Turbulence and CFD models: Theory and applications

Part 3

1. **The Reynolds stress model**

2. **Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress**

3. **Transition models – Review of the** $\gamma - Re_\theta$ **model**

Part 3

1. **The Reynolds stress model**

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3. Transition models – Review of the $\gamma - Re_\theta$ model

The Reynolds stress model

- The extra term appearing in the exact Navier-Stokes RANS/URANS equations is known as the Reynolds stress tensor,

$$\nabla \cdot (\bar{\mathbf{u}}) = 0$$

$$\frac{\partial \bar{\mathbf{u}}}{\partial t} + \nabla \cdot (\bar{\mathbf{u}}\bar{\mathbf{u}}) = -\frac{1}{\rho}\left(\nabla \bar{p}\right) + \nu \nabla^2 \bar{\mathbf{u}} + \frac{1}{\rho}\nabla \cdot \boldsymbol{\tau}^R$$

- Where τ^R is the Reynolds stress tensor, and it can be written as,

$$\tau^R = -\rho\left(\overline{\mathbf{u}'\mathbf{u}'}\right) = -\begin{pmatrix} \rho\overline{u'u'} & \rho\overline{u'v'} & \rho\overline{u'w'} \\ \rho\overline{v'u'} & \rho\overline{v'v'} & \rho\overline{v'w'} \\ \rho\overline{w'u'} & \rho\overline{w'v'} & \rho\overline{w'w'} \end{pmatrix}$$

- So far, we have modeled this term using the Boussinesq approximation.

- When using Reynolds stress models, we directly solver the Reynolds stress tensor.

The Reynolds stress model

- The Reynolds stress tensor τ^R is the responsible for the increased mixing and larger wall shear stresses.

- Remember, increased mixing and larger wall shear stresses are properties of turbulent flows.

- The RANS/URANS approach to turbulence modeling requires the Reynolds stresses to be appropriately modeled, for example, by using eddy viscosity models (EVM).

- However, it is possible to derive its own governing equations (six new equations in 3D as the tensor is symmetric).

- This approach is known as Reynolds stress models (RSM).

- The RSM models are also known as second-order closure (SOC), second-moment closure (SMC), differential stress models (DSM), and stress-transport models (STM).

- Probably, the RSM is the most physically sound RANS/URANS approach as it avoids the use of hypothesis/assumptions to model the Reynolds stress tensor.

- However, it is computationally expensive, and less robust than EVM.

    - It can be unstable if proper boundary conditions and initial conditions are not used.

    - And, as you may guess, it is heavily modeled.

The Reynolds stress model

- The RSM models are more general than EVM models.

- They potentially have better accuracy than the EVM model.

- However, this does not mean that they are better than EVM models.

- RSM models perform better in situations where the EVM models have poor performance,

  - Flows with strong curvature or swirl (cyclone separators and flows with concentrated vortices).

  - Flows in corners with secondary motions.

  - Very complex 3D interacting flows.

  - Highly anisotropic flows.

  - Flows with massive separation.

  - Flows with adverse pressure gradient.

  - Complex shock wave interactions.

- In general, RSM models can be considered in non-equilibrium conditions, that is, when production is not equal to dissipation (any of the previous situations).

- That is, the ratio of production to dissipation is far from one.

The Reynolds stress model

- Let us recall the exact Reynolds stress transport equations,

$$\frac{\partial \tau_{ij}}{\partial t} + \bar{u}_k \frac{\partial \tau_{ij}}{\partial x_k} = -\tau_{ik} \frac{\partial \bar{u}_j}{\partial x_k} - \tau_{jk} \frac{\partial \bar{u}_i}{\partial x_k} + \epsilon_{ij} - \Pi_{ij} + \frac{\partial}{\partial x_k} \left[\nu \frac{\partial \tau_{ij}}{\partial x_k} + C_{ijk} \right]$$

Production term – Notice that it does not require modeling, it is computed from the mean gradients

- Where the following terms require modeling,

$$\epsilon_{ij} = 2\nu \overline{\frac{\partial u_i'}{\partial x_k} \frac{\partial u_j'}{\partial x_k}}$$ ⟵ Dissipation tensor

$$\Pi_{ij} = \overline{\frac{p'}{\rho} \left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i} \right)}$$ ⟵ Pressure-strain correlation tensor

$$\rho C_{ijk} = \rho \overline{u_i' u_j' u_k'} + \overline{p' u_i'} \delta_{jk} + \overline{p' u_j'} \delta_{ik}$$ ⟵ Turbulent transport tensor

- The most critical term when using RSM models is the pressure-strain term.
- RSM models differ by how this term is modeled.

The Reynolds stress model

- The dissipation tensor of the Reynolds stress equations can be modeled as follows

$$\epsilon_{ij} = \frac{2}{3}\epsilon\delta_{ij}$$

- Where ϵ denotes the dissipation rate of turbulence kinetic energy,

$$\epsilon = \nu\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}}$$

- Basically, we are assuming that the small scales are isotropic.

- The use of this assumption avoids the need for employing a dissipation transport equation for each component of the Reynolds stress tensor.

- Which results in a reduction in the number of transport equation to be solved and thus the computational cost.

- It is clear that ϵ needs to be modeled.

- For this we use a similar approach to the one used in the two-equations models presented in the previous lectures.

- Most of the time, the turbulent dissipation rate transport equation ϵ is solved.

The Reynolds stress model

- The turbulent transport tensor of the Reynolds stress equations is also a tensor and can be modeled as follows,

$$C_{ijk} = \frac{\partial}{\partial x_k} \left[\frac{\mu_t}{\sigma} \frac{\partial}{\partial x_k} \left(\overline{u_i' u_j'} \right) \right]$$

- Using this approach [1], the turbulent transport tensor is modeled using a gradient-diffusion model (this is the easiest and most robust approach).

- And alternative approach is the one proposed by Daly and Harlow [2],

$$C_{ijk} = C_s \frac{2}{3} \frac{k^2}{\epsilon} \left[\frac{\partial \tau_{jk}}{\partial x_i} + \frac{\partial \tau_{ik}}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_k} \right]$$

- Have in mind that there are more complex forms to model the turbulent transport tensor, but they are not very robust for industrial applications.

[1] F. S. Lien, M. A. Leschziner. Assessment of Turbulent Transport Models Including Non-Linear RNG Eddy-Viscosity Formulation and Second-Moment Closure. 1994.
[2] B. J. Daly, F. H. Harlow. Transport Equations in Turbulence. 1970.

The Reynolds stress model

- The modeling of the pressure-strain term is critical. It contains complex correlations that are difficult to measure.

- Major difference between RSM models is due to the approach taken to model this term.

- The pressure-strain tensor can be decomposed as follows,

$$\Pi_{ij} = \underbrace{\Pi_{ij,1}}_{\text{Slow pressure strain term}} + \underbrace{\Pi_{ij,2}}_{\text{Fast pressure strain term}}$$

- To most widely used approach to model this term is the LRR [1] method, which is given as follows,

$$\Pi_{ij,1} = -C_1 \frac{\epsilon}{k}\left(\tau_{ij} - \frac{2}{3}\delta_{ij}k\right) \qquad \Pi_{ij,2} = -C_2\left(P_{ij} - \frac{2}{3}\delta_{ij}P\right)$$

$$P_{ij} = -\tau_{ki}\frac{\partial \overline{u}_j}{\partial x_k} - \tau_{kj}\frac{\partial \overline{u}_i}{\partial x_k} \qquad P = \frac{1}{2}P_{kk} \quad \longleftarrow \quad \text{Capital P stand for production not pressure}$$

[1] B. E. Launder, G. J. Reece, W. Rodi. Progress in the Development of a Reynolds-Stress Turbulence Closure. 1975.

- The **solvable** RSM equations of the LRR model are given as follows,

$$\frac{\partial \tau_{ij}}{\partial t} + \bar{u}_k \frac{\partial \tau_{ij}}{\partial x_k} = -\tau_{ik}\frac{\partial \bar{u}_j}{\partial x_k} - \tau_{jk}\frac{\partial \bar{u}_i}{\partial x_k} + \epsilon_{ij} - \Pi_{ij} + \frac{\partial}{\partial x_k}\left[\nu\frac{\partial \tau_{ij}}{\partial x_k} + C_{ijk}\right]$$

- With the following auxiliary relationships,

$$C_{ijk} = \frac{\partial}{\partial x_k}\left[\frac{\mu_t}{\sigma}\frac{\partial}{\partial x_k}\left(\overline{u_i' u_j'}\right)\right]$$

$$\Pi_{ij} = -C_1\frac{\epsilon}{k}\left(\tau_{ij} - \frac{2}{3}\delta_{ij}k\right) - C_2\left(P_{ij} - \frac{2}{3}\delta_{ij}P_k\right)$$

If you compare this term with the original formulation of the LRR method, you will notice that this term has been further simplified

$$P_{ij} = -\tau_{ki}\frac{\partial \overline{u}_j}{\partial x_k} - \tau_{kj}\frac{\partial \overline{u}_i}{\partial x_k}$$

$$P = \frac{1}{2}P_{kk}$$

$$\epsilon_{ij} = \frac{2}{3}\epsilon\delta_{ij} \qquad \text{and} \qquad \epsilon = \nu\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}}$$

- And the following closure coefficients,

$$\sigma = 0.82 \qquad C_1 = 1.8 \qquad C_2 = 0.6$$

- With the following auxiliary relation for the kinematic eddy viscosity (which is used in the turbulent transport tensor of the Reynolds [1]),

$$\mu_t = \rho C_\mu \frac{k^2}{\epsilon}$$

- We also need to solve an additional transport equation for the turbulent dissipation rate ϵ,

$$\nabla_t \epsilon + \nabla \cdot (\bar{\mathbf{u}} \epsilon) = C_{\epsilon_1} \frac{\epsilon}{k} \tau^R : \nabla \bar{\mathbf{u}} - C_{\epsilon_2} \frac{\epsilon^2}{k} + \nabla \cdot \left[\left(\nu + \frac{\nu_t}{\sigma_\epsilon} \right) \nabla \epsilon \right]$$

- This equation is used to compute the turbulent dissipation rate in the solvable Reynolds stress equation and in the turbulent eddy viscosity auxiliary equation.

[1] F. S. Lien, M. A. Leschziner. Assessment of Turbulent Transport Models Including Non-Linear RNG Eddy-Viscosity Formulation and Second-Moment Closure. 1994.

The Reynolds stress model

- The Reynolds stress model (RSM) [1, 2, 3, 4] is the most elaborate type of RANS turbulence model.

- It abandons the isotropic eddy-viscosity hypothesis.

- But still it is computed as a scalar quantity.

- The RSM closes the RANS equations by solving transport equations for the Reynolds stresses, together with an equation for the turbulent dissipation rate or the specific dissipation rate.

- This means that five additional transport equations are required in 2D flows, and seven additional transport equations are solved in 3D.

- Then, the Reynolds stresses are inserted directly into the momentum equations.

- If additional scalars are present (temperature, passive scalars, and so on), three additional equations need to be added.

- If the turbulent kinetic energy equation is needed for specific terms, it is obtained by taking the trace of the Reynolds tress tensor.

- The most used versions of the RSM are the LRR [3] and the SSG [5], both based on the turbulent dissipation rate equation.

[1] M. M. Gibson, B. E. Launder. Ground Effects on Pressure Fluctuations in the Atmospheric Boundary Layer. 1978.
[2] B. E. Launder. Second-Moment Closure: Present... and Future?. 1989.
[3] B. E. Launder, G. J. Reece, W. Rodi. Progress in the Development of a Reynolds-Stress Turbulence Closure. 1975.
[4] B. J. Daly, F. H. Harlow. Transport Equations in Turbulence. 1970.
[5] C. G. Speziale, S. Sarkar, T. B. Gatski. Modelling the Pressure-Strain Correlation of Turbulence: An Invariant Dynamical Systems Approach. 1991.

The Reynolds stress model

- The RSM might not always yield results that are clearly superior to EVM models.

- However, the use of the RSM is a must when the flow features of interest are the result of anisotropy in the Reynolds stresses.

- Among the examples are cyclone flows, highly swirling flows in combustors, rotating flow passages, and the stress-induced secondary flows in ducts.

- Despite its apparent superiority over EVM models, the RSM is not widely used.

- Also, the RSM is not widely validated as other EVM models.

- There are also algebraic version of the RSM models that solve two equations.

  - Explicit Algebraic Reynolds Stress Model [1, 2], which can be seen as an extension of the $k - \epsilon$ and $k - \omega$ family models.

[1] W. Rodi. A New Algebraic Relation for Calculating Reynolds Stress. 1976.
[2] S. Girimaji. Fully Explicit and Self-Consistent Algebraic Reynolds Stress Model. 1996.

The Reynolds stress model

- The RSM model can be used with wall functions or a wall resolving approach.

- The wall boundary conditions for the solution variables are all taken care of by the wall functions implementation.

- Therefore, when using commercial solvers (Fluent in our case) you do not need to be concerned about the boundary conditions at the walls.

- If you are using a wall resolving approach, all Reynolds stresses must approach zero in an asymptotic way at the wall.

- The freestream values can be estimated as follows,

$$\overline{u_1'^2} = k$$

$$\overline{u_2'^2} = \overline{u_3'^2} = \frac{1}{2}k$$

$$\overline{u_i'u_j'} = 0 \quad (i \neq j)$$

$$\overline{u'^2} : \overline{v'^2} : \overline{w'^2} \approx 4 : 2 : 3$$
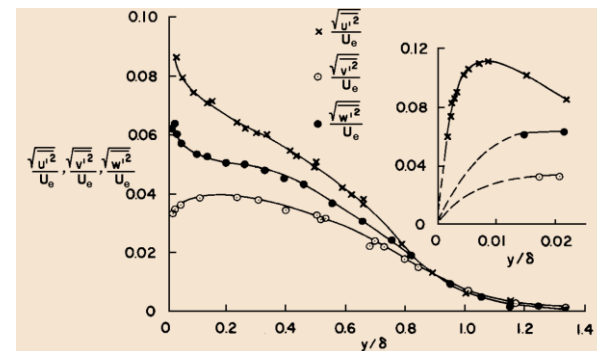
Notice that the Reynolds stress are anisotropic

- The boundary condition for turbulent dissipation rate or specific dissipation rate are determined in the same manner as for the two-equations turbulence models.

