Turbulence and CFD models: Theory and applications

Roadmap to Lecture 10

1. ~~SRS simulations~~
2. ~~LES equations – Filtered Navier-Stokes equations~~
3. **Sub-grid scale models for LES**
4. DES brief review
5. A few mesh resolution guidelines and rough estimates for LES/DES simulations
6. Final remarks on LES/DES turbulence models

Sub-grid scale models for LES

- As for RANS/URANS turbulence models, in LES there are many different models to approximate the apparent stress tensor that appears in the governing equations as a consequence of the filtering operation.
- We will call this tensor the sub-grid scale stress tensor or τ^{SGS}.
- Just to name a few models to approximate the sub-grid scale stress tensor:
    - Smagorinsky, dynamic Smagorinsky-Lilly, Deardoff, WALE, Germano dynamic model, Algebraic WMLES S-Omega Model Formulation, one equation kinetic energy transport (standard and dynamic), Bardina model, and so on.
- As for RANS, the sub-grid scale models are based on the Boussinesq hypothesis and the gradient diffusion hypothesis.
- Most of the models are algebraic (or zero equations).
    - Meaning that they do not use additional equations to model the stress tensor.
- Also, as they are algebraic, they do not explicitly compute the modeled TKE.

Sub-grid scale models for LES

LES sub-grid scale models – The Smagorinsky model

- Let us introduce the Smagorinsky model [1].
- In this model, the sub-grid scale stress tensor τ^{SGS} is modeled as follows,

$$\tau^{SGS} = -2\nu_{SGS}\overline{\mathbf{S}}$$

- Where $\overline{\mathbf{S}}$ is the strain-rate tensor, and ν_{SGS} is the sub-grid scale eddy viscosity.
- As for RANS models, our task is to somehow compute the turbulent viscosity, or sub-grid scale eddy viscosity ν_{SGS} .

References:
[1] J. Smagorinsky. General circulation experiments with primitive equations. I. The basic experiment, Monthly Weather Review, Vol. 91, pp. 99-164, 1963.

Sub-grid scale models for LES

LES sub-grid scale models – The Smagorinsky model

- In the Smagorinsky model, the sub-grid scale stress tensor τ^{SGS} is modeled as follows,

$$\tau^{SGS} = -2\nu_{SGS}\overline{\mathbf{S}}$$

- Where $\overline{\mathbf{S}}$ is the resolved strain-rate tensor, and is defined as follows,

$$\overline{\mathbf{S}} = \frac{1}{2}\left(\nabla\overline{\mathbf{u}} + \nabla\overline{\mathbf{u}}^{\mathrm{T}}\right)$$

- And ν_{SGS} is the sub-grid scale eddy viscosity, and is computed using the following relationship,

$$\nu_{SGS} = \left(C_S\Delta\right)^2\left|\overline{\mathbf{S}}\right|$$

Sub-grid scale models for LES

LES sub-grid scale models – The Smagorinsky model

- In the sub-grid scale eddy viscosity relation,
    - Δ is the filter width (proportional the cell spacing).
    - C_s is the Smagorinsky constant (or coefficient).
    - and $|\overline{\mathbf{S}}|$ is defined as (magnitude of the strain rate tensor),

$$|\overline{\mathbf{S}}| = \left(2\overline{\mathbf{S}} : \overline{\mathbf{S}}\right)^{1/2}$$

- The filter width Δ is usually computed as follows,

$$(\Delta_x \Delta_y \Delta_z)^{1/3} = \text{Cell volume}^{1/3}$$

- Obviously, this filter width approximation is accurate for uniform hexahedral cell.
- Depending on the cell shape different formulations are available.

Sub-grid scale models for LES

LES sub-grid scale models – The Smagorinsky model

- Notice that the sub-grid scale eddy viscosity relation resembles a mixing length formulation (refer to the Prandtl mixing length formulation),

Smagorinsky formulation

$$\nu_{SGS} = l_m^2 \left| \overline{\mathbf{S}} \right|$$

$$l_m = C_s \Delta$$

Prandtl mixing length formulation

$$\nu_t = l_m^2 \left| \frac{\partial \bar{u}}{\partial y} \right|$$

$$l_m = \kappa y$$

- In these relations, l_m represents a characteristics mixing length.
- Based on dimensional grounds, the Smagorinsky model can be written as follows,

$$\nu_{SGS} = \left(C_S \Delta \right)^2 \left| \overline{\mathbf{S}} \right|$$

Sub-grid scale models for LES

LES sub-grid scale models – The Smagorinsky model

- In the Smagorinsky model, close to the walls, a non-zero sub-grid scale viscosity can be incorrectly overpredicted.
- Therefore, the Smagorinsky model can be further improved by adding a damping function [1,2] to let the turbulent viscosity exponentially damp to zero close to the walls,

$$\nu_{SGS} = \left[C_S \Delta \underbrace{\left(1 - e^{y^+/25} \right)} \right]^2 \left| \overline{\mathbf{S}} \right|$$

Van Driest damping function [3]

- Where y^+ is the dimensionless wall distance normal to wall, and it can be computed as described in reference [4].

References:
[1] P. Moin, J. Kim. Numerical investigation of turbulent channel flow. J. Fluid Mech. 118, 341-377, 1982.
[2] P. S. Granville. A Modified Van Driest Formula for the Mixing Length of Turbulent Boundary Layers in Pressure Gradients. J. Fluids Eng., 111(1): 94-97. Mar 1989.
[3] E. Van Driest. On turbulent flow near a wall. J. Aerospace Sci., 23, 1007-1011, 1956.
[4] P. Tucker. Differential equation based length scales to improve DES and RANS simulations. AIAA Paper 2003-3968, 2003.

LES sub-grid scale models – The Smagorinsky model

- The Smagorinsky model is the oldest LES model, but because of its simplicity, it is widely used.
- It is not a particularly good choice for wall-bounded flows, but for shear free flows it can be quite adequate.
- In the Smagorinsky model, the value of the constant (or coefficient) C_s is of the order $\mathcal{O}(10^{-1})$.
- The values found in the literature can range anywhere from 0.06 to 0.35.
- Many of the drawbacks of the Smagorinsky model are due to the fact that the value of this coefficient can depend on the flow conditions.
- To overcome the drawbacks of the Smagorinsky model, more refined models have been developed.
    - Such as dynamic models that automatically compute the value of the coefficient in space and time.

LES sub-grid scale models – The Smagorinsky model

- In the Smagorinsky model, the original value of the constant (or coefficient) C_s is equal to 0.18.
- The value of the coefficient C_s was first suggested by Lilly [1].
- Different authors have found similar values [2,3].
- The Smagorisky coefficient C_s can be approximated in terms of the Kolmogorov constant C_K as follows,

$$C_s \approx \frac{1}{\pi}\left(\frac{3}{2}C_K\right)^{-3/4}$$

- Where if C_K is equal to 1.4, then C_s is equal to 0.18.
- It is worth stressing that the value of this coefficient is not universal.
- For wall bounded flows a value of 0.1 is recommended, for isotropic turbulence a value of 0.18 can be used (the same value found by Lilly using isotropic turbulence data), and for shear free flows a value of 0.24 is usually a good choice.

References:

[1] D. Lilly. The representation of small-scale turbulence in numerical simulation experiments. Proc. IBM Scientific Computing Symposium on Environmental Sciences, IBM, White Plains, NT, 1967.

[2] V. Canuto, Y. Cheng. Determination of the Smagorinsky-Lilly constant Cs. Phys. Fluids, 9, 1368-1378, 1997.

[3] F. Nicoud, F. Ducros. Subgrid-scale stress modelling based on the square of the velocity gradient tensor. Flow, Turb. Combustion, 62. 1999.

Sub-grid scale models for LES

LES sub-grid scale models – The Smagorinsky model

- Let us study the rationale behind the Smagorinsky model.
- On dimensional grounds, the sub-grid scale turbulent viscosity has the following units,

$$\nu_{SGS} = \frac{m^2}{s}$$

- Using the length scale and the velocity, the sub-grid scale turbulent viscosity can be expressed as follows,

$$\nu_{SGS} \propto lU$$

- However, this relation of proportionality is not Galilean invariant.
- Moreover, we need to provide a mean to compute the length scales.
- Therefore, it should be a better way.

LES sub-grid scale models – The Smagorinsky model

- By using the Boussinesq hypothesis, we can compute the sub-grid scale turbulent viscosity using the strain rate, as follows,

$$\nu_{SGS} = l^2 \left| \overline{\mathbf{S}} \right| \rightarrow \frac{m^2}{s}$$

- This relation is dimensionally consistent and Galilean invariant.
- At this point, we need estimate the integral length scale.
- In LES simulations, the length scales are determined by the cell size. Therefore,

$$l = C_s \Delta$$

- Where Δ is the filter width, that can be approximated as follows,

$$\Delta = \text{Cell volume}^{1/3}$$

LES sub-grid scale models – The Smagorinsky model

- At this point, the Smagorinsky model can be written as follows,

$$\nu_{SGS} = (C_S \Delta)^2 \left| \overline{\mathbf{S}} \right|$$

- Notice that as the mesh gets finer, the model produces less sub-grid scale turbulent viscosity (it approaches to a DNS).
- The Smagorisky coefficient C_s can be seen as a scaling factor of the turbulent eddies.
- That is, it is used to scale down the eddies size so there are no larger than the filter width.
- Obviously, to respect this behavior the value of the coefficient C_s has to be less than 1 and larger than 0.
- As you can see, this model resembles the Prandtl mixing length formulation.
- This formulation is also based on the idea that the cells are hexahedral (same applies for all LES turbulence models).
- Nevertheless, LES models can be used with general unstructured polyhedral cells.
- But have in mind that irregular cells can add significant numerical diffusion.

LES sub-grid scale models – The Smagorinsky model

- This model does not provide any means to compute sub-grid scale turbulent kinetic energy or k_{SGS} (the modelled turbulent kinetic energy).
- Again, using dimensional analysis we can derive a relation for k_{SGS}, as follows,

$$k_{SGS} = \left(\frac{\nu_{SGS}}{C_S \Delta} \right)^2$$

Recall that the dimensions of the turbulent kinetic energy are m^2/s^2

- Another alternative to compute k_{SGS} has been proposed in references [1,2,3], as follows,

$$k_{SGS} = C_I \Delta^2 \left| \overline{\mathbf{S}} \right|^2$$

- Where C_I is approximately equal to 0.10 (this approach is widely use in atmospheric flows).

References:
[1] G. Erlebacher, M. Hussaini, C. Speziale, T. Zang. Toward the large-eddy simulation of compressible turbulent flows. J. Fluid Mech. 238, 155–185, 1992.
[2] P. Moin, K. Squires, K., W. Cabot, S. Lee. A dynamic subgrid-scale model for compressible turbulence and scalar transport. Phys. Fluids A 3(11), 2746–2757, 1991.
[3] A. Yoshizawa. Statistical theory for compressible turbulent shear flows, with the application to subgrid modeling. Phys. Fluids 29(7), 2152–2164, 1986.

LES sub-grid scale models – The Smagorinsky model

- Finally, the rate of transfer of energy to the unresolved or residual motion (production term), is equal to,

$$P^{SGS} = -\tau^{SGS}\overline{\mathbf{S}} = 2\nu^{SGS}\overline{\mathbf{S}} : \overline{\mathbf{S}} = \nu^{SGS} \left|\overline{\mathbf{S}}\right|^2$$

- Where $\left|\overline{\mathbf{S}}\right|$ is the characteristic filtered rate of strain,

$$\left|\overline{\mathbf{S}}\right| = \left(2\overline{\mathbf{S}} : \overline{\mathbf{S}}\right)^{1/2}$$

- For the Smagorinsky model (or for any other eddy-viscosity model with $\nu^{SGS} > 0$), this energy transfer is everywhere from the filtered motions to the residual motions.
- That is, the energy transfer follows one direction, from resolved scales to unresolved scales.

