m - R_m(\alpha_m) \le 0$$

$$\phi_{db} = Y_b - R_b(\alpha_b) \le 0$$

- **Friction**

$$\phi_s = |f| + \mu\sigma_2 \le 0$$

sliding

$$\dot{\gamma}_m = \nu\dot{\lambda} \quad , \quad \dot{\lambda} \ge 0$$

$\nu = f/|f|$

4. Continuum damage-friction model

Layered micro-model

(Gambarotta e Lagomarsino, 1997)

Evolution of the internal variables

$\sigma_2 \geq 0$

Opened interface

$$\phi_{dm} = \tfrac{1}{2} c_{mn}\sigma_2^2 + \tfrac{1}{2} c_{mt}\tau^2 - R_m(\alpha_m) \leq 0$$

$$\phi_{db} = \tfrac{1}{2} c_{bt}\tau^2 - R_b(\alpha_b) \leq 0$$

$$\begin{Bmatrix} \dot\phi_{dm} \\ \dot\phi_{db} \end{Bmatrix} = -\begin{bmatrix} R'_m & 0 \\ 0 & R'_b \end{bmatrix} \begin{Bmatrix} \dot\alpha_m \\ \dot\alpha_b \end{Bmatrix} + \begin{Bmatrix} c_{mn}\sigma_2\dot\sigma_2 + c_{mt}\tau\dot\tau \\ c_{bt}\tau\dot\tau \end{Bmatrix} \leq \mathbf{0}$$

$$\{\dot\phi_{dm} \quad \dot\phi_{db}\}\{\dot\alpha_m \quad \dot\alpha_b\}^t = \mathbf{0} \qquad \{\dot\alpha_m \quad \dot\alpha_b\}^t \geq \mathbf{0}$$

$\sigma_2 < 0$

Closed interface

$$\phi_{dm} = \tfrac{1}{2}\frac{\gamma_m^2}{c_{mt}\alpha_m^2} - R_m(\alpha_m) \leq 0$$

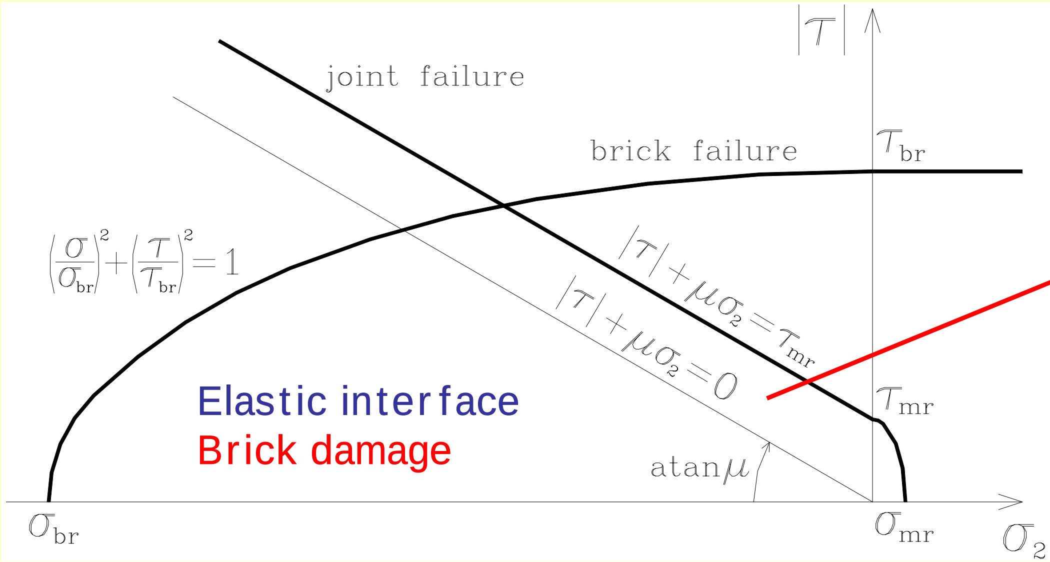
$$\phi_s = \left|\tau - \frac{\gamma_m}{c_{mt}\alpha_m}\right| + \mu\sigma_2 \leq 0$$

$$\phi_{db} = \tfrac{1}{2} c_{bn}\sigma_2^2 + \tfrac{1}{2} c_{bt}\tau^2 - R_b(\alpha_b) \leq 0$$

$$\begin{Bmatrix} \dot\phi_{dm} \\ \dot\phi_s \\ \dot\phi_{db} \end{Bmatrix} = \begin{bmatrix} -\frac{\gamma_m^2}{c_{mt}\alpha_m^3} - R'_m & \frac{\nu\gamma_m}{c_{mt}\alpha_m^2} & 0 \\ \frac{\nu\gamma_m}{c_{mt}\alpha_m^2} & \frac{-1}{c_{mt}\alpha_m} & 0 \\ 0 & 0 & R'_b \end{bmatrix} \begin{Bmatrix} \dot\alpha_m \\ \dot\lambda \\ \dot\alpha_b \end{Bmatrix} + \begin{Bmatrix} 0 \\ \nu\dot\tau + \mu\dot\sigma_2 \\ c_{bn}\sigma_2\dot\sigma_2 + c_{bt}\tau\dot\tau \end{Bmatrix} \leq \mathbf{0}$$

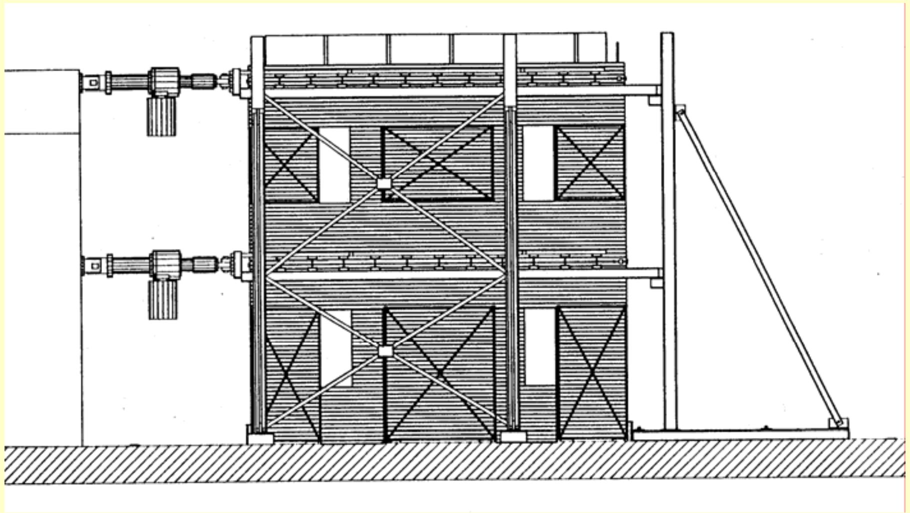
$$\{\dot\phi_{dm} \quad \dot\phi_s \quad \dot\phi_{db}\}\{\dot\alpha_m \quad \dot\lambda \quad \dot\alpha_b\}^t = \mathbf{0} \qquad \{\dot\alpha_m \quad \dot\lambda \quad \dot\alpha_b\}^t \geq \mathbf{0}$$

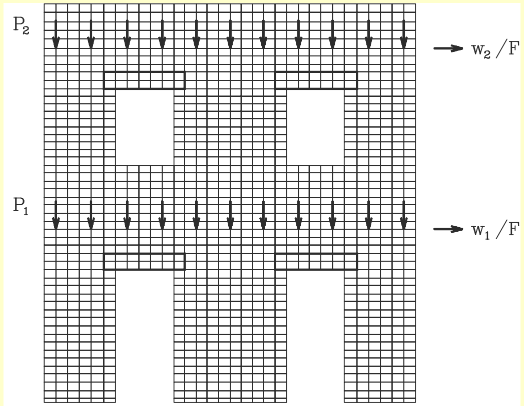
Limit states



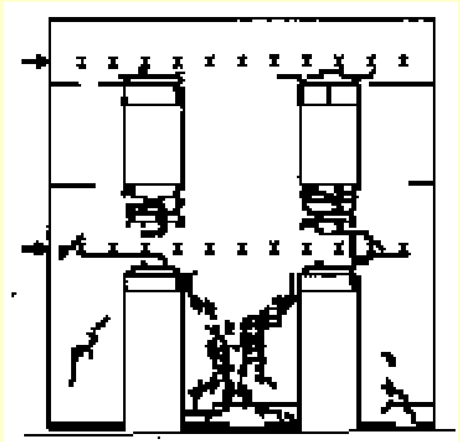
Damage in the interface and brick units

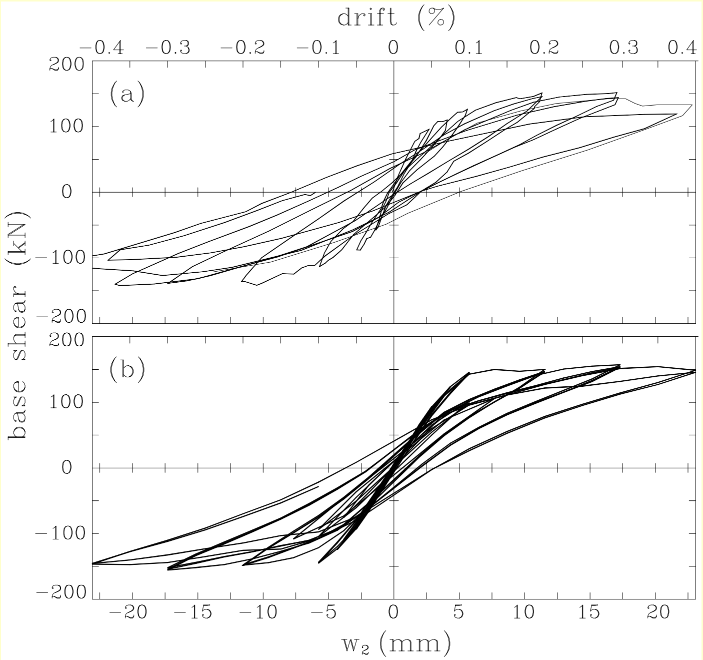
4. Large shear walls – simulation of experimental results





Crack pattern
(Magenes *et al*)



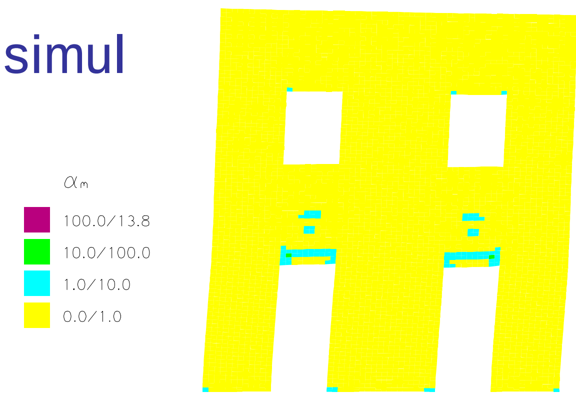


Cyclic response of the *door wall*: a) experimental; b) numerical simulation.

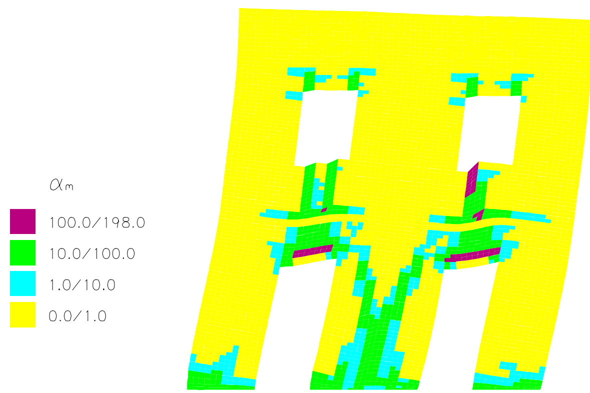
exp

DRIFT 0.1%

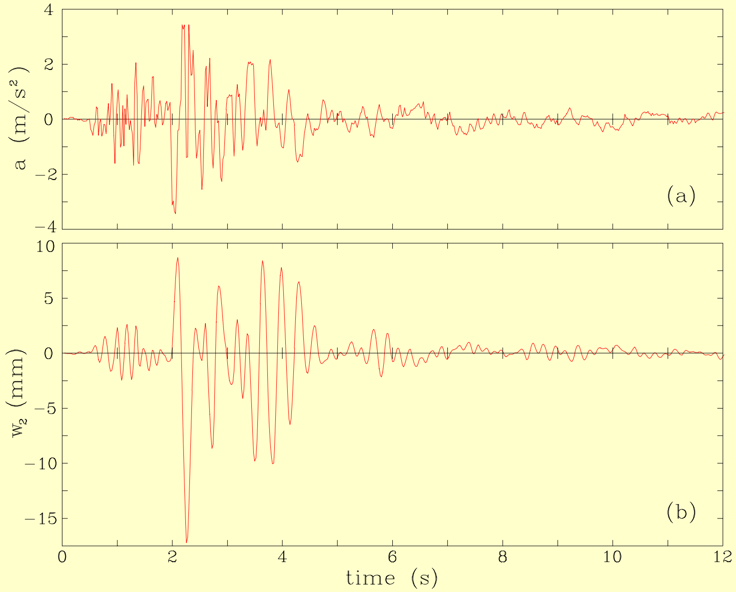
simul



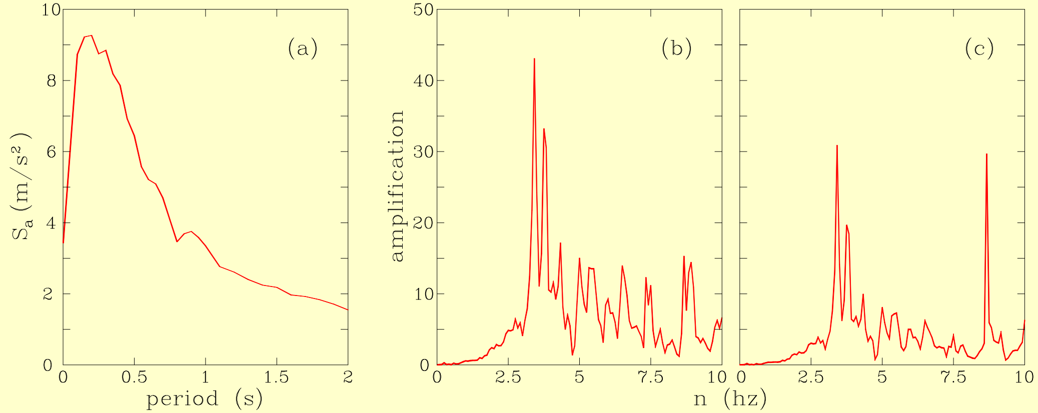
DRIFT 0.3%



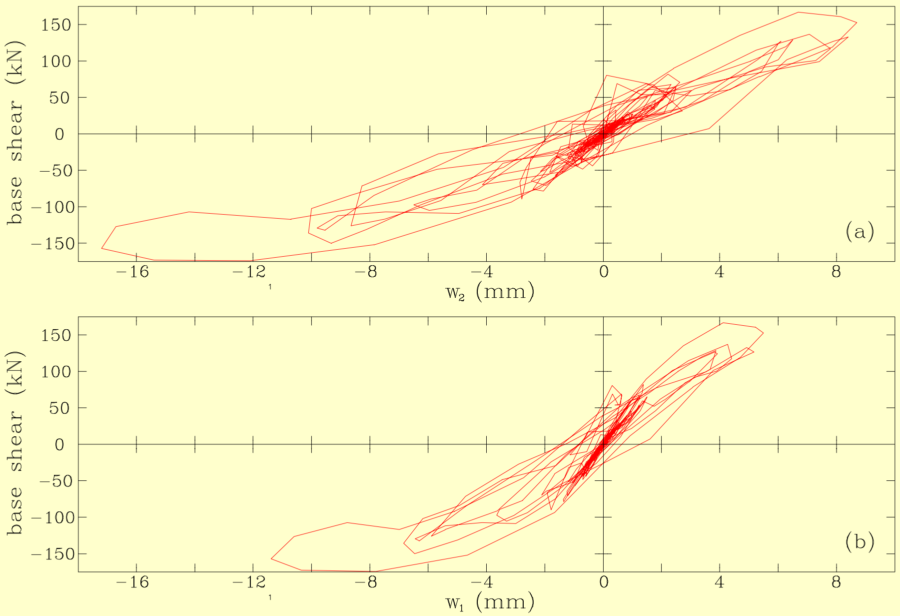
4. Large shear walls – dynamic response to ground motion



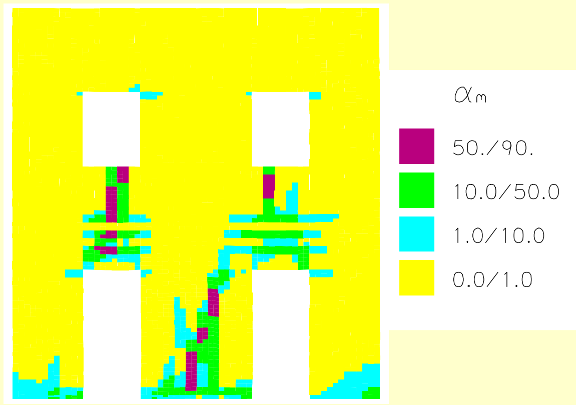
(a) Acceleration time history applied at the base of the wall.
(b) Displacement time history on the second floor.