Part 3

1. ~~The Reynolds stress model~~

2. **Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress**

3. Transition models – Review of the $\gamma - Re_\theta$ model

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Now that we have addressed the main EVM, let us talk about the budgets of turbulence kinetic energy, dissipation rate, and Reynolds stresses.

- We have seen so far that the transport equations of the turbulent quantities can be expressed in the following way,

$$\underbrace{\nabla_t \phi}_{\text{Transient term}} + \underbrace{\nabla \cdot \bar{\mathbf{u}} \phi}_{\text{Convection}} = \underbrace{P^\phi}_{\text{Production}} + \underbrace{\epsilon^\phi}_{\text{Dissipation}} + \underbrace{D^\phi}_{\text{Diffusion}} + \underbrace{S^\phi}_{\text{Source terms}}$$

- Where ϕ represents the transported turbulent quantity.

- At the same time, each term can be decomposed into sub-terms.

- For example, the pressure-strain tensor in the RSM model can be decomposed into the following contributions,

$$\Pi_{ij} = \underbrace{\Pi_{ij,1}}_{\text{Slow pressure strain term}} + \underbrace{\Pi_{ij,2}}_{\text{Fast pressure strain term}}$$

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

$$\underbrace{\nabla_t \phi}_{\text{Transient term}} + \underbrace{\nabla \cdot \bar{\mathbf{u}} \phi}_{\text{Convection}} = \underbrace{P^\phi}_{\text{Production}} + \underbrace{\epsilon^\phi}_{\text{Dissipation}} + \underbrace{D^\phi}_{\text{Diffusion}} + \underbrace{S^\phi}_{\text{Source terms}}$$

- The budget data represent the contribution of each term to the whole turbulence process (production, dissipation, transport, redistribution, and diffusion).

- Detailed budgets for the turbulence kinetic energy, production rate, dissipation rate, diffusion rate, and Reynolds stresses are usually computed using DNS or LES data of canonical flows or experimental measurements.

- This data reveal a lot of about the turbulent characteristics of the flow and the profiles of the transported quantities.

- In wall bounded flows, all the terms in the budget become important close to the wall.

- The budget data is used for model development, to test existing closure models, and to calibrate closure coefficients.

- This is hardcore model validation.

- These budgets are usually plotted in normalized units.

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Before continuing, recall that the exact and resolved Reynolds stresses can be computed as follows,

$$\tau^R = \tau^R_{ij} = -\rho \left(\overline{\mathbf{u}'\mathbf{u}'} \right) = - \begin{pmatrix} \rho\overline{u'u'} & \rho\overline{u'v'} & \rho\overline{u'w'} \\ \rho\overline{v'u'} & \rho\overline{v'v'} & \rho\overline{v'w'} \\ \rho\overline{w'u'} & \rho\overline{w'v'} & \rho\overline{w'w'} \end{pmatrix}$$

- Where the fluctuating quantifies can be computed as follows,

$$\phi' = \phi - \overline{\phi}$$

- In our discussion, the resolved Reynolds stresses are those computed using Reynolds stress models.

- And the exact Reynolds stresses are those computed using the fluctuation quantities via SRS simulations (LES or DNS).

- That is, using SRS simulations we are computing the average of the product of the fluctuating velocities or $\overline{\mathbf{u}'\mathbf{u}'}$.

- Which is very different to the product of the average of the fluctuating velocities or $\overline{\mathbf{u}}'\overline{\mathbf{u}}'$.

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Also, recall that the modelled Reynolds stresses can be computed as follows (Boussinesq hypothesis),

$$\tau_{ij}^{R} = -\rho\overline{u_i' u_j'} = \begin{pmatrix} 2\mu S_{11} - \frac{2}{3}k & 2\mu S_{12} & 2\mu S_{13} \\ 2\mu S_{21} & 2\mu S_{22} - \frac{2}{3}k & 2\mu S_{23} \\ 2\mu S_{31} & 2\mu S_{32} & 2\mu S_{33} - \frac{2}{3}k \end{pmatrix}$$

Note:

$$S_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$$

- This second rank tensor represents the modelled Reynolds stresses using the Boussinesq hypothesis.

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

$$\underbrace{\nabla_t \phi}_{\text{Transient term}} + \underbrace{\nabla \cdot \bar{\mathbf{u}} \phi}_{\text{Convection}} = \underbrace{P^\phi}_{\text{Production}} + \underbrace{\epsilon^\phi}_{\text{Dissipation}} + \underbrace{D^\phi}_{\text{Diffusion}} + \underbrace{S^\phi}_{\text{Source terms}}$$

- When using RANS/URANS models it is possible to plot the budgets. However, we can only plot the data computed by the model.

  - If you are running steady simulations:

    - Using two equations models, such as the $k - \epsilon$ and the $k - \omega$ family of turbulence models, you cannot plot the resolved Reynolds stresses as they are not computed by the models (everything is modeled using the Boussinesq hypothesis).

    - Using the standard Spalart-Allmaras model, you cannot plot the turbulent kinetic energy, neither the resolved Reynolds stresses. They are not computed by the models.

    - Same applies for different turbulence models. You can only access the information that is directly computed by the model.

    - However, using the Reynolds stress models, you can plot the resolved Reynolds stresses for steady and unsteady simulations.

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

$$\underbrace{\nabla_t \phi}_{\text{Transient term}} + \underbrace{\nabla \cdot \bar{\mathbf{u}} \phi}_{\text{Convection}} = \underbrace{P^\phi}_{\text{Production}} + \underbrace{\epsilon^\phi}_{\text{Dissipation}} + \underbrace{D^\phi}_{\text{Diffusion}} + \underbrace{S^\phi}_{\text{Source terms}}$$

- When using RANS/URANS models it is possible to plot the budgets. However, we can only plot the data computed by the model.

  - If you are running unsteady simulations:

    - As for steady simulations, you can only access the information that is directly computed by the model.

    - However, if you compute the unsteady statistics (which is strongly recommended), you will be able to compute the fluctuating quantities; therefore, the Reynolds stresses and the kinetic turbulent energy.

    - To compute the fluctuations of the transported quantities, you need to compute the instantaneous values and the mean values,

    $$\phi' = \phi - \overline{\phi}$$

    - An remember the following averaging rule,

    $$\overline{\phi' \psi'} \neq \bar{\phi}' \bar{\psi}'$$

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- To stress this point, if you are running URANS or scale resolving simulations (SRS), and if you want to compute the Reynolds stresses, and if this information is not available directly from the solver, you need to compute the average of the product of the fluctuations.

- And before computing the average of the product of the fluctuations, you need to compute the fluctuations.

- Same applies for the computation of the budgets.

- Therefore, before running the simulation, remember to set the unsteady statistics and define the products of the fluctuating quantities (or the term appearing in the budget).

- By the default, these quantities might not be computed by the solver.

- All the correlations appearing in the Reynolds stress tensor or the terms appearing in the budget of the governing equations, are computed from the instantaneous fields.

- Recall that the Reynolds stress tensor read as,

$$
\tau^R = \tau_{ij}^R = -\rho \left(\overline{\mathbf{u}'\mathbf{u}'} \right) = -
\begin{pmatrix}
\rho \overline{u'u'} & \rho \overline{u'v'} & \rho \overline{u'w'} \\
\rho \overline{v'u'} & \rho \overline{v'v'} & \rho \overline{v'w'} \\
\rho \overline{w'u'} & \rho \overline{w'v'} & \rho \overline{w'w'}
\end{pmatrix}
$$

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- The budget of the **Reynolds stresses** can be computed using the exact Reynolds stress transport equation.

$$\underbrace{\frac{\partial \tau_{ij}^R}{\partial t}}_{1} + \underbrace{\bar{u}_k \frac{\partial \tau_{ij}^R}{x_k}}_{2} = \underbrace{-\left(\tau_{ik}^R \frac{\partial \bar{u}_j}{\partial x_k} + \tau_{jk}^R \frac{\partial \bar{u}_i}{\partial x_k} \right)}_{3} + \underbrace{2\nu \overline{\frac{\partial u_i'}{\partial x_k} \frac{\partial u_j'}{\partial x_k}}}_{4} + \cdots$$

$$\cdots + \underbrace{\frac{1}{\rho} \left(\overline{u_i' \frac{\partial p'}{\partial x_j}} + \overline{u_j' \frac{\partial p'}{\partial x_i}} \right)}_{5} + \underbrace{\frac{\partial}{\partial x_k} \left(\nu \frac{\partial \tau_{ij}^R}{\partial x_k} \right)}_{6} + \underbrace{\frac{\partial}{\partial x_k} \left(\overline{u_i' u_j' u_k'} \right)}_{7}$$

1. Transient stress rate of change term.
2. Convective term.
3. Production term.
4. Dissipation rate.
5. Turbulent stress transport related to the velocity and pressure fluctuations.
6. Rate of viscous stress diffusion (molecular).
7. Diffusive stress transport resulting from the triple correlation of velocity fluctuations.

- So, if you have good quality data, all these terms can be computed.

- Usually, this data comes from DNS simulations or experiments.

- You can also use data coming from SRS models (LES, DES, SAS, URANS).

- However, have in mind that turbulence models (e.g., EVM) add approximations to the terms appearing in the exact governing equations, so you will be plotting the modelled terms.

- To plot the resolved terms, you need to avoid the use of approximations.

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- The budget of the **turbulent kinetic energy** can be computed using the exact turbulent kinetic energy transport equation.

- By taking the trace ($i = j$) of the Reynolds stress equation we obtain,

This term is negative

$$\underbrace{\frac{\partial \tau_{ii}}{\partial t}}_{1} + \underbrace{\bar{u}_k \frac{\partial \tau_{ii}}{\partial x_k}}_{2} = \underbrace{2\tau_{ij} \frac{\partial \bar{u}_i}{\partial x_j}}_{3} + \underbrace{\epsilon_{ii}}_{4} + \underbrace{\frac{\partial}{\partial x_k} \left(\nu \frac{\partial \tau_{ii}}{\partial x_k} \right)}_{5} + \underbrace{\frac{2}{\rho} \left(\overline{u'_i \frac{\partial p'}{\partial x_i}} \right)}_{6} + \underbrace{\frac{\partial}{\partial x_k} (\overline{u'_i u'_i u'_k})}_{7}$$

1. Transient rate of change term.
2. Convective term.
3. Production term arising from the product of the Reynolds stress and the velocity gradient.
4. Dissipation rate.

5. Rate of viscous stress diffusion (molecular).
6. Turbulent transport associated with the eddy pressure and velocity fluctuations.
7. Diffusive turbulent transport resulting from the triple correlation of velocity fluctuations.

- So, if you have good quality data, all these terms can be computed.

- Usually, this data comes from DNS simulations or experiments.

- You can also use data coming from SRS models (LES, DES, SAS, URANS).

- However, have in mind that turbulence models (e.g., EVM) add approximations to the terms appearing in the exact governing equations, so you will be plotting the modelled terms.

- To plot the resolved terms, you need to avoid the use of approximations.

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- The exact turbulent kinetic energy transport equation, can also be written as,

$$\frac{\partial k}{\partial t} + \bar{u}_j \frac{\partial k}{\partial x_j} = \tau_{ij} \frac{\partial \bar{u}_i}{\partial x_j} - \epsilon + \frac{\partial}{\partial x_j} \left[\nu \frac{\partial k}{\partial x_j} - \frac{1}{2} \overline{u_i' u_i' u_j'} - \frac{1}{\rho} \overline{p' u_j'} \right]$$

- Where we have used the following relations,

$$k = \frac{1}{2} \overline{u_i' u_i'} = \frac{1}{2} \left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right) \qquad \tau_{ii} = \tau_{ii}^R = - \left(\overline{\mathbf{u}' \mathbf{u}'} \right)^{\mathrm{tr}} = - \left(\overline{u_i' u_j'} \right) = -2k$$

- The dissipation rate (per unit mass) ϵ is given by the following relation,

$$\epsilon_{ii} = \epsilon = \nu \overline{\frac{\partial u_i'}{\partial x_j} \frac{\partial u_i'}{\partial x_j}}$$

- The budget of the turbulence dissipation rate can be computed using the exact turbulence dissipation transport equation.

$$\frac{\partial \epsilon}{\partial t} + \overline{u}_j \frac{\partial \epsilon}{\partial x_j} = -2\nu \frac{\partial \overline{u}_i}{\partial x_j} \left(\overline{\frac{\partial u'_i}{\partial x_k} \frac{\partial u'_j}{\partial x_k}} + \overline{\frac{\partial u'_k}{\partial x_i} \frac{\partial u'_k}{\partial x_j}} \right) - 2\nu \frac{\partial^2 \overline{u}_i}{\partial x_k \partial x_j} \overline{u'_k \frac{\partial u'_i}{\partial x_j}}$$

$$\underbrace{}_{1} \quad \underbrace{}_{2} \quad \underbrace{}_{3} \quad \underbrace{}_{4}$$

$$-2\nu \overline{\frac{\partial u'_i}{\partial x_k} \frac{\partial u'_i}{\partial x_m} \frac{\partial u'_k}{\partial x_m}} - 2\nu^2 \overline{\frac{\partial^2 u'_i}{\partial x_k \partial x_m} \frac{\partial^2 u'_i}{\partial x_k \partial x_m}}$$

$$\underbrace{}_{5} \quad \underbrace{}_{6}$$

$$+\nu \frac{\partial \left(\frac{\partial \epsilon}{\partial x_j} \right)}{\partial x_j} - \nu \frac{\partial \left(\overline{u'_j \frac{\partial u'_i}{\partial x_m} \frac{\partial u'_i}{\partial x_m}} \right)}{\partial x_j} - 2\frac{\nu}{\rho} \frac{\partial \left(\overline{\frac{\partial p'}{\partial x_m} \frac{\partial u'_j}{\partial x_m}} \right)}{\partial x_j}$$

$$\underbrace{}_{7} \quad \underbrace{}_{8} \quad \underbrace{}_{9}$$

1. Transient rate of change term.
2. Convective term.
3. Production term that arises from the product of the gradients of the fluctuating and mean velocities.
4. Production term that generates additional dissipation based on the fluctuating and mean velocities.
5. Dissipation (destruction) associated with eddy velocity fluctuating gradients.
6. Dissipation (destruction) arising from eddy velocity fluctuating diffusion.
7. Viscous diffusion.
8. Diffusive turbulent transport resulting from the eddy velocity fluctuations.
9. Dissipation of turbulent transport arising from eddy pressure and fluctuating velocity gradients.