Sub-grid scale models for LES

LES sub-grid scale models – The Smagorinsky model

- However, experimental studies and DNS simulations [1,2,3,4] have shown that energy is also transferred from smaller scales to larger scales, albeit at a much lower rate.
- This reverse process is known as backscatter and is not easily captured using models based on the Boussinesq hypothesis.
- For the Smagorinsky model in order to be able to capture the backscatter, one correction could be to reduce to zero the sub-grid scale viscosity ν^{SGS} in regions where the backscatter is expected.
  - However, this implies knowing a-priori the backscatter regions.
- Also, specific models have been developed to account for backscatter, such as, scale similarity models based on the triple decomposition.
- In the triple decomposition, the cross-stress term **C**, and the sub-grid scale Reynolds stress **R** are both associated with the backscatter.

References:
[1] S. Pope. Turbulent Flows. Cambridge University Press. 2014.
[2] P. Bernard, J. Wallace. Turbulent flow. Analysis, measurement, and prediction. Wiley, 2002.
[3] C. Baily, G. Compte-Bellot. Turbulence. Springer, 2015.
[4] P. Bernard. Turbulent fluid flow. Wiley, 2019.

Sub-grid scale models for LES

LES sub-grid scale models – Dynamic Smagorinsky-Lilly model

- One of the largest deficiencies of the Smagorinsky model is that the coefficient C_s needs to be calibrated.
- Germano et al. [1] and Lilly [2], conceived a procedure in which the Smagorinsky model constant is dynamically computed in space and time based on the information provided by the resolved scales of motion.
- The concept of the dynamic procedure is to apply a second filter (called the test filter) to the equations of motion.
- The new filter width is usually equal to twice the grid filter width.
    - Both filters produce a resolved flow field.
- The difference between the two resolved fields is the contribution of the small scales whose size is in between the grid filter and the test filter.
- The information related to these scales is used to compute the model constant,

$$C_s\left(\mathbf{x}, t\right)$$

References:
[1] M. Germano, U. Piomelli, P. Moin, W. H. Cabot. Dynamic Subgrid-Scale Eddy Viscosity Model. In Summer Workshop. Center for Turbulence Research, Stanford, CA. 1996.
[2] D. K. Lilly. A Proposed Modification of the Germano Subgrid-Scale Closure Model. Physics of Fluids. 4. 633–635. 1992.

Sub-grid scale models for LES

LES sub-grid scale models – Dynamic Smagorinsky-Lilly model

- In the dynamic Smagorinsky model, the sub-grid scale eddy viscosity is computed as follows,

$$\nu_{SGS} = C_s(\mathbf{x}, t)\,\Delta^2 \left|\overline{\mathbf{S}}\right|$$

- The use of the second filter leads to the so-called sub-test scale stresses,

$$\tau_{ij}^{ST} = \widehat{\overline{u_i u_j}} - \hat{\bar{u}}_i \hat{\bar{u}}_j$$

- The sub-test scale stresses are related to the SGS stresses via the Germano identity [1] as follows,

$$\hat{L}_{ij} = \tau_{ij}^{ST} - \hat{\tau}_{ij}^{SGS} = \widehat{\bar{u}_i \bar{u}_j} - \widehat{\bar{u}}_i \widehat{\bar{u}}_j$$

- Where $\hat{L}_{ij}$ denotes the Leonard stresses associated with the test filter.
- It represents the contribution to the Reynolds stresses by the scales whose length is contained between the filter width Δ and the test filter width $\hat{\Delta} = 2\Delta$.

References:
[1] M. Germano. Turbulence: the filtering approach. J. Fluid Mech. Vol. 238. 1992

Sub-grid scale models for LES

LES sub-grid scale models – Dynamic Smagorinsky-Lilly model

- If we express the sub-test scale stresses and SGS stresses using the eddy viscosity approach, we obtain,

$$\hat{L}_{ij} - \frac{\delta_{ij}}{3} L_{kk} = -2C_s M_{ij}$$

- Where,

$$M_{ij} = \hat{\Delta}^2 \left| \hat{\bar{S}} \right| \hat{\bar{S}}_{ij} - \left[\widehat{\Delta^2 \left| \bar{S} \right| \bar{S}_{ij}} \right]$$

- At this point, C_s can be computed using the least-squares minimization proposed by Lilly [1],

$$C_s \left(\mathbf{x}, t \right) = -\frac{1}{2} \frac{L_{ij} M_{ij}}{M_{ij}^2}$$

References:
[1] D. K. Lilly. A Proposed Modification of the Germano Subgrid-Scale Closure Model. Physics of Fluids. 4. 633–635. 1992.

LES sub-grid scale models – Dynamic Smagorinsky-Lilly model

- As for the standard Smagorinsky model, this model does not provide any means to compute sub-grid scale turbulent kinetic energy k_{SGS} .
- Based on dimensional grounds, k_{SGS} can be computed as follows,

$$k_{SGS} = \left(\frac{\nu_{SGS}}{C_{DS}\Delta} \right)^2$$

- Where C_{DS} is the dynamic coefficient computed by the model.
- Usually, C_{DS} is allowed to vary between 0 and 0.25.
- The dynamic Smagorinsky-Lilly model performs much better than standard Smagorinsky model.
- However, from a computational point of view, it is much more expensive.
- Generally speaking, this model is recommended over the standard Smagorinsky model.

LES sub-grid scale models – The WALE model

- Nicoud and Ducros [1] developed the WALE (Wall-Adapting Local Eddy-viscosity) model.
- This model is designed to overcome many of the deficiencies of the Smagorinsky model without adding significant new complexities.
- This method is based on invariants of the velocity-gradient tensor (instead of the strain-rate tensor used in the Smagorinsky model).
- This model was found to have the correct near-wall behavior and to vanish in laminar flows.
- It is also known to work with transitional flows.
- Vreman [2], also proposed a family of algebraic eddy viscosity models based on the second invariant of the velocity gradient tensor (similar to the WALE model), which also has these properties.

References:
[1] F. Nicoud, F. Ducros. Subgrid-scale stress modelling based on the square of the velocity gradient tensor. Flow, Turb. Combustion, 62. 1999.
[2] A. Vreman. Comment on “Inapplicability of the dynamic Clark model to the large eddy simulation of incompressible turbulent channel flows”. Phys. Fluids 15, 2003.

Sub-grid scale models for LES

LES sub-grid scale models – The WALE model

- In this model, the sub-grid scale eddy viscosity is computed as follows,

$$\nu_{SGS} = (C_W \Delta)^2 \frac{\left(S_{ij}^d S_{ij}^d\right)^{3/2}}{\left(\bar{S}_{ij}\bar{S}_{ij}\right)^{5/2} + \left(S_{ij}^d S_{ij}^d\right)^{5/4}}$$

- Where S_{ij}^d (filtered velocity gradient tensor) is defined as follows,

$$S_{ij}^d = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_k}\frac{\partial \bar{u}_k}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_k}\frac{\partial \bar{u}_k}{\partial x_i}\right) - \frac{1}{3}\delta_{ij}\frac{\partial \bar{u}_k}{\partial x_l}\frac{\partial \bar{u}_l}{\partial x_k}$$

Invariant, traceless, symmetric tensor

- And C_w is the model coefficient, which values can go anywhere between 0.3 to 0.6.

References:
[1] F. Nicoud, F. Ducros. Subgrid-scale stress modelling based on the square of the velocity gradient tensor. Flow, Turb. Combustion, 62. 1999.

LES sub-grid scale models – The WALE model

- The WALE model is built up from the invariant, traceless, symmetric tensor defined from the filtered velocity gradient tensor S_{ij}^d .

$$S_{ij}^d = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_k}\frac{\partial \bar{u}_k}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_k}\frac{\partial \bar{u}_k}{\partial x_i}\right) - \frac{1}{3}\delta_{ij}\frac{\partial \bar{u}_k}{\partial x_l}\frac{\partial \bar{u}_l}{\partial x_k}$$

- Note that this tensor includes the contributions of strain S and rotation W.
- By substituting $\partial \bar{u}_i / \partial x_j$ into S_{ij}^d , and after a lot algebra, we obtain the following expression,

$$S_{ij}^d S_{ij}^d = \frac{1}{6}\left(S^2 S^2 + \Omega^2 \Omega^2\right) + \frac{2}{3} S^2 \Omega^2 + 2 I \bar{S}_{ik} \bar{S}_{kj} \bar{W}_{jl} \bar{W}_{li}$$

$$\frac{\partial \bar{u}_i}{\partial x_j} = \bar{S}_{ij} + \bar{W}_{ij} \qquad S^2 = \bar{S}_{ij}\bar{S}_{ij} \qquad \Omega^2 = \bar{W}_{ij}\bar{W}_{ij}$$

- Clearly, both strain S and rotation W play a role in determining the magnitude of $S_{ij}^d S_{ij}^d$.

Sub-grid scale models for LES

LES sub-grid scale models – The WALE model

- The particular choice of terms in the denominator of ν_{SGS} prevents it from being zero at all locations in a flow.

$$\nu_{SGS} = (C_W \Delta)^2 \frac{\left(S_{ij}^d S_{ij}^d\right)^{3/2}}{\left(\bar{S}_{ij} \bar{S}_{ij}\right)^{5/2} + \left(S_{ij}^d S_{ij}^d\right)^{5/4}}$$

- The term containing $\bar{S}_{ij}$ remains finite at solid surfaces in the presence of shearing.
- While at locations where $\bar{S}_{ij}$ itself might be zero it is unlikely that the term containing S_{ij}^d will also be zero.
- This prevents singularities in the eddy viscosity prediction and enforces the correct near-wall behavior.

Sub-grid scale models for LES

LES sub-grid scale models – The WALE model

- The particular choice of terms in the denominator of ν_{SGS} prevents it from being zero at all locations in a flow.

$$\nu_{SGS} = (C_W \Delta)^2 \frac{\left(S_{ij}^d S_{ij}^d\right)^{3/2}}{\left(\bar{S}_{ij}\bar{S}_{ij}\right)^{5/2} + \left(S_{ij}^d S_{ij}^d\right)^{5/4}}$$

- Following a similar construction as in the Smagorinksy model, the eddy viscosity in the WALE model is derived from the product of a squared length scale and a term with time dependence t^{-1}.
- Since ν_{SGS} is to be proportional to $\left(S_{ij}^d S_{ij}^d\right)^{3/2}$ (a factor that has time dependency t^{-6}) in order to arrive at an expression for ν_{SGS} with time unit proportional to t^{-1}, this term must be divided by a term with dimension t^{-5}.
- As you can see, dimensional analysis is all over turbulence modeling.

Sub-grid scale models for LES

LES sub-grid scale models – The WALE model

- As for the previous models, this model does not provide any means to compute sub-grid scale turbulent kinetic energy k_{SGS} .
- Again, based on dimensional grounds, k_{SGS} can be computed as follows,

$$k_{SGS} = \left(\frac{\nu_{SGS}}{C_W \Delta} \right)^2$$

- Where C_W is the model coefficient and is usually between $0.3 < C_W < 0.6$ [1].
- This model performs much better than the standard Smagorinsky model.
- Also, its performance is comparable to that of the dynamic Smagorinsky-Lilly but with a lower computational cost.
- This model is recommended over the standard Smagorinsky model.

References:
[1] F. Nicoud, F. Ducros. Subgrid-scale stress modelling based on the square of the velocity gradient tensor. Flow, Turb. Combustion, 62. 1999.