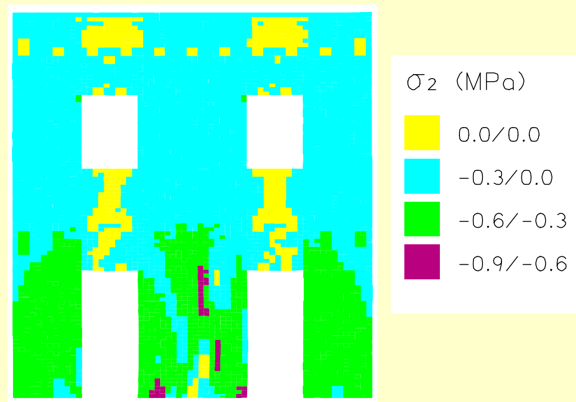


(a) Acceleration response spectrum of the input base motion. Amplification function with respect to the base of the wall: (b) first floor displacement, (c) second floor displacement.



Cyclic response of the large scale wall: (a) second floor; (b) first floor.

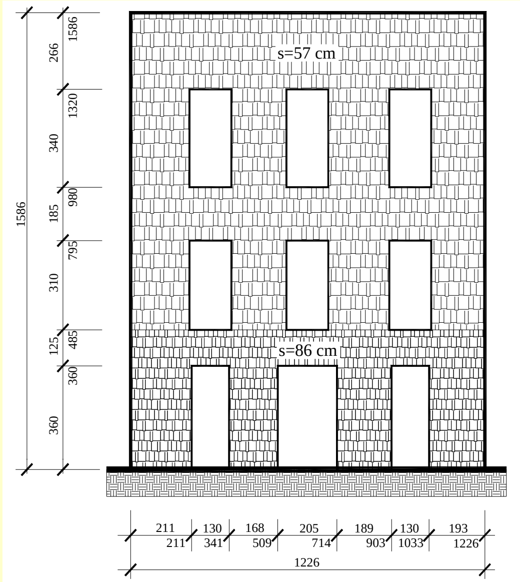


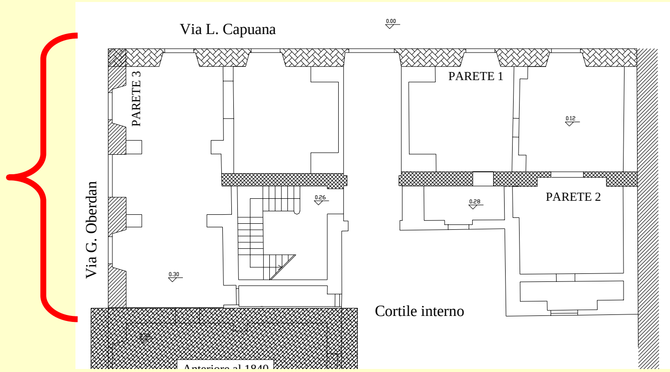


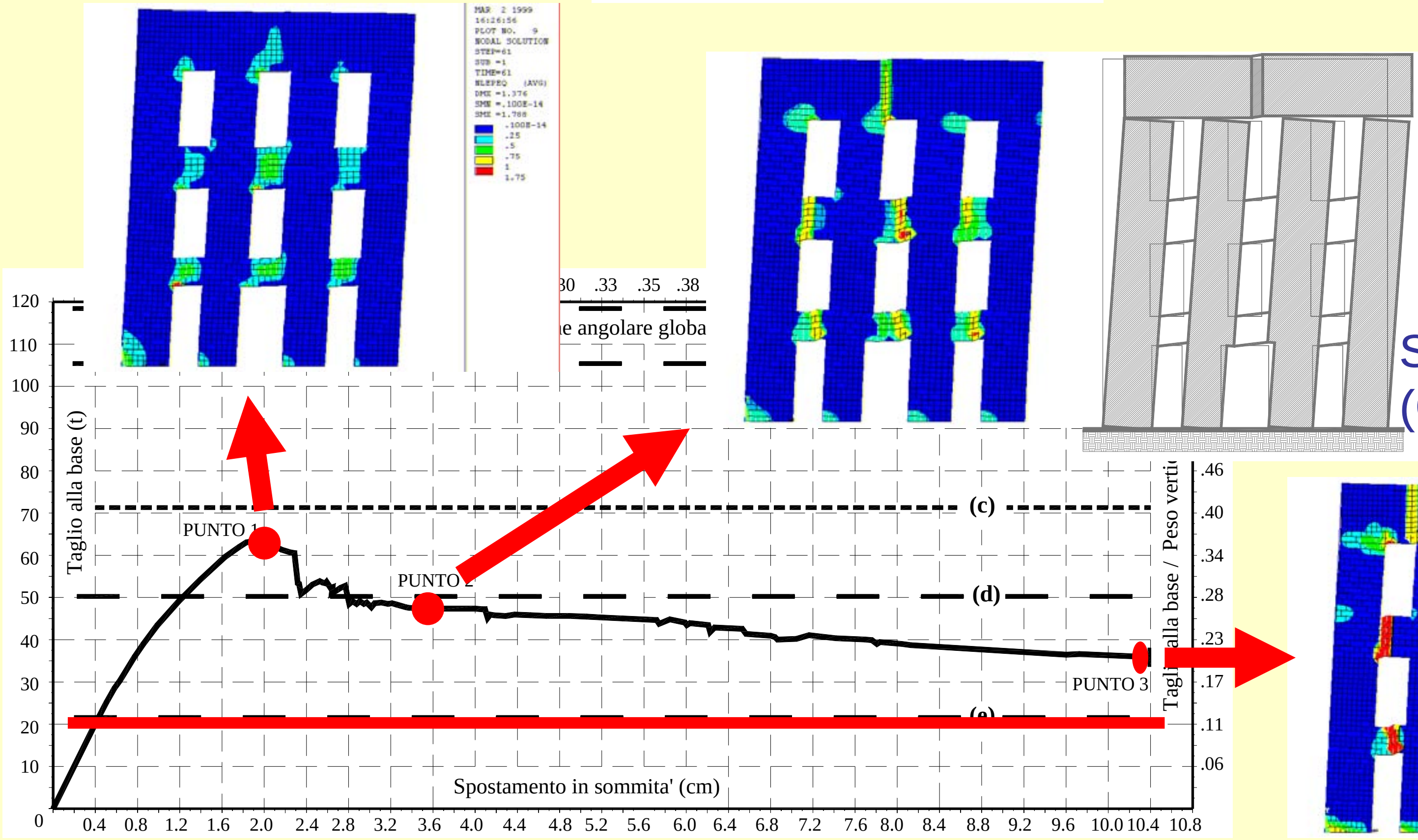
4. Large shear walls – response to horizontal forces

Brencich etal, 2001
Masonry building in Catania
GNDT

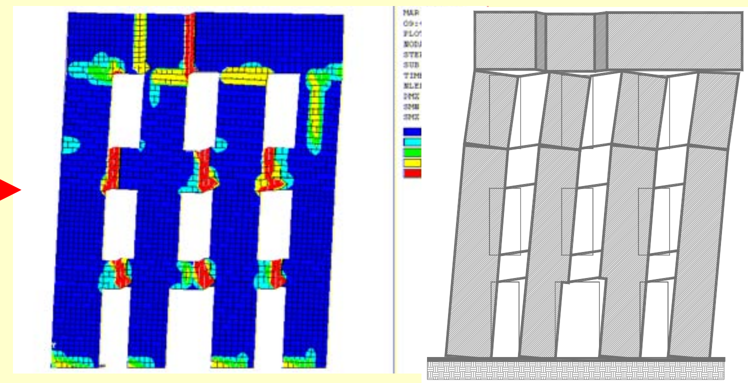
Horizontal forces superimposed on vertical dead loads



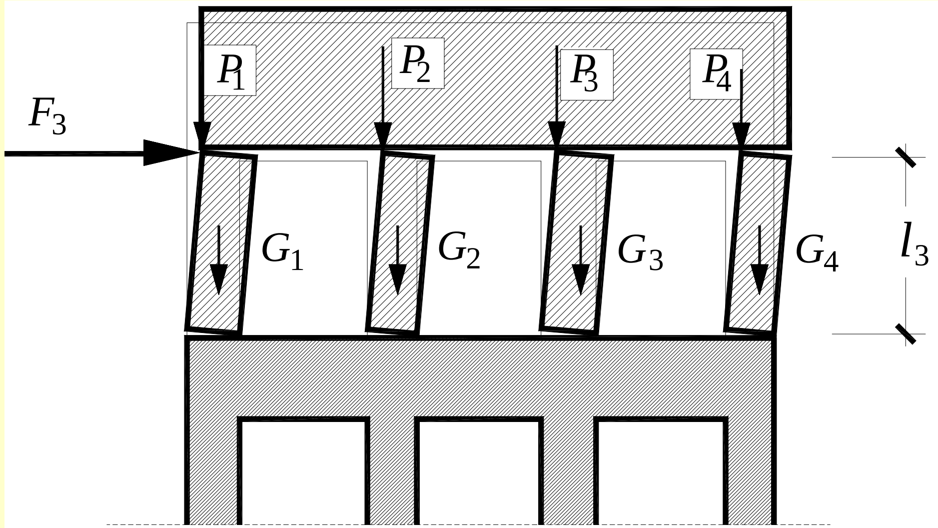


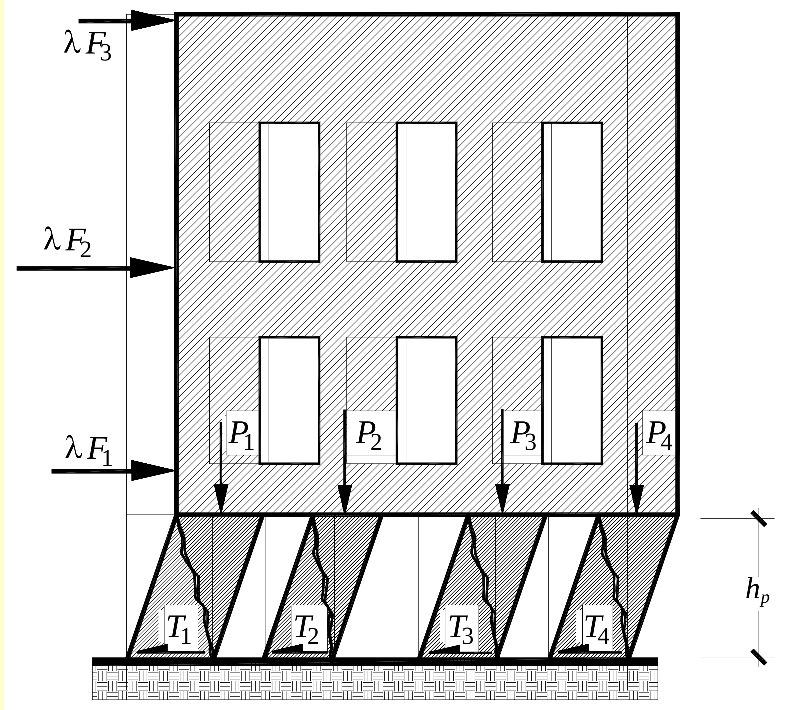


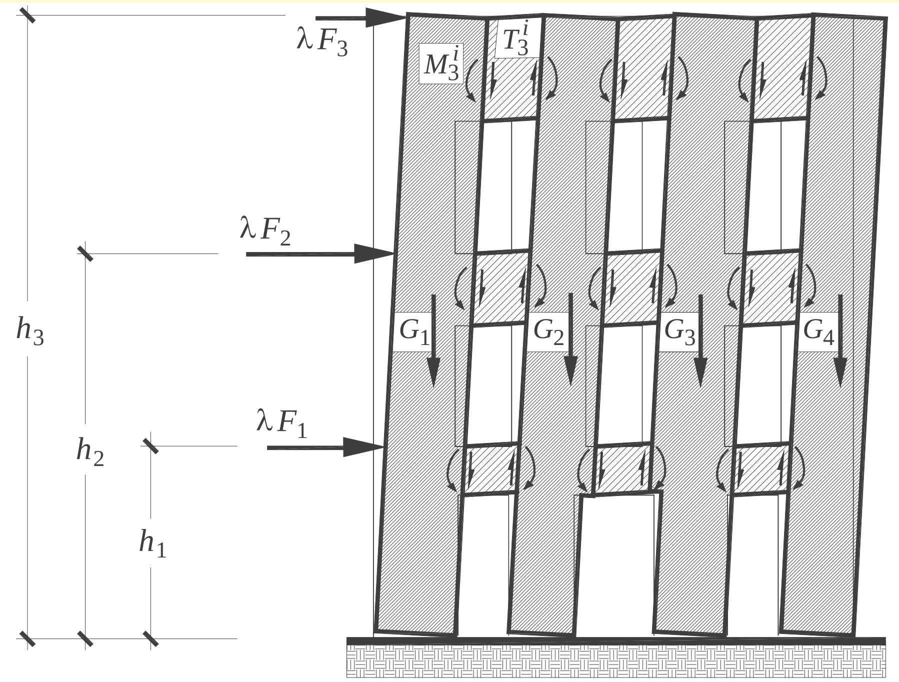
Simplified collapse mechanism (Como e Grimaldi)

