
Turbulence and CFD models: Theory and applications

Roadmap to Lecture 4

1. Practical turbulence estimates

Practical turbulence estimates

Introduction

- In Lecture 3, Kolmogorov scales, Taylor scales, and integral scales were introduced.
- We then explored the concepts of energy spectrum, energy cascade, integral length scales, and grid length scales.
- We also studied the basic concepts of turbulence near the wall, we introduced the Law of the Wall, and the non-dimensional quantity y^+ .
- Finally, we took a glimpse to a turbulence model.
- At this point, the question is,
 - How can we use this information?
 - How can we get an initial estimate of the variables related to the turbulence model?
 - How can we estimate the meshing requirements?
- Hereafter, we will give some standard practices on how to get turbulence estimates for initial conditions, boundary conditions, and mesh resolution.
- Have in mind that many commercial applications have these relationships already implemented, so you do not need to do these computations manually (which are trivial).

Practical turbulence estimates

Introduction

- Using everything we have learned so far, we can get global estimates for the following variables:
 - Eddies velocity, eddies size, and time scales (integral, Taylor, and Kolmogorov).
 - Rough estimates of the number of grid points or cells needed to resolve integral length scales.
 - Energy dissipation rate ϵ .
 - Turbulent kinetic energy k .
 - Turbulent kinematic viscosity μ_t .
 - Turbulent intensity I_t .
 - Wall distance units y^+ .
- Remember, we will compute initial estimates for global quantities.
- If you want to get the local values, you will need to run a simulation where you will resolve a transport equation for these variables.
- I cannot stress this enough; we will compute rough estimates which are fine for initial conditions, boundary conditions, or generating an initial mesh.

Practical turbulence estimates

Introduction

- Let us first compute the integral eddy length scale, turbulence intensity, turbulent kinetic energy, and turbulent dissipation.
- We will use the **LIKE** acronym [1] to describe the workflow that we will use to compute these practical estimates.

- **L** = l = integral eddy length scale
- **I** = I_t = turbulence intensity
- **K** = k = turbulent kinetic energy
- **E** = ϵ = turbulent dissipation

- We already know many relations from Lectures 2 and 3.
- We will introduce a few new equations.
- Many of these relationships can be derived from dimensional analysis.
- Remember to always check the dimensional groups.

Practical turbulence estimates

A reminder about the units

Derived quantity	Symbol	Dimensional units	SI units
Velocity	u	LT^{-1}	m/s
Density	ρ	ML^{-3}	kg/m ³
Kinematic viscosity	ν	L^2T^{-1}	m ² /s
Dynamic viscosity	μ	$ML^{-1}T^{-1}$	kg/m-s
Energy dissipation rate per unit mass	ϵ	L^2T^{-3}	m ² /s ³
Turbulent kinetic energy per unit mass	k	L^2T^{-2}	m ² /s ²
Length scales	l	L	m
Wavelength	κ	L^{-1}	1/m
Intensity	I_t	-	-
Stress	τ	$ML^{-1}T^{-2}$	kg/(m-s ²) or N/m ² or Pa

Practical turbulence estimates

Integral eddy length scale – LIKE

- Usually, the integral length scales l are represented by a characteristic dimension of the domain,

$$l \sim x_{char}$$

- That is, the system characteristic length places a limit on the maximum integral eddy length.
- In practice, this limit is not reached nor there is a typical value.
- Therefore, conservative approximations are often used based on a percentage of the system characteristic length.
- For example,
 - If you are simulating the flow about a cylinder, you can say that the largest eddies are about 70% of the cylinder diameter.
 - If you are simulating the flow in a pipe, you can say that the largest eddies are about 80% the diameter of the pipe.
 - If you are simulating the flow about an airfoil (with no large flow separation), you can say that the largest eddies are about the 20% the airfoil thickness.

Practical turbulence estimates

Integral eddy length scale – LIKE

- You will find often the following relationships in the literature.
 - For internal flows (pipes and ducts), where D is the diameter or height,

$$l \approx 0.07D$$

- For boundary layers over surfaces, where δ is the turbulent boundary layer thickness,

$$l \approx 0.4\delta$$

where the boundary layer thickness can be approximated using the following correlation (among many available in the literature),

$$\delta \approx \frac{0.37x}{Re_x^{1/5}}$$

Practical turbulence estimates

Integral eddy length scale – LIKE

- You will find often the following relationships in the literature.
 - For grid generated turbulence in wind tunnels, where S is the grid spacing,

$$l \approx 0.2S$$

- The following is a personal estimate that I often use,

$$l \approx 0.7h_b$$



- Where h_b is the blockage height in the direction of the incoming flow.
- For example:
 - Airfoil thickness, if it is aligned with the flow or at a low AOA.
 - If the airfoil is at a high AOA, the blockage height.
 - Frontal area of a body (cylinder, truck, and so on).
- This relationship can be use for internal and external flows

Practical turbulence estimates

Integral eddy length scale – LIKE

- If you have a rough estimate for k and ϵ , for example, from experimental measurements, you can compute the integral length scales as follows.
 - Based on dimensional analysis, Taylor suggests [1,2] that the integral length scales can be approximated as follows,

$$l \approx \frac{k^{3/2}}{\epsilon}$$

Use this estimate if you are interested in the largest integral length scale

- This form is the recommended one to compute the ratio of integral length scales to grid length scales R_l .
- This equation can also be used to get preliminary estimates for boundary and initial conditions.

[1] G. I. Taylor. Statistical theory of turbulence. Proceedings of the Royal Society of London. 1935.

[2] D. Wilcox. Turbulence Modeling for CFD. DCW Industries Inc., 2010.

Practical turbulence estimates

Integral eddy length scale – LIKE

- If you have a rough estimate for k and ϵ , for example, from experimental measurements, you can compute the integral length scales as follows.
 - The previous estimate has been improved over the years based on experimental and numerical data, as explained by Wilcox [1,2],

$$l = C_{\mu} \frac{k^{3/2}}{\epsilon} \quad \text{where} \quad C_{\mu} = 0.09$$

Use this estimate if you are interested in the average integral length scale
This is the preferred correlation

- This relation is often used in the $k - \epsilon$ and $k - \omega$ turbulence models.
- In the $k - \epsilon$ turbulence model, the closure coefficient C_{μ} is also used to compute the turbulent viscosity, as follows,

$$\nu_t = \frac{C_{\mu} k^2}{\epsilon}$$

[1] D. Wilcox. Turbulence Modeling for CFD. DCW Industries Inc., 2010.

[2] W. Jones, B. Launder. The prediction of laminarization with a two-equation model of turbulence. Intl. J. Heat and Mass Transfer, 15, 301-314. 1972.

Practical turbulence estimates

Integral eddy length scale – LIKE

- If you have a rough estimate for k and ϵ , for example, from experimental measurements, you can compute the integral length scales as follows.
 - In the $k - \omega$ turbulence model, the closure coefficient C_μ is used in the transport equation of the TKE.
 - And depending on the version of the model, it is also used in limiter functions to compute the turbulent viscosity.
 - The $k - \epsilon$ turbulence model and the $k - \omega$ turbulence model are related thru the following relationship,

$$\epsilon = \beta^* \omega k \quad \text{where} \quad C_\mu = \beta^* = 0.09$$

- And recall that the integral length scales can be computed as follows [1,2],

$$l = C_\mu \frac{k^{3/2}}{\epsilon}$$

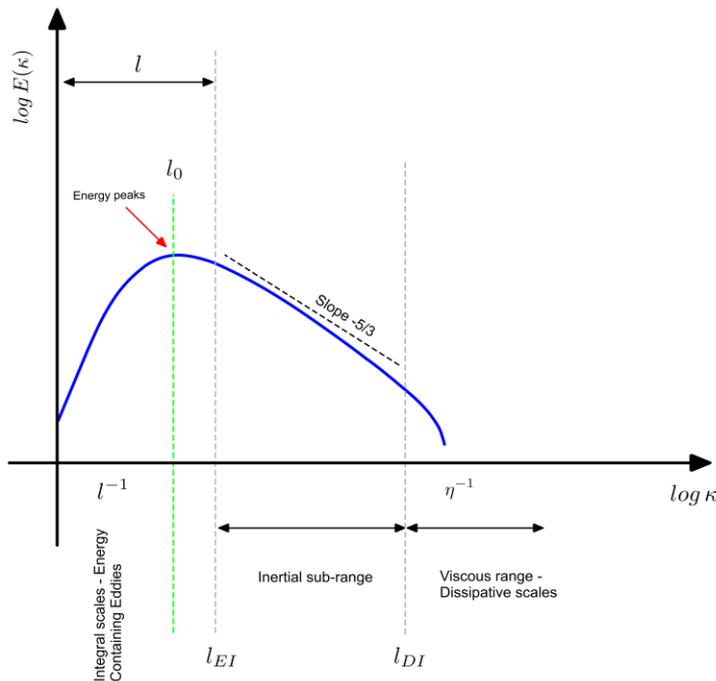
[1] D. Wilcox. Turbulence Modeling for CFD. DCW Industries Inc., 2010.

[2] W. Jones, B. Launder. The prediction of laminarization with a two-equation model of turbulence. Intl. J. Heat and Mass Transfer, 15, 301-314. 1972.

Practical turbulence estimates

Integral eddy length scale – LIKE

- We can talk about a maximum integral scale and an average integral scale.
- In reference to the energy spectrum plot below,
 - The maximum integral scale is where energy peaks or l_0 .
 - The average integral scale is the average length scale of all eddies contained in the energy containing regime. That is, the region containing all the characteristic eddies \bar{l} .
 - To some extension, we can say that these scales are interchangeable without loss of generality.



$$l \approx \frac{k^{3/2}}{\epsilon}$$

Maximum integral length scale

$$l = C_\mu \frac{k^{3/2}}{\epsilon}$$

Avera integral length scale

Practical turbulence estimates

Turbulence intensity – LIKE

- The turbulence intensity (also called turbulence level) is often abbreviated as follows,

$$I \quad I_t \quad T_u \quad T' \quad T_L$$

- The turbulence intensity can be computed as follows,

$$I = \frac{u_{\text{RMS}}}{\bar{u}}$$

← Intensity of velocity fluctuations

← Mean velocity – Freestream velocity

- The intensity of the velocity fluctuations (turbulence strength) is defined by the root mean square (RMS) of the velocity fluctuations,

$$u_{\text{RMS}} = \sqrt{\frac{1}{3} \left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right)}$$

←

- It gives a measure of the dispersion of the velocity fluctuations squared (or normal Reynolds stresses).
- It is nothing else but the standard deviation of the fluctuations.

Practical turbulence estimates

Turbulence intensity – LIKE

- The normal Reynolds stress components $\overline{u'^2}$, $\overline{v'^2}$, $\overline{w'^2}$, can also be regarded as the kinetic energy per unit mass of the fluctuating velocity in the three spatial directions.
- If we sum the normal Reynolds stresses and multiply by 0.5, we obtain the turbulent kinetic energy,

$$k = \frac{1}{2} \left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right) = \frac{1}{2} \overline{u'_i u'_i}$$

- The normal Reynolds stresses can be normalized relative to the mean flow velocity, as follows,

$$\hat{u} \equiv \frac{\sqrt{\overline{u'^2}}}{\bar{u}} \quad \hat{v} \equiv \frac{\sqrt{\overline{v'^2}}}{\bar{u}} \quad \hat{w} \equiv \frac{\sqrt{\overline{w'^2}}}{\bar{u}}$$

← These three quantities are known as relative intensities.