Sub-grid scale models for LES

LES sub-grid scale models – TKE one equation model

- As for RANS/URANS models that depends on the TKE equation, sub-grid scale models has been developed based on the TKE [1,2,3,4].
- In this family of sub-grid scale models an additional transport equation for k_{SGS} is solved.
- In this model, the sub-grid scale eddy viscosity is computed as follows,

$$\nu_t = C_k h \sqrt{k_{SGS}}$$

- Where *h* is the grid spacing or filter width and C_k is the model constant.
- Note that as basis for the velocity scale of the eddy viscosity we use,

$$k_{SGS} = \frac{1}{2}\tau_{kk}$$

- Note that in our notation the filter width or grid spacing *h* is the same as Δ ,

$$h \equiv \Delta$$

References:
[1] U. Schumann. Subgrid Scale Model for Finite Difference Simulations of Turbulent Flows in Plane Channel and Annuli. J. Comp. Phys., 1975
[2] A. Yoshizawa, K. Horiuti. A Statistically Derived Subgrid-Scale Kinetic Energy Model for the Large-Eddy Simulation of Turbulent Flows. J. Phys. Soc. Japan, 1985.
[3] W. Kim, S. Menon. Application of the localized dynamic subgrid-scale model to turbulent wall-bounded flows. Technical Report AIAA-97-0210. January 1997.
[4] L. Davidson, S. Peng. Hybrid LES-RANS Modeling: a One Equation SGS Model Combined with a k-omega Model for Predicting Recirculating Flows. Int. J. Num. Meth. Fluids, 2003.

LES sub-grid scale models – TKE one equation model

- The filtered TKE equation used in this model can be formulated as follows,

$$\frac{\partial k_{sgs}}{\partial t} + \frac{\partial \bar{u}_j k_{sgs}}{\partial x_j} = 2hC_k \sqrt{k_{sgs}} \bar{S}_{ij} \bar{S}_{ij} - C_\epsilon \frac{k_{sgs}^{3/2}}{h} + \frac{\partial}{\partial x_j} \left[\left(\nu + C_k h \sqrt{k_{sgs}} \right) \frac{k_{sgs}}{\partial x_j} \right]$$

- As for the RANS/URANS TKE equation, this equation has production, dissipation, and transport terms.
- Note that since this is a one-equation model, the dissipation takes the following form,

$$\epsilon = \frac{k_{sgs}^{3/2}}{h}$$

LES sub-grid scale models – TKE one equation model

- The filtered TKE equation used in this model can be formulated as follows,

$$\frac{\partial k_{sgs}}{\partial t} + \frac{\partial \bar{u}_j k_{sgs}}{\partial x_j} = 2hC_k\sqrt{k_{sgs}}\bar{S}_{ij}\bar{S}_{ij} - C_\epsilon \frac{k_{sgs}^{3/2}}{h} + \frac{\partial}{\partial x_j}\left[\left(\nu + C_k h\sqrt{k_{sgs}}\right)\frac{k_{sgs}}{\partial x_j}\right]$$

- The length scale *h* is related to the grid spacing or filter width, and can be estimated as follows,

$$h = (\Delta x \Delta y \Delta z)^{1/3}$$

$$h = \min\left(\Delta x, \Delta y, \Delta z\right)$$

This choice has been found to be more effective in some circumstances [1]

- Note that these relations are formulated for hexahedral meshes.
- For general unstructured meshes there are different formulations.
- In LES simulations, the performance of the filter operator strongly depends on the cell type.
- It is strongly recommended to use hexahedral meshes with low growth rate.

- In this model, usual values for the constants are,

$$C_k = 0.07 \qquad C_\epsilon = 1.05$$

References:
[1] L. Davidson, S. Peng. Hybrid LES-RANS Modeling: a One Equation SGS Model Combined with a k-omega Model for Predicting Recirculating Flows. Int. J. Num. Meth. Fluids, 2003.

Sub-grid scale models for LES

LES sub-grid scale models – TKE one equation model

- One of the advantages of using the TKE one equation sub-grid scale model, is that we can compute k_{SGS} .
- Then, among many things, this quantity can be used to assess the quality of the LES simulations.
- A good LES simulation aims at resolving 80% of the turbulent energy spectrum.
- A way to measure the quality of a LES simulation is by using the Pope criterion [1, 2], which is a simple measure of the fraction of the turbulent kinetic energy in the resolved motions. This criterion is defined as follows,

$$M(\mathbf{x},t) = \frac{k_{SGS}(\mathbf{x},t)}{k_{SGS}(\mathbf{x},t) + k_{RES}(\mathbf{x},t)} \qquad \text{where} \qquad k_{RES} = 0.5\left(u'^2 + v'^2 + w'^2\right)$$

where k_{SGS} is the turbulent kinetic energy of the SGS eddies and k_{RES} is the kinetic energy of the resolved eddies (determined by the grid size).

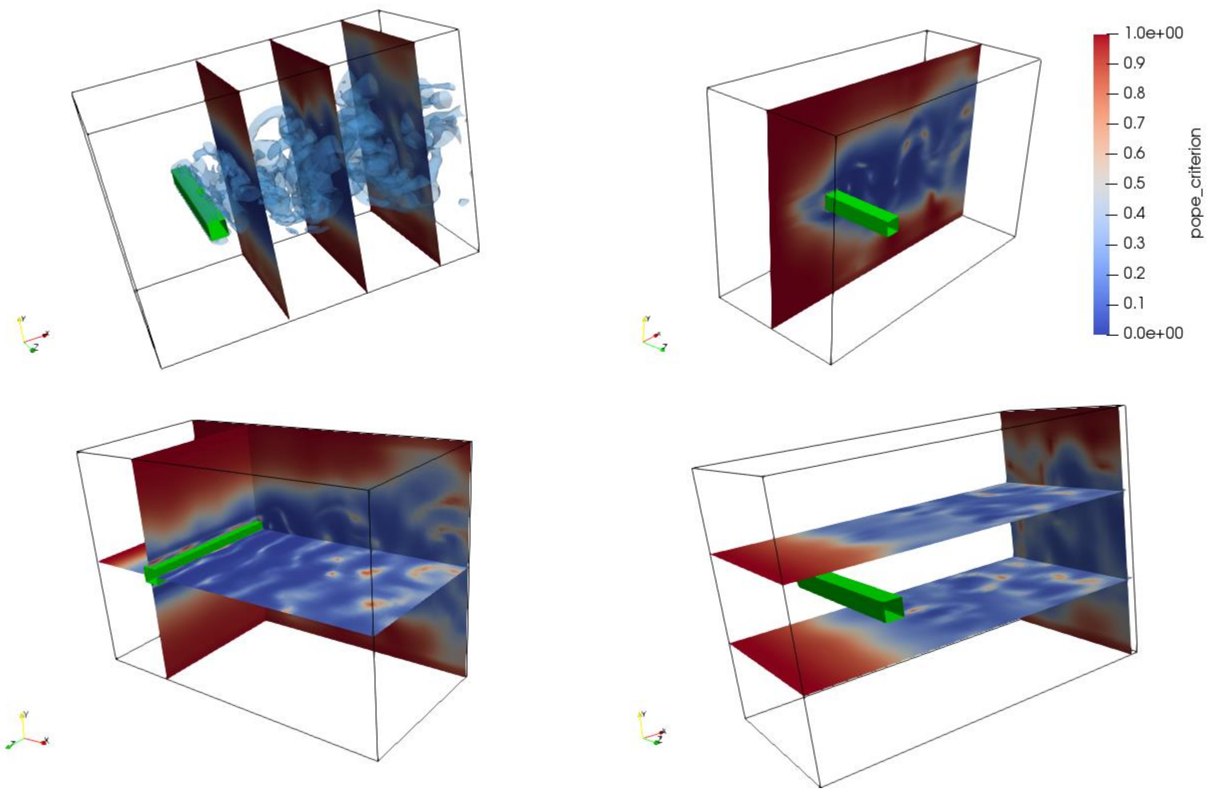
- It is clear that to compute M, k_{SGS} is required, and not all sub-grid scales models compute this quantity.

[1] S. Pope. Turbulent Flows. Cambridge University Press. 2014.
[2] S. Pope. Ten questions concerning the large-eddy simulation of turbulent flows. New J. of Physics. 2004.

Sub-grid scale models for LES

LES sub-grid scale models – TKE one equation model

- In the Pope criterion, the values of M are between 0 and 1.
- A value of M = 0 corresponds to a DNS simulation, and a value of M = 1 corresponds to a RANS simulation.
- A good quality LES should have a value of M between 0.6 and 0.8.



- The Pope criterion is a global field. It is computed in the whole domain.
- The Pope criterion can be visualized by plotting it in several cut planes of the domain.
- It can also be visualized using iso-surfaces.
- You can compute the Pope criterion using the instantaneous values or the mean values.

Final remarks

- A very good alternative to the Smagorinsky model is the WALE model (Wall-Adaptive Local Eddy Viscosity model).
    - The WALE model overcomes many of the drawbacks of the Smagorinsky model and retains its simplicity.
    - The WALE model predicts accurately the flow near the walls.
    - It also predicts transition, but you need very fine meshes.
    - The model constants (or coefficients) can depend on the flow conditions.
- Dynamic models where the Smagorinsky coefficient is dynamically computed in function of space and time offer superior performance, but at a slightly higher computational cost.

Sub-grid scale models for LES

Final remarks

- Some more advanced LES models introduce transport effects by solving an equation for the turbulent kinetic energy and using double filtering to find out more information about the sub-grid scales.
    - Clearly, these models are more expensive than the algebraic models.
    - However, they give information about the modeled TKE.
    - These models are also a good choice.
    - One equation TKE LES models (standard and dynamic) are often available in many CFD solvers.
    - The one equation TKE LES models (standard or dynamic) are recommended over the Smagorinsky and WALE models.

Roadmap to Lecture 10

1. ~~SRS simulations~~
2. ~~LES equations – Filtered Navier-Stokes equations~~
3. ~~Sub-grid scale models for LES~~
4. **DES brief review**
5. A few mesh resolution guidelines and rough estimates for LES/DES simulations
6. Final remarks on LES/DES turbulence models

DES brief review

- An alternative to LES models, is the use of Detached-Eddy simulation (DES) models.
- Detached-Eddy Simulation (DES) is a hybrid technique first proposed by Spalart et al. [1] for prediction of turbulent flows at high Reynolds numbers (refer also to [2,3]).
- The development of this technique was motivated by estimates which indicate that the computational costs of applying Large-Eddy Simulation (LES) to complete configurations such as an airplane, submarine, or road vehicle are prohibitive.
- The high cost of LES when applied to complete configurations at high Reynolds numbers arises because of the resolution required in the boundary layers, an issue that remains even with fully successful wall-layer modeling.
  - The boundary layer must be resolved in the spanwise and streamwise directions as well.
- In Detached-Eddy Simulation (DES), the aim is to combine the most favorable aspects of the two techniques, *i.e.*, application of RANS models for predicting the attached boundary layers and LES for resolution of time-dependent, three-dimensional large eddies.
- The scaling cost of the method is then favorable since LES is not applied to resolve the relatively small structures that cover the boundary layer.

References:
[1] P. Spalart, W. Jou, M. Stretlets, S. Allmaras. Comments on the Feasibility of LES for Wings and on the Hybrid RANS/LES Approach. AFOSR Conf. 1997.
[2] P. Spalart. Strategies for turbulence modelling and simulations. International Journal of Heat and Fluid Flow 21 (2000).
[3] K. D. Squires. Detached-eddy simulation: Current status and perspectives. Direct and Large-Eddy Simulation V. ERCOFTAC Series, vol 9. Springer, 2004.