- So, if you have good quality data, all these terms can be computed.

- Usually, this data comes from DNS simulations or experiments.

- You can also use data coming from SRS models (LES, DES, SAS, URANS).

- However, have in mind that turbulence models (e.g., EVM) add approximations to the terms appearing in the exact governing equations, so you will be plotting the modelled terms.

- To plot the resolved terms, you need to avoid the use of approximations.

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- To plot the budgets of the turbulence quantities, you need to compute the instantaneous and averaged values.

- You can get the data needed to plot these budgets using steady (not recommended) and unsteady solvers.

- With steady solvers, it is relatively easy as you just need to get the information of the last iteration.

- With unsteady solvers, it is much more cumbersome, as you need to run the simulation until reaching a statistically steady behavior.

- You also need to compute the unsteady statistics (it is also possible to compute the steady statistics).

Sampling location

Mean solution or steady solution

Velocity magnitude (m/s) - Mean
0   0.2   0.4   0.6   0.8   1   1.2

Time: 4600.000000

Unsteady solution

Velocity magnitue (m/s) - Instantaneous
0   0.2   0.4   0.6   0.8   1   1.2

http://www.wolfdynamics.com/training/turbulence/zaki1.avi
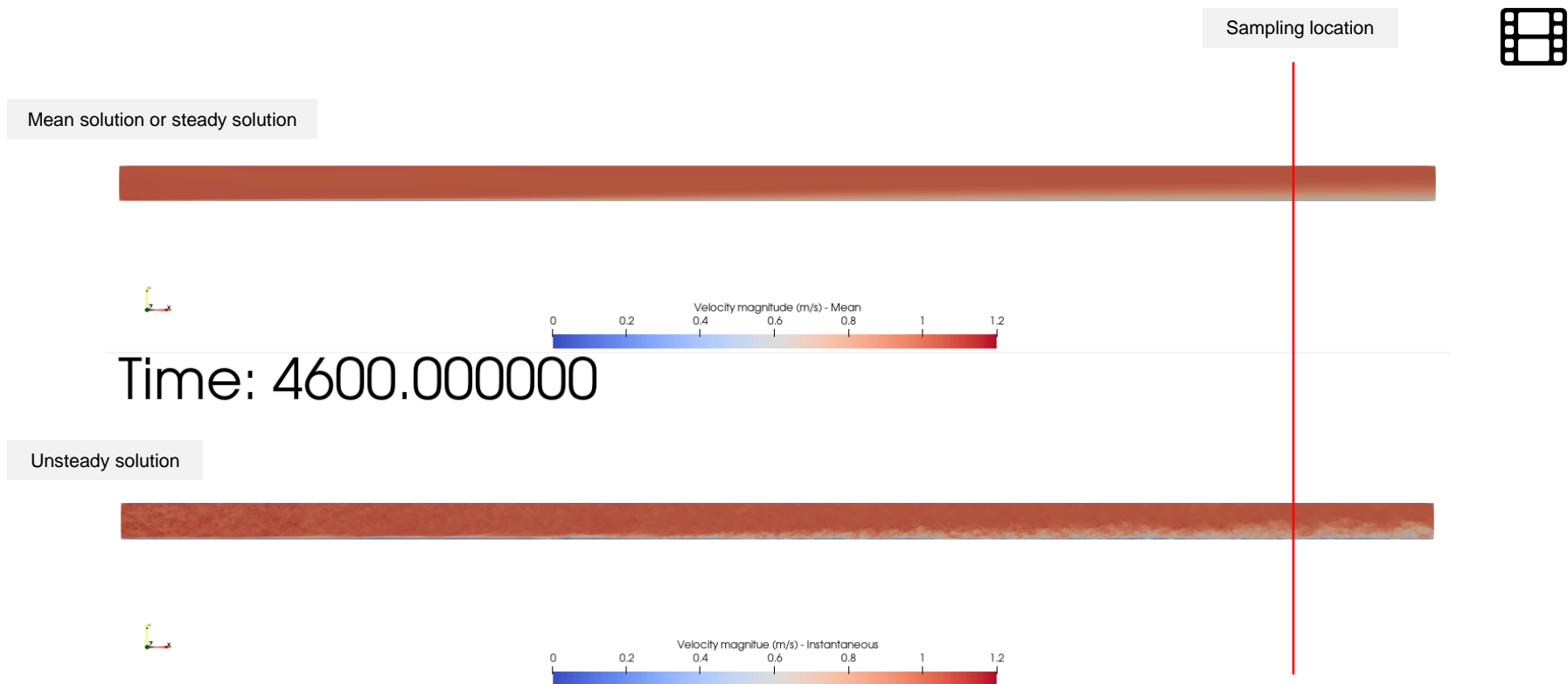
Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

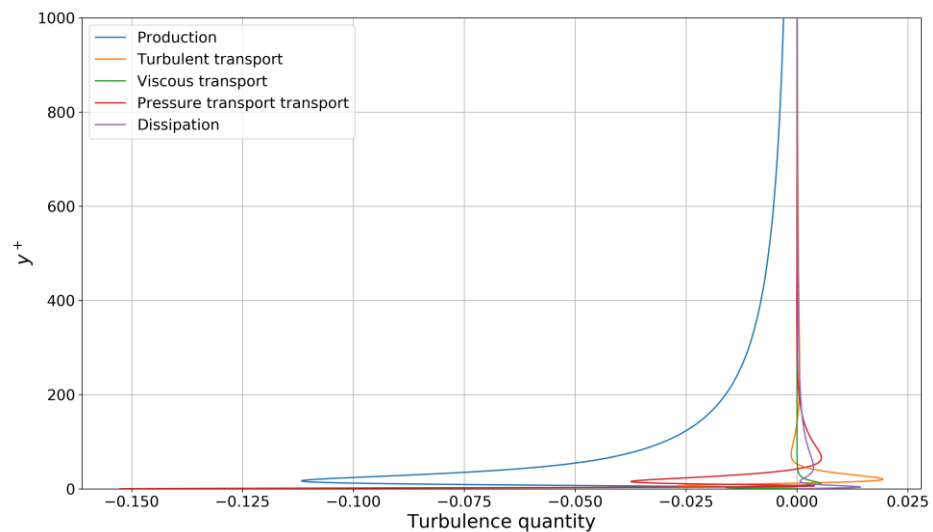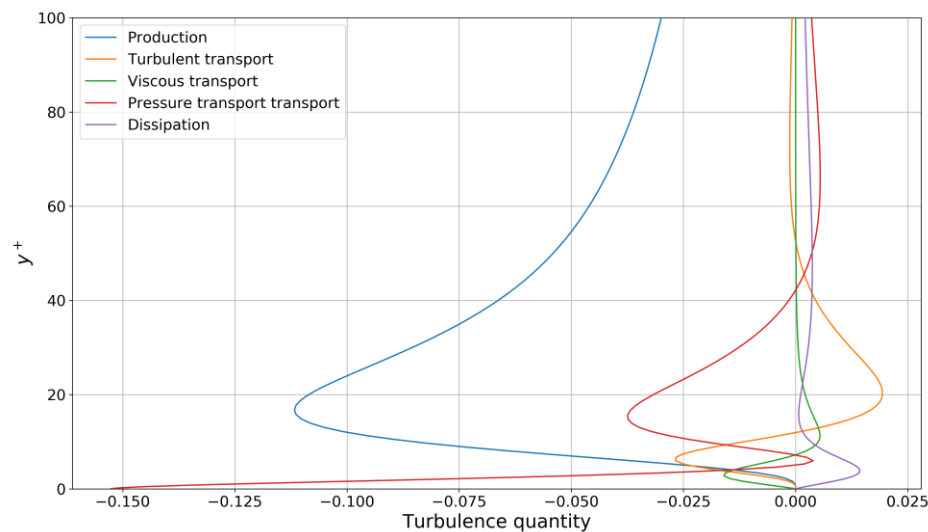- Direct numerical simulation of turbulent channel flow up to Re_tau = 5200 [1, 2].
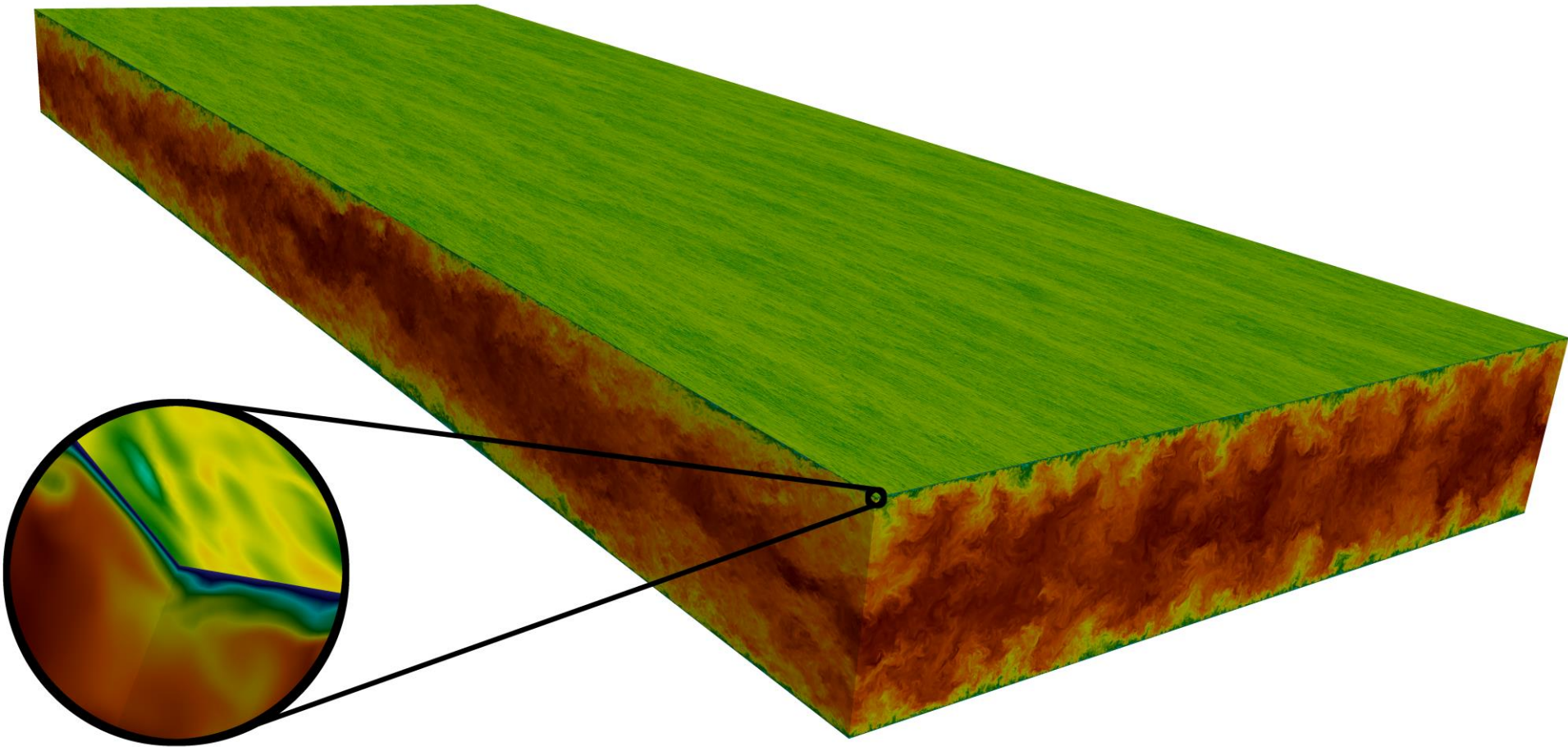
[1] M. Lee, R. Moser, Direct numerical simulation of turbulent channel flow up to Re_tau = 5200, 2015, Journal of Fluid Mechanics, vol. 774, pp. 395-415.
[2] Data source: https://turbulence.oden.utexas.edu/channel2015/

- Turbulent budget of the turbulent kinetic energy.