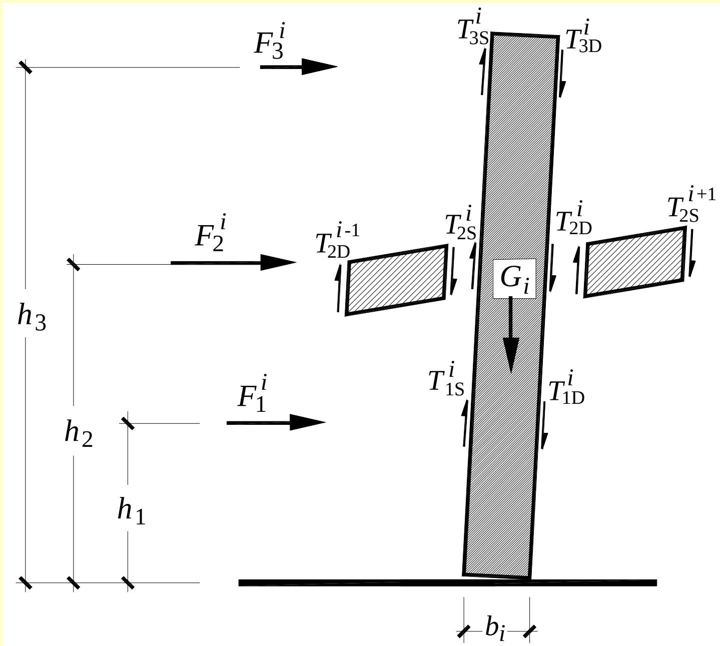


4. Large shear walls – simplified approaches

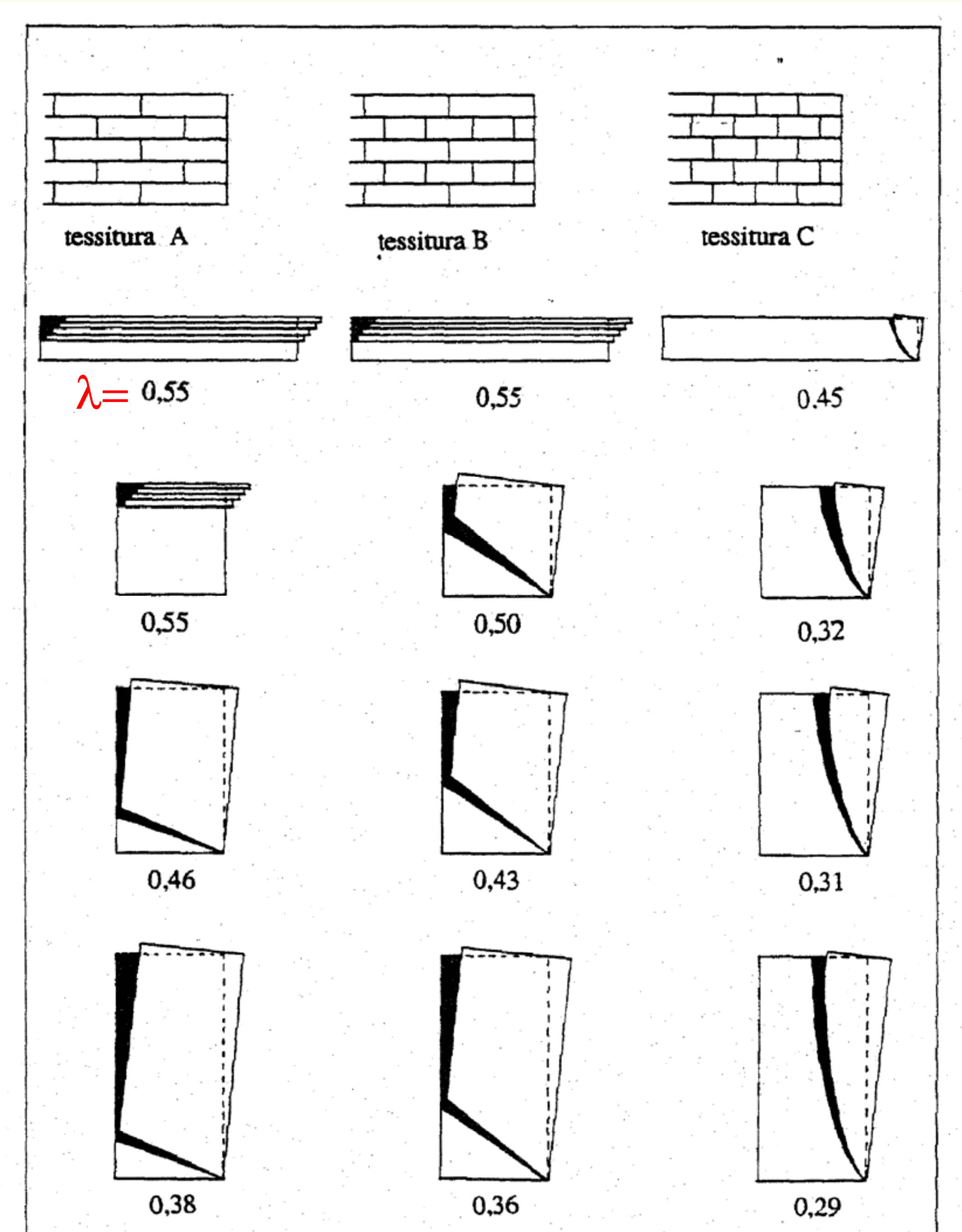








4. Shear walls – influence of the unit shape and bond pattern



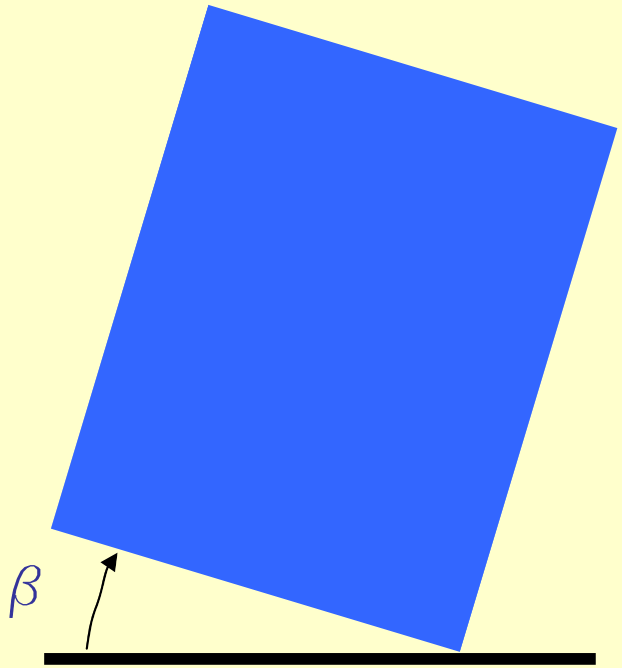
$\lambda = \tan\beta$

Experimental results
Dry block masonry
(Giuffrè *et al.*)

Collapse mechanisms
and limit slope angle β

For varying:

- Bond pattern
- Wall slenderness



4. Shear walls – continuum models
homogenization of elastic brick and damaging interfaces

Luciano e Sacco, 1997

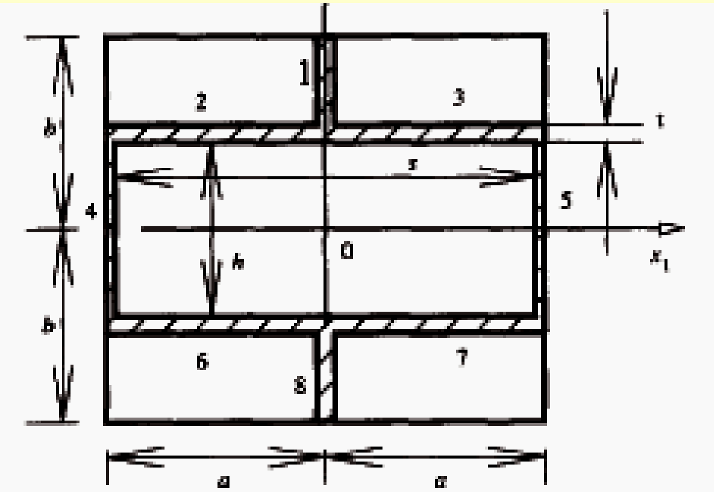


Fig. 1. Repetitive unit cell for the regular masonry material.

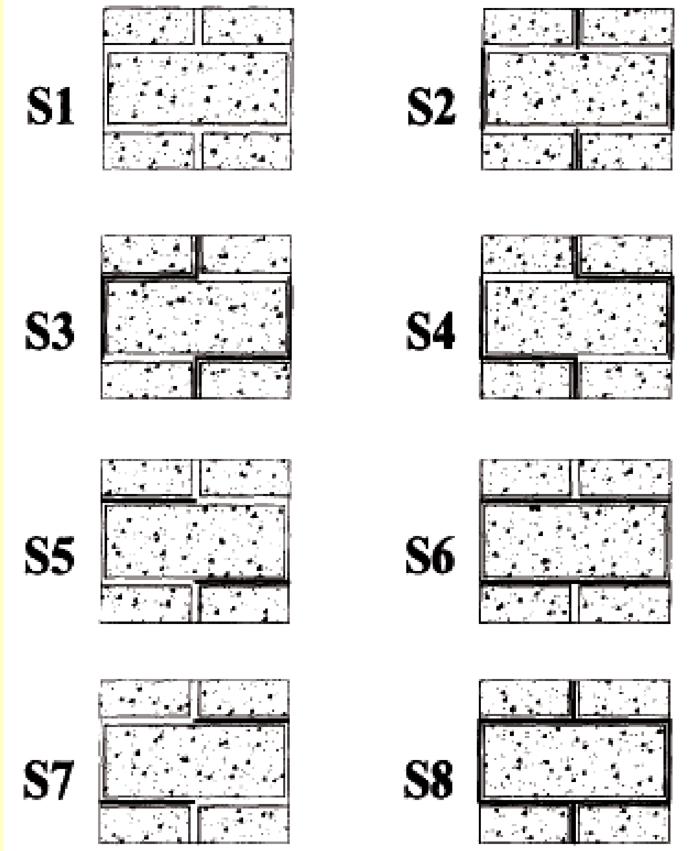
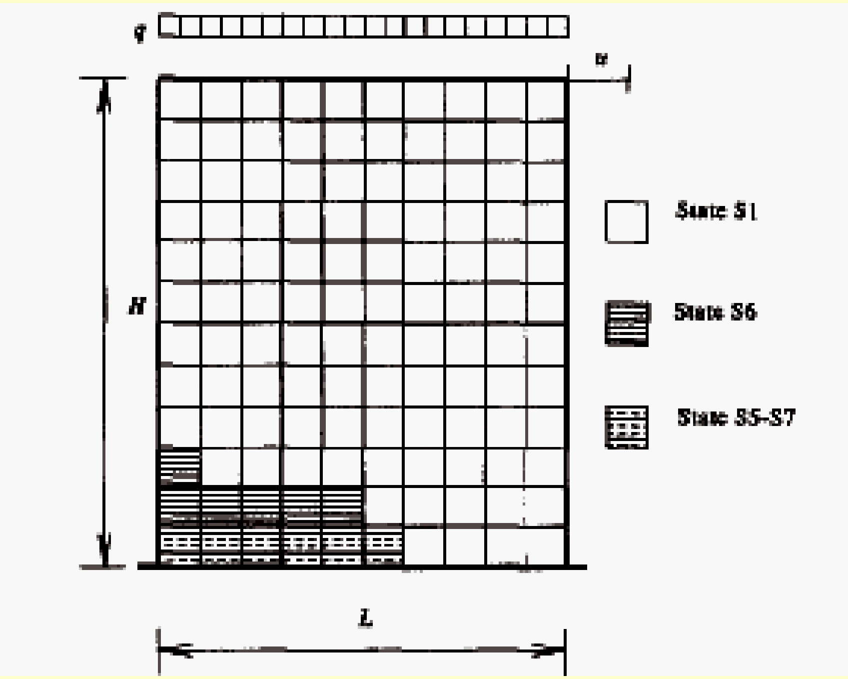
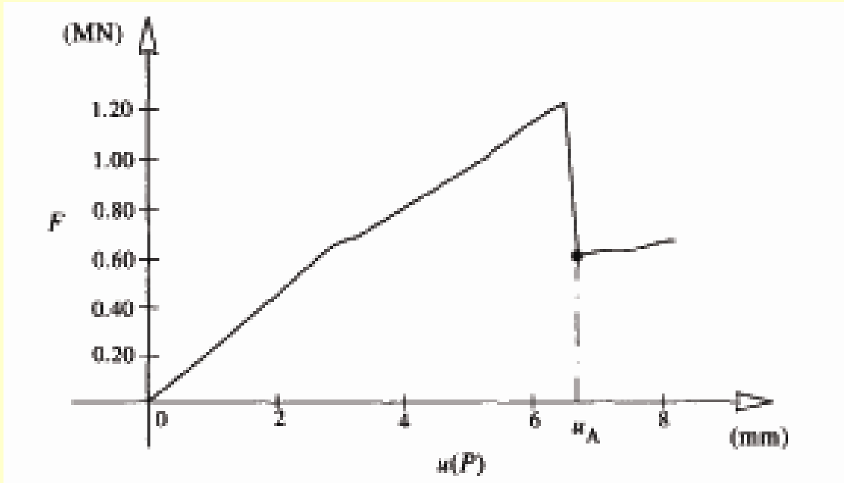
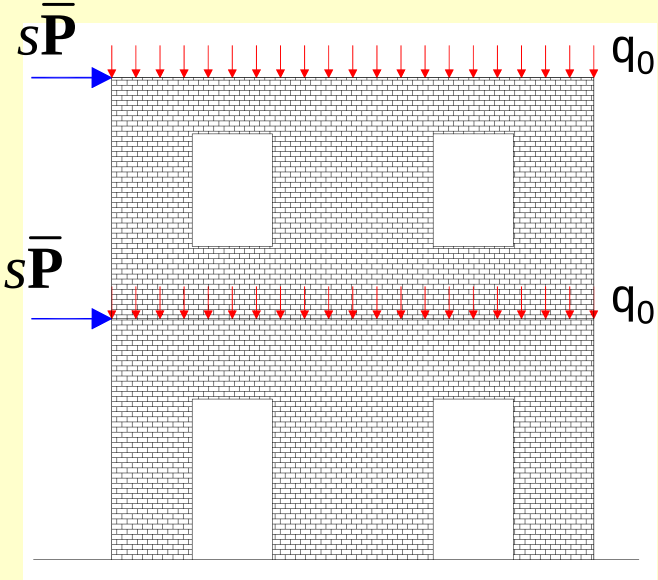


Fig. 2. Possible damaged states of old masonry material.





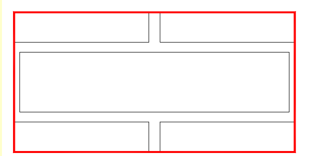
4. Shear walls – Multiscale limit analysis – influence of the bond pattern



Admissible macro-stress fields (Suquet, 1983)

Macro Σ, E

micro σ, ε



$$S^{hom} = \left\{ \Sigma \middle| \begin{cases} \Sigma = \dfrac{1}{A}\displaystyle\int_{\partial\mathcal{E}} \mathbf{x}\otimes\mathbf{t}\,ds \\ div\boldsymbol{\sigma} = 0 \quad \forall\mathbf{x}\in\mathcal{E} \\ \|\boldsymbol{\sigma}\|\mathbf{n} = \mathbf{0} \quad \forall\mathbf{x}\in\mathcal{J} \\ \boldsymbol{\sigma}\mathbf{n} \ \text{anti-periodico su} \ \partial\mathcal{E} \\ \boldsymbol{\sigma}(\mathbf{x})\in S^{\alpha} \quad \forall\mathbf{x}\in\mathcal{E}^{\alpha},\ \alpha\text{=b, m} \\ \boldsymbol{\sigma}(\mathbf{x})\in S^{i} \quad \forall\mathbf{x}\in\mathcal{J} \end{cases} \right\}$$

Dual kinematic definition of S^{hom}

- Alpa Monetto, 1994, Alpa, Gambarotta et al 1996
  De Buhan, De Felice, 1997
  S^b, S^m unbounded, S^i Coulomb

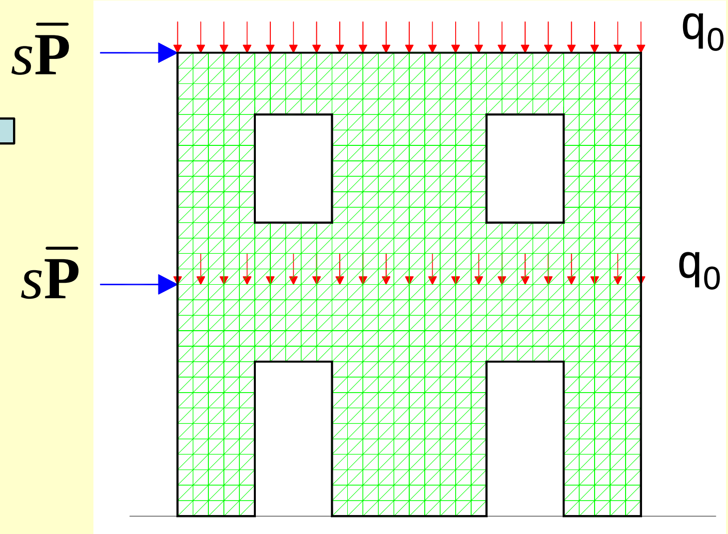
- Milani et al, 2005
  S^b, S^m Mohr-Coulomb – cut-off
  S^i not active

Lower bound

$$\begin{cases} s_L = \max(s_s), \\ \mathbf{C}\Sigma_V = \mathbf{c}, \\ \mathbf{Q}\Sigma_V - s_s\bar{\mathbf{q}} = \mathbf{q}_0, \\ \mathbf{Y}^T\Sigma_V \le \mathbf{y}. \end{cases}$$

Upper bound

$$\begin{cases} s_U = \min(s_k) = \min\left(-\mathbf{q}_{0I}^{\mathrm{T}}\mathbf{a} + \mathbf{z}^{\mathrm{T}}\dot{\lambda}\right), \\ \mathbf{Ba} - \mathbf{Z}^{\mathrm{T}}\dot{\lambda} = \mathbf{0} \\ \mathbf{Aa} = \mathbf{0}, \\ \bar{\mathbf{q}}_I^{\mathrm{T}}\mathbf{a} = 1, \\ \dot{\lambda} \ge \mathbf{0}. \end{cases}$$