DES brief review

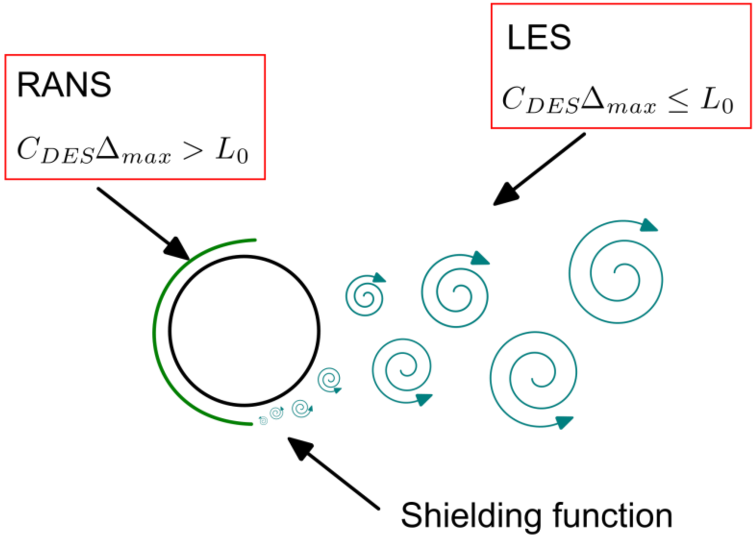
- DES models formulations consist in blending RANS and LES models.
- DES models can be built on top of any RANS model (usually Spalart-Allmaras or $k-\omega$ SST).
- In DES models the switch between RANS and LES is based on a criterion similar to,

$$C_{DES}\Delta_{max} > L_0 \rightarrow \mathrm{RANS}$$
$$C_{DES}\Delta_{max} \leq L_0 \rightarrow \mathrm{LES}$$

where

$$L_0 = \frac{k^{1.5}}{\epsilon} = \frac{k^{0.5}}{0.09\,\omega}$$

- In the previous equations, Δ_{max} is the maximum edge length of the local cell.



A shielding function can be used to avoid the resolved structures from entering into the boundary layer regions. This is referred to as delayed DES or DDES.

DES brief review

- Due to the direct impact of the grid spacing on the RANS model, DES models require more carefully crafted meshes to avoid inappropriate behavior.
- The original DES model is straightforward and simple [1,2].
- Nevertheless, the user requires not only a basic understanding of the model behavior, but also has to follow intricate grid generation guidelines to avoid undefined simulation behavior somewhere between RANS and LES.
- Problematic behavior of standard DES has been reported by Menter and Kuntz [3], who demonstrated that an artificial separation could be produced for an airfoil simulation when refining the max cell edge length (h_{max}) inside the wall boundary layer below a critical value of $h_{max}/\delta < 0.5\text{-}1$, where δ is the local boundary layer thickness.
- This effect was termed Grid Induced Separation (GIS) [3,4] as the separation depends on the grid spacing and not the flow physics.
- GIS is obviously produced by the effect of a sudden grid refinement which changes the DES model from RANS to LES, without balancing the reduction in eddy-viscosity by resolved turbulence content.
- The switch from the RANS to the LES model inside wall boundary layers is not desirable.

References:

[1] P. Spalart, W. Jou, M. Stretlets, S. Allmaras. Comments on the Feasibility of LES for Wings and on the Hybrid RANS/LES Approach. AFOSR Conference (1997).

[2] P. Spalart. Strategies for turbulence modelling and simulations. International Journal of Heat and Fluid Flow 21 (2000).

[3] F. Menter, M. Kuntz, R. Langtry. Ten years of experience with the SST turbulence model. 4th International Symposium on Turbulence Heat and Mass Transfer (2003).

[4] P. Spalart, S. Deck, M. Shur, K. Squires, M. Strelets, A. Travin. A new version of detached-eddy simulation, resistant to ambiguous grid densities. Theor. Comput. Fluid Dyn. 20 (2006).

DES brief review

- GIS can in principle be avoided by shielding the RANS model from the DES formulation for wall boundary layers.
- This was proposed by Menter and Kuntz [1], who used the blending functions of the SST model for that purpose.
- Later, Spalart et al. [2] proposed a more generic formulation of the shielding function, which depends only on the eddy-viscosity and the wall distance. It can therefore, in principle, be applied to any eddy-viscosity based DES model.
- The resulting formulation was termed **Delayed Detached Eddy Simulation (DDES)** [2].
- While the shielding function developed in [2] was considered generic, it was essentially calibrated for the Spalart-Allmaras (SA) one-equation RANS model.
- The **DDES** approach was later extended to a two-equation (Menter $k-\omega$) formulation by Strelets (2001).

References:
[1] F. Menter, M. Kuntz, R. Langtry. Ten years of experience with the SST turbulence model. 4th International Symposium on Turbulence Heat and Mass Transfer (2003).
[2] P. Spalart, S. Deck, M. Shur, K. Squires, M. Strelets, A. Travin. A new version of detached-eddy simulation, resistant to ambiguous grid densities. Theor. Comput. Fluid Dyn. 20 (2006).
[3] M. Strelets. Detached Eddy Simulation of Massively Separated Flows. 39th Aerospace Sciences Meeting and Exhibit, American Institute of Aeronautics and Astronautics (2001).

DES brief review

- The most recent improvement of the DES formulation is termed **Improved Delayed Detached Eddy Simulation (IDDES)** [1].
- The IDDES formulation incorporates two major enhancements to the original DES method.
  - The first enhancement led to the DDES (Delayed Detached Eddy Simulation) formulation [2] which addresses the issue of GIS by preventing the switch from RANS to LES mode within attached boundary layers due solely to the specifics of the grid design.
  - The second enhancement was the introduction of a wall-modeled LES functionality [1].
  - In general, the IDDES formulation provides a wall-modeled LES (WMLES) response if resolved turbulent content is supplied as an inflow (or initial) condition, and resorts to a DDES response otherwise.
  - The wall-modeled LES behavior is intended to be active only when the inflow conditions include resolved turbulent content, and the grid is fine enough to at least resolve the largest energy containing boundary layer eddies.
  - The IDDES model features several rather intricate length scales definitions, and blending and shielding functions, which allow using this model both in DDES and WMLES mode.

References:
[1] M. Shur, P. Spalart, M. Strelets, A. Travin. A hybrid RANS-LES approach with delayed-DES and wall-modelled LES capabilities. International Journal of Heat and Fluid Flow, 29 (2008).
[2] P. Spalart, S. Deck, M. Shur, K. Squires, M. Strelets, A. Travin. A new version of detached-eddy simulation, resistant to ambiguous grid densities. Theor. Comput. Fluid Dyn. 20 (2006).

SST DDES Formulation

- The governing equations of the SST DDES model read as [1],

$$\frac{\partial \rho k}{\partial t} + \nabla \cdot (\rho \mathbf{u} k) = \nabla \cdot \left[\left(\mu + \sigma_k \mu_t \right) \nabla k \right] + P_k - \frac{\rho \sqrt{k^3}}{l_{DDES}}$$

$$\frac{\partial \rho \omega}{\partial t} + \nabla \cdot (\rho \mathbf{u} \omega) = \nabla \cdot \left[\left(\mu + \sigma_\omega \mu_t \right) \nabla \omega \right] + 2 \left(1 - F_1 \right) \rho \sigma_{\omega 2} \frac{\nabla k \cdot \nabla \omega}{\omega} + \alpha \frac{\rho}{\mu_t} P_k - \beta \rho \omega^2$$

$$\mu_t = \rho \frac{a_1 \cdot k}{\max \left(a_1 \cdot \omega, F_2 \cdot S \right)}$$

- This model features several blending and shielding functions.
- Many of the constant used in this model are computed by a blend function between the respective constants of the $k - \epsilon$ and $k - \omega$ models.

References:
[1] F. Menter, M. Kuntz, R. Langtry. Ten years of experience with the SST turbulence model. 4th International Symposium on Turbulence Heat and Mass Transfer (2003).

SST DDES Formulation

- The blending functions, shielding functions, and constants used read as [1],

$$F_1 = \tanh(\arg_1^4)$$

$$\arg_1 = \min\left(max\left(\frac{\sqrt{k}}{C_\mu \omega d_\omega}, \frac{500\nu}{d_\omega^2 \omega} \right), \frac{4\rho\sigma_{\omega 2} k}{CD_{k\omega} d_\omega^2} \right)$$

$$CD_{k\omega} = \max\left(2\rho\sigma_{\omega 2} \frac{\nabla k \cdot \nabla \omega}{\omega}, 10^{-10} \right)$$

$$F_2 = \tanh\left(\arg_2^2\right)$$

$$\arg_2 = \max\left(\frac{2\sqrt{k}}{C_\mu \omega d_\omega}, \frac{500\nu}{d_\omega^2 \omega} \right)$$

$$P_k = \min\left(\mu_t S^2, 10 \cdot C_\mu \rho k \omega\right)$$

References:

[1] F. Menter, M. Kuntz, R. Langtry. Ten years of experience with the SST turbulence model. 4th International Symposium on Turbulence Heat and Mass Transfer (2003).

SST DDES Formulation

- The blending functions, shielding functions, and constants used read as [1],

$$l_{DDES} = l_{RANS} - f_d \max\left(0, l_{RANS} - l_{LES}\right)$$

$$l_{LES} = C_{DES} h_{\max}$$

$$l_{RANS} = \frac{\sqrt{k}}{C_\mu \omega}$$

$$C_{DES} = C_{DES1} \cdot F_1 + C_{DES2} \cdot (1 - F_1)$$

$$f_d = 1 - \tanh\left[(C_{d1} r_d)^{C_{d2}}\right]$$

$$r_d = \frac{\nu_t + \nu}{\kappa^2 d_\omega^2 \sqrt{0.5 \cdot (S^2 + \Omega^2)}}$$

$$C_\mu = 0.09, \kappa = 0.41, a_1 = 0.31$$

$$C_{DES1} = 0.78, C_{DES2} = 0.61, C_{d1} = 20, C_{d2} = 3$$

References:

[1] F. Menter, M. Kuntz, R. Langtry. Ten years of experience with the SST turbulence model. 4th International Symposium on Turbulence Heat and Mass Transfer (2003).

DES brief review

- Summary of the main characteristics of DES models:
    - DES models [1] are hybrid between RANS and LES.
    - RANS models are used close to the walls, and in the far field LES models are used.
    - The near wall turbulence is not explicitly computed, but fully modeled.
    - The mesh resolution requirements close to the walls are equivalent to those of RANS/URANS.
    - These models work particularly well for detached flows and external aerodynamics.
    - Refrain from using DES models with internal flows.
    - It is strongly is recommended to resolve the boundary layer (only normal to the wall).
        - However, this will impose a restriction on the time-step.
    - It is possible to use more stretching in the stream-wise and span-wise direction than with LES because it is not necessary to resolve eddies located in the wall region.

References:
[1] P. Spalart, W. Jou, M. Stretlets, S. Allmaras. Comments on the Feasibility of LES for Wings and on the Hybrid RANS/LES Approach. AFOSR Conf. 1997.