- An interactive plot is available at the following link:
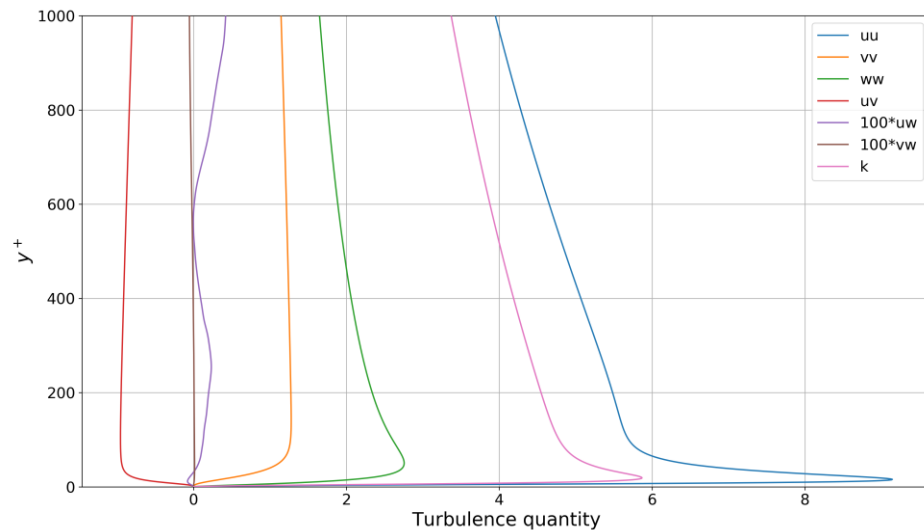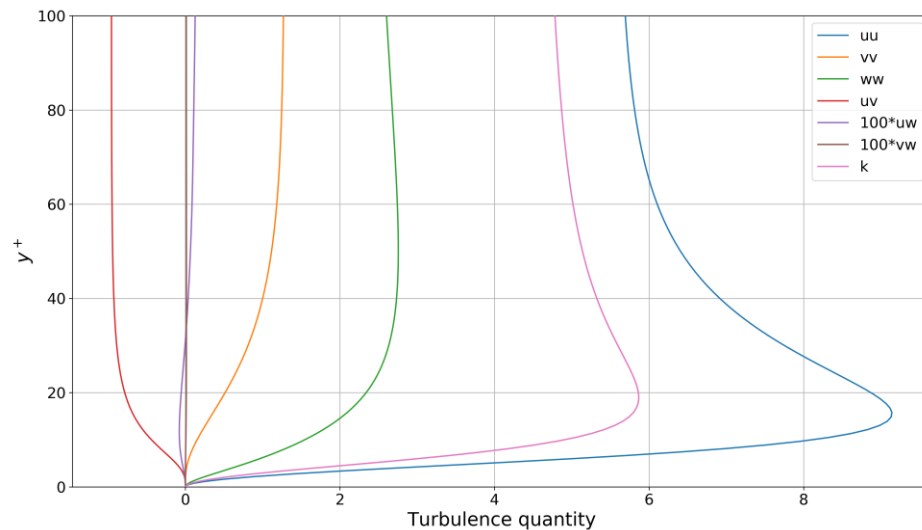 https://nbviewer.org/url/www3.dicca.unige.it/guerrero/turbulence2022/bokeh/lecture6/plots-bokeh.ipynb
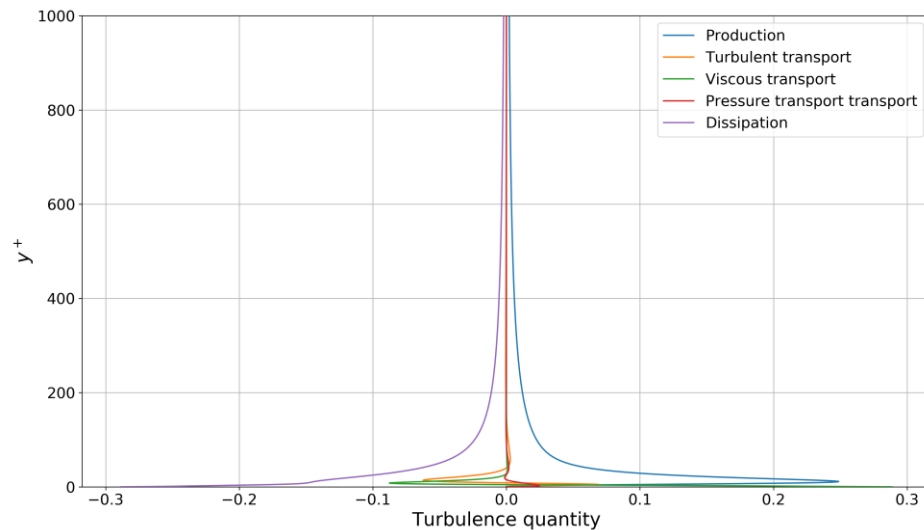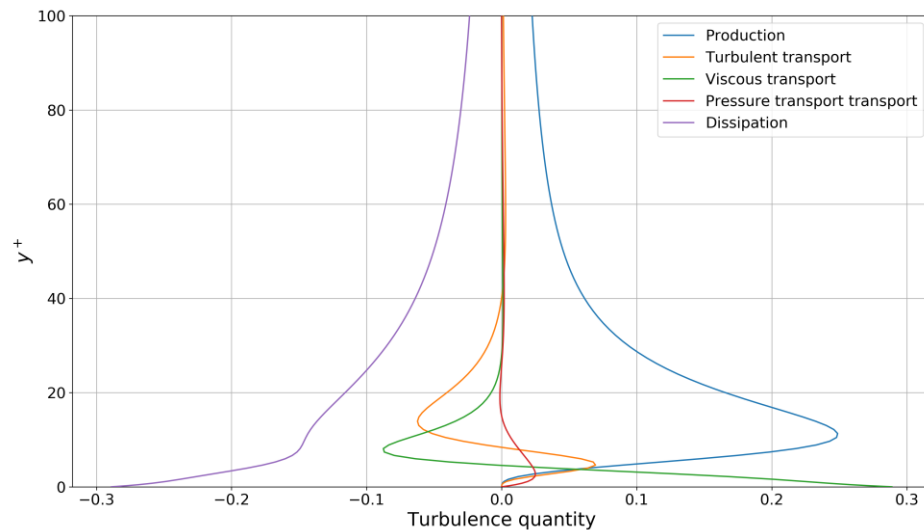
[1] M. Lee, R. Moser, Direct numerical simulation of turbulent channel flow up to Re_tau = 5200, 2015, Journal of Fluid Mechanics, vol. 774, pp. 395-415.
[2] Data source: https://turbulence.oden.utexas.edu/channel2015/

- Turbulent budget of the normal Reynolds stress u'u'.

- An interactive plot is available at the following link:
  https://nbviewer.org/url/www3.dicca.unige.it/guerrero/turbulence2022/bokeh/lecture6/plots-bokeh.ipynb

[1] M. Lee, R. Moser, Direct numerical simulation of turbulent channel flow up to Re_tau = 5200, 2015, Journal of Fluid Mechanics, vol. 774, pp. 395-415.
[2] Data source: https://turbulence.oden.utexas.edu/channel2015/

- Turbulent budget of the normal Reynolds stress v'v'.

- An interactive plot is available at the following link:
  https://nbviewer.org/url/www3.dicca.unige.it/guerrero/turbulence2022/bokeh/lecture6/plots-bokeh.ipynb

[1] M. Lee, R. Moser, Direct numerical simulation of turbulent channel flow up to Re_tau = 5200, 2015, Journal of Fluid Mechanics, vol. 774, pp. 395-415.
[2] Data source: https://turbulence.oden.utexas.edu/channel2015/

- Turbulent budget of the normal Reynolds stress w'w'.

- An interactive plot is available at the following link:
  https://nbviewer.org/url/www3.dicca.unige.it/guerrero/turbulence2022/bokeh/lecture6/plots-bokeh.ipynb
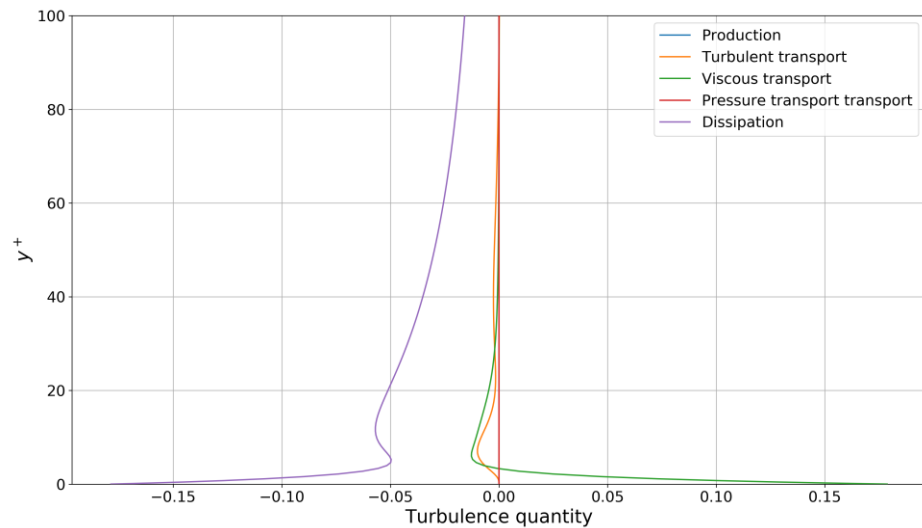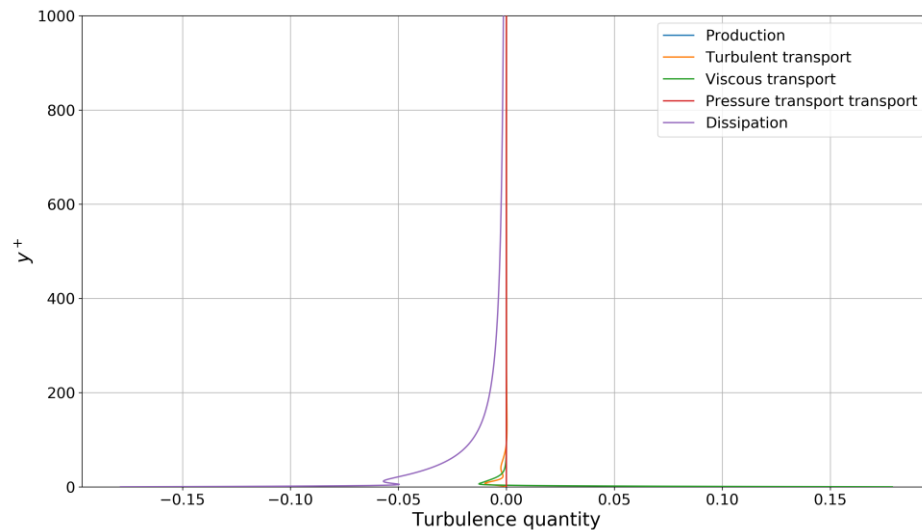
[1] M. Lee, R. Moser, Direct numerical simulation of turbulent channel flow up to Re_tau = 5200, 2015, Journal of Fluid Mechanics, vol. 774, pp. 395-415.
[2] Data source: https://turbulence.oden.utexas.edu/channel2015/

- Turbulent budget of the shear Reynolds stress u'v'.

- An interactive plot is available at the following link:
  https://nbviewer.org/url/www3.dicca.unige.it/guerrero/turbulence2022/bokeh/lecture6/plots-bokeh.ipynb

[1] M. Lee, R. Moser, Direct numerical simulation of turbulent channel flow up to Re_tau = 5200, 2015, Journal of Fluid Mechanics, vol. 774, pp. 395-415.
[2] Data source: https://turbulence.oden.utexas.edu/channel2015/

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulent budget – Variance and covariance of velocity components.

- An interactive plot is available at the following link:
  https://nbviewer.org/url/www3.dicca.unige.it/guerrero/turbulence2022/bokeh/lecture6/plots-bokeh.ipynb

[1] M. Lee, R. Moser, Direct numerical simulation of turbulent channel flow up to Re_tau = 5200, 2015, Journal of Fluid Mechanics, vol. 774, pp. 395-415.
[2] Data source: https://turbulence.oden.utexas.edu/channel2015/