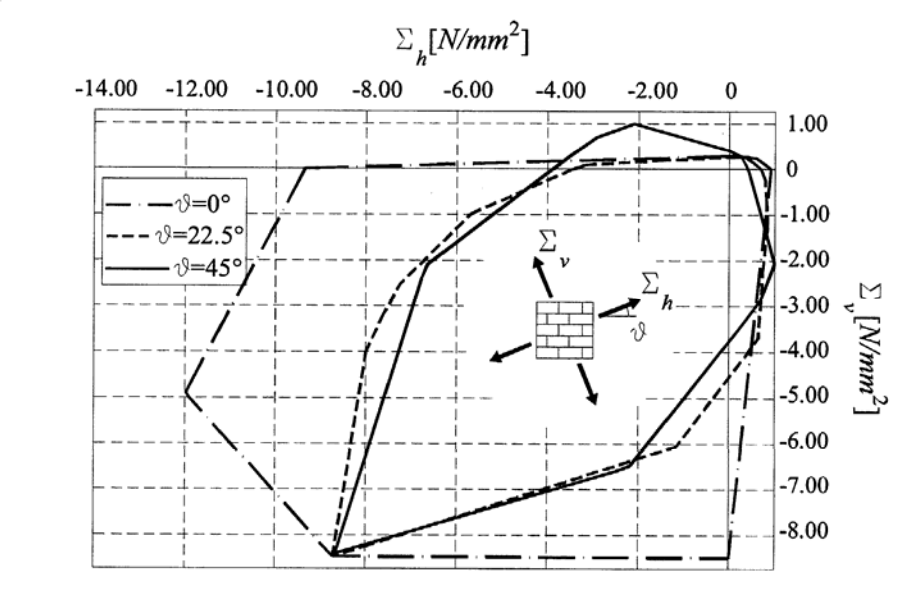


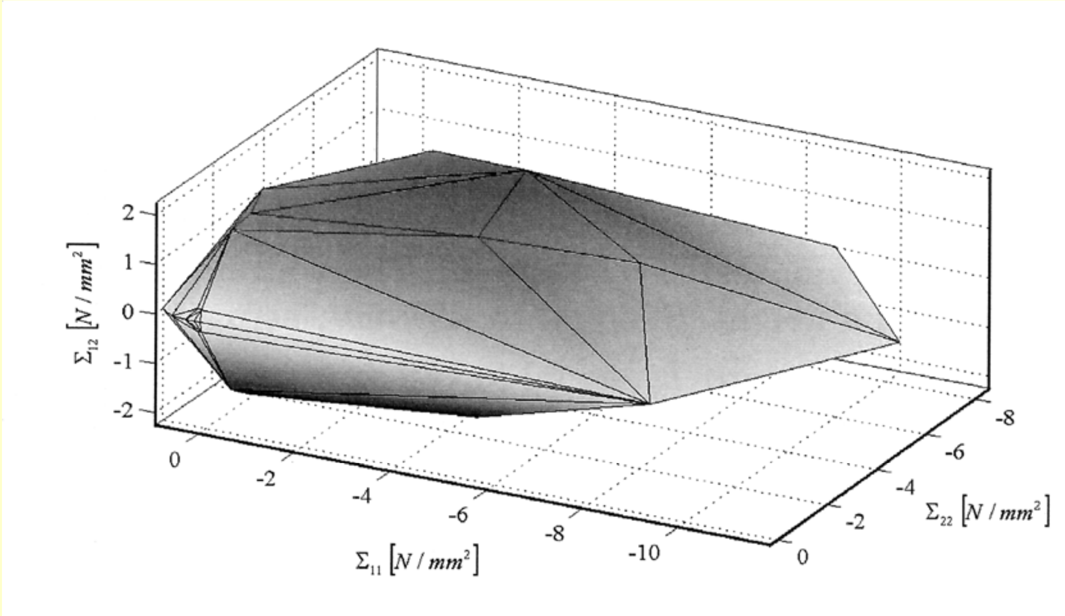
FE discretization

- Equilibrium model
- Compatible model

Anderheggen e Knöpfel, Sloan & coworkers, Pastor et al., Maier & coworkers.... ......

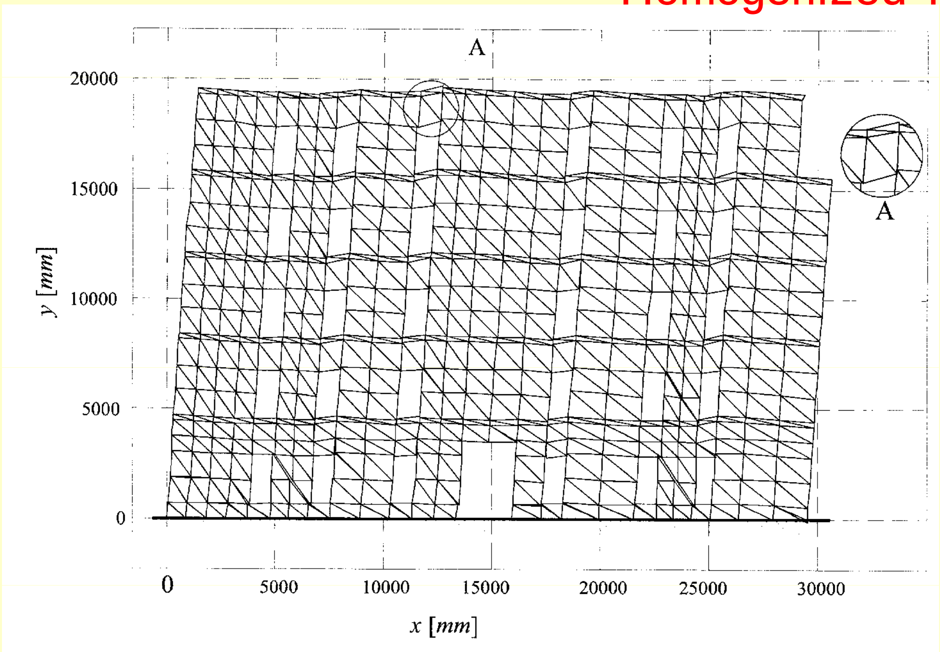
4. Shear walls – Multiscale limit analysis – influence of the bond pattern





Homogenized failure surface

Milani et al., 2005



Collapse mechanism (U.B.) Catania Building

Brencich et al, 2000

4. Shear walls

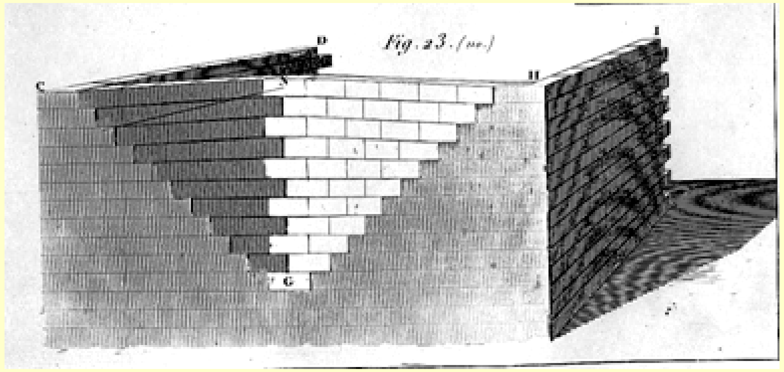
- In-plane model

  non-local continuum model able to take into account the scale effect unit size/structure/size, high gradients of the micro-stress field, regolarization of damage model

  Besdo, Műhlhaus, Rizzi, Trovalusci, Masiani, Salerno.....

  Trovalusci e Masiani, IJSS, 2005

- Out-of-plane models



Elastic models

Cecchi e Sab, 2002, 2004,

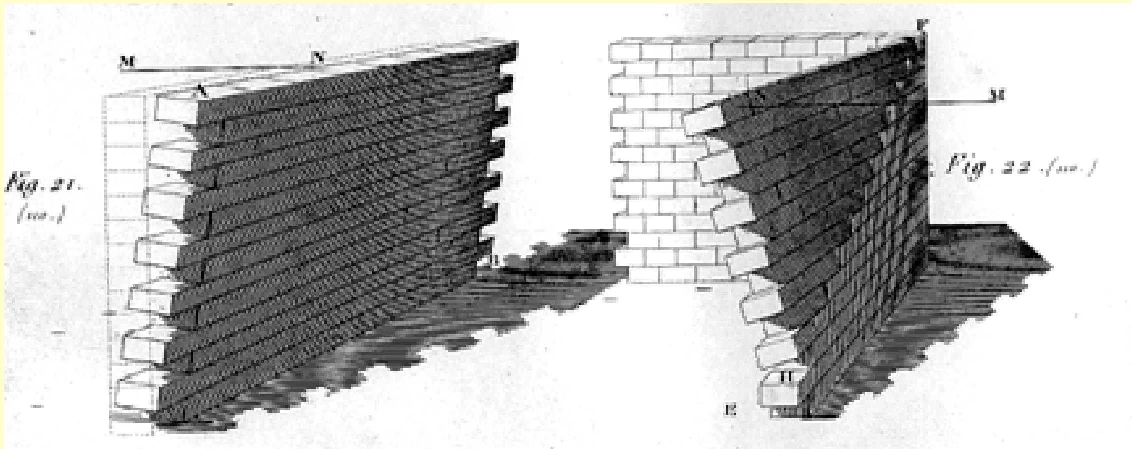
Limit analysis:

Discrete models:

Orduna e Lourenco, 2005

Continuum models:

Sab, 2003, Milani e Tralli, 2005



4. Shear walls

Cecchi et al., 2006

- Out-of-plane collapse - multiscale models

Dissipation Power

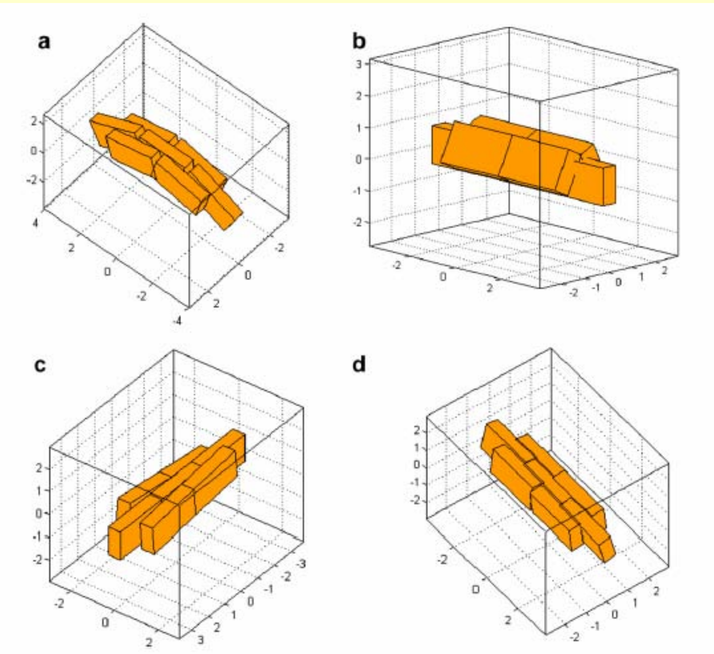
$$\pi = \overline{\mathbf{N}} \cdot \mathrm{sym}(\mathrm{grad}\,\bar{\mathbf{w}}) + \mathbf{T} \cdot (\mathrm{grad}\, w_3 + \boldsymbol{\Omega}\mathbf{e}_3) + \mathbf{M} \cdot \mathrm{sym}(\mathrm{grad}\boldsymbol{\Omega}\mathbf{e}_3).$$

Internal forces

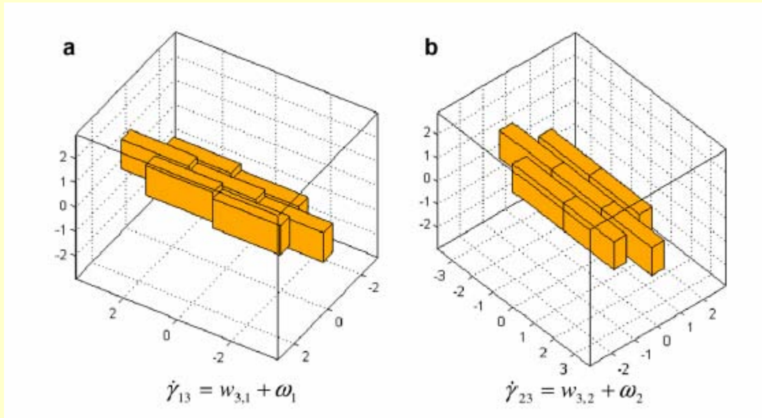
$$\overline{\mathbf{N}} = \frac{1}{2A}\sum_n \mathrm{sym}\,\bar{\mathbf{t}}_p \otimes (\mathbf{g}^b - \mathbf{g}^a)$$

$$\mathbf{T} = \frac{1}{2A}\sum_n \mathbf{t}_{3p}(\mathbf{g}^b - \mathbf{g}^a)$$

$$\mathbf{M} = \frac{1}{2A}\left[\sum_n \mathbf{t}_{3p}\mathrm{sym}[(\mathbf{p} - \mathbf{g}^a)\otimes(\mathbf{g}^a - \mathbf{x}) - (\mathbf{p} - \mathbf{g}^b)\otimes(\mathbf{g}^b - \mathbf{x})] + \sum_n \int_l \mathrm{sym}[d_{3p}\bar{\mathbf{t}}(\xi) - t_3(\xi)\bar{\mathbf{d}}_p] \otimes (\mathbf{g}^b - \mathbf{g}^a)\right]$$



Flexural & Torsional Mechanisms



Shearing Mechanisms

Elementary deformations of the Representative Volume Element

4. Shear walls

- Out-of-plane collapse - multiscale models

Cecchi et al., 2006

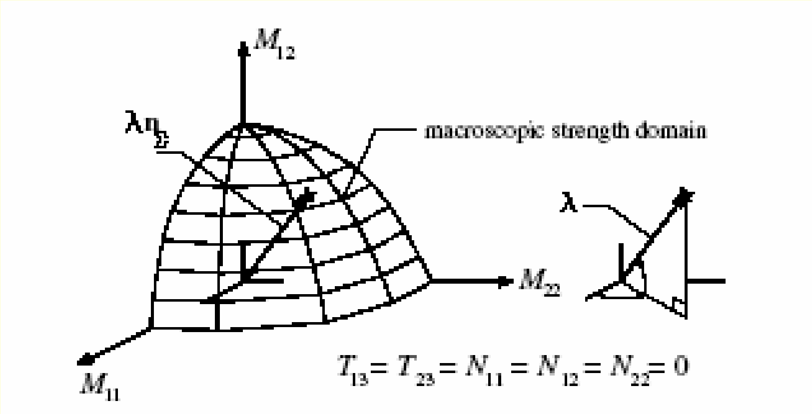
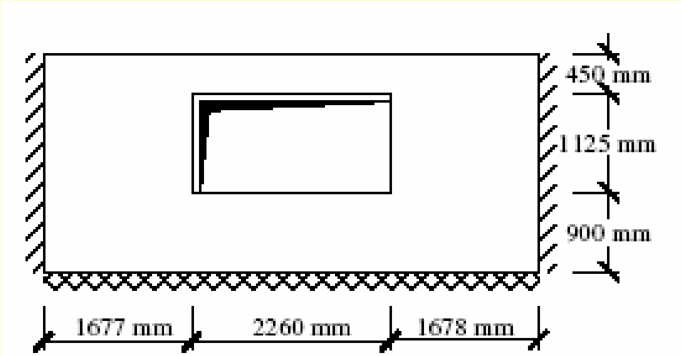
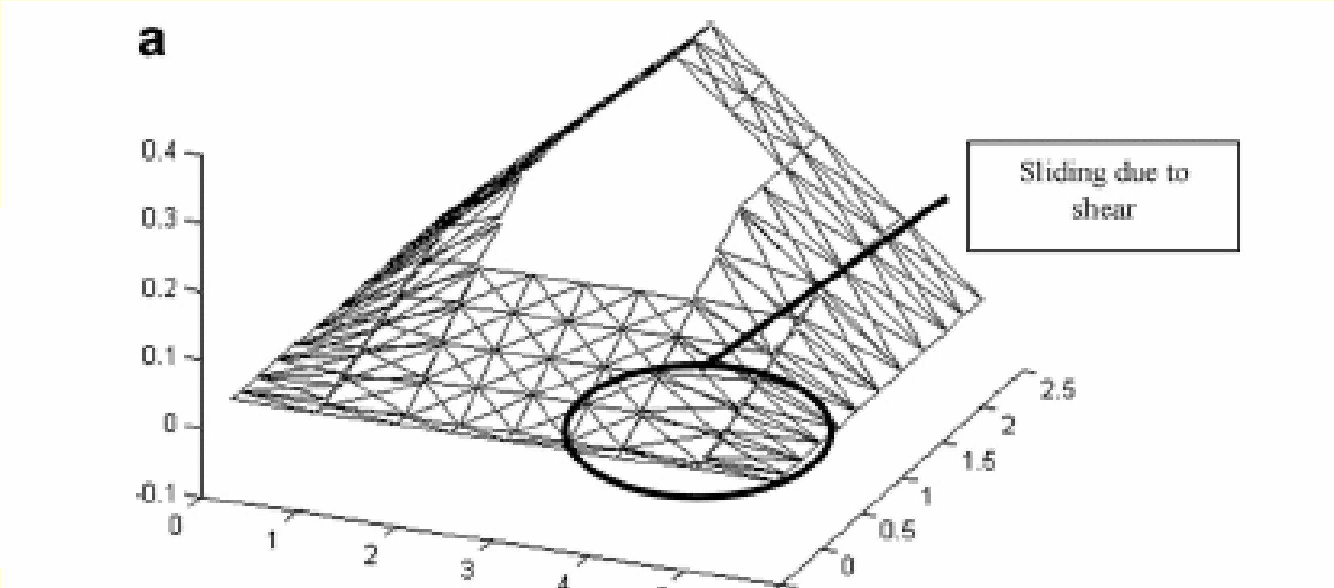


Fig. 8. Meaning of n_Σ in the special 3D case $\Sigma = (M_{11}, M_{12}, M_{22})$.