Practical turbulence estimates

Turbulence intensity – LIKE

- For anisotropic turbulence (*i.e.*, the normal components of the Reynolds stress tensor are different), a rough but useful estimate to the normal Reynolds stress components is the following,

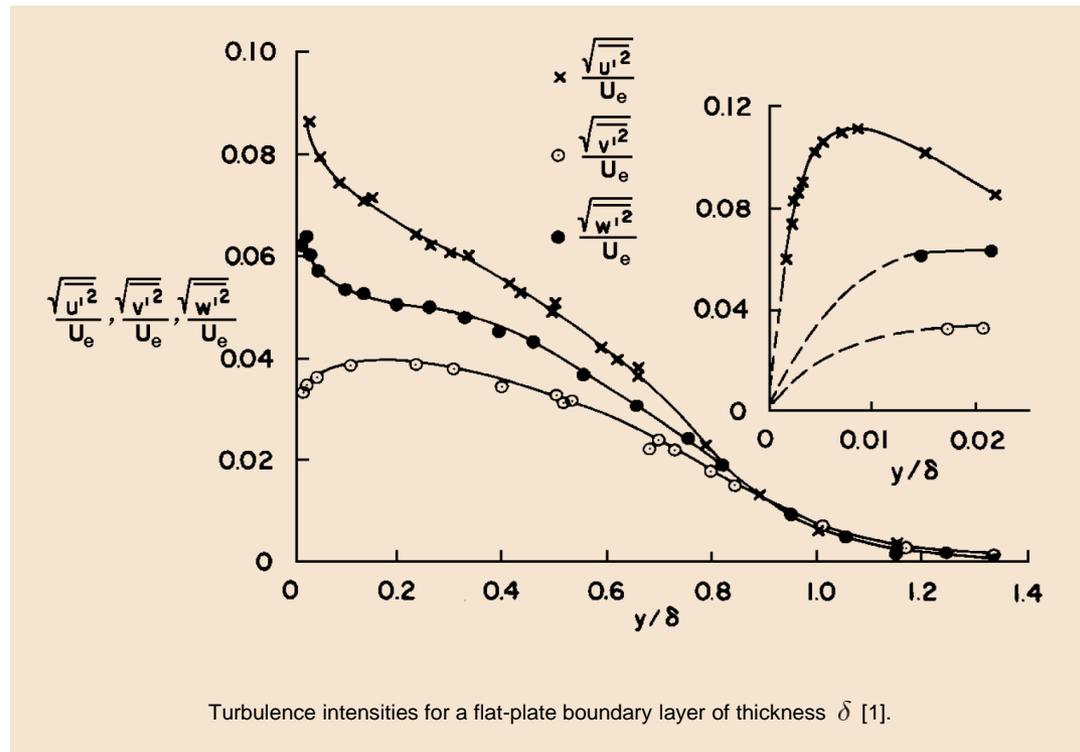
$$\overline{u'^2} : \overline{v'^2} : \overline{w'^2} \approx 4 : 2 : 3$$

Correlation based on flat-plate boundary layer

- And recall that,

$$I = \frac{u_{\text{RMS}}}{\bar{u}}$$

$$u_{\text{RMS}} = \sqrt{\frac{1}{3} \left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right)}$$



Practical turbulence estimates

Turbulence intensity – LIKE

- For the case of isotropic turbulence, the normal components of the Reynolds stress tensor are the same.

$$\overline{u'^2} = \overline{v'^2} = \overline{w'^2}$$

- Then, the turbulence intensity can be computed as follows,

$$I = \sqrt{\frac{\frac{2}{3}k}{\bar{u}^2}} = \frac{\sqrt{\frac{2}{3}k}}{\bar{u}}$$

- And recall that,

$$I = \frac{u_{\text{RMS}}}{\bar{u}} \quad u_{\text{RMS}} = \sqrt{\frac{1}{3} (\overline{u'^2} + \overline{v'^2} + \overline{w'^2})}$$

$$\left\{ \begin{array}{l} k = \frac{1}{2} (\overline{u'^2} + \overline{v'^2} + \overline{w'^2}) \\ k = \frac{3}{2} \overline{u'^2} \end{array} \right. \quad \leftarrow \text{For isotropic turbulence}$$

Practical turbulence estimates

Turbulence intensity – LIKE

- To use the previous relations, you need to have some form of measurements or previous experience to base the estimate on.
- The turbulence intensity can also be estimated using empirical correlations.
- For instance, if you are working with pipes, there are many correlations that are expressed in the form of a power law,

$$I = C_1 Re_h^{-C_2}$$

Hydraulic Reynolds number

- One widely used correlation is the following one,

$$I = 0.16 Re_h^{-1/8}$$

- Remember, there are many forms of these correlations (for smooth and rough pipes).

Practical turbulence estimates

Turbulence intensity – LIKE

- If you are working with external aerodynamics, it might be a little bit more difficult to get rule of thumb estimates.
- However, the following estimates are acceptable,

	Low	Medium	High
I_t	1.0 %	5.0 %	10.0 %

Low turbulence intensity (1%): external flow around cars, ships, submarines, and aircrafts. Very high-quality wind-tunnels can also reach low turbulence levels, typically below 1.0%.

Medium turbulence intensity (5%): flows in not-so-complex devices like large pipes, fans, ventilation flows, wind tunnels, low speed flows, and fully-developed internal flows. Typical values are between 2.0% and 7.0%.

High turbulence intensity (10%): high-speed flow inside complex geometries like heat-exchangers and rotating machinery (turbines and compressors). Typical values are between 10.0% and 20.0%.

Practical turbulence estimates

Turbulent kinetic energy – LIKE

- The turbulent kinetic energy k is also known as TKE.
- If you have some estimates of the velocity fluctuations or normal Reynolds stresses, the TKE (per unit mass) can be computed as follows,

$$k = \frac{1}{2} \left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right)$$

- Otherwise, you can use a rule of thumb turbulence intensity estimate and compute TKE as follows,

$$k = \frac{3}{2} (\bar{u}I)^2 \quad \text{where } \bar{u} \text{ is the freestream velocity}$$

- Instead of using \bar{u} , you can also use a value slightly higher, which we will call turbulent freestream value u_{tur} ,

Arbitrary constant to correct velocity fluctuations

$$u_{tur} \approx C\bar{u} = \frac{11}{10}\bar{u}$$

Practical turbulence estimates

Energy dissipation rate – LIKE

- Once TKE is known, together with a crude estimate of the integral eddy length scale, the energy dissipation rate ϵ (per unit mass) can be computed as follows,

$$\epsilon = C_{\mu} \frac{k^{3/2}}{l} \quad \text{where} \quad C_{\mu} = 0.09$$

- You can compute the specific dissipation rate once you know ϵ and l , as follows,

$$\omega = \frac{\epsilon}{\beta^* k} \quad \text{where} \quad \beta^* = \frac{9}{100}$$

- You can compute the integral eddy length scale from specific dissipation rate as follows,

$$l = \frac{k^{1/2}}{\omega}$$

Practical turbulence estimates

Turbulent viscosity

- So far, we computed estimates for the integral eddy length scale, turbulence intensity, turbulent kinetic energy and energy dissipation rate.
- In the previous lecture, we saw that in turbulence modeling there is an extra ingredient, turbulent viscosity ν_t .
- We can also get an estimate for this quantity. How do we estimate it depends on the turbulent model.
- For example, the $k - \omega$ model computes the turbulent viscosity as follows,

$$\nu_t = \frac{k}{\omega}$$

Where you can compute ω from ϵ as follows,

$$\omega = \frac{\epsilon}{\beta^* k} \quad \beta^* = \frac{9}{100}$$

- As usual, check the dimensional groups.
- Remember, the turbulent viscosity is not a physical quantity.
- Also, the turbulent viscosity is larger than the laminar viscosity (molecular viscosity). Usually, two or more orders of magnitude

Practical turbulence estimates

Turbulent viscosity

- Based purely on dimensional analysis and in function of the integral length scales and turbulent dissipation rate, the turbulent viscosity can be estimated as follows,

$$\nu_t \sim k^{1/2} l_0 \sim \frac{k^2}{\epsilon}$$

- And recall the following relationships,

$$k \sim (\epsilon l_0)^{2/3} \implies \epsilon \sim \frac{k^{3/2}}{l_0} \implies l_0 \sim \frac{k^{3/2}}{\epsilon}$$

- As usual, check the dimensional groups.

Practical turbulence estimates

Turbulent viscosity

- We can also use a rule of thumb to get a fast estimate of the turbulent viscosity.
- Using the turbulence intensity I_t , we can get an estimate for the viscosity ratio μ_t/μ as follows,

	Low	Medium	High
I_t	1.0 %	5.0 %	10.0 %
μ_t/μ	1-2	10	100

Low turbulence intensity (1%): external flow around cars, ships, submarines, and aircrafts. Very high-quality wind-tunnels can also reach low turbulence levels, typically below 1.0%.

Medium turbulence intensity (5%): flows in not-so-complex devices like large pipes, fans, ventilation flows, wind tunnels, low speed flows, and fully-developed internal flows. Typical values are between 2.0% and 7.0%.

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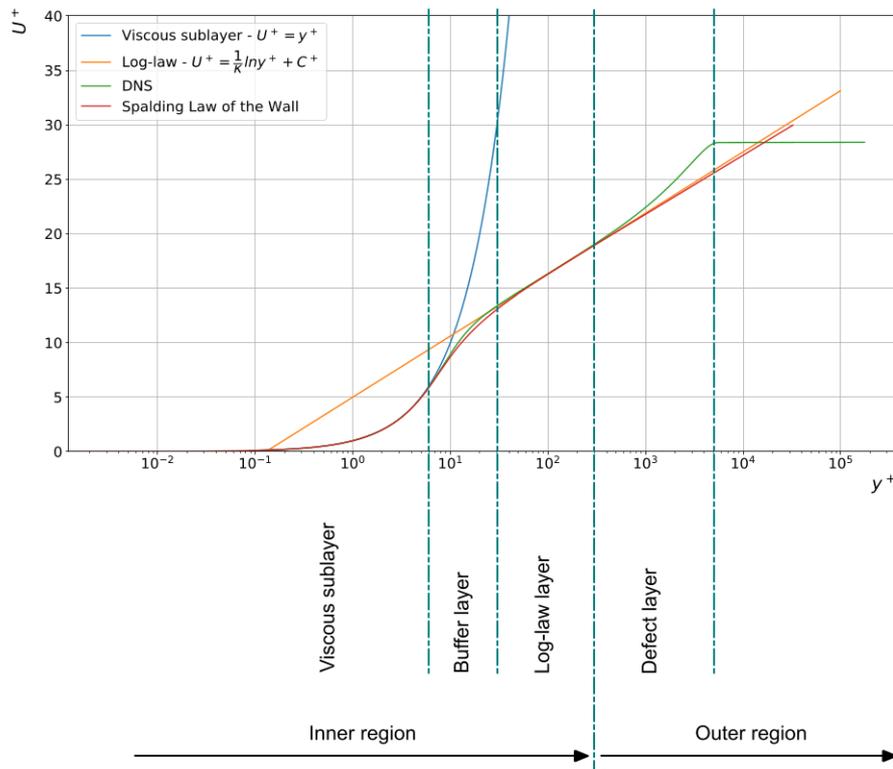
- We usually use these estimates when dealing with external aerodynamics.

Estimation of y^+ and wall distance units

Practical turbulence estimates

Estimation of y^+

- Let us revisit the definition of y^+ .
- y^+ is the non-dimensional distance normal to the wall.
- It is very similar to the Reynolds number, but it is non-dimensionalized using the shear velocity U_τ .



$$y^+ = \frac{\rho \times U_\tau \times y}{\mu} = \frac{U_\tau \times y}{\nu}$$

$$U_\tau = \sqrt{\frac{\tau_w}{\rho}} \quad \leftarrow \text{wall shear stresses}$$

$$u^+ = \frac{U}{U_\tau}$$

In the previous equations:

- y is the distance normal to the wall.
- U_τ is the shear velocity,
- u^+ relates the mean velocity to the shear velocity.

Practical turbulence estimates

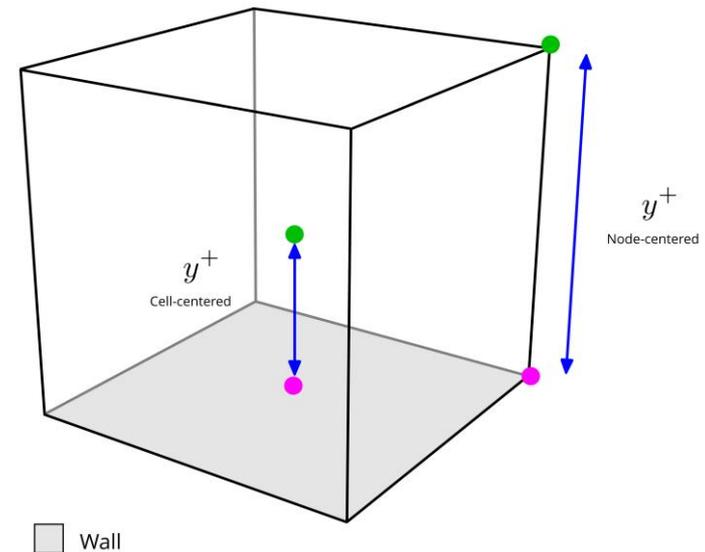
Estimation of y^+

- y^+ is a very important quantity in wall bounded turbulence modeling.
- We can use y^+ to estimate the mesh resolution near the wall before running the simulation.
 - Have in mind that we do not know a-priori the wall shear stresses at the walls.
 - Therefore, we need to use correlations to get a rough estimate and generate the initial mesh.
 - The initial mesh is then generated according to the chosen near the wall treatment, namely, wall resolving, wall functions, or y^+ insensitive.
 - Then, we run a precursor simulation to validate the mesh or to get a better estimate of y^+ and determine if we need a finer or coarser mesh.
 - It is an iterative process, and it can be very time consuming as it might require remeshing and rerunning the simulation to satisfy the near the wall treatment.