- **Let us take a look at the data from the following reference:**

  - **N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987**

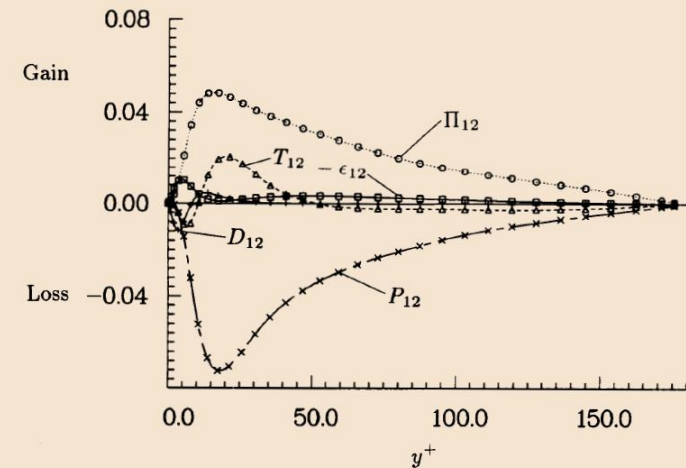- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].
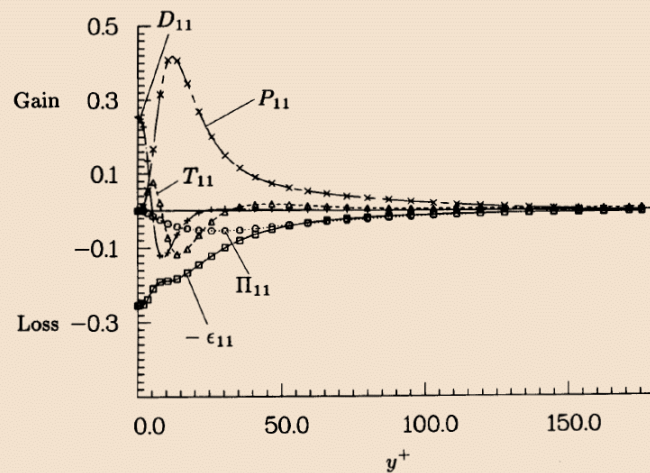
Figure 1. Terms in the budget of $\overline{u_1' u_1'}$ in wall coordinates. P_{11} = Production; T_{11} = Turbulent transport; D_{11} = Viscous diffusion; ϵ_{11} = Dissipation rate; Π_{11} = Velocity pressure-gradient term.
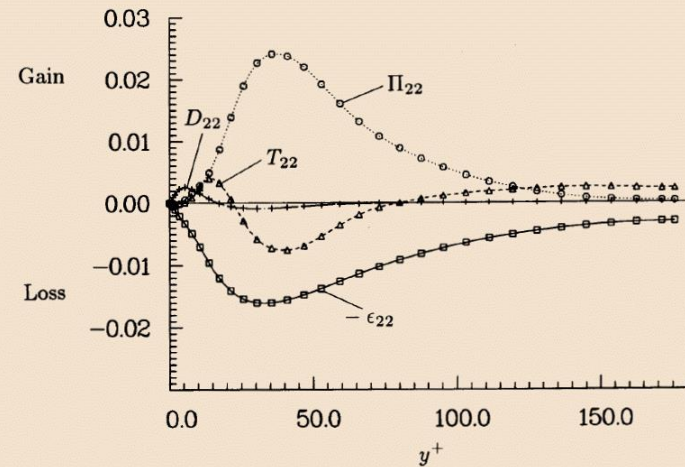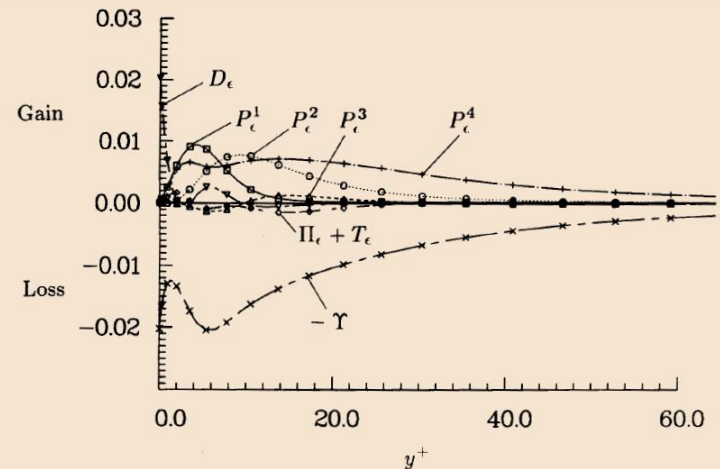
Figure 2. Terms in the budget of $\overline{u_2' u_2'}$ in wall coordinates. T_{22} = Turbulent transport; D_{22} = Viscous diffusion; ϵ_{22} = Dissipation rate; Π_{22} = Velocity pressure-gradient term.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].
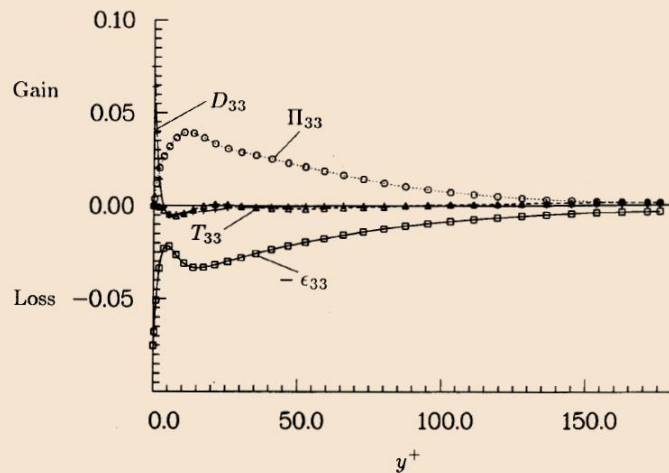
Figure 3.   Terms in the budget of $\overline{u_3' u_3'}$ in wall coordinates. T_{33} = Turbulent transport; D_{33} = Viscous diffusion; ϵ_{33} = Dissipation rate; Π_{33} = Velocity pressure-gradient term.

Figure 4.   Terms in the budget of $\overline{u_1' u_2'}$ in wall coordinates. P_{12} = Production; T_{12} = Turbulent transport; D_{12} = Viscous diffusion; ϵ_{12} = Dissipation rate; Π_{12} = Velocity pressure-gradient rate.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

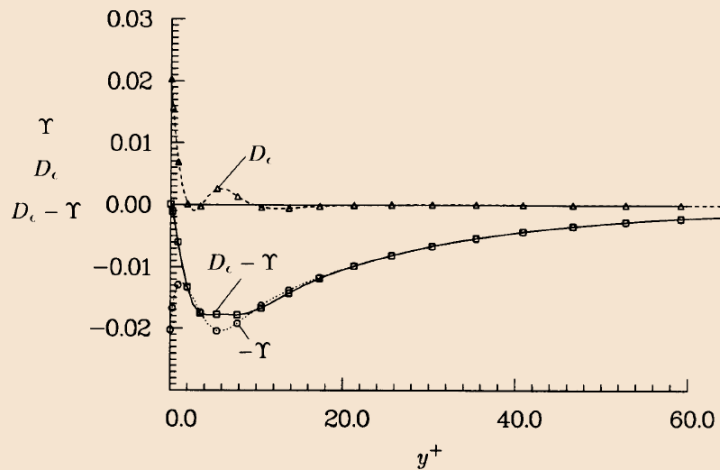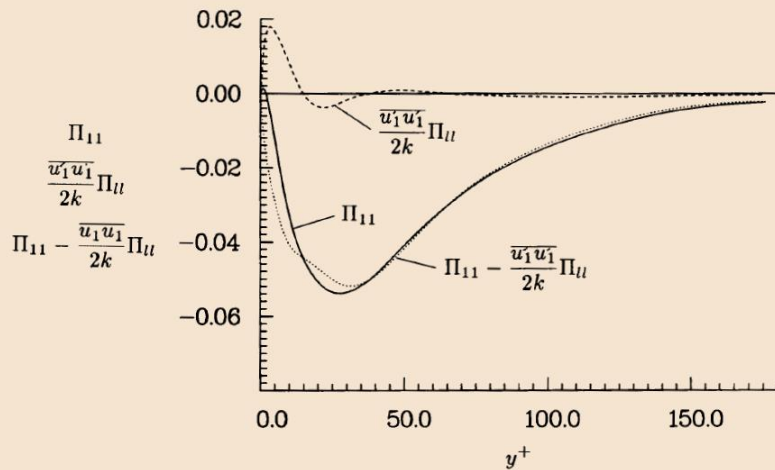- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].
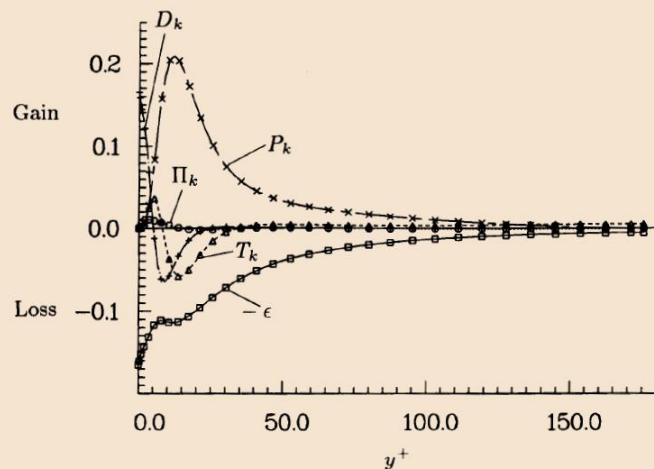
Figure 5. Terms in the budget of the turbulence kinetic energy, k, in wall coordinates. P_k = Production; T_k = Turbulent transport; D_k = Viscous diffusion; ϵ_k = Dissipation rate; Π_k = Velocity pressure-gradient term.

Figure 6. Terms in the budget of the dissipation rate of the turbulence kinetic energy, ϵ, in wall coordinates. P_ϵ^1 = Production by mean velocity gradient; P_ϵ^2 = Mixed production; P_ϵ^3 = Gradient production; P_ϵ^4 = Turbulent production; T_ϵ = Turbulent pransport; D_ϵ = Viscous diffusion; Υ = Dissipation rate; Π_ϵ = Pressure transport.

- From figure 5,

  - Production of TKE peaks in the buffer layer (about y^+ 15).

  - Dissipation peaks in the viscous sublayer.

  - As we get away from the walls ($y^+ > 60$), production and dissipation are in balance

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

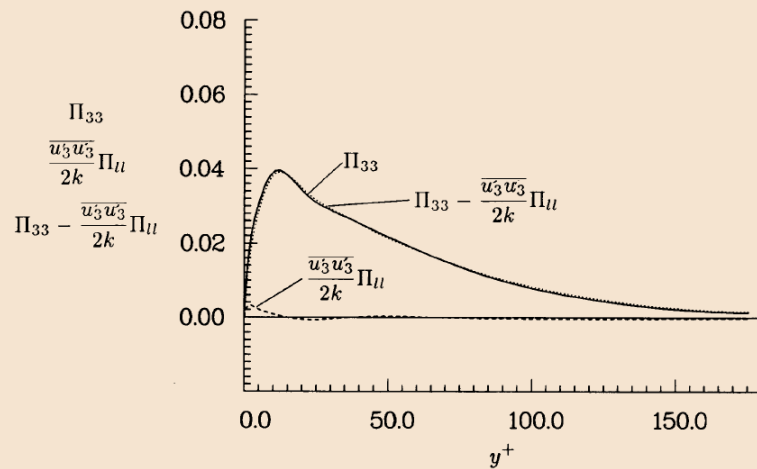Figure 7.   Split of the viscous term into a dissipation rate term, Υ, and a viscous diffusion term, D_ϵ.
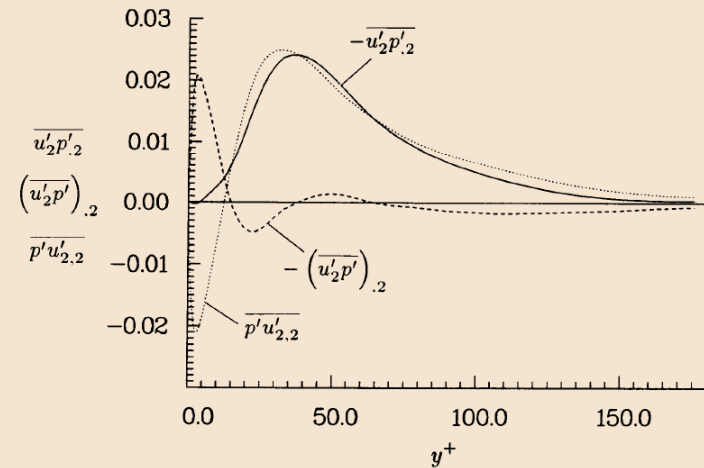
Figure 8.   Split of the velocity pressure-gradient term into a pressure transport term, $(pu'_2)_{,y^+}$, and a pressure-strain term, $\overline{pu'_{2,y^+}}$.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].
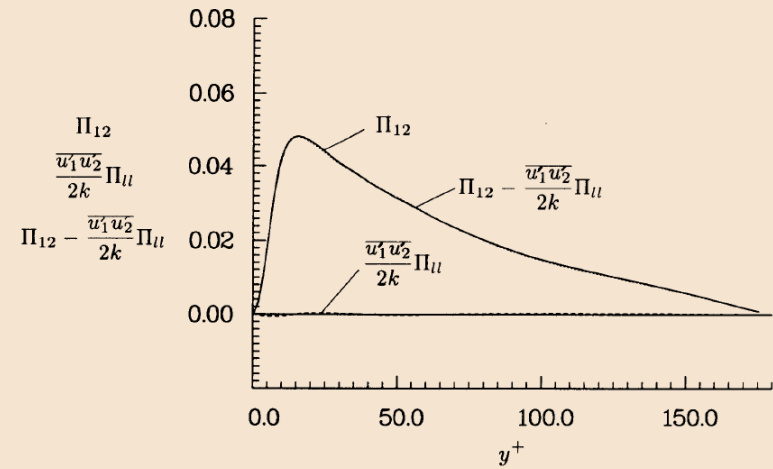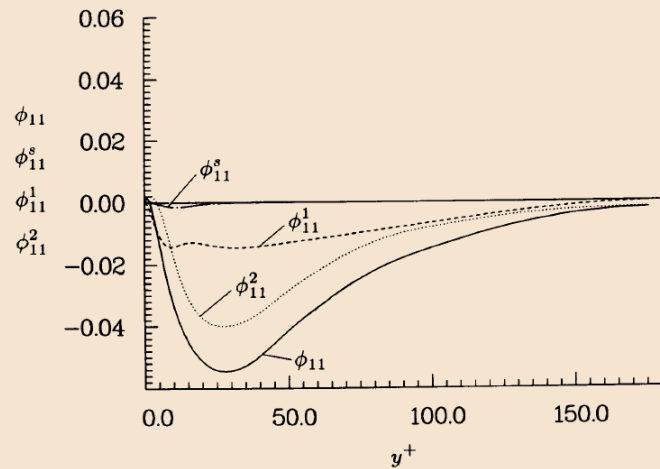

Figure 9. Split of the velocity pressure-gradient term, Π_{11}, into a pressure transport term, $\overline{u_1' u_1'}/2k\,\Pi_{ll}$, and a redistributive term, $\Pi_{11} - \overline{u_1' u_1'}/2k\,\Pi_{ll}$.
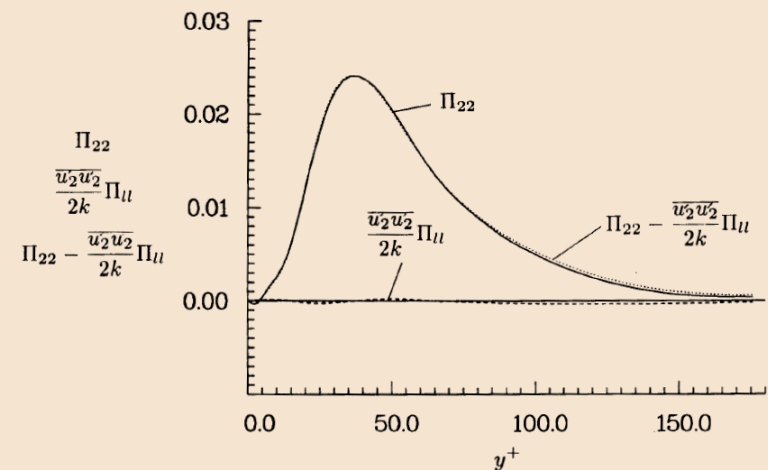
Figure 10. Split of the velocity pressure-gradient term, Π_{22}, into a pressure transport term, $\overline{u_2' u_2'}/2k\,\Pi_{ll}$, and a redistributive term, $\Pi_{22} - \overline{u_2' u_2'}/2k\,\Pi_{ll}$.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

Figure 11. _____ Split of the velocity pressure-gradient term, Π_{33}, into a pressure transport term, $\overline{u_3' u_3'}/2k\,\Pi_{ll}$, and a redistributive term, $\Pi_{33} - \overline{u_3' u_3'}/2k\,\Pi_{ll}$.