Out-of-plane Collapse

Perforated shear wall





Collapse Mechanism

5. Domes

S. Pietro Dome in Roma

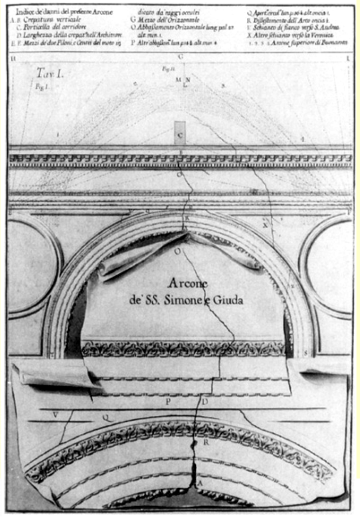
Michelangelo

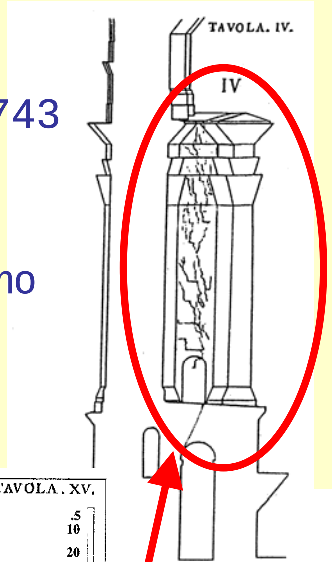
Della Porta e Fontana, 1590

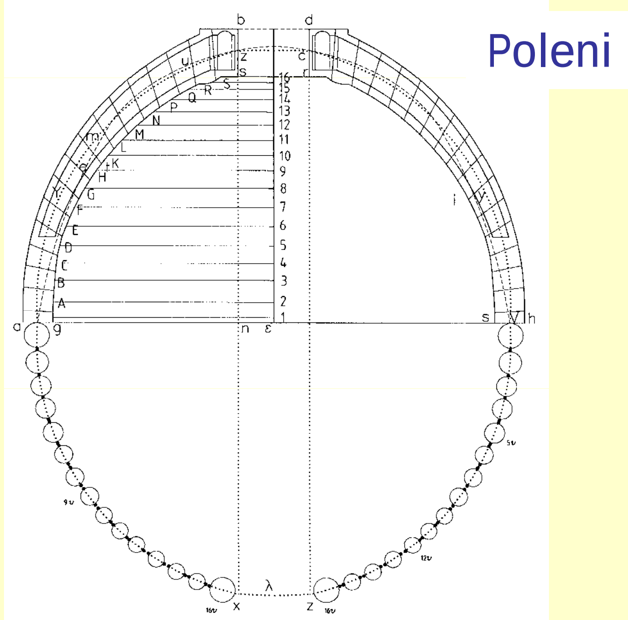
Boscovich, Le Seur, Jacquier, 1743

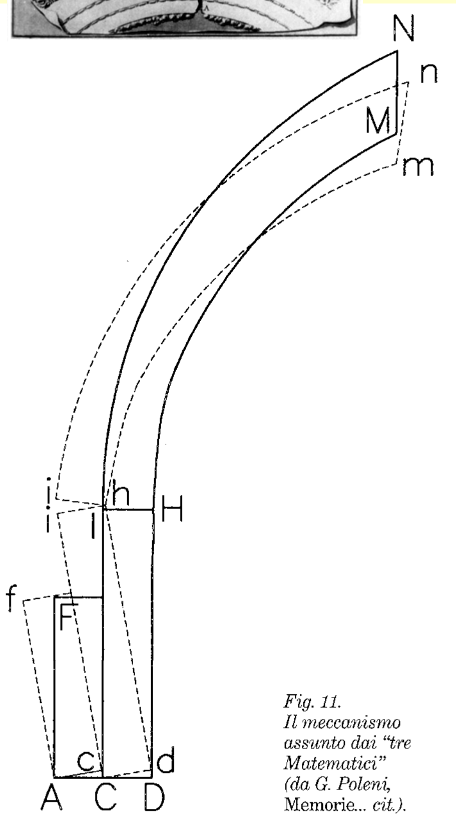
Poleni, 1748 – Vanvitelli

Burri, Beltrami, Di Stefano, Como

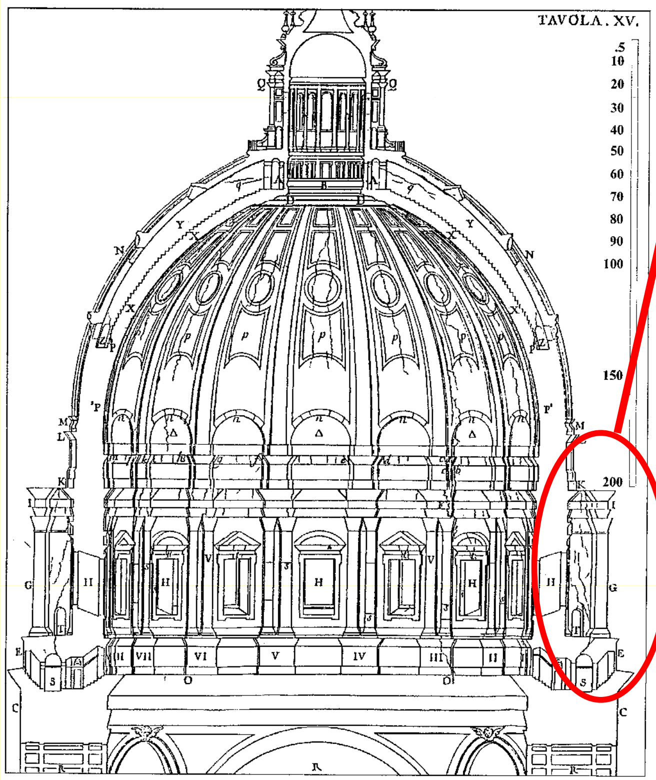


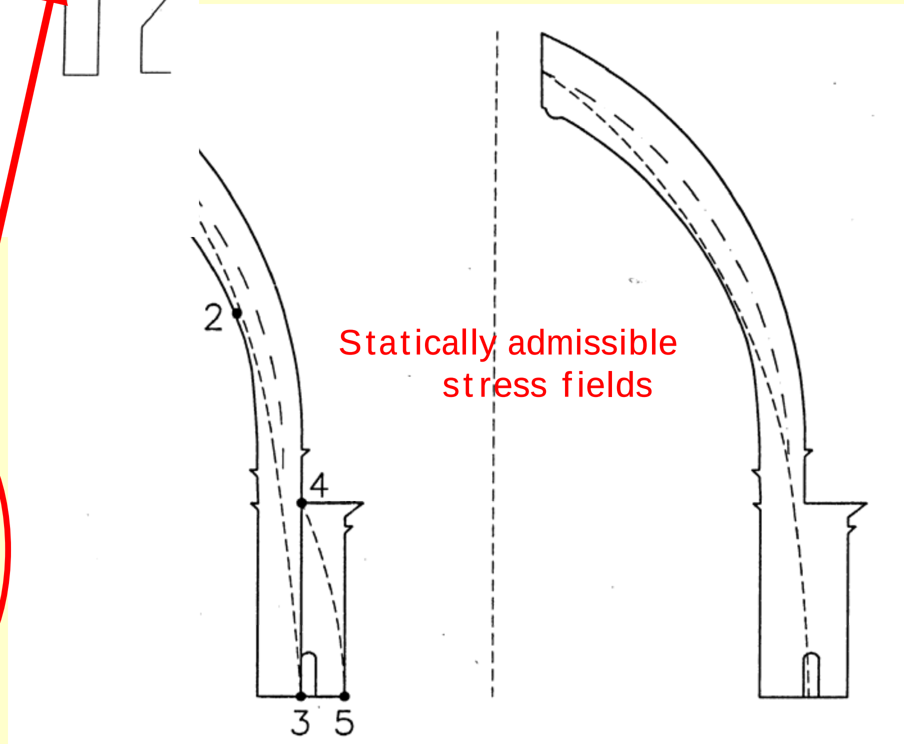






Collapse mechanism by the "Tre Mattematici"

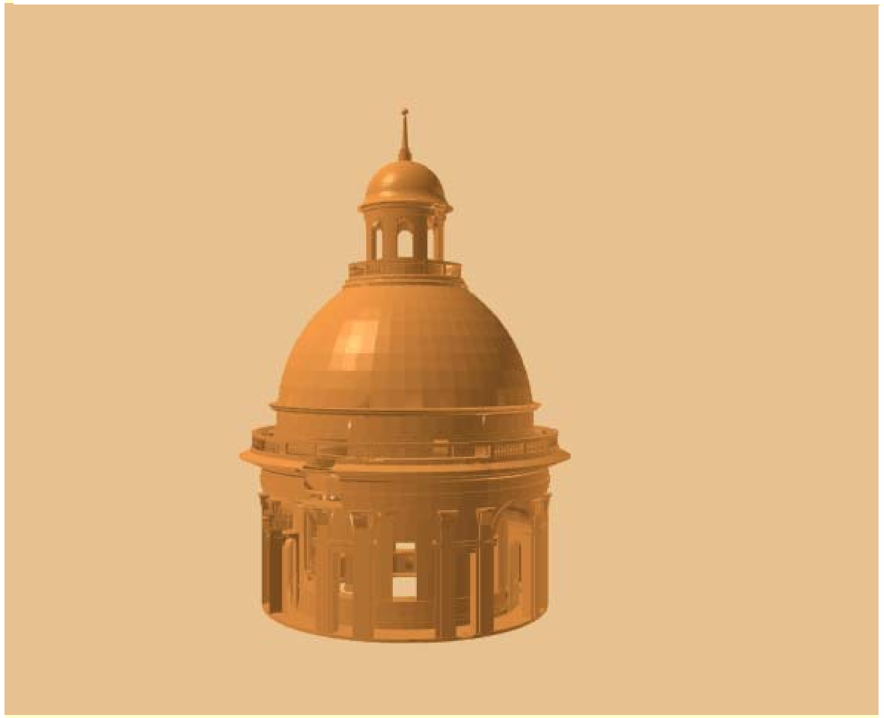


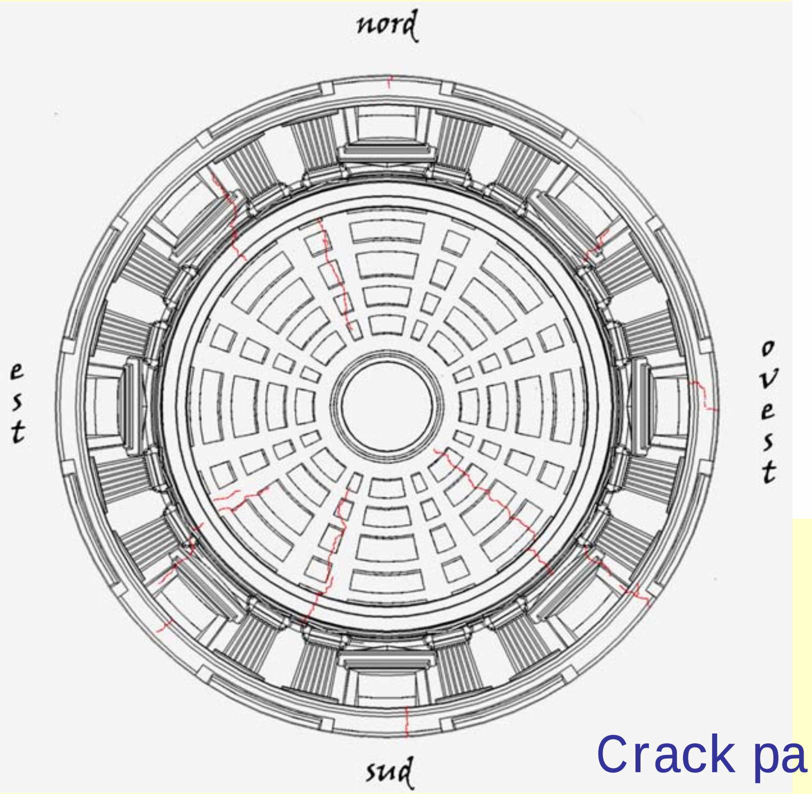


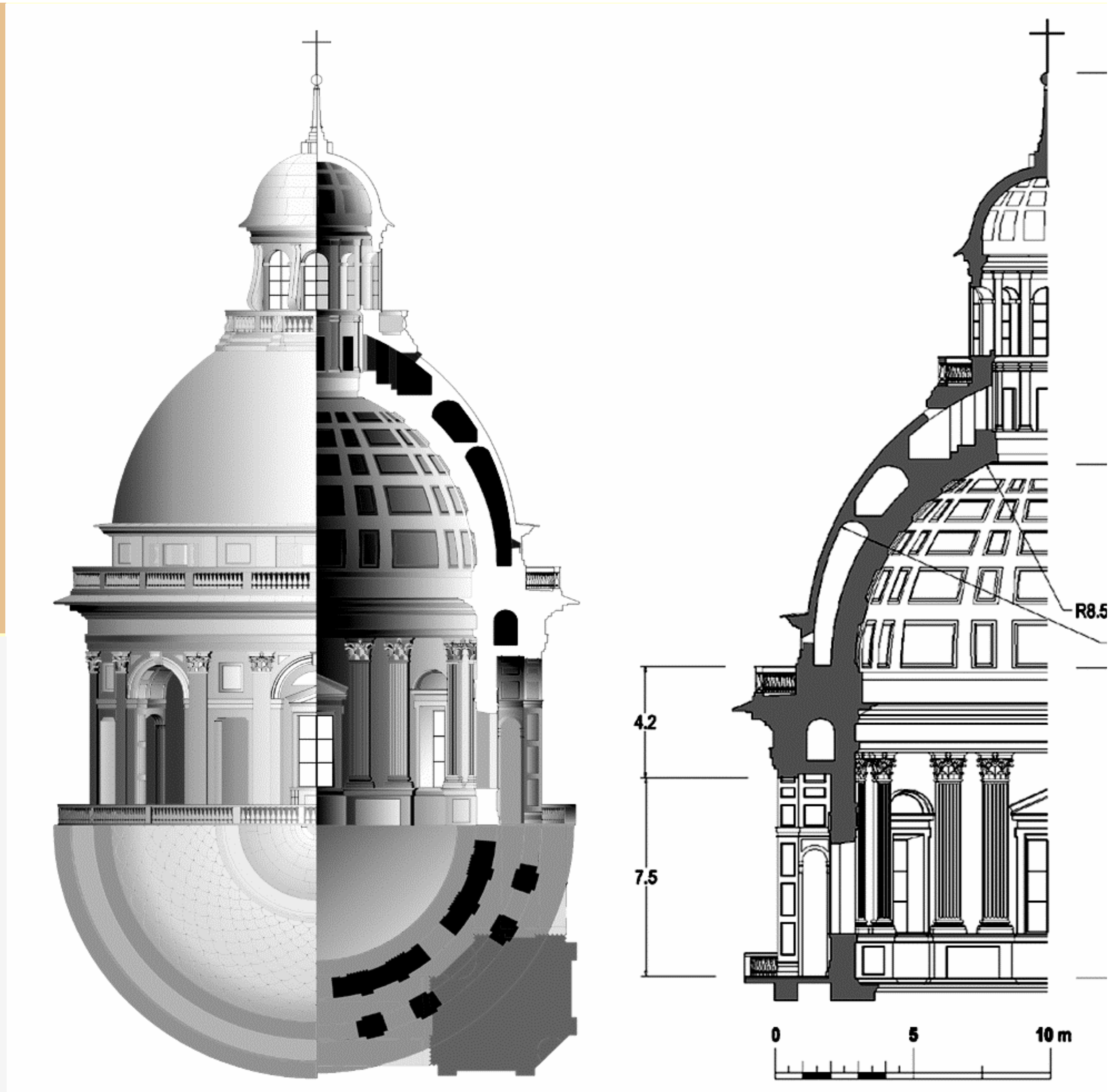
Least abutment thrust, Como

Elastic NTR solution Como

Dome-drum interaction: Basilica di Carignano in Genova (G. Alessi, 1540-1600)