DES brief review

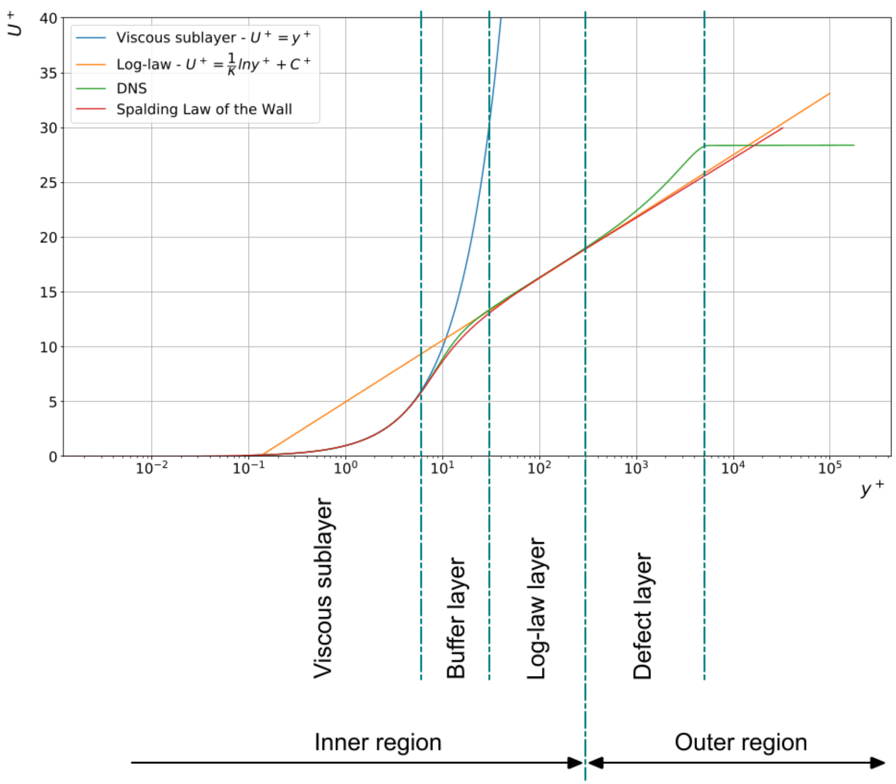
- Summary of the main characteristics of DES models:
  - In DES, it is critical to analyze the location of the LES/RANS interface.
    - The goal is to be in RANS mode in the wall regions and in LES mode in the free flow.
  - With DES we can use larger CFL numbers in comparison to LES.
  - When using DES models, the user requires not only a basic understanding of the model behavior, but also has to follow intricate grid generation guidelines to avoid undefined simulation behavior somewhere between RANS and LES.
  - If you are planning to use a DES model, it is recommended to use the DDES formulation.
  - DES formulations based on the Spalart-Allmaras and the $k-\omega$ SST turbulence models, they both perform well when dealing with external aerodynamics.
  - However, it is recommended to use the Salart-Allmaras formulation as it has been widely validated, and many of closure coefficients and blending functions have been explicitly calibrated for the Spalart-Allmaras formulation.

Roadmap to Lecture 10

1. ~~SRS simulations~~
2. ~~LES equations – Filtered Navier-Stokes equations~~
3. ~~Sub-grid scale models for LES~~
4. ~~DES brief review~~
5. **A few mesh resolution guidelines and rough estimates for LES/DES simulations**
6. Final remarks on LES/DES turbulence models

A few mesh resolution guidelines and rough estimates for LES/DES simulations

- Let us recall the velocity profile near the wall (Law of the wall) and the definition of y^+.
- y^+ is the non-dimensional distance normal to the wall.
- It is very similar to the Reynolds number, but it is non-dimensionalized using the shear velocity U_τ and computed using the distance normal to the wall.



$$y^+ = \frac{\rho \times U_\tau \times y}{\mu} = \frac{U_\tau \times y}{\nu}$$

$$U_\tau = \sqrt{\frac{\tau_w}{\rho}}$$ ← wall shear stresses

$$u^+ = \frac{U}{U_\tau}$$

In the previous equations:

- y is the distance normal to the wall.
- U_τ is the shear velocity,
- u^+ relates the mean velocity to the shear velocity.

y^+ wall distance units normal to the wall

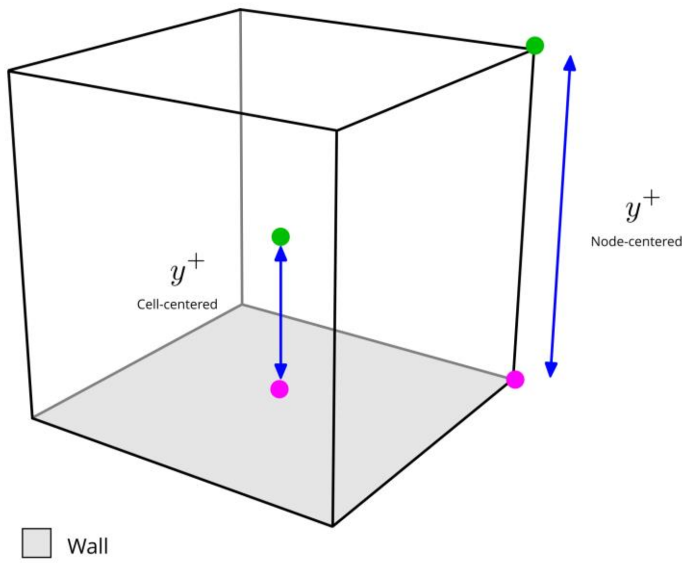
- y^+ is very important quantity in wall bounded turbulence modeling.
- We can use y^+ to estimate the mesh resolution near the wall before running the simulation.
- Have in mind that we do not know a-priori the wall shear stresses at the walls.
- Therefore, we need to use correlations to get a rough estimate and generate the initial mesh.
- The initial mesh is then generated according to the chosen near the wall treatment, namely, wall resolving, wall functions, or y^+ insensitive.
- Then, we run a precursor simulation to validate the mesh or to get a better estimate of y^+ and determine if we need a finer or coarser mesh.
- It is an iterative process, and it can be very time consuming, as it might require remeshing and rerunning the simulation to satisfy the near the wall treatment.

y^+ wall distance units normal to the wall

- y^+ is very important quantity in wall bounded turbulence modeling.
- It is quite difficult to get a uniform y^+ value at the walls.
- We usually try to get a y^+ mean value as close as possible to your target value.
- Also, it is important to monitor that we do not get very high maximum values of y^+ (more than a 1000)
- Values of y^+ up to 300 are fine and maybe up to 600.
- Remember, the maximum value depends on the system Reynolds number.
- y^+ values larger than 300 and up to a 1000 (or even more) are acceptable if they do not covert a large surface (no more than 10% of the total wall area), or if they are not located in critical zones.
- Use common sense when accessing y^+ value.

y^+ wall distance units normal to the wall

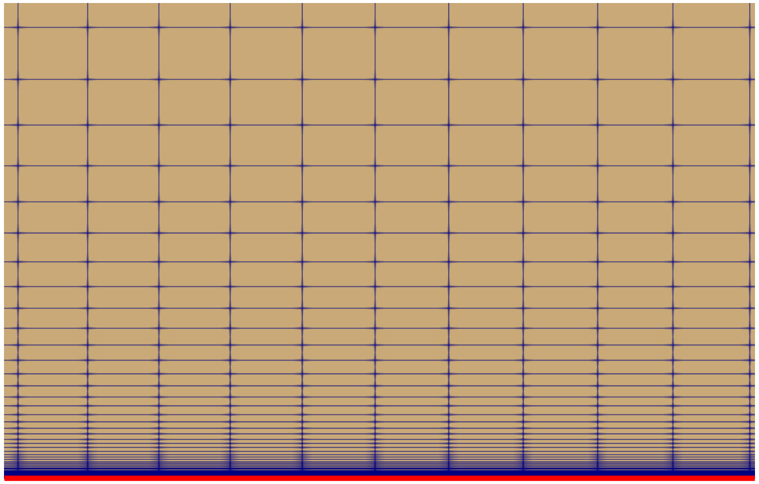
- It is worth mentioning that y^+ can be defined in reference to the cell center or the cell nodes.
- From now on, we are going to assume that we are using a cell-centered solver.
    - Ansys Fluent, StarCCM+, OpenFOAM, code Saturne, SU2, are all cell-centered.
    - It is up to you to check with the solver's documentation to find out with what kind of formulation are you using.
- By the way, y^+ is computed in the same way disregarding of the cell type used.
    - In the cell-centered formulation you need to know the location of the cell center.
    - In the node-centered formulation you need to know the location of the reference node. Usually, the node closest to the surface.



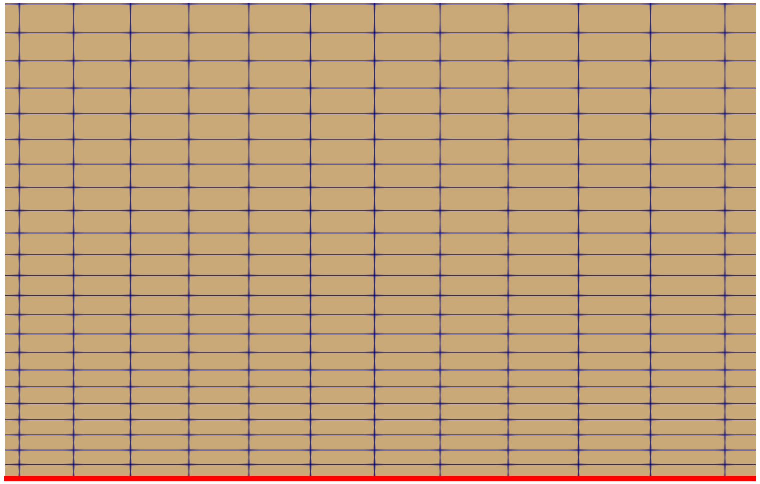
y^+ wall distance units normal to the wall

- At meshing time and estimate the normal distance from the wall to the first cell center (y), we use the well-known y^+ definition.
- Where we set a target y^+ value and then we solve for the quantity y.
    - If you choose a low target y^+ (e.g., less than 10), you will have a mesh that is clustered towards the wall (small value of y).
    - If you choose a large y^+ value (e.g., more than 100), you will have a coarse mesh close to the walls (large value of y).

WALL



Fine mesh towards the walls



WALL

Coarse mesh towards the walls

Note: depending on the implementation you can talk about cell center or node center.

Estimation of y^+ and normal wall distance

- At meshing time, to estimate the normal distance from the wall to the first cell center, we use the well-known y^+ definition,

$$y^+ = \frac{\rho \times U_\tau \times y}{\mu} = \frac{U_\tau \times y}{\nu}$$

- The problem is that at meshing time we do not know the value of the shear velocity,

$$U_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

- So, how do we get an initial estimate of this quantity?
    - We can use correlations, as described in the next slide.

Estimation of y^+ and normal wall distance

- At meshing time, to estimate the normal distance from the wall to the first cell center, you can proceed as follows,

1. $$Re = \frac{\rho \times U \times L}{\mu}$$

Compute the Reynolds number using the characteristic length of the problem.

2. $$C_f = 0.058 \times Re^{-0.2}$$

Compute the friction coefficient using any of the correlations available in the literature. There are many correlations available ranging from pipes to flat plates, for smooth and rough surfaces.

This correlation corresponds to a smooth flat plate case, ideal for external aerodynamics.

3. $$\tau_w = \frac{1}{2} \times C_f \times \rho \times U_\infty^2$$

Compute the wall shear stresses using the friction coefficient computed in the previous step.

4. $$U_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

Compute the shear velocity using the wall shear stresses computed in the previous step.

5. $$y = \frac{\mu \times y^+}{\rho \times U_\tau}$$

Set a target y^+ value and solve for y using the flow properties and previous estimates.

This value represents the distance from the wall to the first cell center.

At this point, you can generate your mesh trying to respect this value.