Practical turbulence estimates

Estimation of y^+

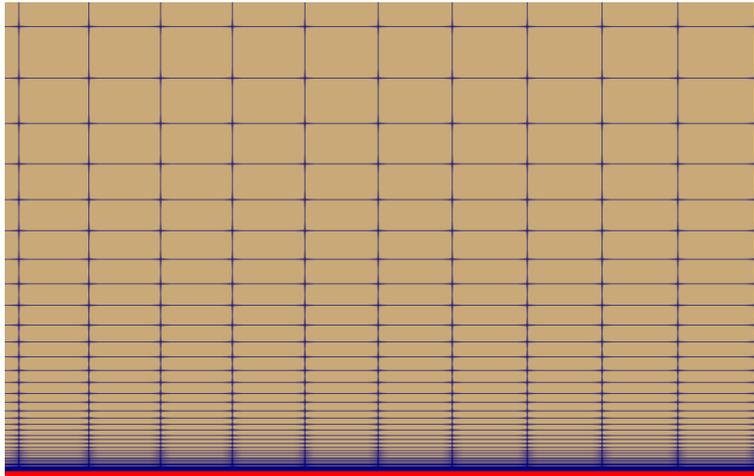
- It is worth mentioning that y^+ can be defined in reference to the cell center or the cell nodes.
- From now on, we are going to assume that we are using a cell-centered solver.
 - Ansys Fluent, StarCCM+, OpenFOAM, code Saturne, SU2, are all cell-centered.
 - It is up to you to check with the solver's documentation to find out the formulation used.
- By the way, y^+ is computed in the same way disregarding of the cell type used.
- In the cell-centered formulation you need to know the location of the cell center.
- In the node-centered formulation you need to know the location of the reference node. Usually, the node closest to the surface.



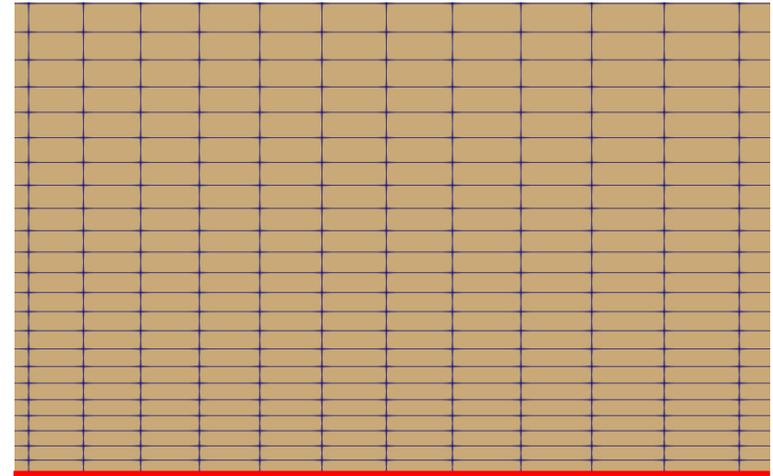
Practical turbulence estimates

Estimation of y^+

- At meshing time, to estimate the normal distance from the wall to the first cell center (y), we use the well-known y^+ definition.
- Where we set a target y^+ value and then we solve for the quantity y .
 - If you choose a low target y^+ (e.g., less than 10), you will have a mesh that is clustered towards the wall (small value of y).
 - If you choose a large y^+ value (e.g., more than 100), you will have a coarse mesh close to the walls (large value of y).



Fine mesh towards the walls



Coarse mesh towards the walls

Practical turbulence estimates

Estimation of y^+

- At meshing time, to estimate the normal distance from the wall to the first cell center (y), we use the well-known y^+ definition,

$$y^+ = \frac{\rho \times U_\tau \times y}{\mu} = \frac{U_\tau \times y}{\nu}$$

- The problem is that at meshing time we do not know the value of the shear velocity,

$$U_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

- So, how do we get an initial estimate of this quantity?
- Solution, we use correlations.

Practical turbulence estimates

Estimation of y^+

- At meshing time, to estimate the normal distance from the wall to the first cell center (y), we can proceed as follows,

- $$Re = \frac{\rho \times U \times L}{\mu}$$

← Compute the Reynolds number using the characteristic length of the problem.
- $$C_f = 0.058 \times Re^{-0.2}$$

← Compute the friction coefficient using any of the correlations available in the literature. There are many correlations available ranging from pipes to flat plates, and for smooth and rough surfaces.
This correlation corresponds to a smooth flat plate case, ideal for external aerodynamics.
- $$\tau_w = \frac{1}{2} \times C_f \times \rho \times U_\infty^2$$

← Compute the wall shear stresses using the friction coefficient computed in the previous step.
- $$U_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

← Compute the shear velocity using the wall shear stresses computed in the previous step.
- $$y = \frac{\mu \times y^+}{\rho \times U_\tau}$$

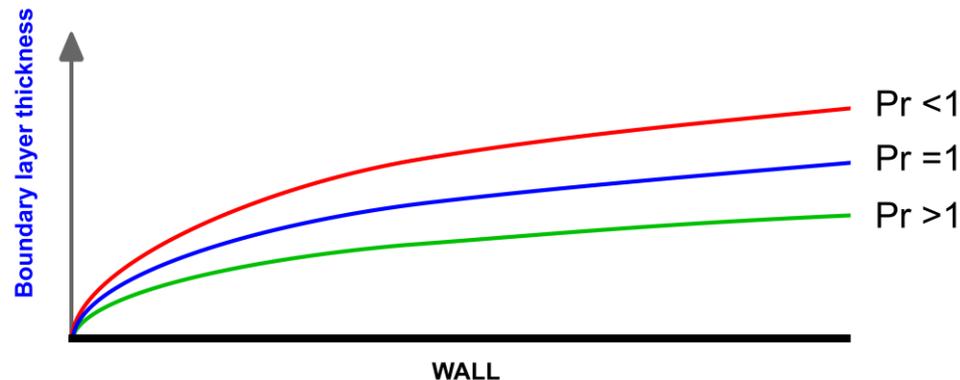
← Set a target y^+ value and solve for y using the flow properties and previous estimates.
This value represents the distance from the wall to the first cell center.
At this point, you can generate your mesh trying to respect this value.

Practical turbulence estimates

y^+ guidelines – Correction of the mesh

- The thickness of the thermal sublayer for a high Prandtl number fluid (e.g., water) is much less than the momentum sublayer thickness.
- Therefore, the mesh requirements close to the walls need to be corrected for the thinner thermal boundary layer.
- The y^+ value can be corrected as follows,

$$y^+ \approx \frac{1}{\sqrt{Pr}}$$



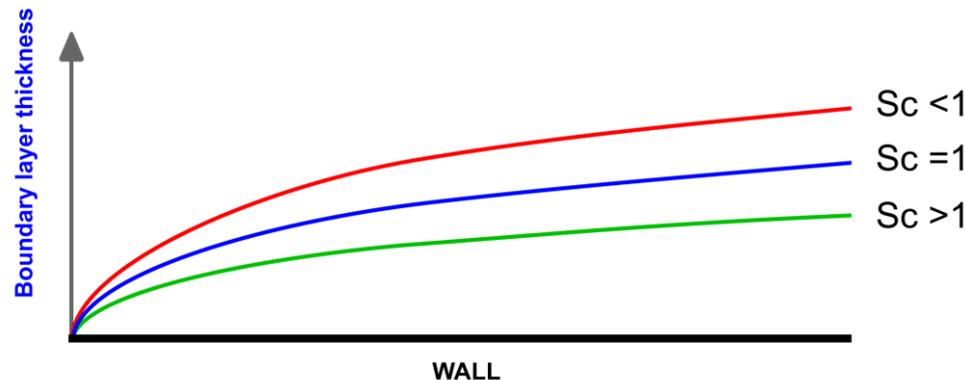
Thermal boundary layer in function of Prandtl number (Pr)

Practical turbulence estimates

y^+ guidelines – Correction of the mesh

- A similar situation exists when working with passive scalars, multispecies transport, and combustion.
- In these cases, we use the Schmidt number (Sc).
- If the Schmidt number is considerably larger than unity, the thickness of the mass diffusion sublayer is much less than the momentum sublayer thickness.
- In this situation, the y^+ value can be corrected as follows,

$$y^+ \approx \frac{1}{\sqrt[3]{Sc}}$$



Mass diffusivity boundary layer in function of Schmidt number (Sc)

Practical turbulence estimates

Short note on the Prandtl number

- Recall that the Prandtl number is defined as follows,

Diagram illustrating the definition of the Prandtl number (Pr) as the ratio of kinematic viscosity (ν) to thermal diffusivity (α), which is also equal to the ratio of dynamic viscosity (μ) times specific heat (c_p) to thermal conductivity (k).

$$Pr = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$$

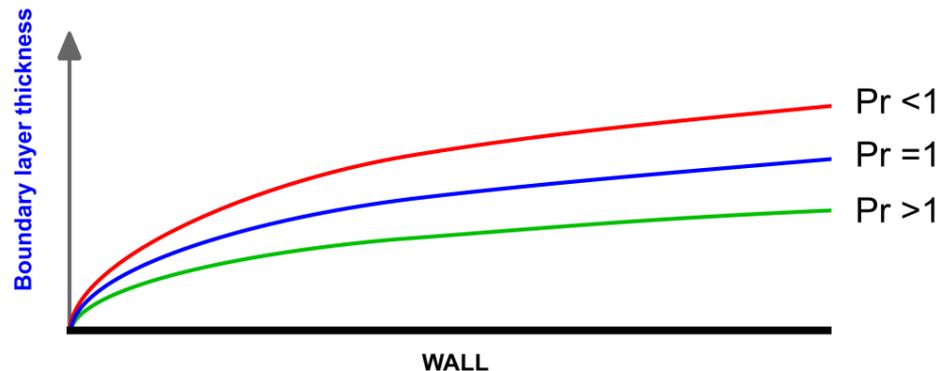
Labels and arrows in the diagram:
- Kinematic viscosity (Momentum diffusivity) points to ν .
- Thermal diffusivity points to α .
- Dynamic viscosity points to μ .
- Specific heat points to c_p .
- Thermal conductivity points to k .

- Note that the Prandtl number contains no length scale and is dependent only on the fluid properties.
- In heat transfer problems, the Prandtl number controls the relative thickness of the momentum and thermal boundary layers.

Practical turbulence estimates

Short note on the Prandtl number

- For Prandtl number equal to 1, the thermal boundary layer is equal to the momentum boundary layer.
- When Pr is less than 1, it means that the heat diffuses quickly compared to the velocity (momentum). The thermal boundary layer is much thicker than the velocity boundary layer.
- When Pr is more than one, the thermal boundary layer is much thinner than the velocity boundary layer.
- The Prandtl numbers of gases are about 1, which indicates that both momentum and heat dissipate through the fluid at about the same rate



Thermal boundary layer in function of Prandtl number (Pr)

$$Pr = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$$

Practical turbulence estimates

Short note on the Schmidt number

- The mass transfer equivalent of the Prandtl number is the Schmidt number.
- The Schmidt number (Sc) is a dimensionless number defined as the ratio of momentum diffusivity (kinematic viscosity) and mass diffusivity and is used to characterize fluid flows in which there are simultaneous momentum and mass diffusion convection processes.

$$Sc = \frac{\nu}{D} = \frac{\mu}{\rho D}$$

Kinematic viscosity (Momentum diffusivity) \longrightarrow ν \longleftarrow Dynamic viscosity

Mass diffusivity \longrightarrow D \longleftarrow Mass diffusivity

\uparrow
Fluid density

- As for Pr, the Schmidt number (Sc) contains no length scale and is dependent only on the fluid properties.
- Sc physically relates the relative thickness of the hydrodynamic layer and mass-transfer boundary layer.
- The behavior of the mass-transfer boundary layer (or concentration boundary layer) in function of Sc is similar to that of the Pr number.

Practical turbulence estimates

Short note on the Lewis number

- The Schmidt number Sc and the Prandtl number Pr can be related using the Lewis number Le .
- Le is used to characterize fluid flows where there is simultaneous heat and mass transfer.
- The Lewis number relates the thickness of the thermal boundary layer in relation to the mass-transfer boundary layer (or concentration boundary layer).
- The Lewis number is defined as,

$$Le = \frac{\alpha}{D} = \frac{k}{\rho D_B c_p} = \frac{Sc}{Pr}$$

Thermal conductivity

Thermal diffusivity

Mass diffusivity

Diffusion coefficient of the binary mixture

- If Le is equal to 1, the thickness of thermal boundary layer and the mass-transfer boundary layer is approximately the same.
- Le is often used when conducting mass-transfer, multi-species, or combustion studies.

Practical turbulence estimates

Short note on the turbulent Pr_t , Sc_t , Le_t

- The previous definitions of Pr , Sc and Le are related to the molecular properties* of the working fluid.
- When dealing with turbulence modeling, we need to define the equivalent turbulent numbers, namely,
 - Turbulent Prandtl number or Pr_t .
 - Turbulent Schmidt number or Sc_t .
 - Turbulent Lewis number or Le_t .
- Something similar to what we have done with the turbulent viscosity ν_t .
- Remember, these turbulent numbers are not physical properties.
- These numbers depends on the turbulent motion of the flow or the eddy diffusivity.