Crack pattern in the inner dome (from below)

Basilica di Carignano: Safe theorem

Statically admiddible states

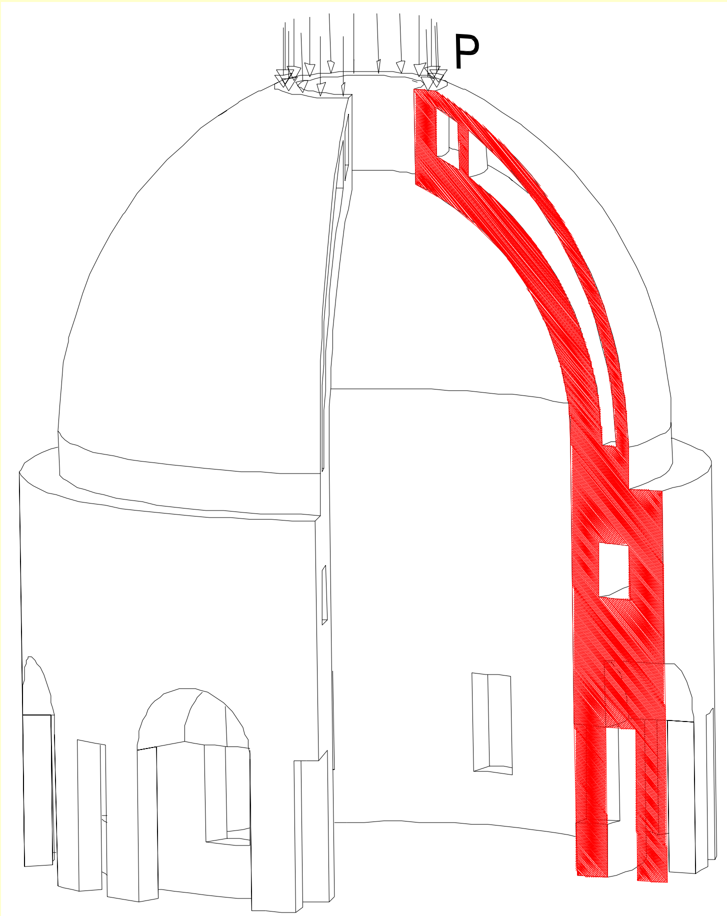
Hypotheses
- NTR material
- Infinite compressive strength
- No sliding failures admitted

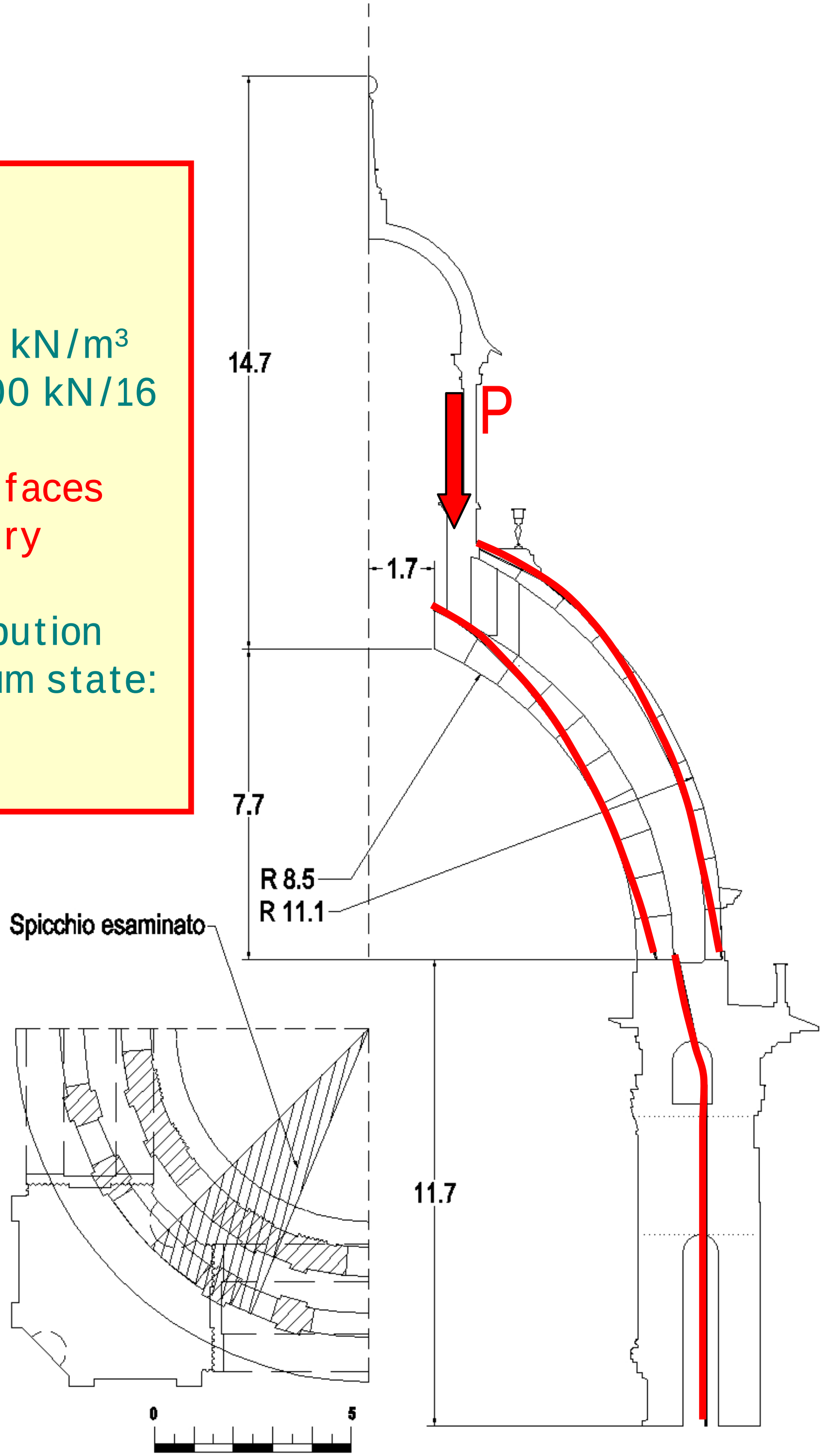
Equilibrium of a slice

Loads:
- masonry weight γ=17 kN/m^3
- lantern weight P=1200 kN/16

Search for thrust surfaces lying within the masonry

Lantern weight distribution for the safe equilibrium state:
85% inner shell
15% outer shell





Gambarotta et al., 2002

Upper Bound Theorem

If $\exists\ \dot{\mathbf{u}} \in KinAdm$ such that:

$$\dot{W} = \int_{\mathscr{B}^-} \mathbf{b} \bullet \dot{\mathbf{u}}^- dv + \int_{\mathscr{B}^+} \mathbf{b} \bullet \dot{\mathbf{u}}^+ dv = \dot{W}_a + \dot{W}_{res} \geq 0$$

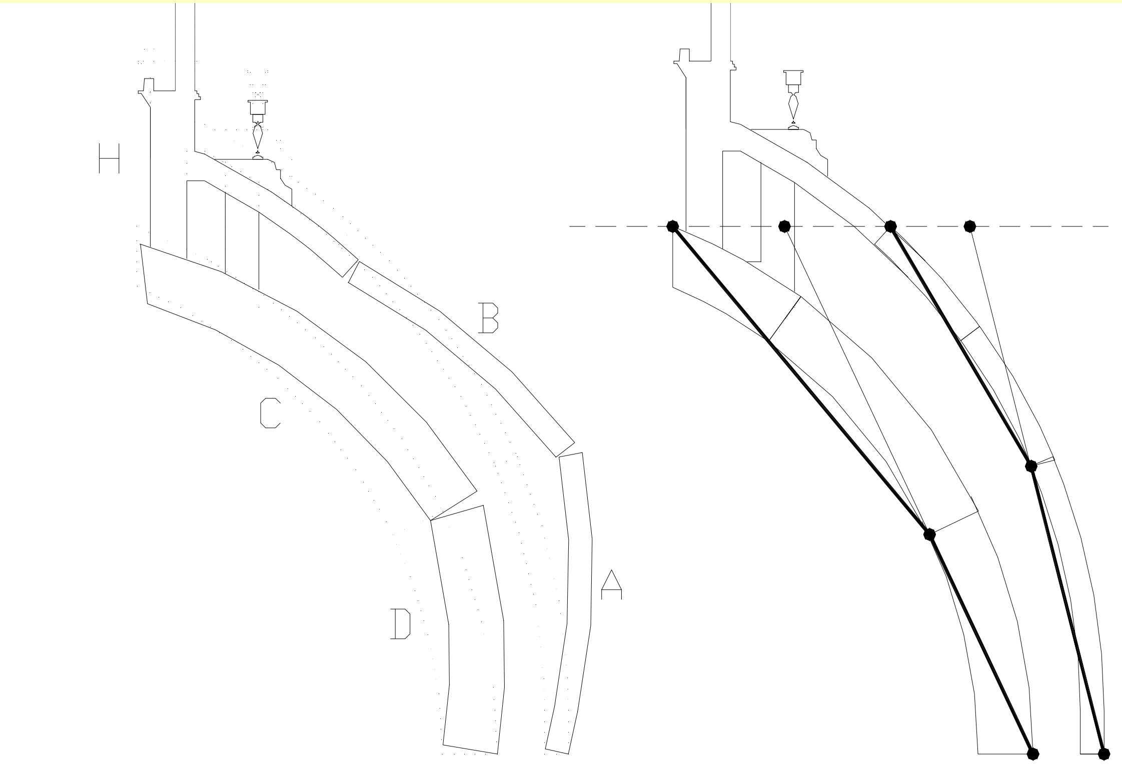
⇒ The structure will collapse

(Romano e Romano, Romano e Sacco, Como)

b - *unit volume weigth*

$\mathbf{u}^+$ - *upward velocity*

$\mathbf{u}^-$ - *downward velocity*

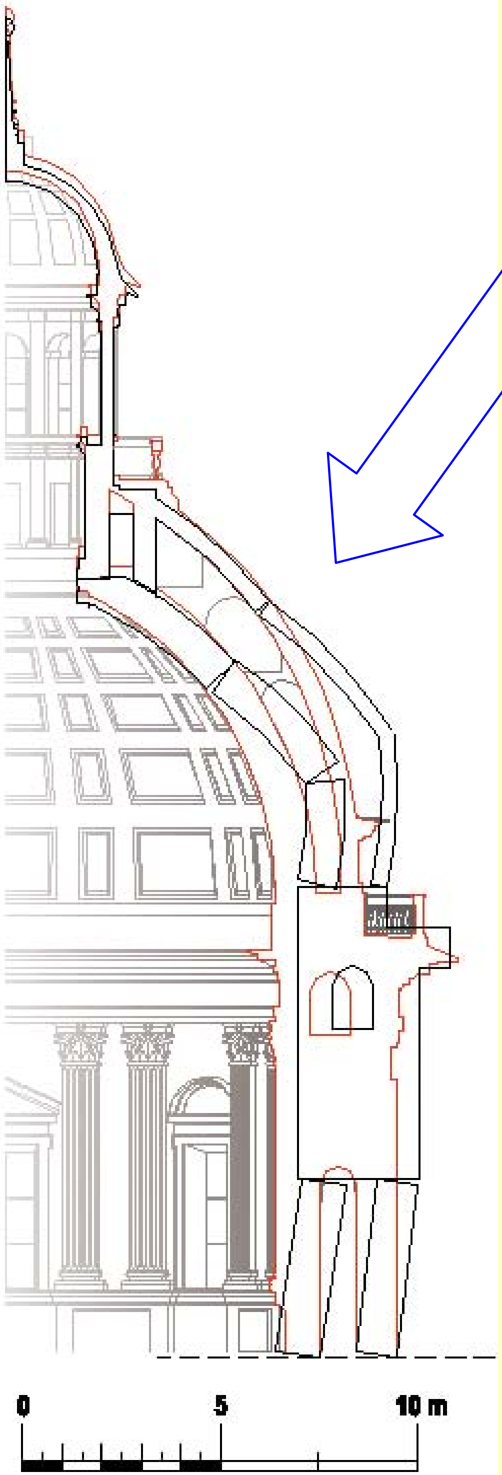


⇐ 1. Local mechanism
Inner and outer domes

$$\eta_1 = \frac{|\dot{W}_{res}|}{\dot{W}_a} \approx 2 > 1$$

⇒ $\dot{W} < 0$

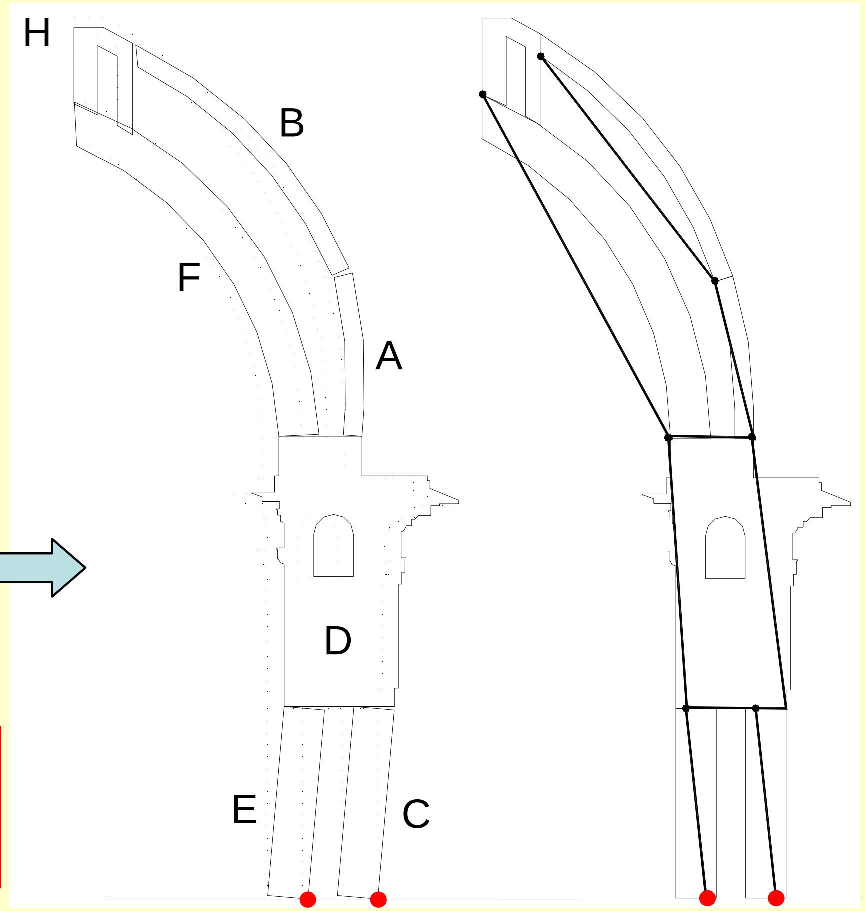
Overall Mechanism domes-drum int.



Global mechanism 1.

$$\eta_2 = \frac{|\dot{W}_{res}|}{\dot{W}_a} \approx 8.5$$

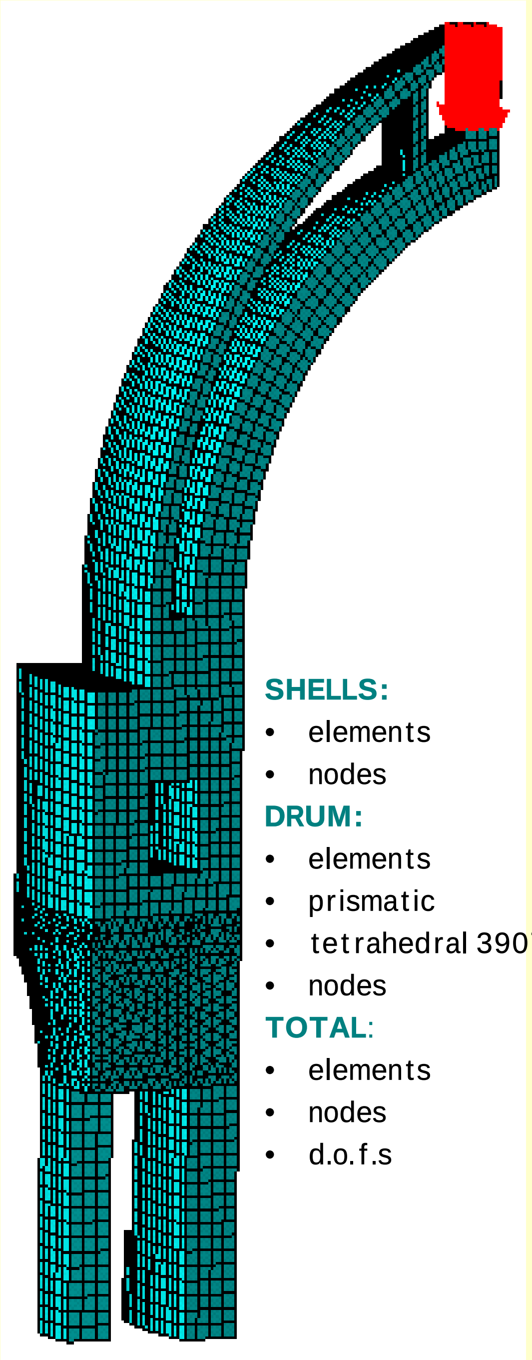
Global mechanism 2.