- You will find a simple calculator for the wall distance estimation in the following link: http://www.wolfdynamics.com/tools.html?id=2

Estimation of y^+ and normal wall distance

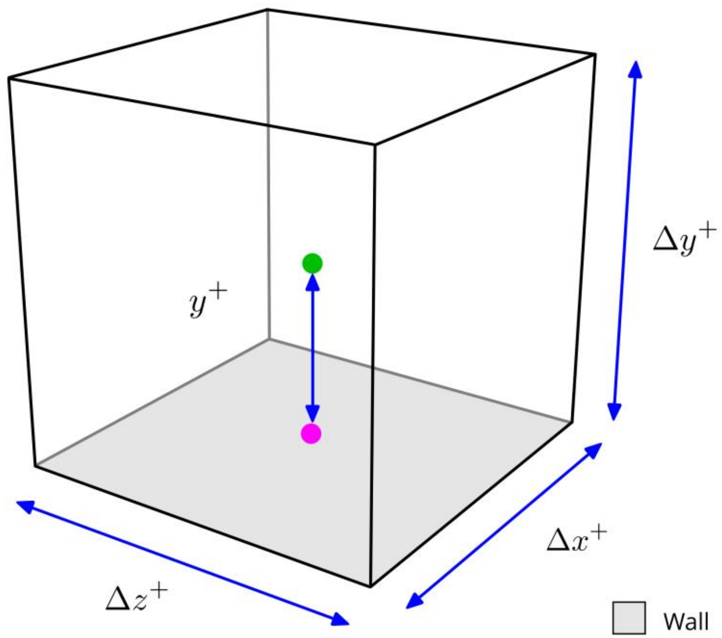
- Remember, the thickness of the thermal sublayer for a high Prandtl number Pr fluids (e.g., water) is much less than the momentum sublayer thickness.
- Therefore, the mesh requirements close to the walls need to be corrected for the thinner thermal boundary layer.
- The y^+ value can be corrected as follows,

$$y^+ \approx \frac{1}{\sqrt{Pr}}$$

- A similar situation exists when working with species transport and using the Schmidt number.
- If the Schmidt number (Sc) is considerably larger than unity, then the thickness of the thermal diffusion sublayer is much less than the momentum sublayer thickness.
- In this situation, the y^+ value can be corrected as follows,

$$y^+ \approx \frac{1}{\sqrt[3]{Sc}}$$

Wall distance units x^+ – y^+ – z^+ and some rough estimates



- Similar to y^+, the wall distance units can be computed in the stream-wise (Δx^+) and span-wise (Δz^+) directions.
- The wall distance units in the stream-wise and span-wise directions can be computed as follows:

$$\Delta x^+ = \frac{U_\tau \Delta x}{\nu} \qquad \Delta z^+ = \frac{U_\tau \Delta z}{\nu}$$

- And recall that y^+ is computed at the cell center (in cell-centered solvers),

$$\Delta y^+ = 2 \times y^+$$

$$(\Delta x^+, \Delta y^+, \Delta z^+) = \left(\frac{x}{l_\tau}, \frac{y}{l_\tau}, \frac{z}{l_\tau}\right)$$

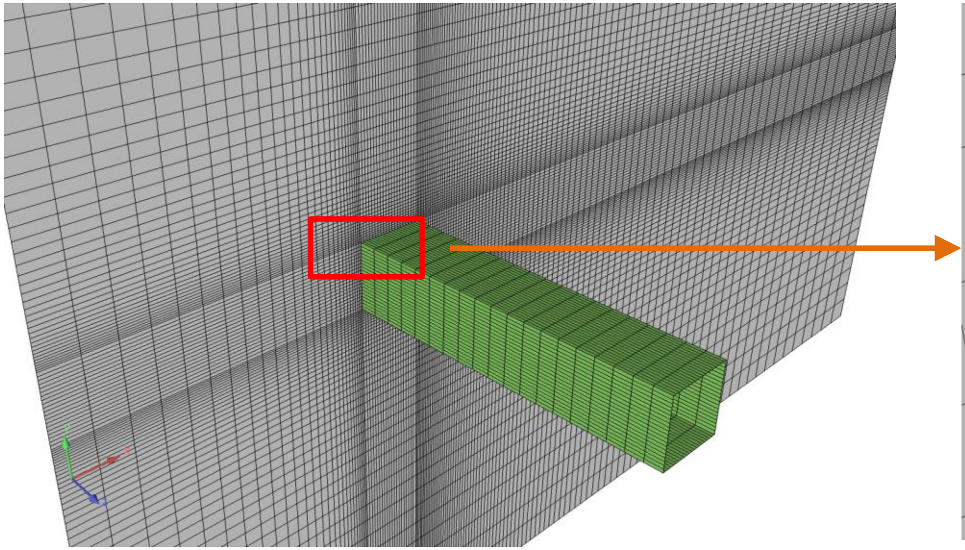
where

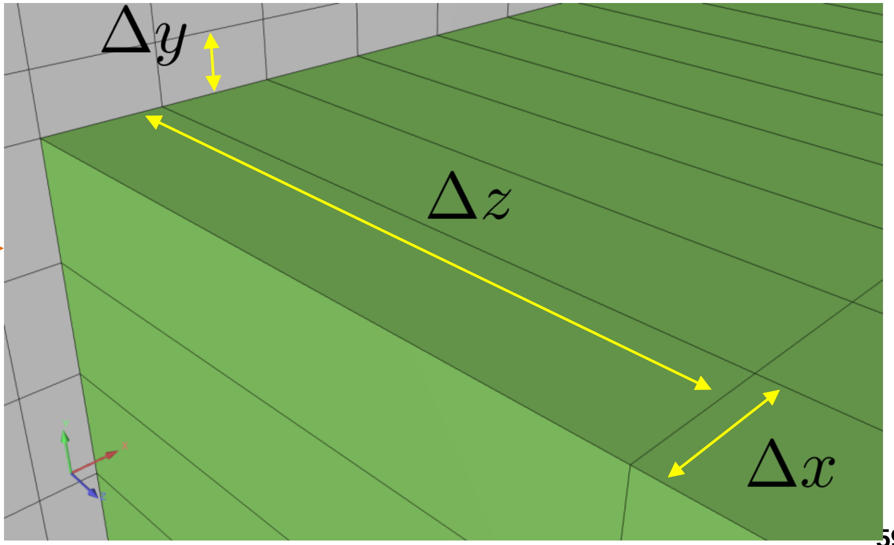
$$l_\tau = \frac{\nu}{U_\tau}$$

Viscous length

Wall distance units x^+ – y^+ – z^+ and some rough estimates

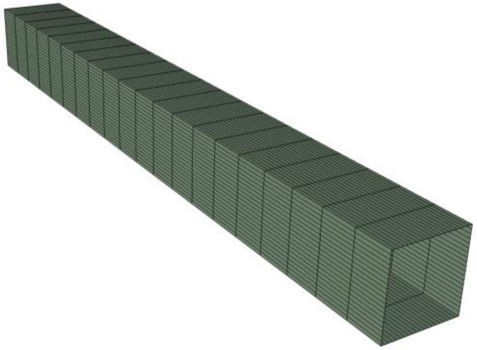
- It is not only about the y^+ value.
- In SRS simulations x^+ and z^+ are also important.
- The x^+ and z^+ values strongly depend on the surface mesh aspect ratio.
- The aspect ratio is defined as the ratio between the longest edge and shortest edge.
- Recommended values of surface mesh aspect ratio for SRS simulations are 10 or less.
- However, have in mind that this is a rough estimate based on experience and good practices.
- Meshing applications and solvers usually have adequate tools for mesh diagnostics.
- Notice that this metric is entirely based on geometrical information.

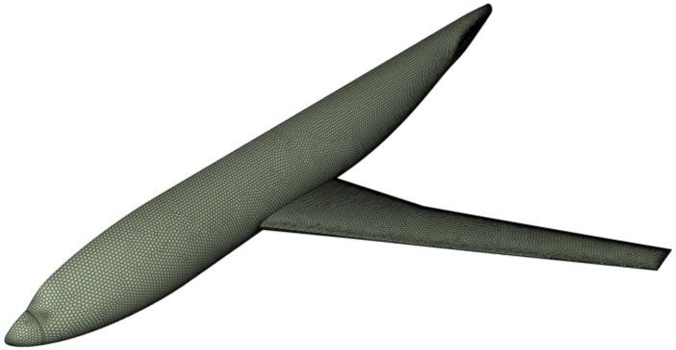


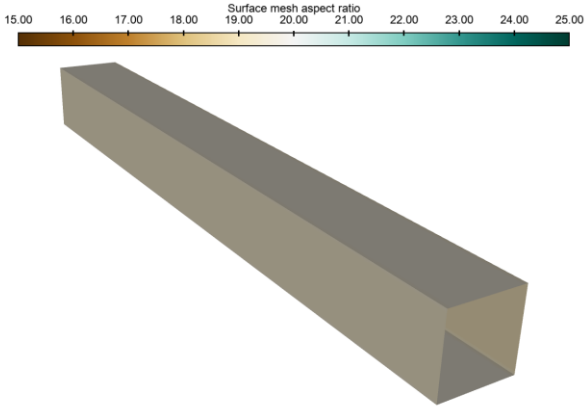


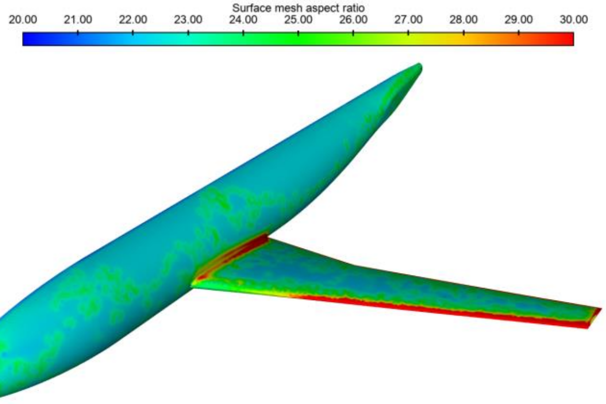
Wall distance units x^+ – y^+ – z^+ and some rough estimates

- For quad elements, the surface aspect ratio can be computed easily.
- For other shapes (triangles, polyhedrons, and so on), it is not very easy to compute.
- Surface mesh aspect ratio has a direct implication when computing the wall distance units x^+ and z^+.









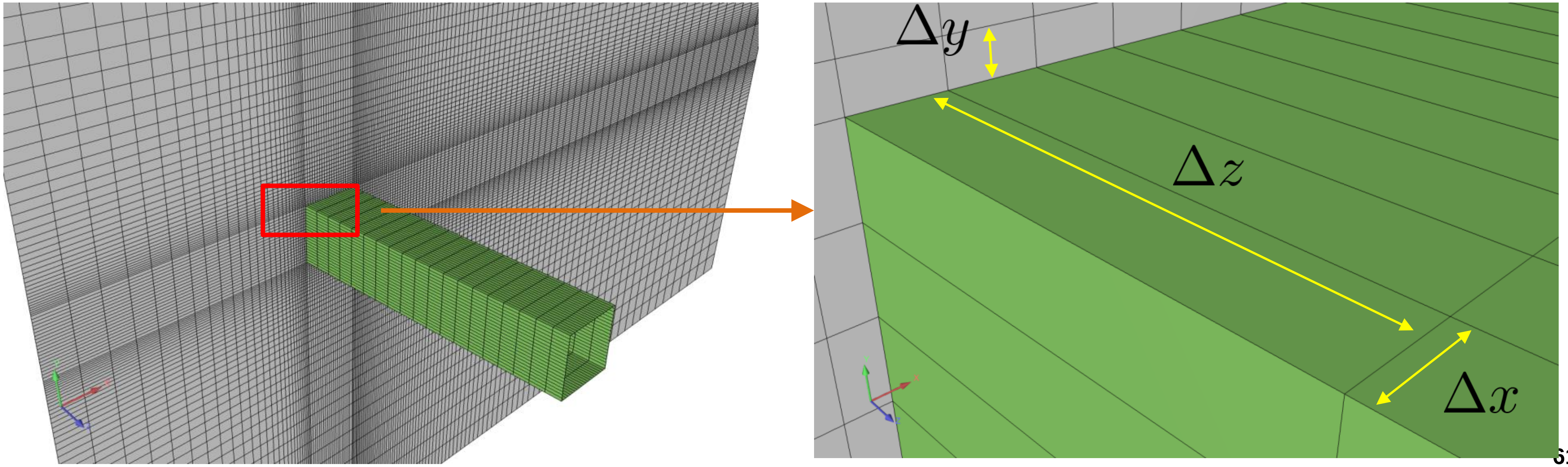
A few mesh resolution guidelines and rough estimates for LES/DES simulations