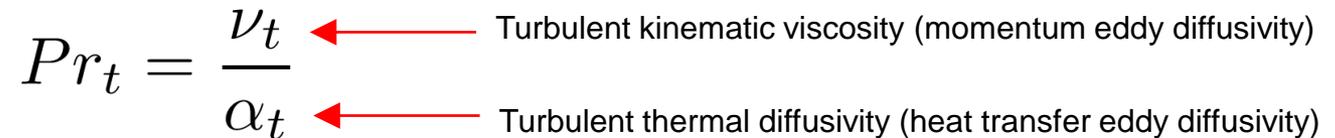
* The molecular properties sometimes are referred to as laminar properties

Practical turbulence estimates

Short note on the turbulent Pr_t , Sc_t , Le_t

- The turbulent Prandtl number (Pr_t) is a non-dimensional quantity defined as the ratio between the momentum eddy diffusivity* and the heat transfer eddy diffusivity.

$$Pr_t = \frac{\nu_t}{\alpha_t}$$



- Pr_t is used when solving the heat transfer problems of turbulent flows.
- From experimental data, Pr_t has been found to have an average value of 0.85.
- But this value highly depends on the flow properties and the flow physics.
- Common values found in literature can go anywhere from 0.7 to 0.9.
- A lot of uncertainty is involved related the value of Pr_t .
- Pr_t has a strong dependence on the molecular Prandtl number among many other parameters.
- In the special case where the Prandtl number and turbulent Prandtl number both equal unity (as in the Reynolds analogy), the velocity profile and temperature profiles are identical.
- When conducting numerical simulations, we need to define the value of this number.

* Eddy diffusivity refers to turbulent motion

Practical turbulence estimates

Short note on the turbulent Pr_t , Sc_t , Le_t

- The turbulent Schmidt number (Sc_t) is a non-dimensional quantity defined as the ratio between the momentum eddy diffusivity and the mass-transfer eddy diffusivity.

$$Sc_t = \frac{\nu_t}{D_t}$$

← Turbulent kinematic viscosity (momentum eddy diffusivity)
← Turbulent mass diffusivity (mass-transfer eddy diffusivity)

- Sc_t is used for solving the mass transfer problem of turbulent flows.
- From experimental and numerical data, Sc_t has been found to go anywhere from 0.1 to 10.
- A lot of uncertainty is involved related the value of Sc_t .
- Some researchers has shown a strong dependency on the Peclet number, with high Sc_t values corresponding to low Peclet numbers and vice versa.
- When conducting numerical simulations, we need to define the value of this number.

Practical turbulence estimates

Short note on the turbulent Pr_t , Sc_t , Le_t

- The turbulent Le number (Le_t) is a non-dimensional quantity defined as the ratio between the turbulent Schmidt number and the turbulent Prandtl number.

$$Le_t = \frac{Sc_t}{Pr_t}$$

 Turbulent Schmidt number
 Turbulent Prandtl number

- In many turbulent combustion modeling studies, Le_t is assumed to be equal to one. This implies that Sc_t and Pr_t are the same.
- The main consequence of this assumption is that the eddy mass diffusivity and eddy thermal diffusivity are the same.
- Scenarios with non-unity Le_t are also possible.
- When conducting numerical simulations, we need to define the value of this number.

Practical turbulence estimates

Short note on the turbulent Pr_t , Sc_t , Le_t

- It is worth mentioning that by combining the previous definitions of Pr_t , Sc_t , and Le_t , we can derive a relation for the turbulent mass diffusivity D_t , as follows,

$$D_t = \frac{\nu_t}{Sc_t}$$

- Evaluating D_t is not easy, as it depends on the turbulent viscosity (hence the turbulence model).
- Matters are worst when Sc_t is not constant. In this case, we need to derive an extra transport equation to compute Sc_t locally.
- For example, the following closure equation is used to compute D_t using the standard $k - \epsilon$ turbulence model and constant Sc_t ,

- Turbulence model constant (0.09)
- The community agrees on this value.
- However, some authors might use a different value.


$$D_t = \frac{C_\mu}{Sc_t} \frac{k^2}{\epsilon}$$

- In this equation, all the uncertainty goes into the choice of the value of Sc_t or Le_t .

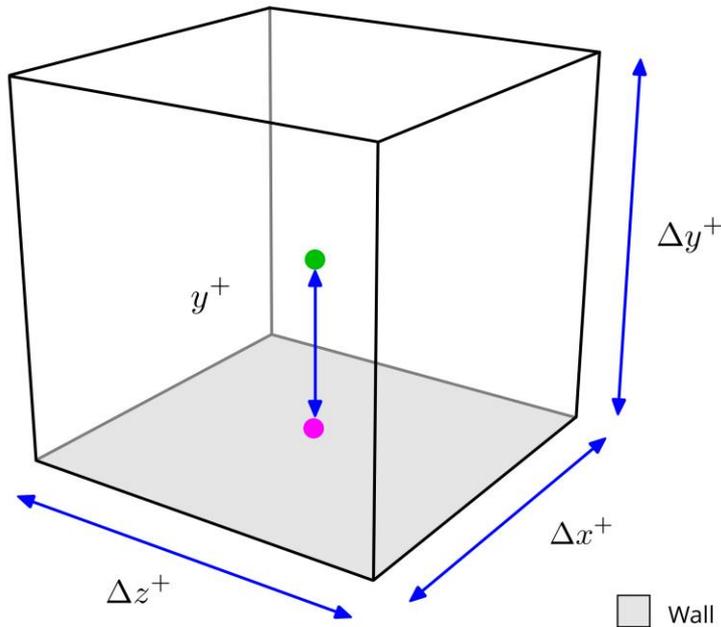
Practical turbulence estimates

y^+ guidelines – Limiting values and solution monitoring

- y^+ always needs to be monitored during the simulation.
 - Have in mind that it is very difficult (if not impossible) to get a uniform y^+ value at the walls.
 - We usually monitor the average y^+ value. If this value covers approximately 80% of the wall, we can take the mesh as a good one.
 - Otherwise, we need to refine or coarse the mesh to get a more uniform distribution of y^+ .
 - It is also important to monitor the maximum values of y^+ . It is not a good practice to have values larger than 1000.
 - Values of y^+ up to 300 are fine.
 - Values of y^+ larger than 300 and up to a 1000 are acceptable if they do not cover a large surface area (no more than 20% of the total wall area), or if they are not located in critical zones.
 - It is also important to monitor the minimum y^+ , as some models might have problems with low y^+ values.
 - Use common sense when accessing the y^+ distribution.

Practical turbulence estimates

Wall distance units $x^+ - y^+ - z^+$



- Similar to y^+ , the wall distance units can be computed in the stream-wise (Δx^+) and span-wise (Δz^+) directions.
- The wall distance units in the stream-wise and span-wise directions can be computed as follows:

$$\Delta x^+ = \frac{U_\tau \Delta x}{\nu} \quad \Delta z^+ = \frac{U_\tau \Delta z}{\nu}$$

- And recall that in our formulation y^+ is computed at the cell center, therefore:

$$\Delta y^+ = 2 \times y^+$$

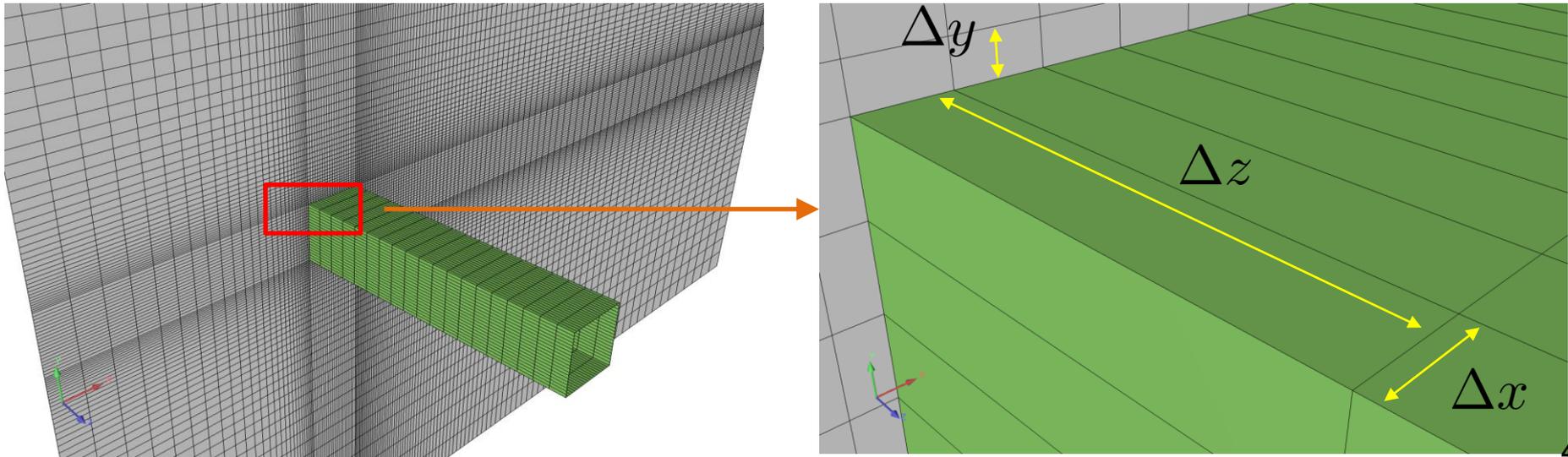
$$(\Delta x^+, \Delta y^+, \Delta z^+) = \left(\frac{x}{l_\tau}, \frac{y}{l_\tau}, \frac{z}{l_\tau} \right)$$

where $l_\tau = \frac{\nu}{U_\tau}$
Viscous length

Practical turbulence estimates

Wall distance units $x^+ - y^+ - z^+$

- It is not only about the y^+ value.
- In SRS simulations, x^+ and z^+ are also important.
- The x^+ and z^+ values strongly depend on the surface mesh aspect ratio.
- The aspect ratio is defined as the ratio between the longest edge and shortest edge.
- Recommended values of surface mesh aspect ratio for SRS simulations are 10 or less (conservatively speaking).
- Meshing applications usually have adequate tools for mesh diagnostics.
- This is a rough estimate based on general guidelines and experience.



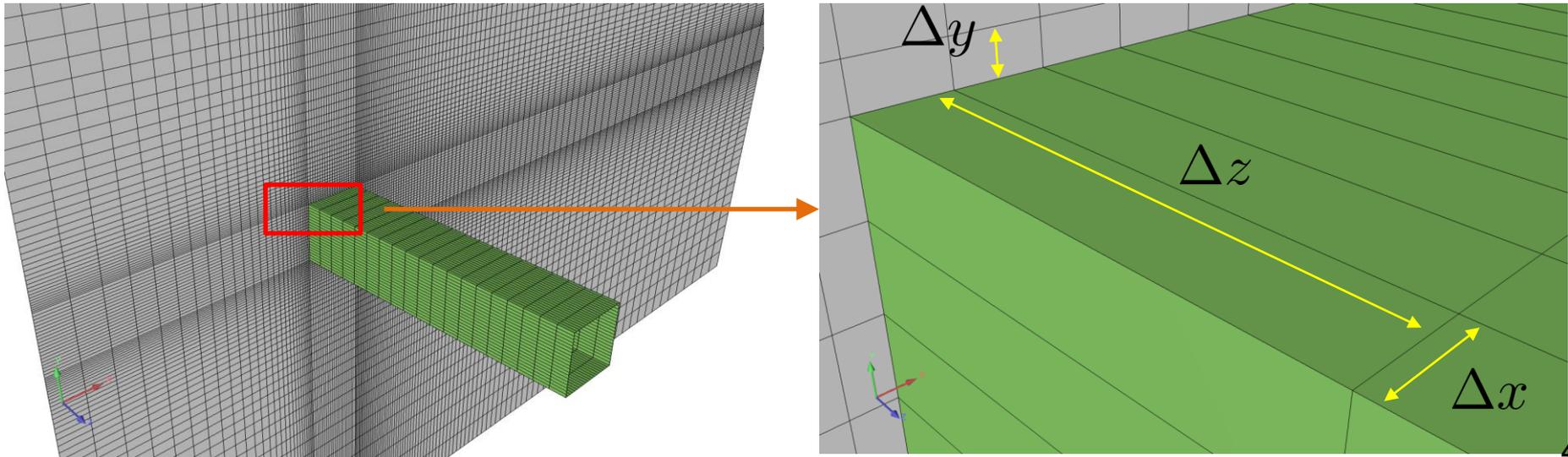
Practical turbulence estimates

Wall distance units $x^+ - y^+ - z^+$

- Besides the surface aspect ratio, you should also check the ratio between surface cell base length and the cell height. This aspect ratio can be approximated as follows,