Figure 12. _____ Split of the velocity pressure-gradient term, Π_{12}, into a pressure transport term, $\overline{u_1' u_2'}/2k\,\Pi_{ll}$, and a redistributive term, $\Pi_{12} - \overline{u_1' u_2'}/2k\,\Pi_{ll}$.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

Figure 13.   Split of pressure-strain term, ϕ_{11}, into a rapid term, ϕ_{11}^1, a return term, ϕ_{11}^2, and a Stokes term, ϕ_{11}^S.
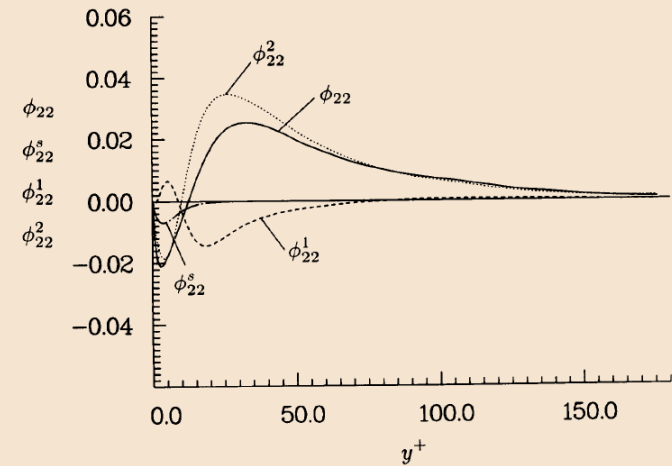
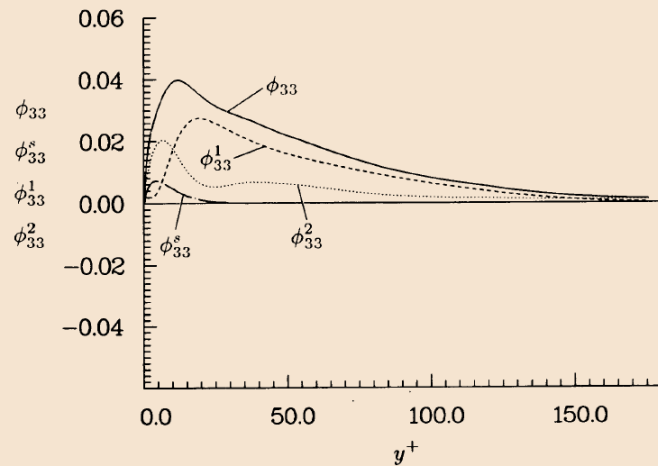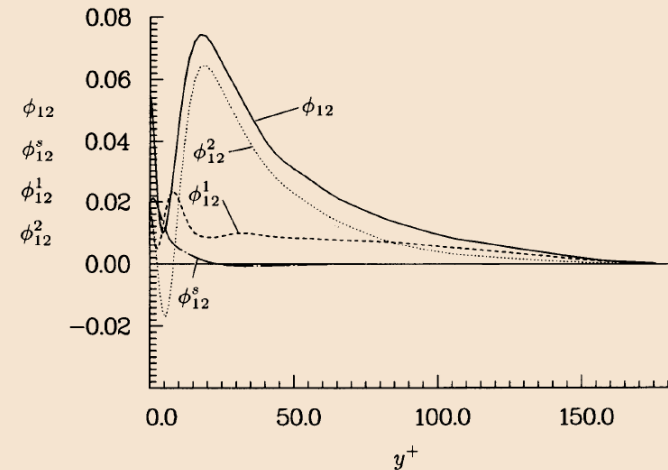Figure 14.   Split of pressure-strain term, ϕ_{22}, into a rapid term, ϕ_{22}^1, a return term, ϕ_{22}^2, and a Stokes term, ϕ_{22}^S.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

Figure 15.   Split of pressure-strain term, ϕ_{33}, into a rapid term, ϕ_{33}^1, a return term, ϕ_{33}^2, and a Stokes term, ϕ_{33}^S.

Figure 16.   Split of pressure-strain term, ϕ_{12}, into a rapid term, ϕ_{12}^1, a return term, ϕ_{12}^2, and a Stokes term, ϕ_{12}^S.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].
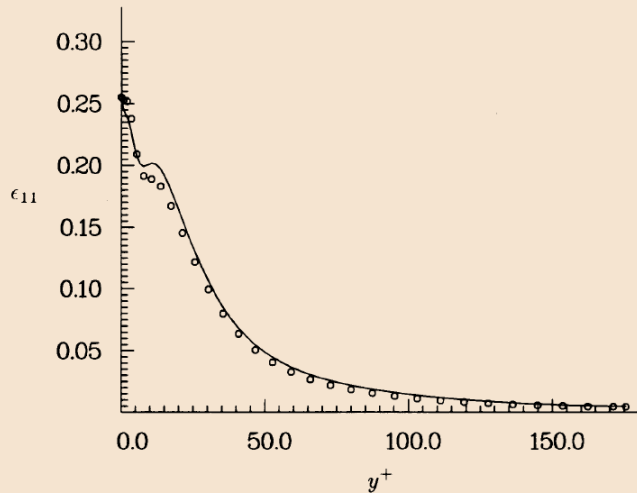
Figure 19. Distribution of ϵ_{11} across the channel. ○ ○ ○ ○ ϵ_{11} term computed from the channel data; —— model, $\epsilon/k \; \overline{u_1' u_1'}$.
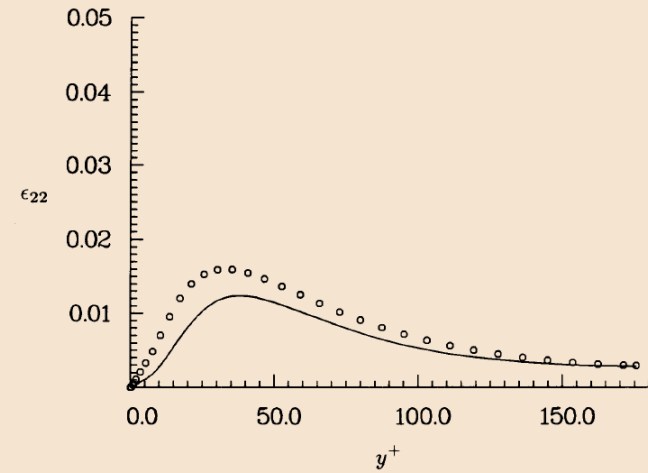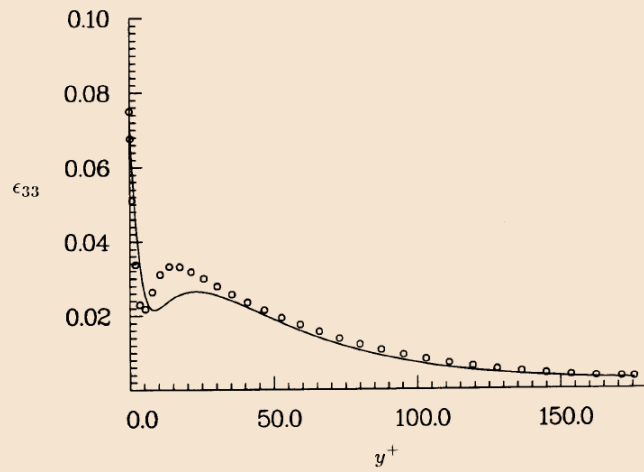
Figure 20. Distribution of ϵ_{22} across the channel. ○ ○ ○ ○ ϵ_{22} term computed from the channel data; —— model, $\epsilon/k \; \overline{u_2' u_2'}$.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

Figure 21. Distribution of ϵ_{33} across the channel. $\circ\circ\circ\circ$ ϵ_{33} term computed from the channel data; —— model, $\epsilon/k\ \overline{u_3' u_3'}$.
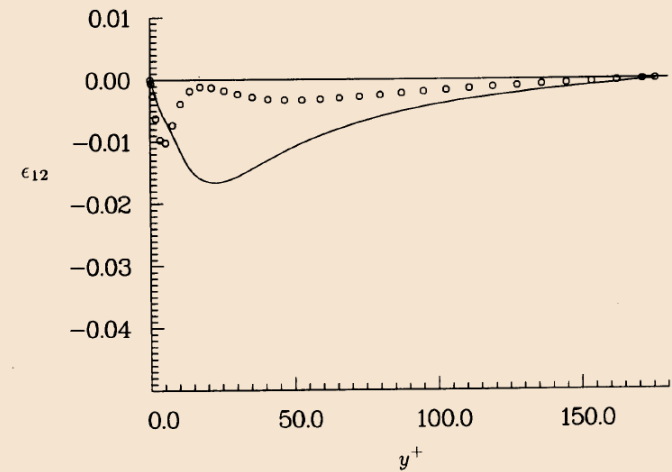
Figure 22. Distribution of ϵ_{12} across the channel. $\circ\circ\circ\circ$ ϵ_{12} term computed from the channel data; —— model, $\epsilon/k\ \overline{u_1' u_2'}$.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].
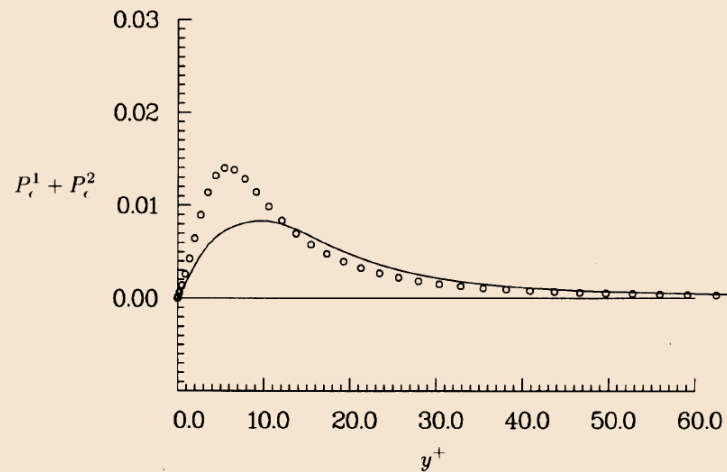

Figure 23.   Distribution of the production term, $P_\epsilon^1 + P_\epsilon^2$, in the budget of ϵ across the channel; o o o o term computed from the channel data; —— model, equation (42).

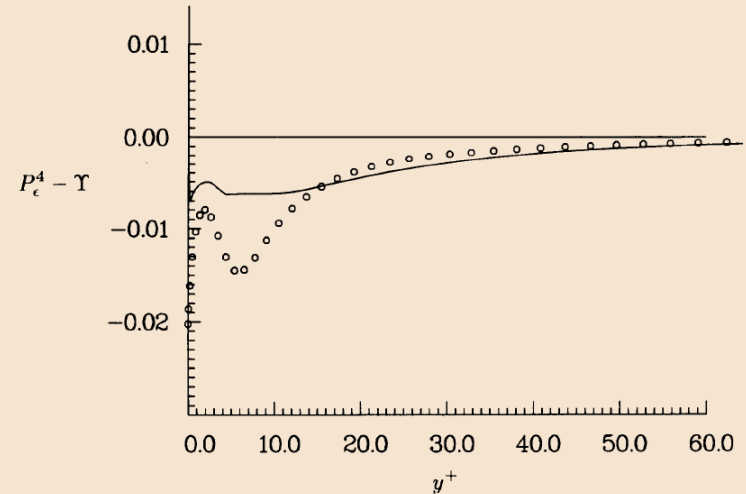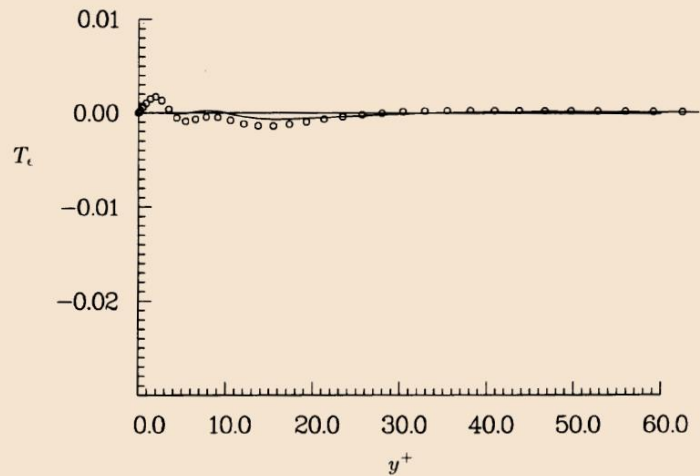Figure 24.   Distribution of the dissipation term, $P_\epsilon^4 - \Upsilon$, in the budget of ϵ across the channel. o o o o term computed from the channel data; —— model, equation (44).