Wall distance units x^+ – y^+ – z^+ and some rough estimates

- Besides the surface aspect ratio, you should also check the ratio between surface cell base length and the cell height. This aspect ratio can be approximated as follows,

$$AR = \frac{\sqrt{\text{Surface cell area}}}{\Delta y}$$

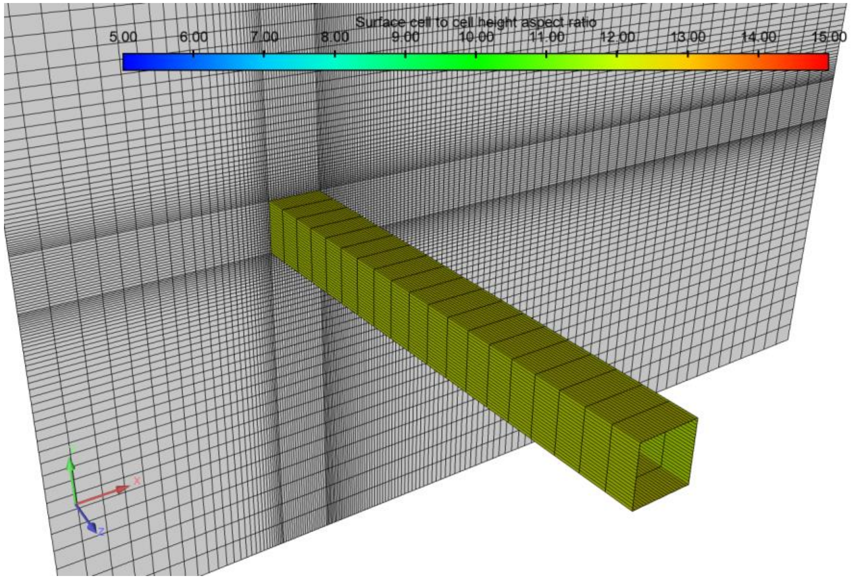
- Recommended values are 20 or less.
- As you can see, this aspect ratio can impose strong requirements on the mesh.
- Again, this is a rough estimate based on experience, and that should be used when conducting SRS simulations

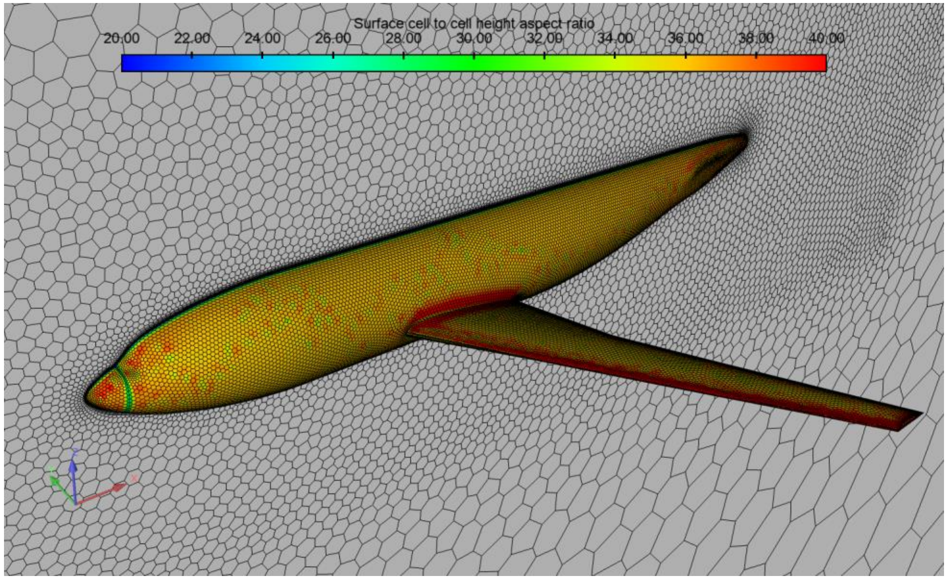


Wall distance units $x^+ – y^+ – z^+$ and some rough estimates

- The contours of surface cell base length and the cell height can be plotted on the surfaces by using the following relation,

$$AR = \frac{\sqrt{\text{Surface cell area}}}{\Delta y}$$





- Notice that this metric is entirely based on geometrical information.

Wall distance units x^+ – y^+ – z^+ and some rough estimates

- The stream-wise and span-wise wall distance units requirements are important when conducting scale-resolving simulations (DES, LES, DNS).
- Typical requirements for LES are (these are recommendations based on different references):

$$\Delta x^+ < 50,\ \Delta z^+ < 50 \quad \text{for} \quad y^+ < 5$$

Wall resolving

$$\Delta x^+ < 4\Delta y^+,\ \Delta z^+ < 4\Delta y^+ \quad \text{for} \quad y^+ \leq 200$$

Wall modeling

Note: it is recommended to use a surface mesh aspect ratio of 10 or less.

- As you can see, these requirements translate in much finer surface meshes and low surface aspect ratios.
- RANS simulations only have requirements respect to y^+. They do not have strict requirements when it comes to the span-wise and stream-wise directions.

Wall distance units x^+ – y^+ – z^+ and some rough estimates

- Let us see how we can approximate the wall distance units in the stream-wise (Δx^+) and span-wise (Δz^+) directions.

$$\Delta x^+ = \frac{U_\tau \Delta x}{\nu} \qquad \Delta z^+ = \frac{U_\tau \Delta z}{\nu}$$

- For simplicity, let us assume that they are the same in both directions (which is not a bad approximation). Then, we can compute Δx^+ and Δz^+ as follows,

$$\Delta x^+ = \Delta z^+ = \frac{\sqrt{\text{Face area}} \times y^+}{y_{\text{cell center}}}$$

- Where we approximated Δx and Δz as follows,

If the surface mesh is uniform and quadrilateral, you can measure the edge length directly from the surface cells →

$$\Delta x = \Delta z = \sqrt{\text{Face area}}$$

← Again, this is a relatively good approximation. Have in mind that there are many more methods.

Wall distance units x^+ – y^+ – z^+ and some rough estimates

- In the previous equations, $y_{\text{cell center}}$ is the distance normal from the surface to the first cell center.
- The quantity $y_{\text{cell center}}$ can be directly obtained from the mesh or the solver (cell wall distance), or it can be inferred from the y^+ value (for non-uniform or industrial meshes), as follows,

$$y_{\text{cell center}} = \frac{y^+ \times \nu}{U_\tau} \qquad U_\tau = \sqrt{\frac{\tau_{wall}}{\rho}}$$

Wall shear stresses.
Computed by the solver.

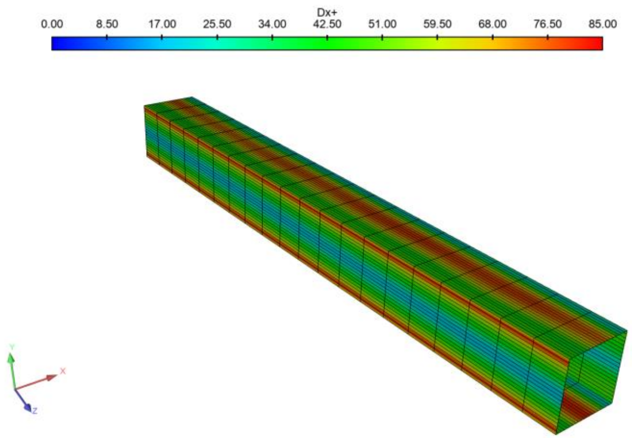
Wall distance units x^+ – y^+ – z^+ and some rough estimates

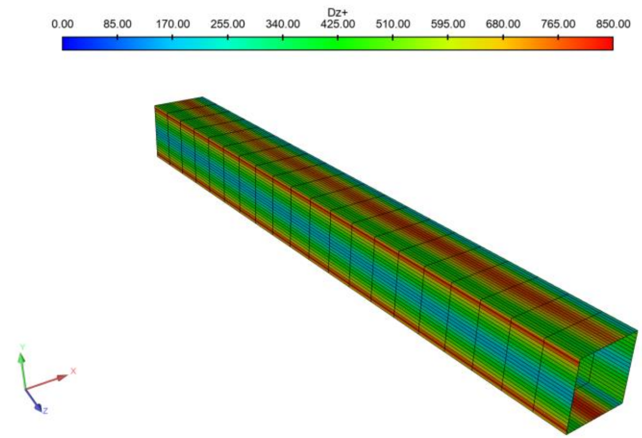
- Summarizing, these are the steps to follow to compute Δx^+ and Δz^+,
  - Compute y^+ (usually computed by the solver).
  - Compute surface mesh face area.
  - Compute $y_{\text{cell center}}$ using y^+, or the geometrical information from the solver.
  - Compute Δx^+ and Δz^+ using the following relationship,

$$\Delta x^+ = \Delta z^+ = \frac{\sqrt{\text{Face area}} \times y^+}{y_{\text{cell center}}} = \frac{\sqrt{\text{Face area}} \times U_\tau}{\nu}$$

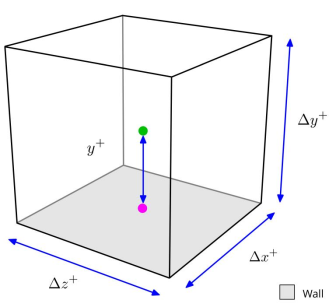
Wall distance units x^+ – y^+ – z^+ and some rough estimates

- Surface representation of Δx^+ and Δz^+.





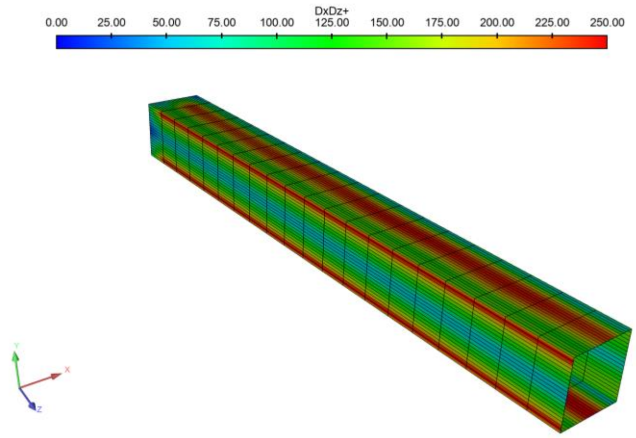
- Hereafter, Δx^+ and Δz^+ were computed using the edges length Δx and Δz, as follows,



$$\Delta x^+ = \frac{U_\tau \Delta x}{\nu} \qquad \Delta z^+ = \frac{U_\tau \Delta z}{\nu}$$

Wall distance units x^+ – y^+ – z^+ and some rough estimates

- Surface representation of Δx^+ and Δz^+.

