Navier-Stokes equations from the general transport equation

- During this discussion, we will use the general transport equation to explain the fundamentals of the finite volume method.

$$\underbrace{\int_{V_P} \frac{\partial \rho \phi}{\partial t} dV}_{\text{Time derivative}} + \underbrace{\int_{V_P} \nabla \cdot (\rho \mathbf{u} \phi) \, dV}_{\text{Convective term}} = \underbrace{\int_{V_P} \nabla \cdot (\Gamma_\phi \nabla \phi) \, dV}_{\text{Diffusive term}} + \underbrace{\int_{V_P} S_\phi (\phi) \, dV}_{\text{Source term}}$$

- But have in mind that starting from the general transport equation we can write down the Navier-Stokes equations (NSE). For example, by setting the variables to,

$$\begin{aligned} \phi &= 1 \\ \Gamma_\phi &= 0 \\ S_\phi &= 0 \end{aligned}$$

- We can obtain the continuity equation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

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- But have in mind that starting from the general transport equation we can write down the Navier-Stokes equations (NSE). For example, by setting the variables to,

$$\begin{aligned} \phi &= u \\ \Gamma_\phi &= \mu \\ S_\phi &= S_u - \frac{\partial p}{\partial x} \end{aligned} \qquad \begin{aligned} \phi &= v \\ \Gamma_\phi &= \mu \\ S_\phi &= S_v - \frac{\partial p}{\partial y} \end{aligned} \qquad \begin{aligned} \phi &= w \\ \Gamma_\phi &= \mu \\ S_\phi &= S_w - \frac{\partial p}{\partial z} \end{aligned}$$

- We can obtain the momentum equations,

$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho \mathbf{u} u) = \nabla \cdot (\mu \nabla u) - \frac{\partial p}{\partial x} + S_u \qquad \frac{\partial \rho v}{\partial t} + \nabla \cdot (\rho \mathbf{u} v) = \nabla \cdot (\mu \nabla v) - \frac{\partial p}{\partial y} + S_v \qquad \frac{\partial \rho w}{\partial t} + \nabla \cdot (\rho \mathbf{u} w) = \nabla \cdot (\mu \nabla w) - \frac{\partial p}{\partial z} + S_w$$

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- But have in mind that starting from the general transport equation we can write down the Navier-Stokes equations (NSE). For example, by setting the variables to,

$$\phi = h$$
$$\Gamma_\phi = k/C_p$$
$$S_\phi = S_h$$

- We can obtain the incompressible energy equation,

$$\frac{\partial \rho h}{\partial t} + \nabla \cdot (\rho \mathbf{u} h) = \nabla \cdot \left(\frac{k}{C_p} \nabla T \right) + S_h$$