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

Figure 25.   Distribution of the turbulent transport term, T_ϵ, in the budget of ϵ across the channel. ○ ○ ○ ○ term computed from the channel data; —— model, equation (45).
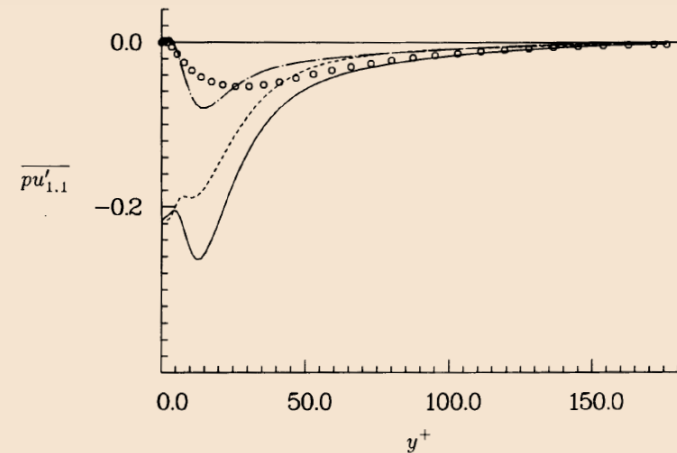
Figure 26.   Pressure-strain term, $\overline{pu'_{1,1}}$, in the budget equation for $\overline{u'_1 u'_1}$ across the channel. ○ ○ ○ ○ term computed from the channel data; —— model, (eq. (46)+eq. (48)+eq. (49)); ---- model, equation (46); —·— model, (eq. (48)+eq. (49)).

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

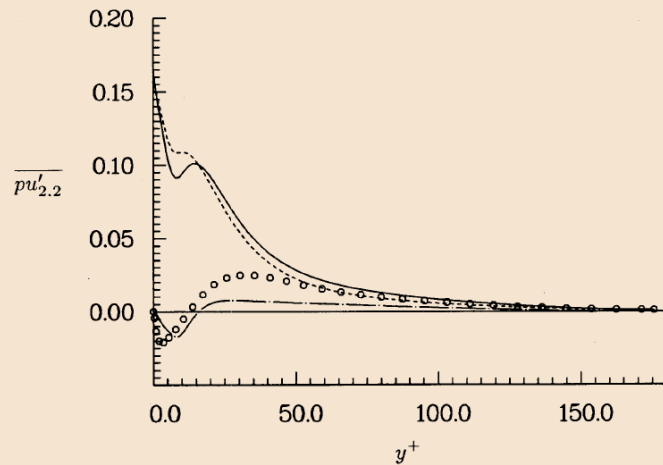- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

Figure 27.   Pressure strain term, $\overline{pu'_{2,2}}$, in the budget equation for $\overline{u'_2 u'_2}$ across the channel. ○ ○ ○ ○ term computed from the channel data; —— model, (eq. (46)+eq. (48)+eq. (49)); – – – – model, equation (46). —·— model, (eq. (48)+eq. (49)).
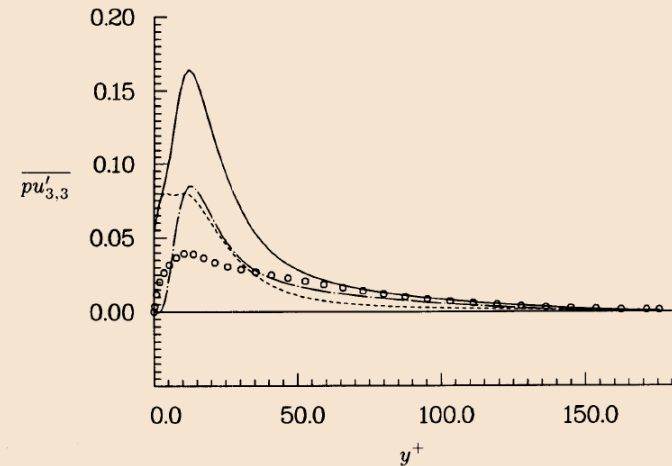
Figure 28.   Pressure strain term, $\overline{pu'_{3,3}}$, in the budget equation for $\overline{u'_3 u'_3}$ across the channel. ○ ○ ○ ○ term computed from the channel data; —— model, (eq. (46)+eq. (48)+eq. (49)); – – – – model, equation (46). —·— model, (eq. (48)+eq. (49)).

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

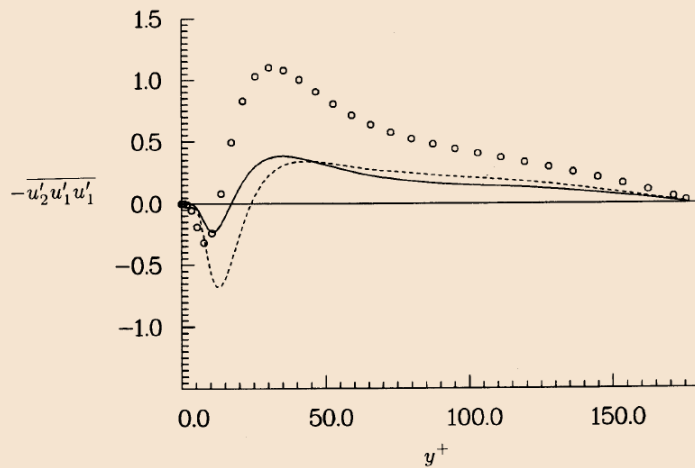

Figure 29.   Triple correlation term $-\overline{u_2' u_1' u_1'}$ across the channel.
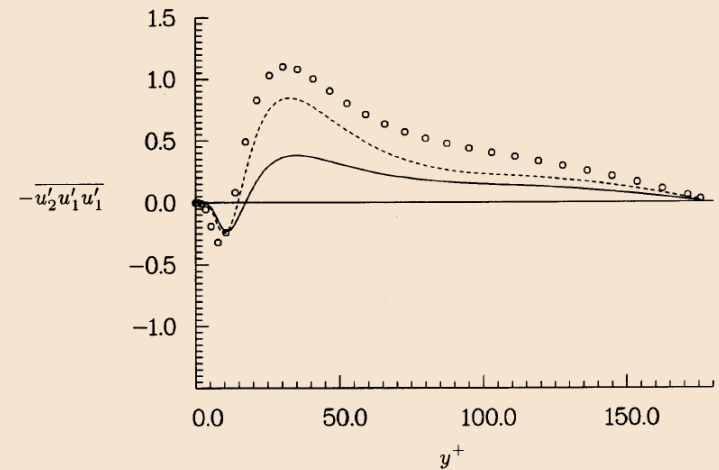o o o o term computed from the channel data; —— model, equation (53); – – – model, equation (54).

Figure 30.   Triple correlation term $-\overline{u_2' u_1' u_1'}$ across the channel.
o o o o term computed from the channel data; —— model, equation (54); – – – model, equation (55).

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

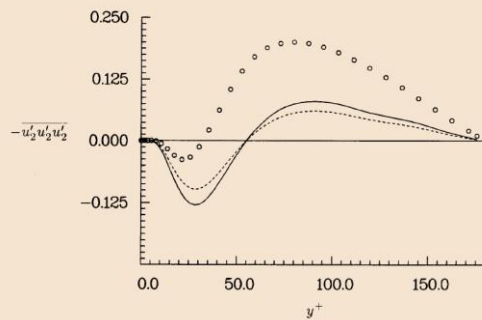

Figure 31.   Triple correlation term $-\overline{u_2'u_2'u_2'}$ across the channel.
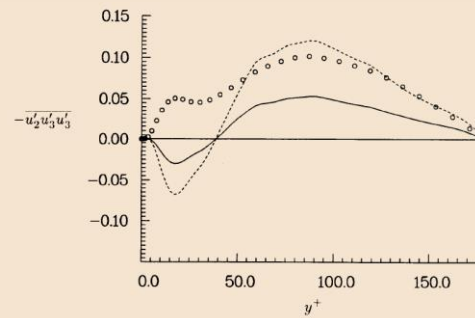∘ ∘ ∘ ∘ term computed from the channel data; —— model, equation (54); - - - - model, equation (55).

Figure 32.   Triple correlation term $-\overline{u_2'u_3'u_3'}$ across the channel.
∘ ∘ ∘ ∘ term computed from the channel data; —— model, equation (54); - - - - model, equation (55).
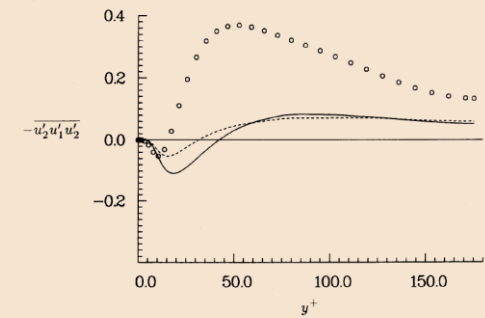
Figure 33.   Triple correlation term $-\overline{u_2'u_1'u_2'}$ across the channel.
∘ ∘ ∘ ∘ term computed from the channel data; —— model, equation (54); - - - - model, equation (55).

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].
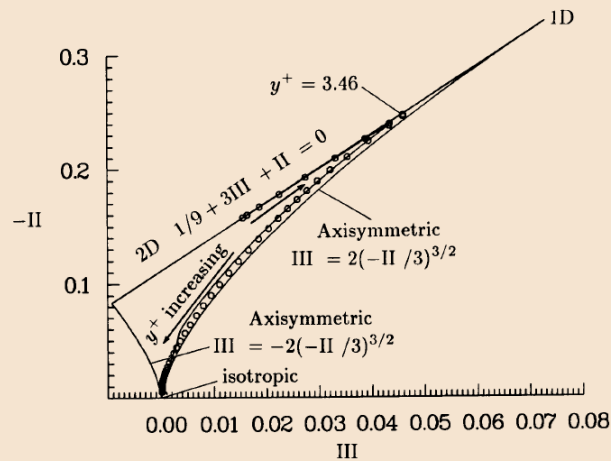

Figure 17.   Anisotropy invariant map. ○ ○ ○ ○ d_{ij} at various y^+ in the channel; —— b_{ij} at various y^+ in the channel.

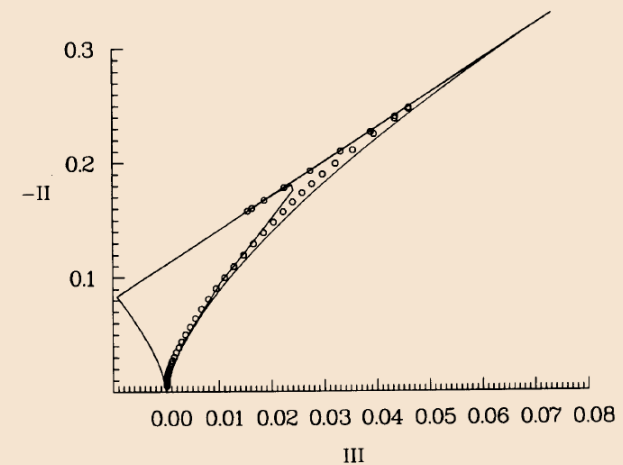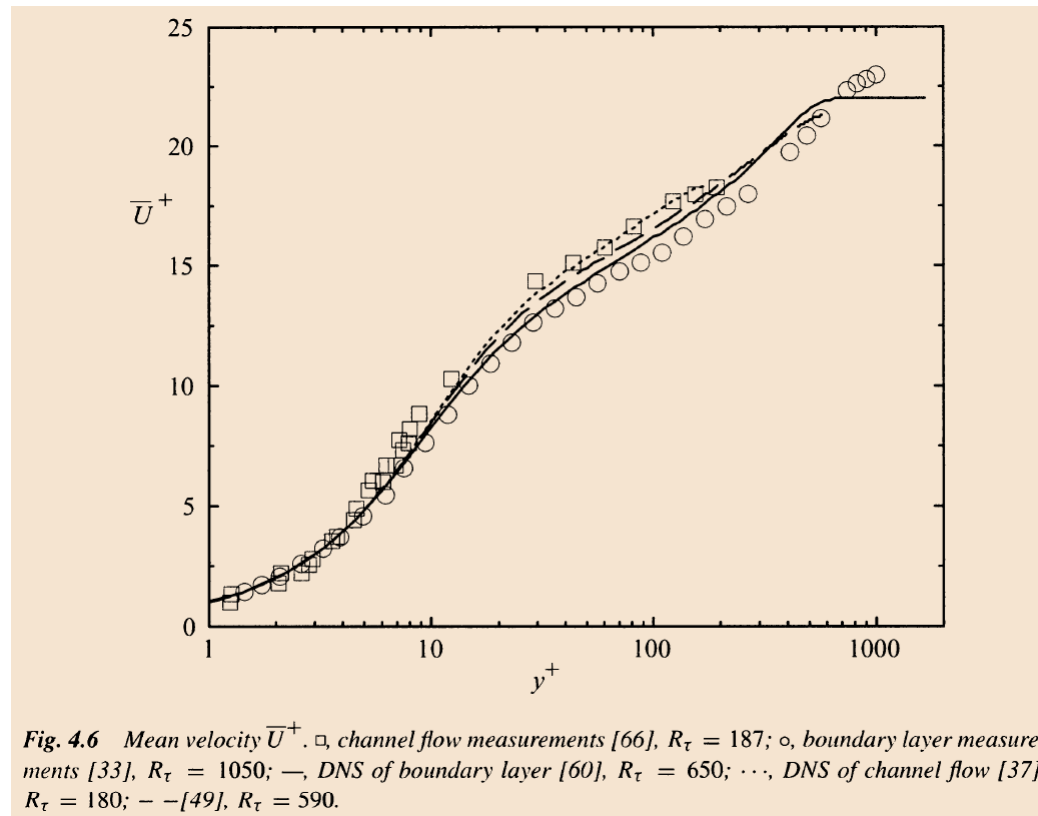Figure 18.   Anisotropy invariant map. ○ ○ ○ ○ d_{ij} at various y^+ in the channel; —— model, equation (40).