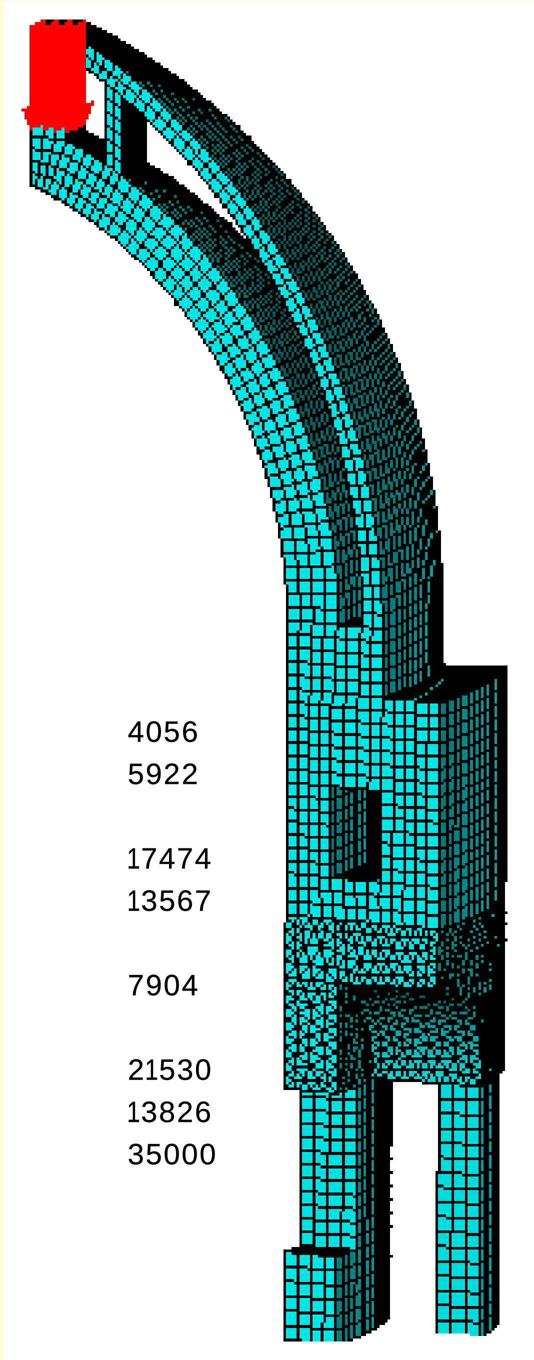


$$\eta_3 = \frac{|\dot{W}_{res}|}{\dot{W}_a} \approx 1.8 \div 7$$

Influence of the column compressive strength on the location of the centre of rotation of the drum slice

FE Model –1/8 slice





SHELLS:

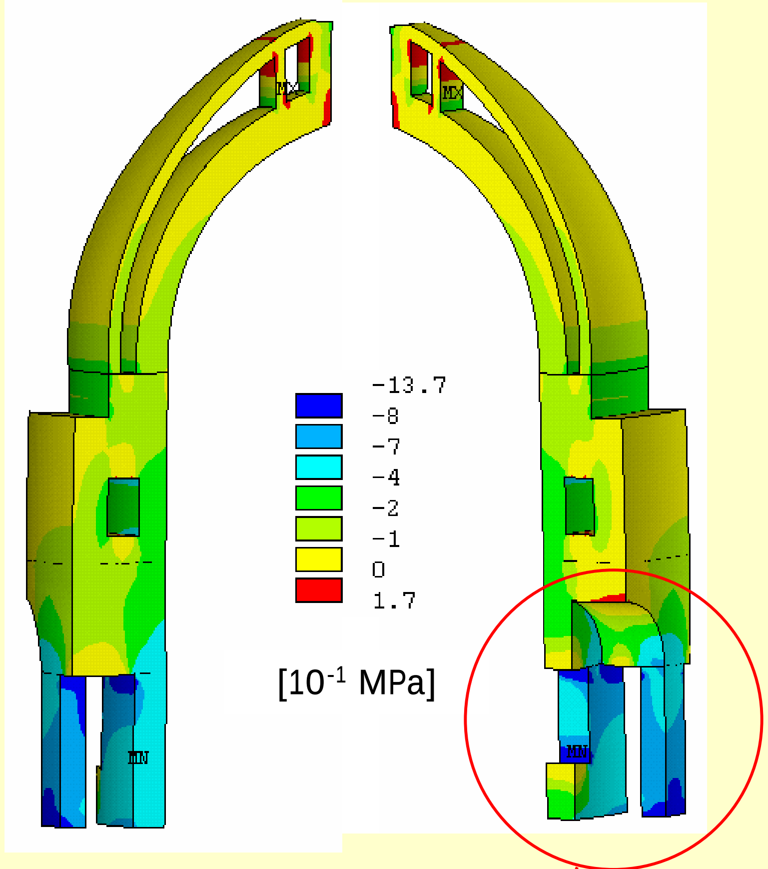
- elements 4056
- nodes 5922

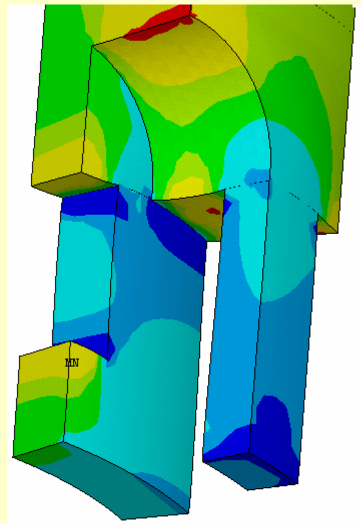
DRUM:

- elements 17474
- prismatic 13567
- tetrahedral 3907
- nodes 7904

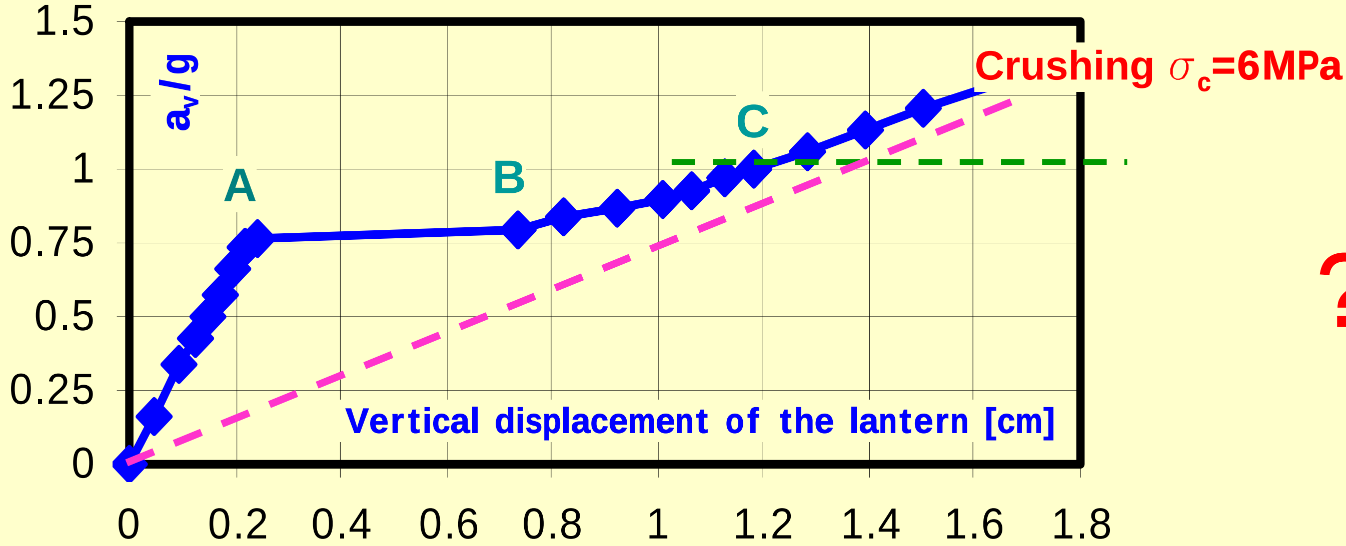
TOTAL:

- elements 21530
- nodes 13826
- d.o.f.s 35000

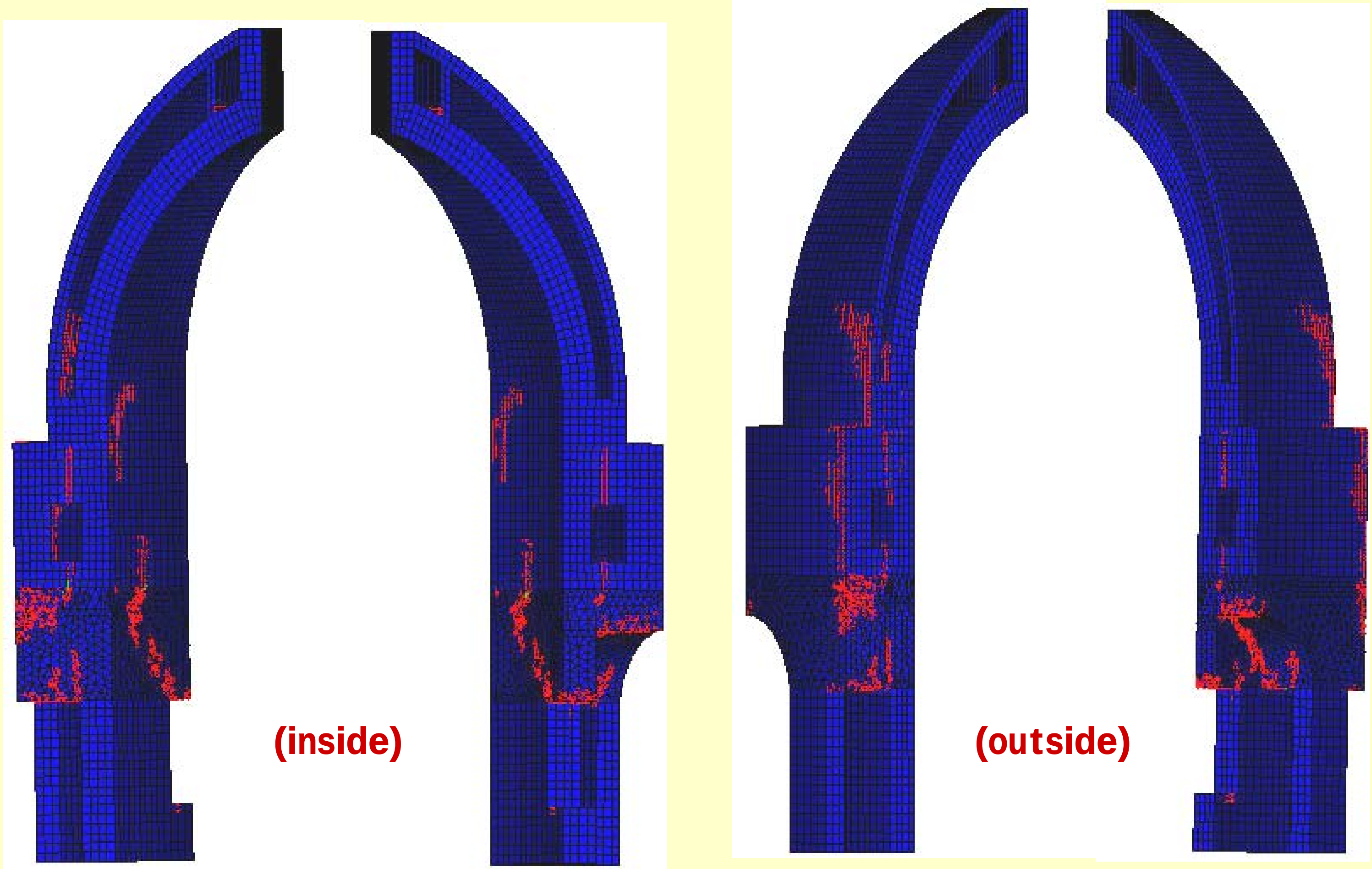




Incremental analysis



?



Crack pattern
State A

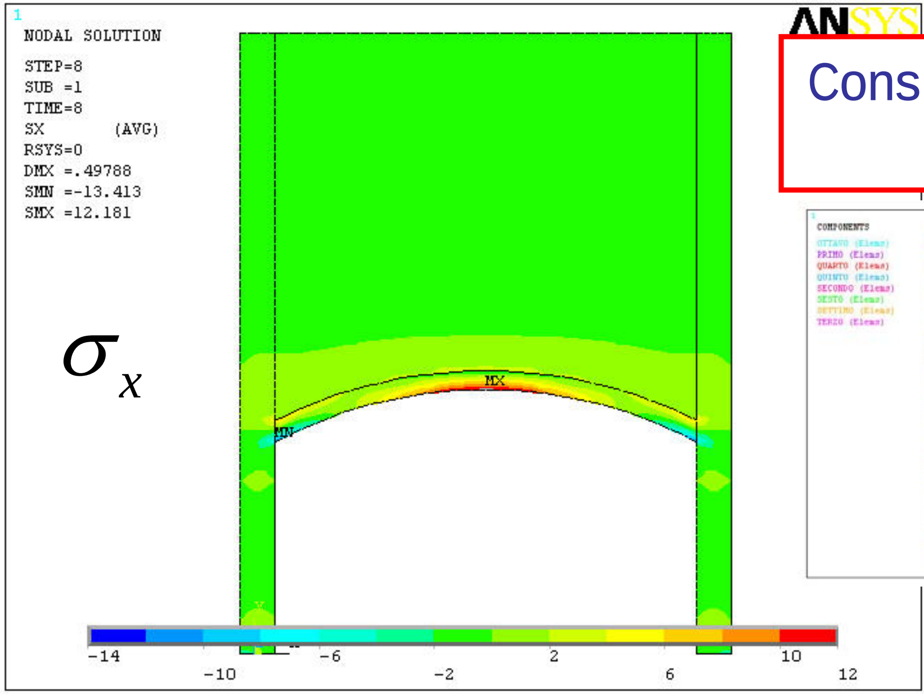
6. Influence of the construction sequence – structural growth

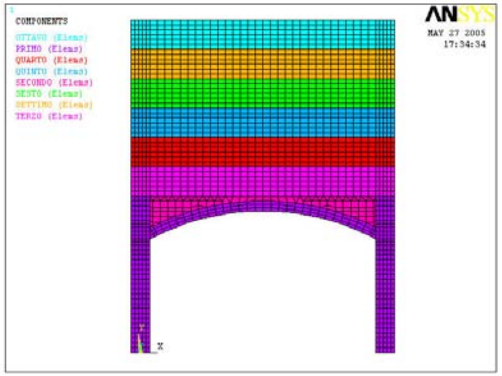
6. Influence of the construction sequence – structural growth

Construction sequence considered

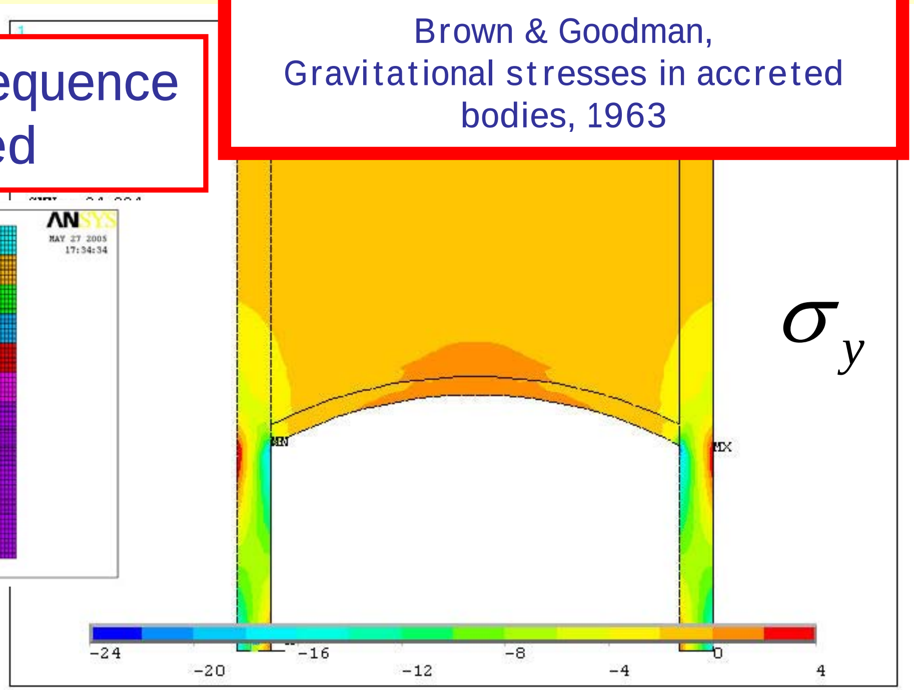
Brown & Goodman,
Gravitational stresses in accreted bodies, 1963