$$AR = \frac{\sqrt{\text{Surface cell area}}}{\Delta y}$$

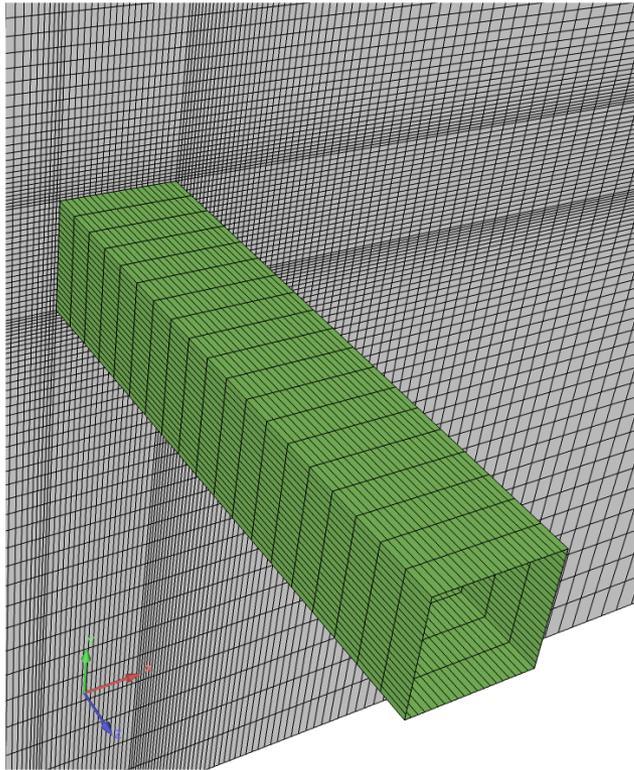
- Recommended values are 20 or less (conservatively speaking).
- As you can see, this aspect ratio can impose strong requirements on the mesh.
- Again, this is a rough estimate based on experience that should be used when conducting SRS simulations



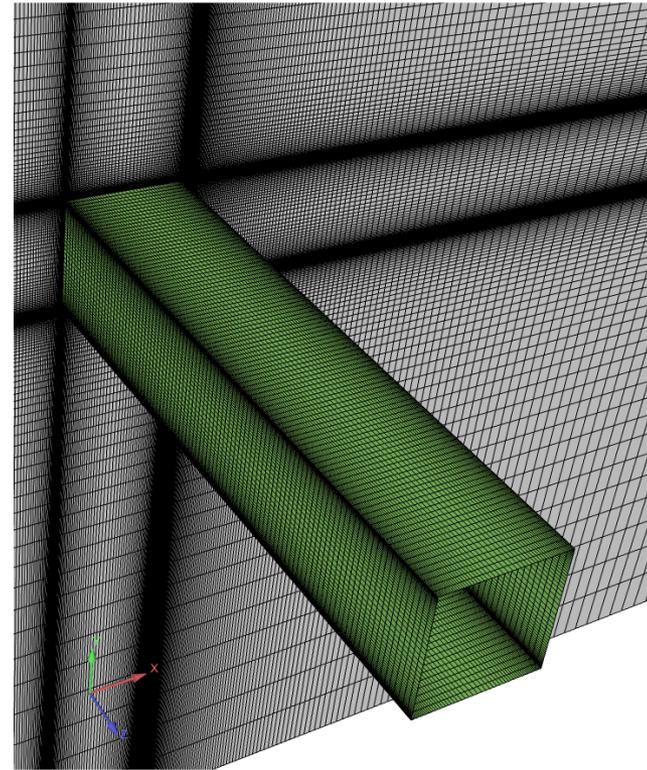
Practical turbulence estimates

Wall distance units $x^+ - y^+ - z^+$

- The wall distance units x^+ and z^+ values strongly depend on the surface mesh aspect ratio.
- After enforcing the two previous geometrical criteria for SRS meshes, your final mesh will be much larger and likely it will resolve 80% of the turbulent spectrum.



Surface mesh with large aspect ratio and large ratio between surface cell base length and the cell height.



Surface mesh with low aspect ratio and low-to-moderate ratio between surface cell base length and the cell height

Practical turbulence estimates

Wall distance units $x^+ - y^+ - z^+$

- The stream-wise and span-wise wall distance units requirements are important when conducting scale-resolving simulations (DES, LES, DNS).
- Typical requirements for LES are (these are recommendations based on different references and personal experience):

$$\Delta x^+ < 50, \Delta z^+ < 50 \quad \text{for} \quad y^+ < 5$$

Wall resolving

$$\Delta x^+ < 4\Delta y^+, \Delta z^+ < 4\Delta y^+ \quad \text{for} \quad 30 \leq y^+ \leq 300$$

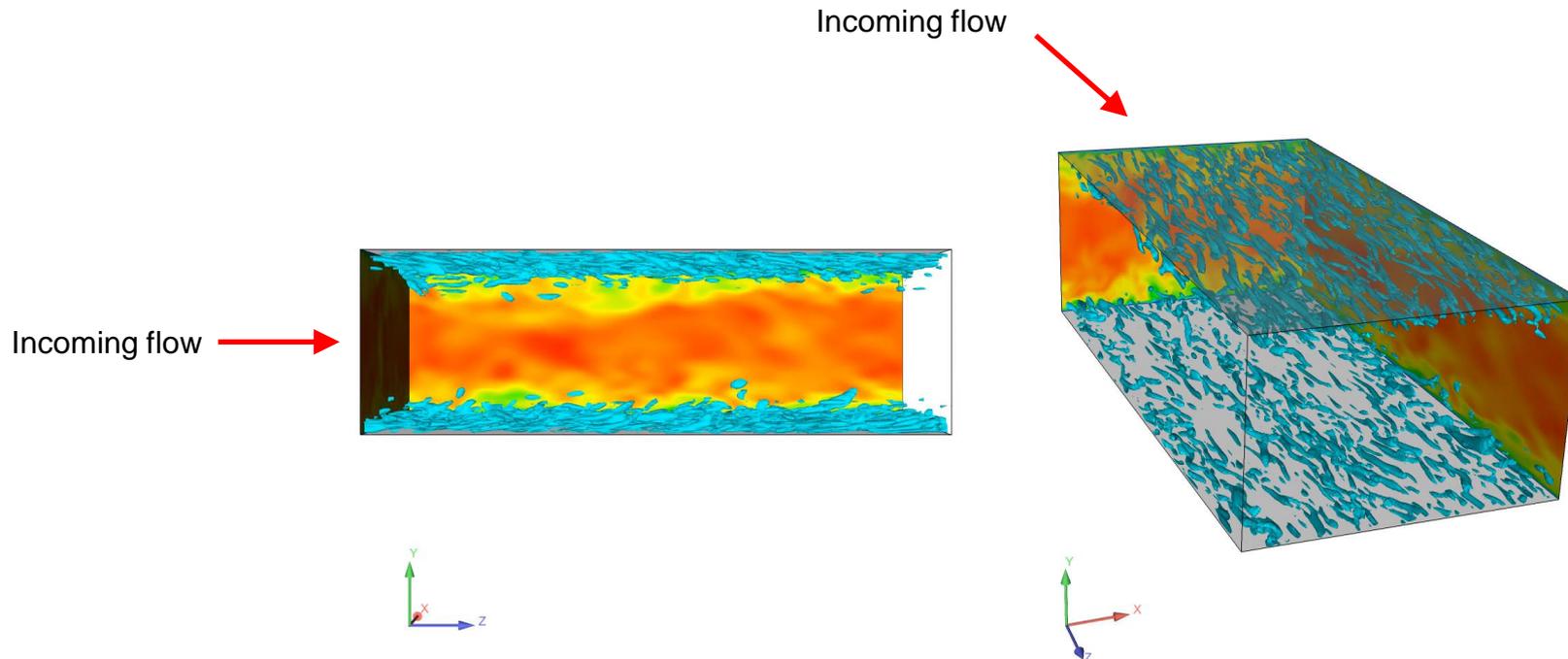
Wall modeling

- As you can see, these requirements translate in much finer surface meshes and low surface aspect ratios.
- We will talk more about this in Lecture 10.
- RANS simulations only have strict requirements respect to y^+ .
 - They do not have strict requirements when it comes to the span-wise and stream-wise directions. Therefore, they are more affordable.

Practical turbulence estimates

Wall distance units $x^+ - y^+ - z^+$

- These strict requirements on SRS simulations are due to the fact that we need to resolve the structures close to the walls (streaks, hairpin vortices, vortex tubes, etc.).
- And this require very fine meshes in all space directions.



Wall bounded flow – Channel flow

<http://www.wolfdynamics.com/training/turbulence/channel1.mp4>

Practical turbulence estimates

Wall distance units – A few mesh resolution guidelines and rough estimates

- The mesh is everything in CFD, and when it comes to turbulence modeling, it is extremely important to have meshes with good quality and acceptable resolution.
- Some general guidelines for meshes to be used with RANS/DES/LES:
 - Resolve well the curvature.
 - Use a mesh growth rate ratio of 1.2 or less.
 - Allow a smooth transition between cells of different sizes (at least 3 cells).
 - Identify the integral scales and try to cluster at least 5 cells in the domain regions where you expect to find these scales.
 - If you have no restrictions, use a wall resolving approach, that is, fine meshes in the direction normal to the walls.
 - It is recommended to avoid hanging nodes (in Lecture 3 we illustrated what they are).

Practical turbulence estimates

Wall distance units – A few mesh resolution guidelines and rough estimates

- The mesh is everything in CFD, and when it comes to turbulence modeling, it is extremely important to have meshes with good quality and acceptable resolution.
- Some guidelines specific to RANS meshes:
 - When it comes to RANS, the most important metric for mesh resolution is the y^+ value.
 - Choose your wall treatment and mesh your domain according to this requirement.
 - If you are doing 3D simulations, there are no strict requirements when it comes to the span-wise and stream-wise directions.
 - But as a rule of thumb rule, you can use Δx^+ and Δz^+ values as high as 300 times the value of Δy^+ and less than a 1000 wall distance units.

Practical turbulence estimates

Wall distance units – A few mesh resolution guidelines and rough estimates

- Some guidelines specific to LES meshes:
 - When it comes to LES meshes, it is recommended to use wall functions.
 - Otherwise, the meshing requirements are similar to those of DNS.
 - Recommended wall distance units values are,

$$\Delta x^+ < 50, \Delta z^+ < 50 \quad \text{for} \quad y^+ < 5$$

Wall resolving

$$\Delta x^+ < 4\Delta y^+, \Delta z^+ < 4\Delta y^+ \quad \text{for} \quad 30 \leq y^+ \leq 300$$

Wall modeling

- If you are doing DNS simulations, the requirements for wall distance units in all directions are in the order of 1.
 - You might be able to go as high as 10 for Δx^+ and Δz^+ .

