On the use of dimensional analysis in turbulence modeling

- By using dimensional analysis, the similarity hypotheses, and a lot of intuition, Kolmogorov derived the following relations that determine the smallest scales in turbulence (Kolmogorov scales),

$$\eta = \left(\frac{\nu^3}{\epsilon} \right)^{1/4} \qquad \tau_\eta = \left(\frac{\nu}{\epsilon} \right)^{1/2} \qquad \upsilon_\eta = (\nu \epsilon)^{1/4}$$

Length scale

Time scale

Velocity scale

- These scales are indicative of the smallest eddies, that is, the scales at which the energy is dissipated in turbulent flows.
- In a similar way, the Taylor microscales can be derived.

On the use of dimensional analysis in turbulence modeling

- Based on dimensional grounds, the different turbulent quantities, namely, turbulent viscosity, integral length scales, dissipation rate and so on, can be computed.
    - Kolmogorov (1942), proposed the use of a second transport equation (specific dissipation rate ω) to compute the turbulent quantities. The eddy viscosity, turbulence length scale, and dissipation can be determined from,

$$\nu_t \sim k/\omega, \qquad l \sim k^{1/2}/\omega, \qquad \epsilon \sim \omega k$$

    - Chou (1945), proposed modeling the exact equation for ϵ. In his formulation, the eddy viscosity and turbulence length scale can be determined from,

$$\nu_t \sim k^2/\epsilon, \qquad l \sim k^{3/2}/\epsilon$$

On the use of dimensional analysis in turbulence modeling

- Based on dimensional grounds, the different turbulent quantities, namely, turbulent viscosity, integral length scales, dissipation rate and so on, can be computed.
  - Rotta (1951, 1968), suggested the use of a transport equation for the turbulence length scale. In his formulation, the eddy viscosity and dissipation can be computed from,

$$\nu_t \sim k^{1/2} l, \qquad \epsilon \sim k^{3/2}/l$$

  - Zeierman *et al.* (1986) and Speziale *et al.* (1990), introduced a transport equation for k and the turbulence dissipation time τ. For these models, the eddy viscosity, turbulence length scale, and dissipation can be determined from,

$$\nu_t \sim k\tau, \qquad l \sim k^{1/2}\tau, \qquad \epsilon \sim k/\tau$$

On the use of dimensional analysis in turbulence modeling

- Regardless of the scales derived or the choice of the turbulent quantities, we see a recurring behavior.
- Specifically, eddy viscosity and length scales are all related on the basis of dimensional arguments.
- Historically, dimensional analysis has been one of the most powerful tools available for deducing and correlating properties of turbulent flows.
- **However, we should always be aware that while dimensional analysis is extremely useful, it unveils nothing about the physics underlying its implied relationships.**
- **The physics is in the choice of the variables and the correctness of the hypotheses taken.**

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