σ_x

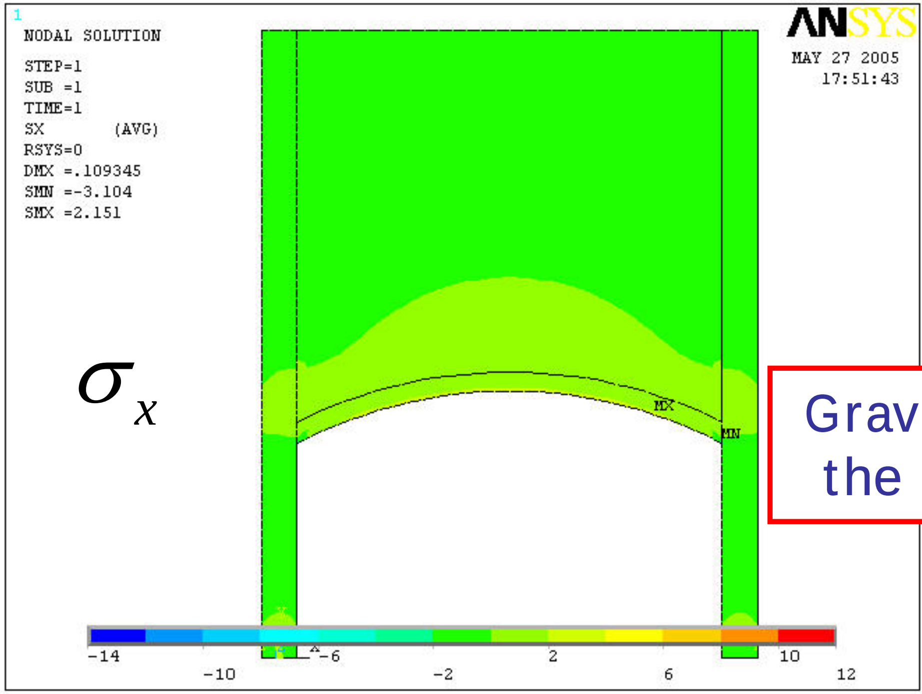




σ_y

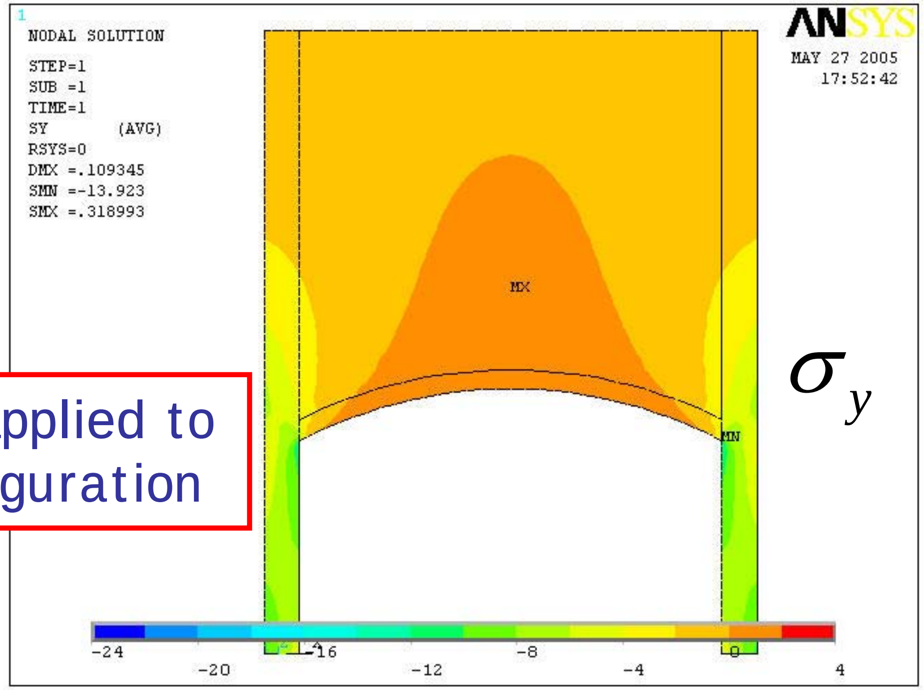


σ_x

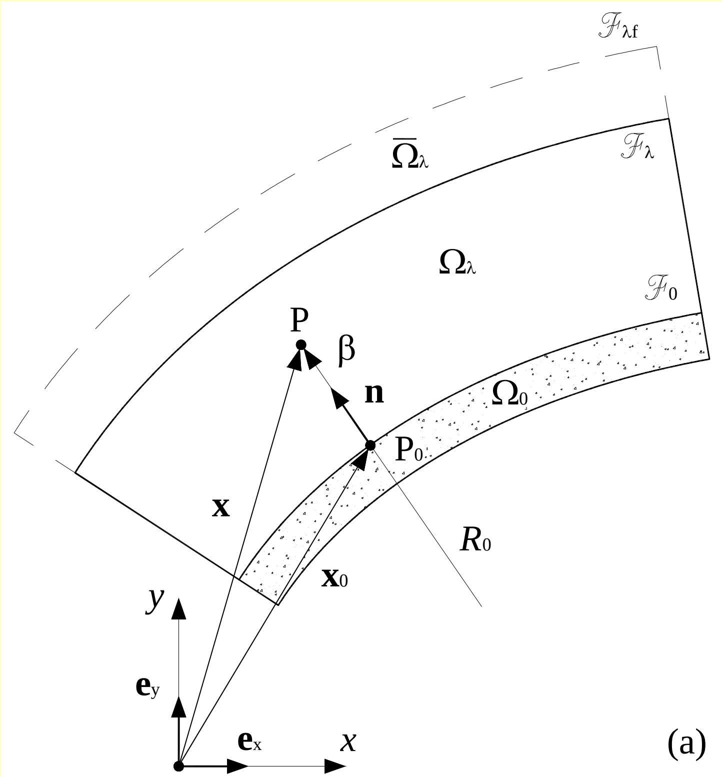


Gravity loads applied to the final configuration

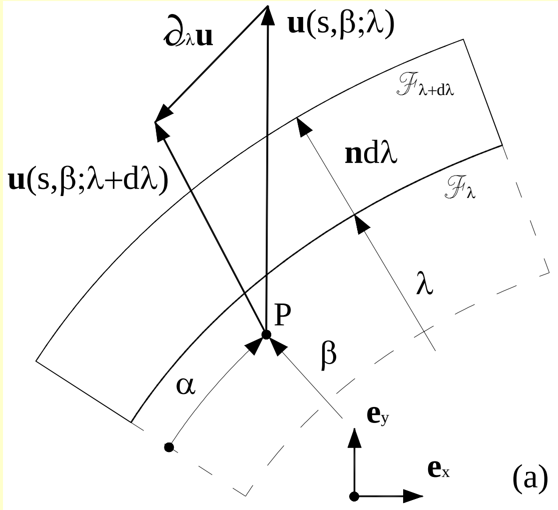
σ_y



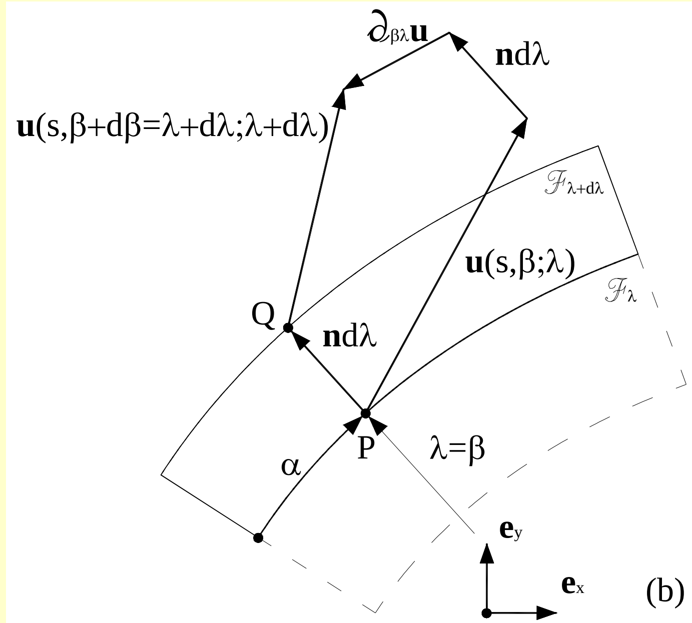
6. Influence of the construction sequence – structural growth



Reference domain



Structural displacement rates



dragged displacement rates

Strain field

$$\mathbf{E}(s,\beta;\lambda) = sym\left(\nabla\bar{\mathbf{u}}_0(s,\beta)\right) + \int_0^\beta sym\left(\nabla\bar{\mathbf{g}}(s,\beta;\lambda)\right)\gamma d\lambda + \\ + sym\left(\nabla\tilde{\mathbf{u}}(s,\beta)\right) + \gamma sym\left(\left(\bar{\mathbf{g}}(s,\beta;\lambda=\beta) - \mathbf{g}(s,\beta;\lambda=\beta)\right)\otimes\nabla\beta\right) + \\ + \int_\beta^\lambda sym\left(\nabla\mathbf{g}(s,\beta;\lambda)\right)\gamma d\lambda.$$

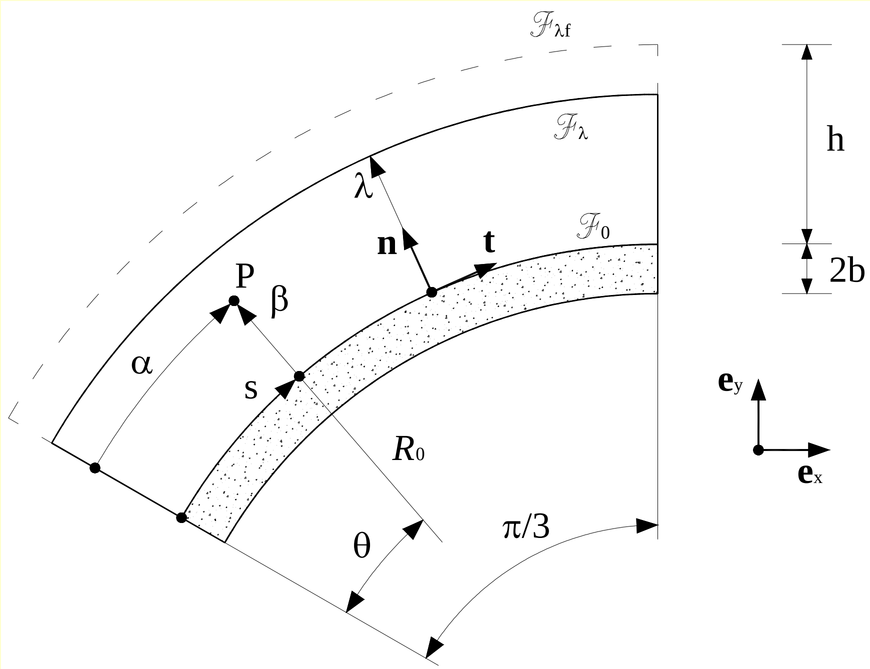
Stess field

$$\mathbf{T}(s,\beta;\lambda=\lambda_f) = \mathbb{C}\left(\mathbf{E}(s,\beta;\lambda=\lambda_f) - \mathbf{E}_t(s,\beta;\lambda=\beta)\right) = \mathbb{C}\int_\beta^{\lambda_f} sym\left(\nabla\left(\mathbf{g}(s,\beta,\lambda)\right)\right)\gamma d\lambda$$

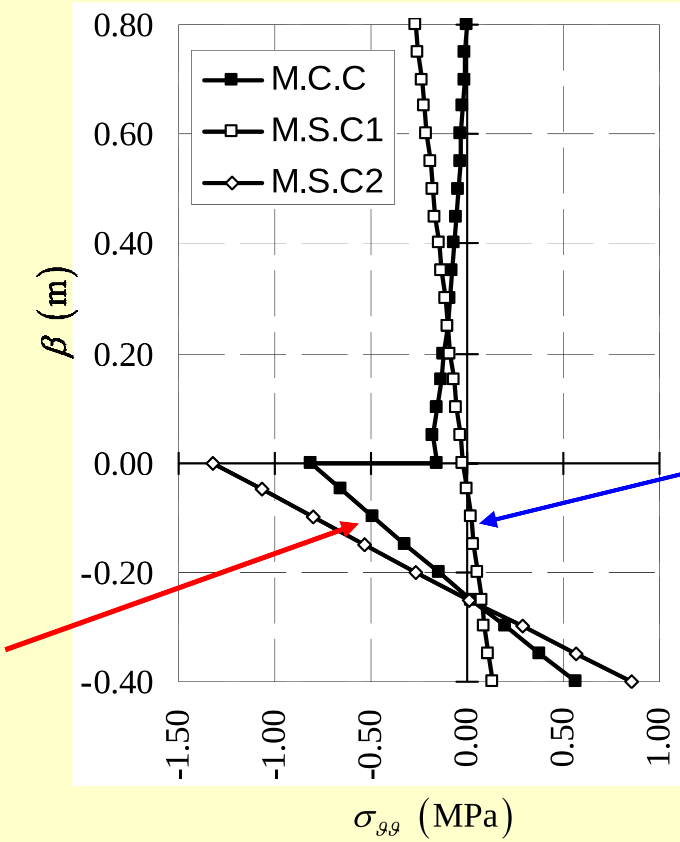
Bacigalupo et al., 2007

6. Influence of the construction sequence – structural growth

Example: Triumphal arch

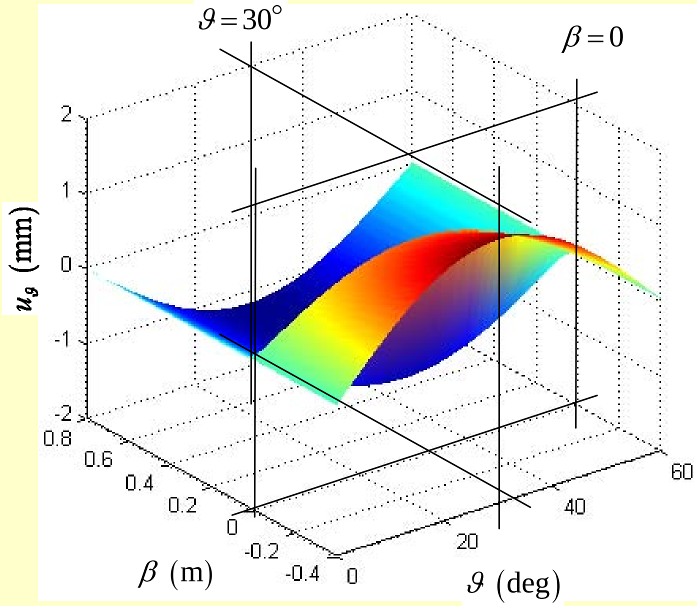


Normal stresses at springing



Growth ignored

Growth included



Displacement field
Tangential component

Bacigalupo et al., 2007

7. Problems & prospects

- Discrete & Continuum models:
  regular versus random masonry pattern (thickness??, real masonry);
  homogenization: size effect –> unit – RVE – wall size;
  interface model: brick unit – mortar layer interaction;
  cohesion: strain localization, non-unique incremental solution

- Damage-frictional models seem to be necessary to understand the masonry wall response to orizontal varying forces. What is the role of perturbations to the reference state due to settlement, construction sequence etc?

- NTR based model are simple and efficient when static loads inducing moderate axial forces are considered. Can comparable simple models be found for high compressive axial forces and time varying loads?

- The fill and spandrel walls notably increase the load carrying capacity of arches and masonry bridges: how this effect can be simply included in assessment procedures?

- Incremental analysis (the reference state often is not well described) or Limit analysis (masonry is far from to be ductile)?

- What simplified procedures for the seismic assessment of buildings and bridges?

- Mechanical decay in the long term.

- etc. etc……………………………………