Practical turbulence estimates

Wall distance units – A few mesh resolution guidelines and rough estimates

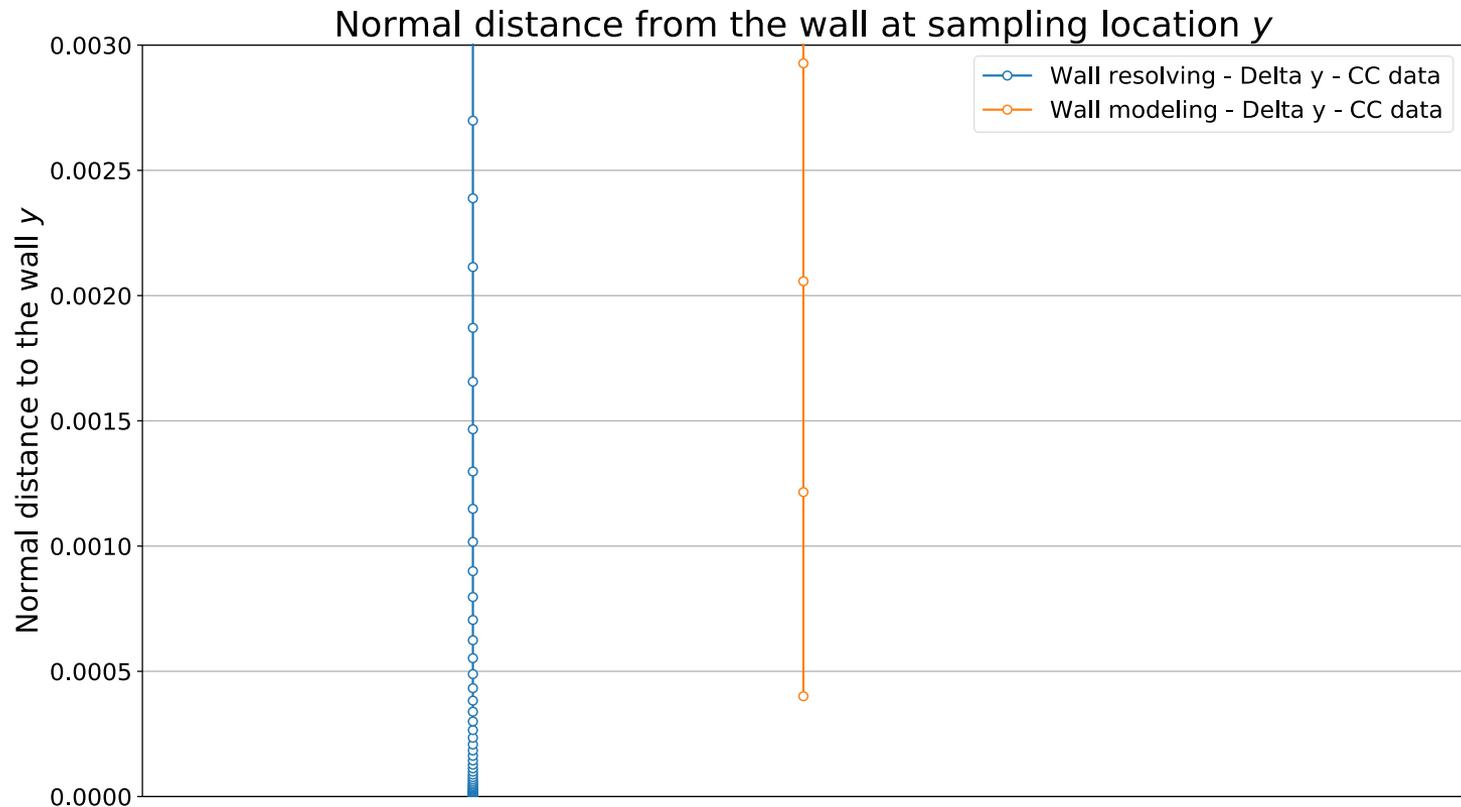
- Some guidelines specific to DES meshes:
 - Usually in DES simulations wall resolving meshes are used close to the walls with requirements similar to RANS.
 - That is, there are no specific requirements on the streamwise and spanwise wall units values.
 - However, it is extremely important to resolve well the integral length scales, as in LES simulations.
 - What is very tricky in DES, is how to control the transition from the RANS mesh close to the walls to the fine-medium mesh far from the walls.
 - You might encounter a mismatch between the RANS mesh and the LES mesh, because the RANS mesh tends to be coarser in the streamwise and spanwise directions.
 - This transition, if not well controlled, can cause grid induced separation and stability problems.

Profiles close to the walls – Dependence on the y^+ value

Practical turbulence estimates

Profiles close to the walls – Dependence on the y^+ value

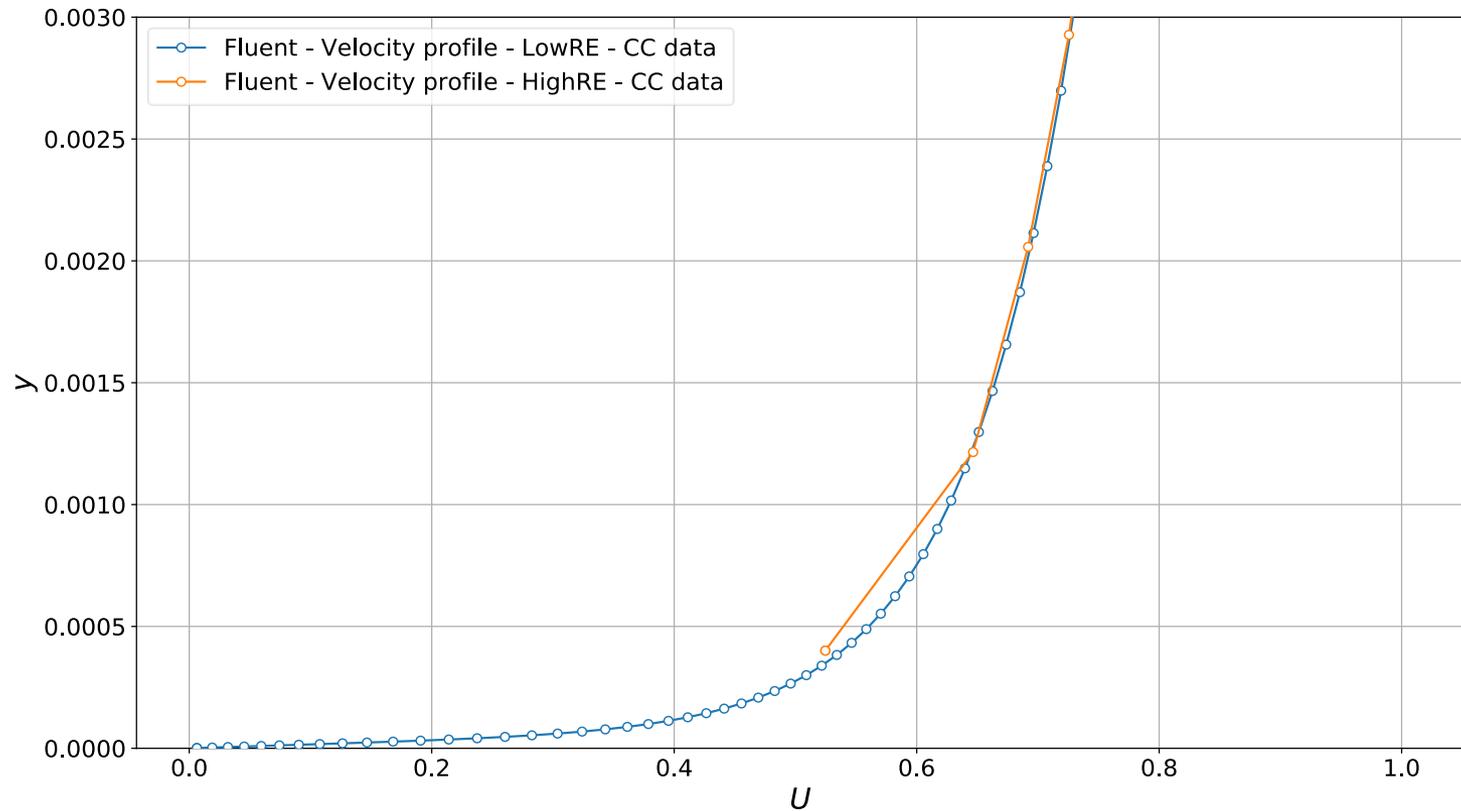
- Distance normal to the wall y (in meters).
- Each circle represents a cell center.



Practical turbulence estimates

Profiles close to the walls – Dependence on the y^+ value

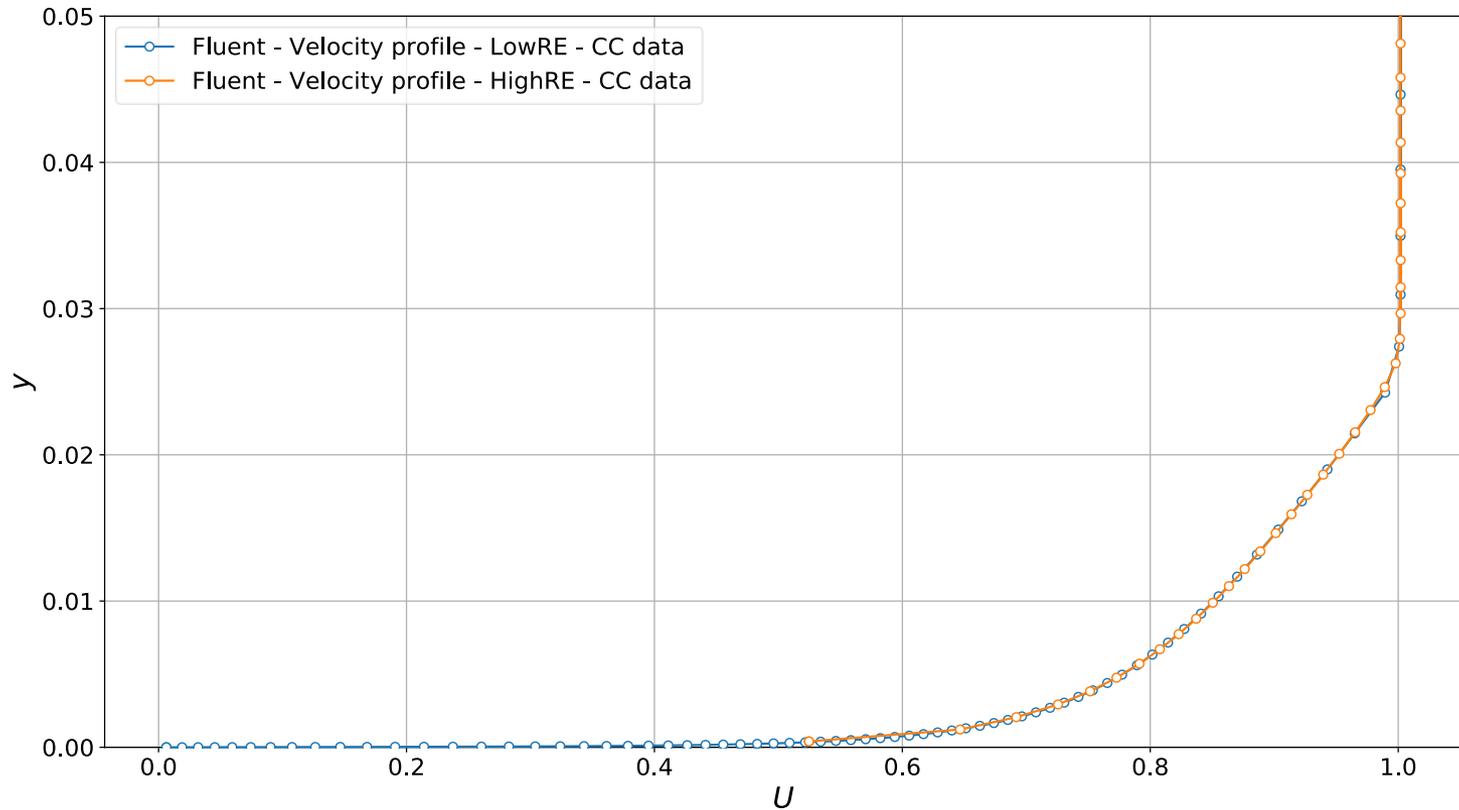
- Velocity profile close to the wall – y vs. U
- Each circle represents a cell center.



Practical turbulence estimates

Profiles close to the walls – Dependence on the y^+ value

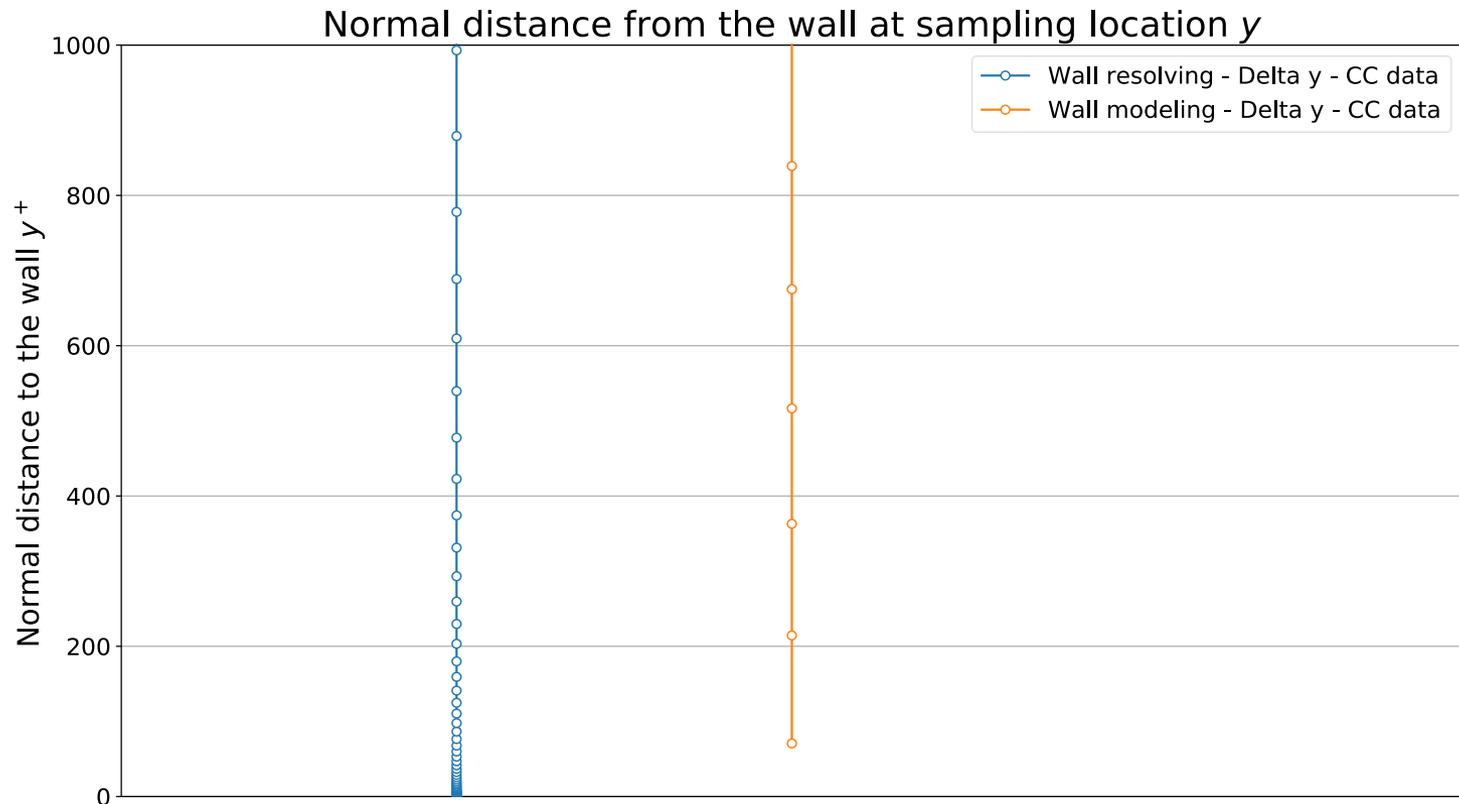
- Velocity profile close to the wall – y vs. U
- Each circle represents a cell center.