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- These budgets can be obtained from experimental measurements or DNS simulations.

- The turbulence kinetic energy, dissipation rate, and Reynolds stress budgets provide valuable guidelines for model developers, model testing, and model validation.

- This is used for hardcore model development and validation.

- Getting this data can be very time consuming and computationally expensive

- Also, postprocessing this data can be challenging and time consuming.

- When conducting SRS simulations and if you are interested in plotting the budgets, never forget to compute the unsteady statistics.

- You will also need to compute the correlations and any additional term of the budget of the governing equations.

- Of all the plots we have seen so far, including those in the previous lectures, maybe the most important plots are the following,

Fig. 4.6 Mean velocity $\overline{U}^+$. □, channel flow measurements [66], $R_\tau = 187$; ○, boundary layer measurements [33], $R_\tau = 1050$; —, DNS of boundary layer [60], $R_\tau = 650$; ···, DNS of channel flow [37], $R_\tau = 180$; − −[49], $R_\tau = 590$.

Nondimensional velocity profile.
Comparison using experimental and numerical data. Images reproduced from reference [1].

[1] P. Bernard, J. Wallace. Turbulent Flow. Analysis, Measurement and Prediction. 2002.

- Of all the plots we have seen so far, including those in the previous lectures, maybe the most important plots are the following,

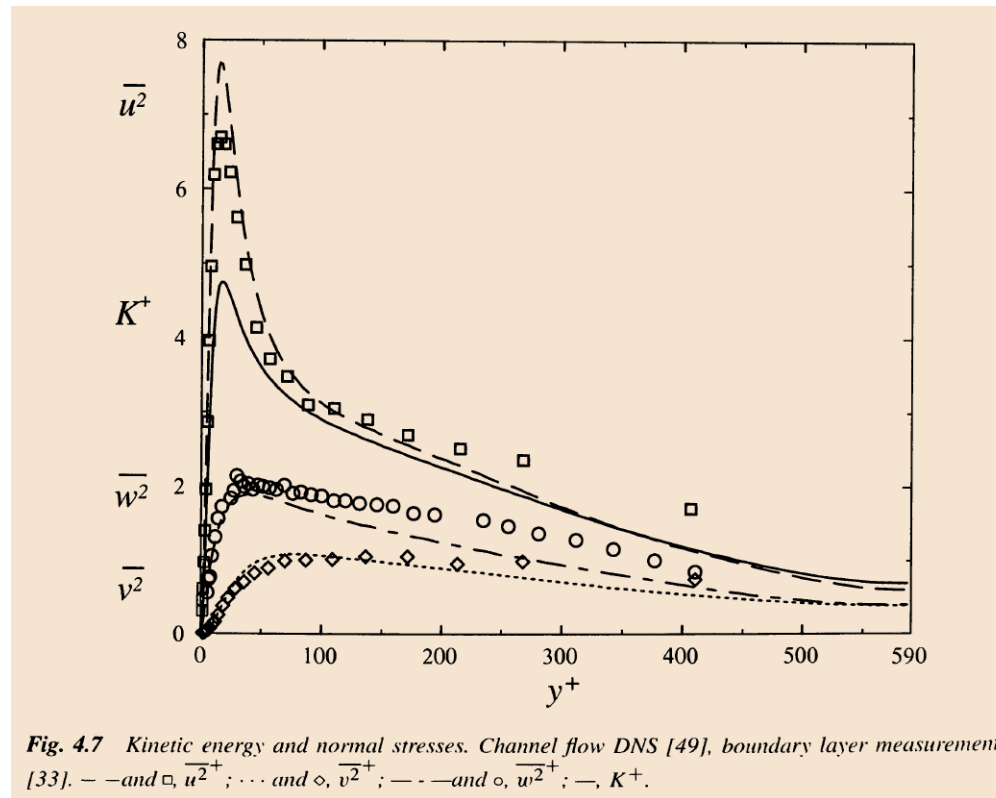**Fig. 4.7** *Kinetic energy and normal stresses. Channel flow DNS [49], boundary layer measurements [33].* $-$ $-$ *and* $\square$, $\overline{u^2}^+$; $\cdots$ *and* $\diamond$, $\overline{v^2}^+$; $-\cdot-$ *and* $\circ$, $\overline{w^2}^+$; $-$, K^+.
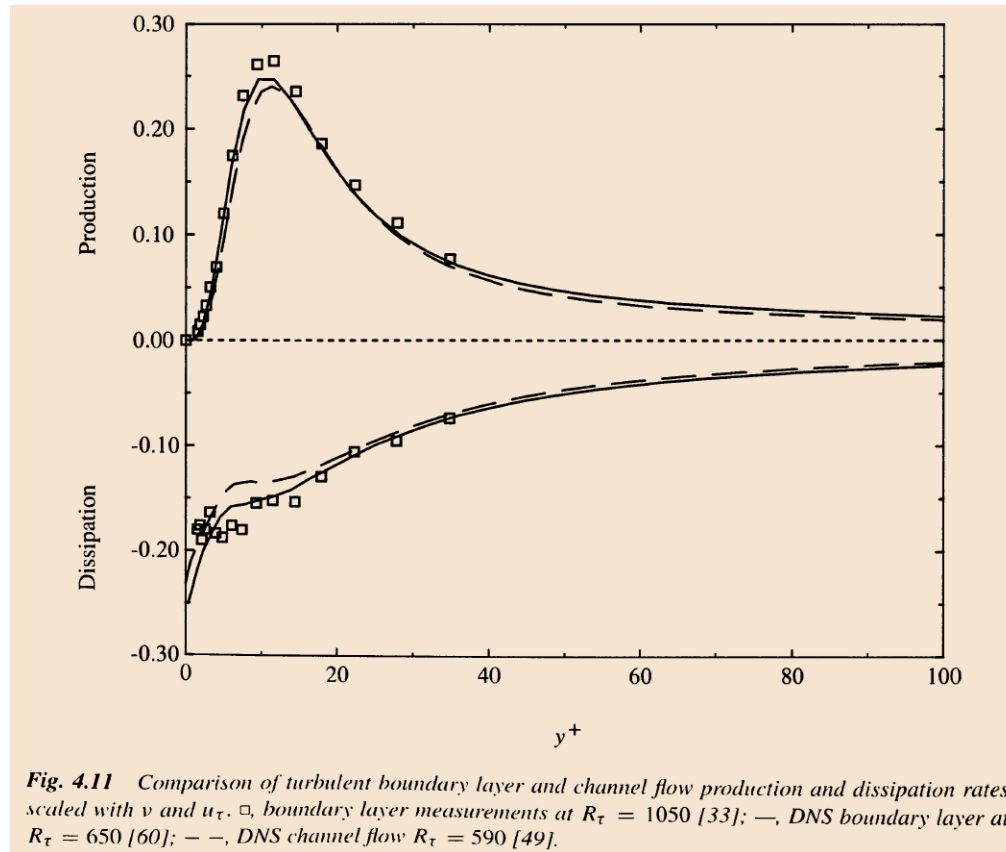
Kinetic energy and normal Reynolds stresses.
TKE is available if you use a turbulence model that solves an equation for TKE, same applies for the normal Reynolds stresses. If these quantities are not computed by the solver, you will need to compute the unsteady statistics.
Comparison using experimental and numerical data. Images reproduced from reference [1].

[1] P. Bernard, J. Wallace. Turbulent Flow. Analysis, Measurement and Prediction. 2002.

- Of all the plots we have seen so far, including those in the previous lectures, maybe the most important plots are the following,

Fig. 4.11 *Comparison of turbulent boundary layer and channel flow production and dissipation rates scaled with v and u_τ. □, boundary layer measurements at $R_\tau = 1050$ [33]; —, DNS boundary layer at $R_\tau = 650$ [60]; − −, DNS channel flow $R_\tau = 590$ [49].*

Turbulent kinetic energy budget (production and dissipation). Most of the solvers will compute these quantities.
Comparison using experimental and numerical data. Images reproduced from reference [1].

[1] P. Bernard, J. Wallace. Turbulent Flow. Analysis, Measurement and Prediction. 2002.

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- All the previous plots were generated by computing the unsteady statics and sampling data in the domain, *e.g.*, a line normal to the wall, cut-planes, point probes, and so on.

- Do not forget to set the unsteady statistics of each term appearing in the exact equations (the budgets).

- To compute the budgets, it is strongly recommended to use SRS simulations, such as LES and DNS.

Sampling lines – L1, L2, L3

Time (s) = 100.0020
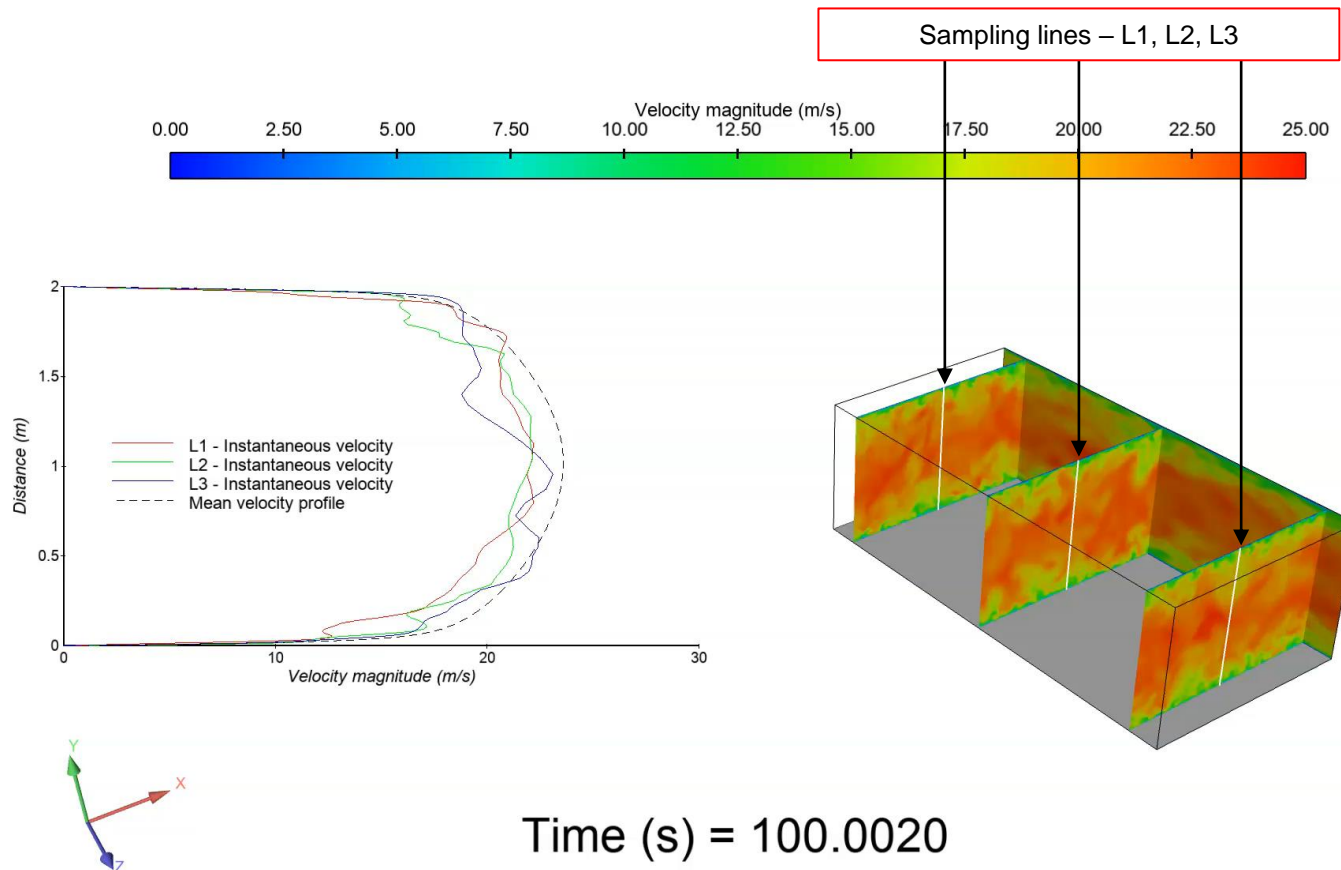
Incomplete list of turbulence databases

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- As getting this data can be very time consuming and computationally expensive, there are many well curated databases where this data is already available.

- The following, is an incomplete list of turbulence databases:

  - http://turbulence.pha.jhu.edu/

  - http://cfd.mace.manchester.ac.uk/

  - http://turbulence.oden.utexas.edu/

  - https://torroja.dmt.upm.es/turbdata/

  - https://www.rs.tus.ac.jp/t2lab/db/index.html

  - https://warwick.ac.uk/fac/sci/eng/staff/ymc/research/dns_database/

  - http://thtlab.jp/DNS/dns_database.html

  - http://www.tfd.chalmers.se/~lada/projects/databases/proright.html

  - https://ctr.stanford.edu/research-data

  - https://turbase.cineca.it/init/routes/#/logging/welcome

  - https://www.mech.kth.se/~pschlatt/DATA/

  - https://jaxa-dns-database.jaxa.jp/

  - https://wmles.umd.edu/validation/