Practical turbulence estimates

Profiles close to the walls – Dependence on the y^+ value

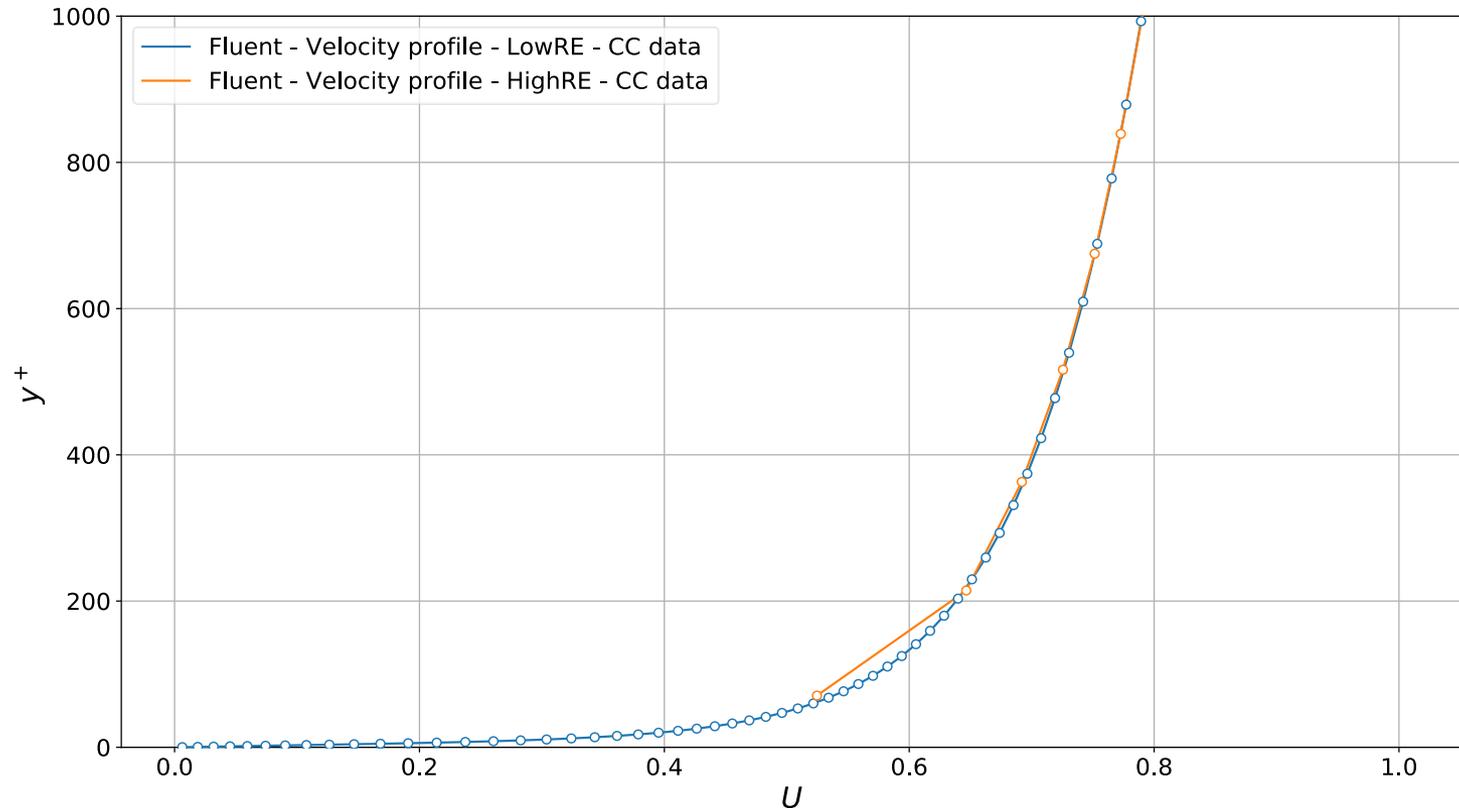
- Non-dimensional distance normal to the wall y^+ .
- Each circle represents a cell center.



Practical turbulence estimates

Profiles close to the walls – Dependence on the y^+ value

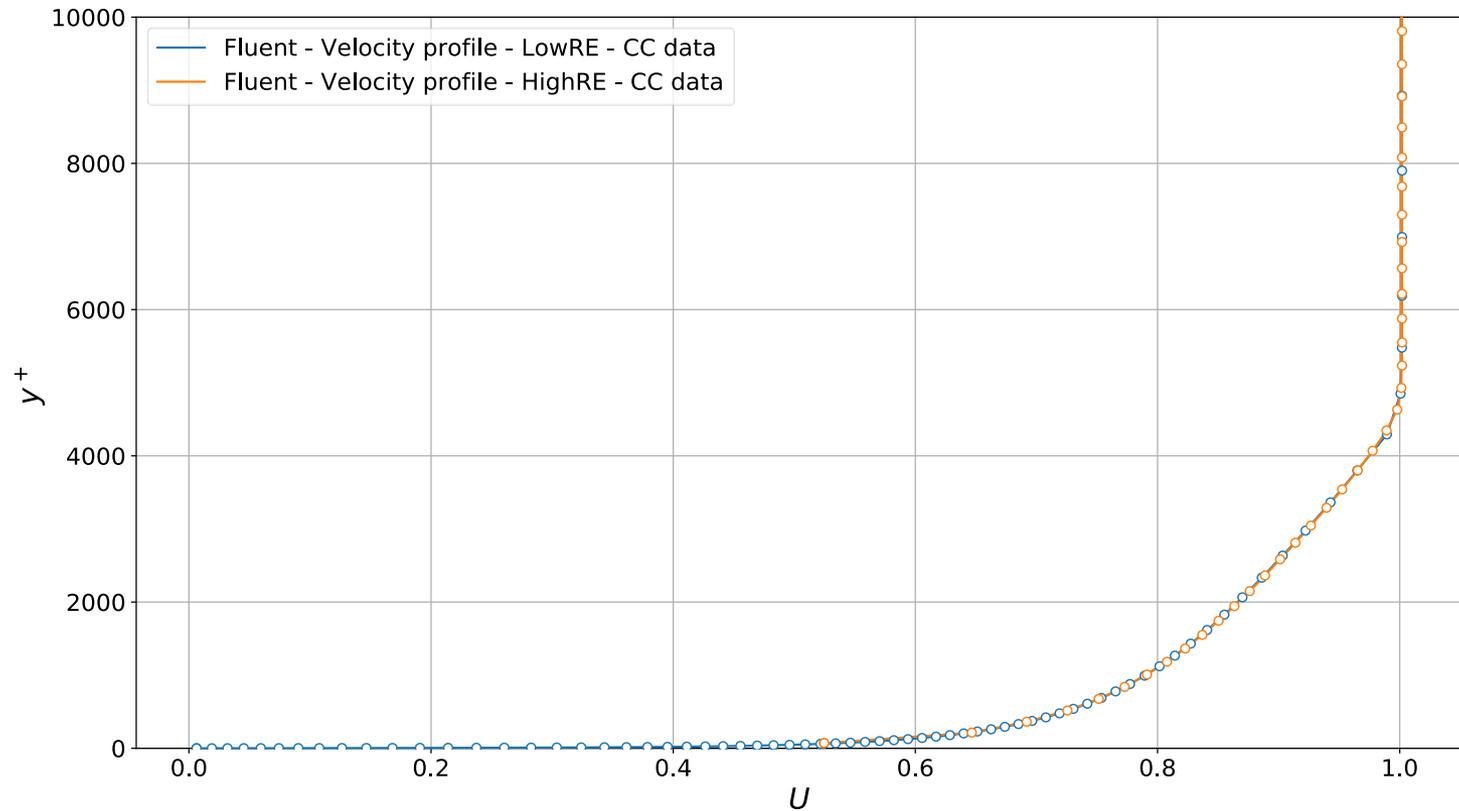
- Velocity profile close to the wall – y^+ vs. U
- Each circle represents a cell center.



Practical turbulence estimates

Profiles close to the walls – Dependence on the y^+ value

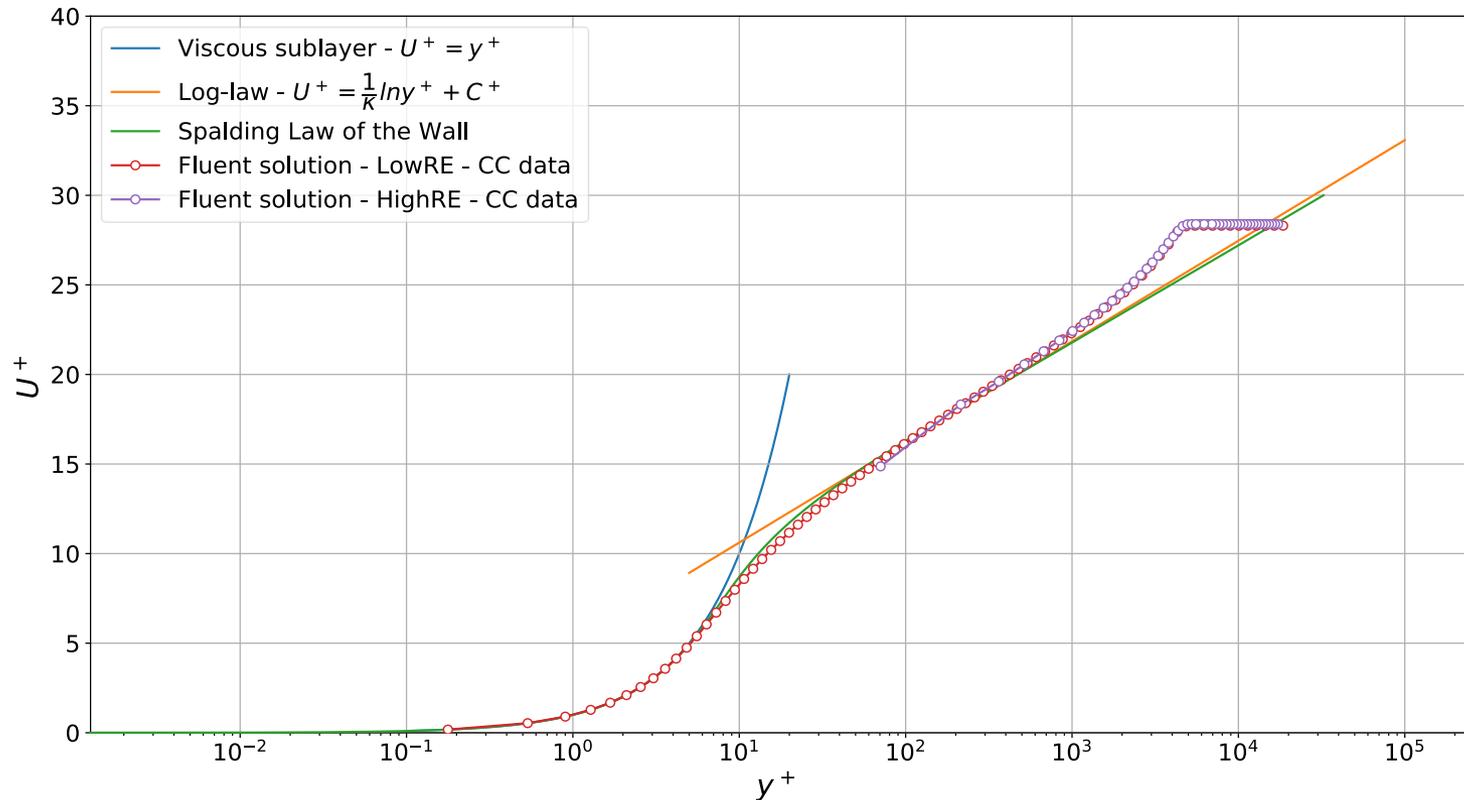
- Velocity profile close to the wall – y^+ vs. U
- Each circle represents a cell center.



Practical turbulence estimates

Profiles close to the walls – Dependence on the y^+ value

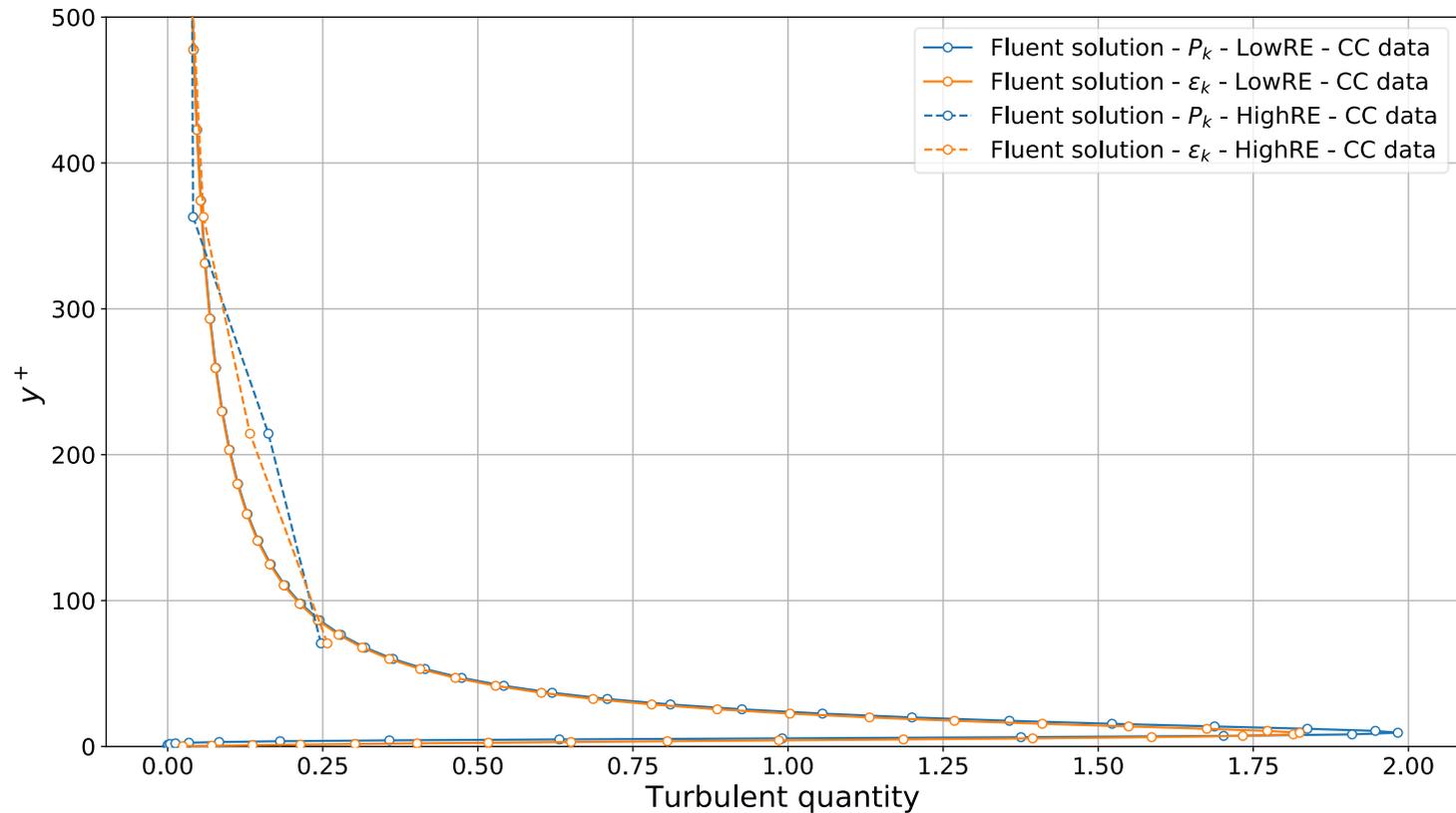
- Normalized velocity profile close to the wall – U^+ vs. y^+
- Each circle represents a cell center.



Practical turbulence estimates

Profiles close to the walls – Dependence on the y^+ value

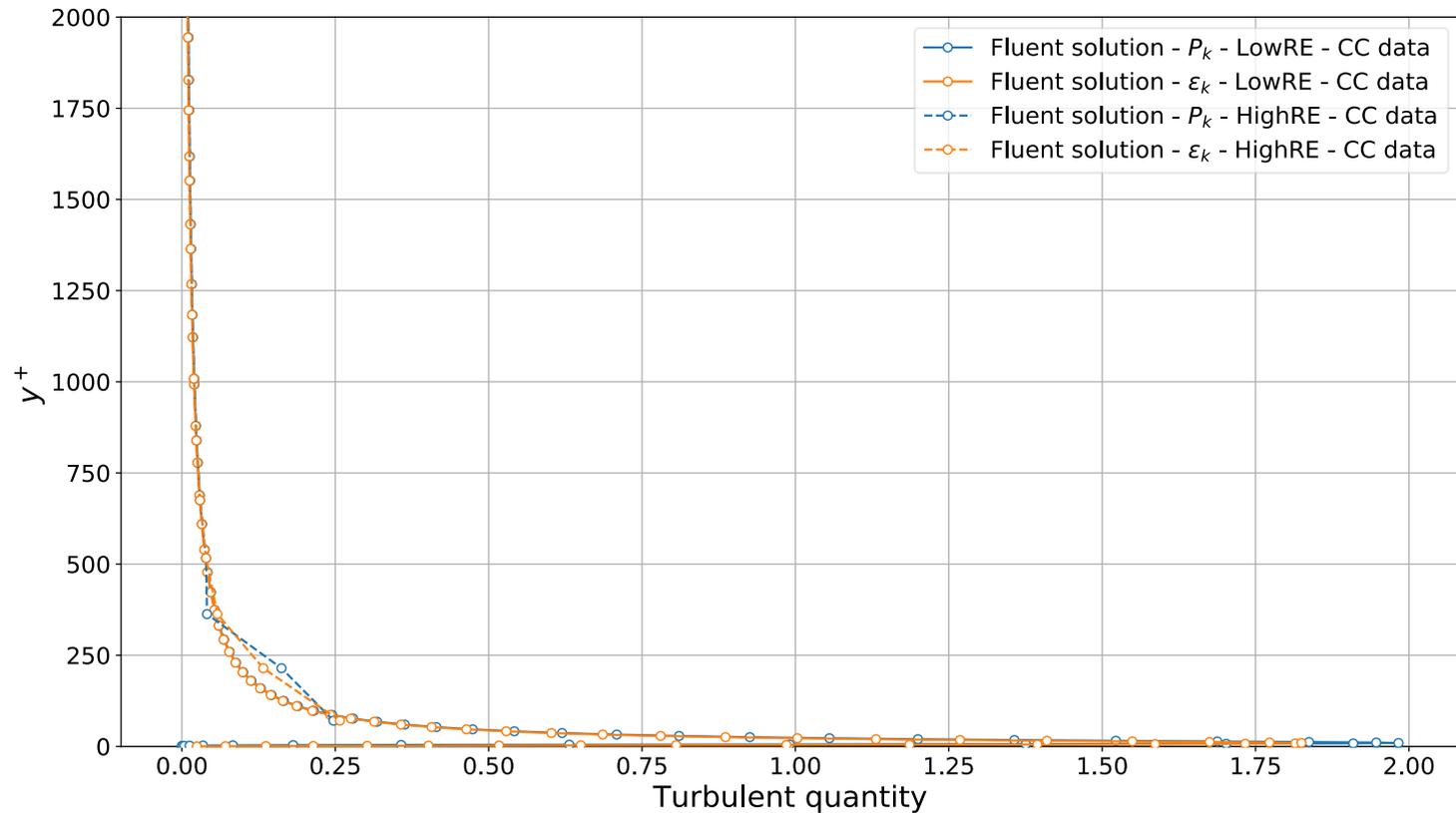
- Turbulent production and dissipation close to the walls.
- Each circle represents a cell center.



Practical turbulence estimates

Profiles close to the walls – Dependence on the y^+ value

- Turbulent production and dissipation close to the walls.
- Each circle represents a cell center.



Summary of turbulence length scales

Practical turbulence estimates

Summary of turbulence length scales

- The Kolmogorov scales are summarized as follows,

$$\eta = \left(\frac{\nu^3}{\epsilon} \right)^{1/4}$$

Length scale

$$\tau_\eta = \left(\frac{\nu}{\epsilon} \right)^{1/2}$$

Time scale

$$v_\eta = (\nu\epsilon)^{1/4}$$

Velocity scale

- The Taylor microscales are summarized as follows,

$$\lambda = \left(\frac{10\nu k}{\epsilon} \right)^{1/2}$$

Length scale

$$\tau_\lambda = \left(\frac{15\nu}{\epsilon} \right)^{1/2}$$

Time scale

$$u_\lambda = \frac{\lambda}{\tau_\lambda}$$

Velocity scale

Practical turbulence estimates

Summary of turbulence length scales

- Taylor [1] suggests that the integral length scales can be approximated as follows,

$$l \approx \frac{k^{3/2}}{\epsilon}$$

- In the $k - \epsilon$ turbulence model, this estimate has been improved using experimental and numerical data [2,3],

$$l = C_\mu \frac{k^{3/2}}{\epsilon} \quad \text{where} \quad C_\mu = 0.09$$

- The previous relation can be expressed in function of ω as follows,

$$l = \frac{k^{1/2}}{\omega} \quad \text{where} \quad \omega = \frac{\epsilon}{\beta^* k} \quad \text{and} \quad \beta^* = \frac{9}{100}$$

[1] G. I. Taylor. Statistical theory of turbulence. Proceedings of the Royal Society of London. 1935.

[2] D. Wilcox. Turbulence Modeling for CFD. DCW Industries Inc., 2010.

[3] W. Jones, B. Launder. The prediction of laminarization with a two-equation model of turbulence. Intl. J. Heat and Mass Transfer, 15, 301-314. 1972.

Practical turbulence estimates

Summary of turbulence length scales

- The eddies turnover time is the ratio between the integral length scales and the velocity $k^{1/2}$ (the measure of the velocity fluctuations around the mean), and it can be computed as follows,

$$\tau_{turnover} \sim \frac{l}{k^{1/2}} \qquad \tau_{turnover} = \frac{C_\mu k}{\epsilon}$$

- The eddy turnover time is a measure of the time it takes an eddy to interact with its surroundings.
- The integral eddy velocity is the ratio of its integral length scale and its turnover time,

$$u_l = \frac{l}{\tau_{turnover}} = k^{1/2}$$

- If you assume isotropic turbulence then, $u_l = \left(\frac{2k}{3}\right)^{1/2}$

Practical turbulence estimates

Summary of turbulence length scales

- The different length scales can be related as follows,

$$\frac{l_0}{\eta} \sim Re_T^{3/4}$$

$$\frac{\lambda}{l_0} = \sqrt{10} Re_T^{-1/2}$$

$$\frac{\lambda}{\eta} = \sqrt{10} Re_T^{1/4}$$

$$\lambda = \sqrt{10} \eta^{2/3} l_0^{1/3}$$

- Support equations:

$$Re_T = \frac{k^{1/2} l_0}{\nu} = \frac{k^2}{\epsilon \nu}$$

$$l_0 = \frac{k^{3/2}}{\epsilon}$$

$$k = \frac{1}{2} \left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right)$$

$$\lambda = \left(\frac{10 \nu k}{\epsilon} \right)^{1/2}$$

Practical turbulence estimates

Summary of Reynolds numbers

- The different Reynolds numbers, based on different length scales, can be summarized as follows,

$$Re = \frac{UL}{\nu}$$

Flow Reynolds number

$$Re_\lambda = \frac{u'\lambda}{\nu}$$

Taylor Reynolds number

$$Re_\eta = \frac{\eta u_\eta}{\nu} = 1$$

Kolmogorov Reynolds number

- The turbulent Reynolds number is related to the integral scales. It is a few order of magnitude lower than the system Reynolds number (on the order of 100 to 10000 times).

$$Re_T = \frac{k^{1/2}l_0}{\nu} = \frac{k^2}{\epsilon\nu}$$

- The turbulence and Taylor Reynolds numbers can be related as follows

$$Re_\lambda = \left(\frac{20}{3} Re_T \right)^{1/2}$$