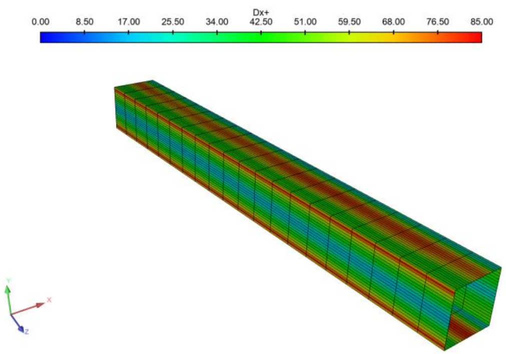


- Alternatively, Δx^+ and Δz^+ can be computed using the area of the surface elements, as follows,

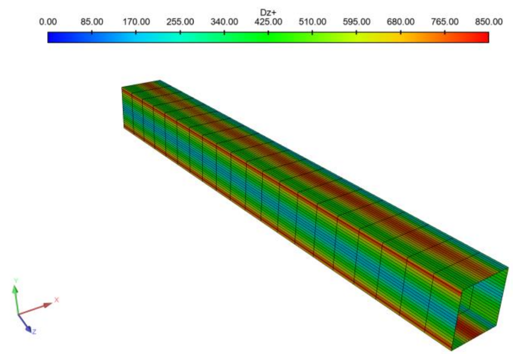
$$\Delta x^+ = \Delta z^+ = \frac{\sqrt{\text{Face area}} \times y^+}{y_{\text{cell center}}} = \frac{\sqrt{\text{Face area}} \times U_\tau}{\nu}$$

Wall distance units x^+ – y^+ – z^+ and some rough estimates

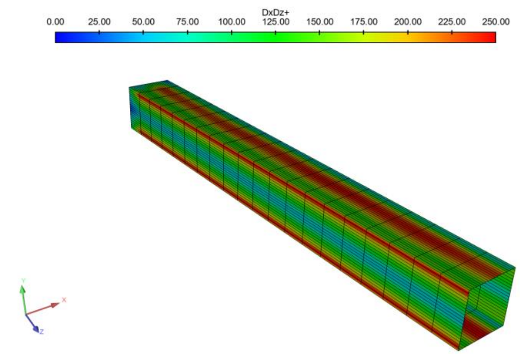
- Independently of the method used, when computing Δx^+ and Δz^+ it is recommended to use mean quantities.



$$\Delta x^+ = \frac{U_\tau \Delta x}{\nu}$$



$$\Delta z^+ = \frac{U_\tau \Delta z}{\nu}$$



$$\Delta x^+ = \Delta z^+ = \frac{\sqrt{\text{Face area}} \times y^+}{y_{\text{cell center}}}$$

- Recall that typical requirements for LES simulations are,

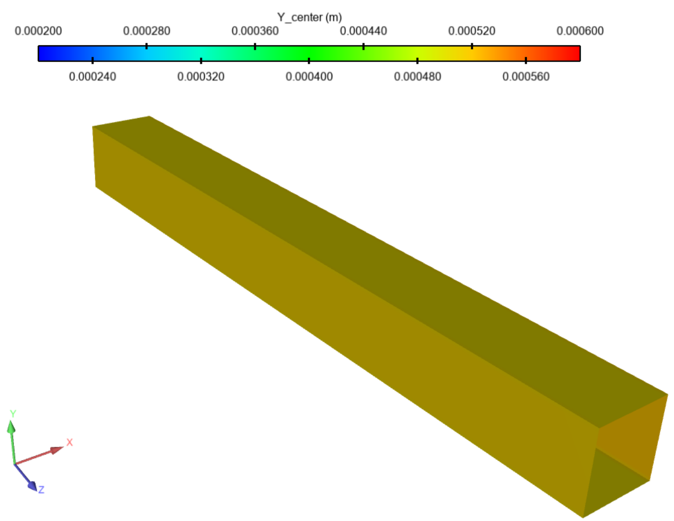
$$\Delta x^+ < 50, \; \Delta z^+ < 50 \qquad \text{for} \qquad y^+ < 5$$ Wall resolving

$$\Delta x^+ < 4\Delta y^+, \; \Delta z^+ < 4\Delta y^+ \qquad \text{for} \qquad y^+ \leq 200$$ Wall modeling

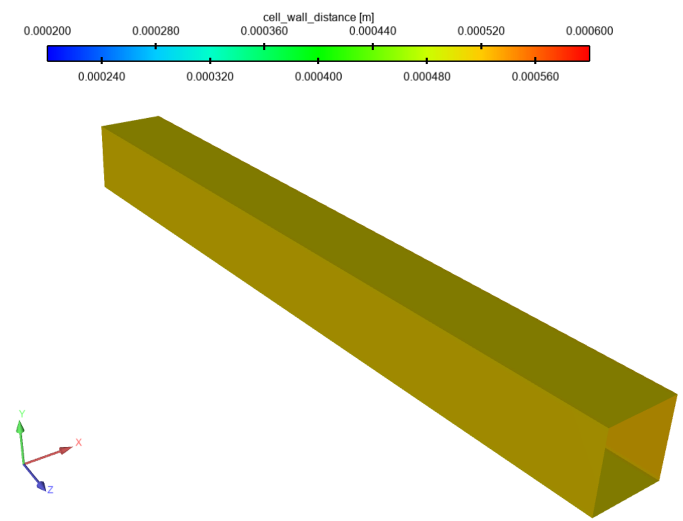
Note: it is recommended to use a surface mesh aspect ratio of 10 or less.

Wall distance units x^+ – y^+ – z^+ and some rough estimates

- Surface representation of $y_{\text{cell center}}$ or the distance normal from the surface to the first cell center,



Computed using y^+. Therefore, based on the wall shear stresses

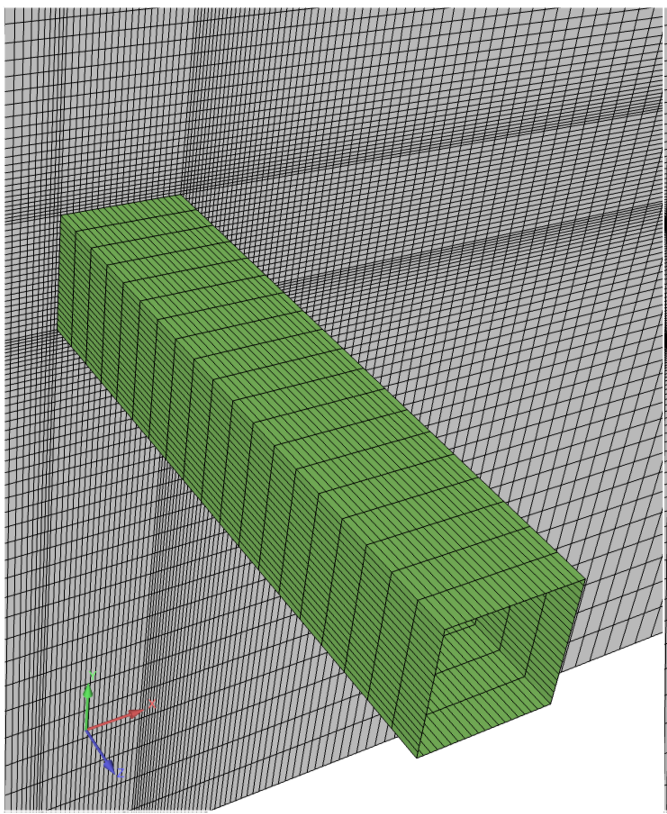


Using geometrical information from the solver

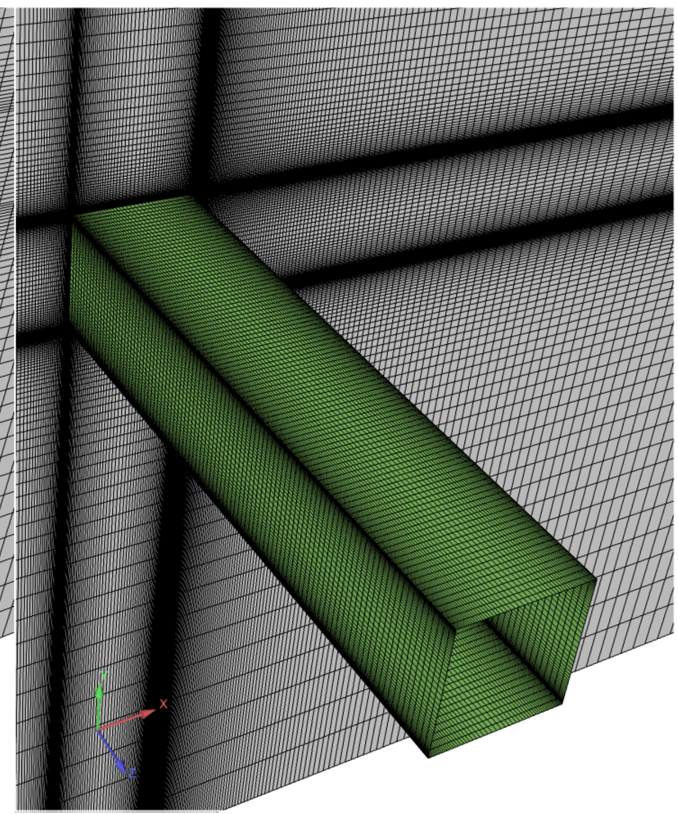
- If you used the previous equation for computing $y_{\text{cell center}}$, this field is based on the wall shear stresses.
- Therefore, even if the mesh is uniform, the surface representation is not uniform.
- If the information of the first cell center normal to the wall is available (geometrical information), it might be a better approximation.

Wall distance units x^+ – y^+ – z^+ and some rough estimates

- The wall distance units x^+ and z^+ values strongly depend on the surface mesh aspect ratio.
- Finer meshes in the spanwise and streamwise directions will likely results in better results as the values of the wall distance units x^+ and z^+ are smaller (same principle as with y^+)



Surface mesh with large aspect ratio and large ratio between surface cell base length and the cell height.

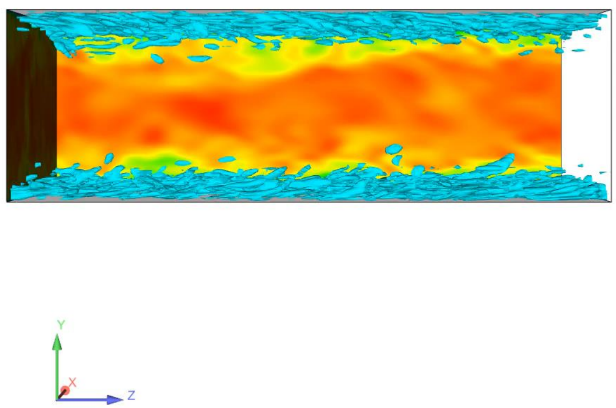


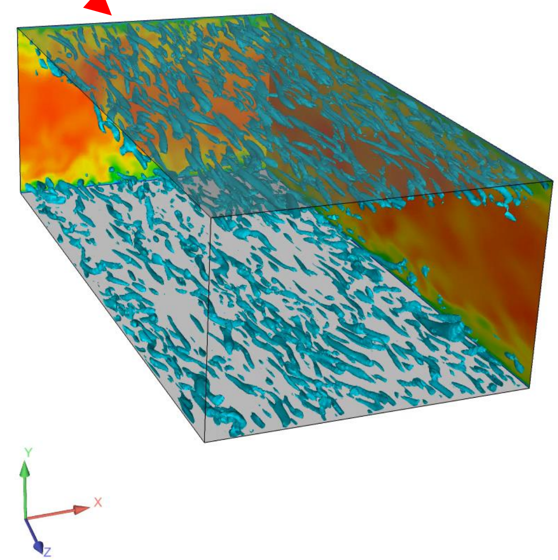
Surface mesh with low aspect ratio and low-to-moderate ratio between surface cell base length and the cell height

Wall distance units x^+ – y^+ – z^+ and some rough estimates

Incoming flow

Incoming flow





Wall bounded flow – Channel flow
http://www.wolfdynamics.com/training/turbulence/channel1.mp4

- In wall resolving LES simulations, typical wall distance units requirements to resolve the structures close to the walls (streaks), are as follows,

$$\Delta x^+ < 50, \Delta z^+ < 50 \quad \text{for} \quad y^+ < 5$$

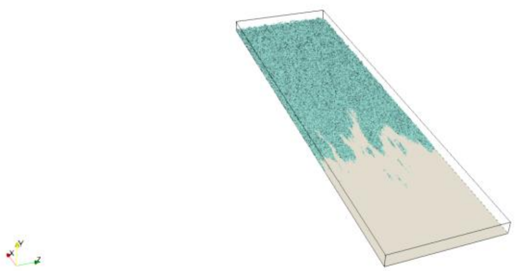
Wall resolving

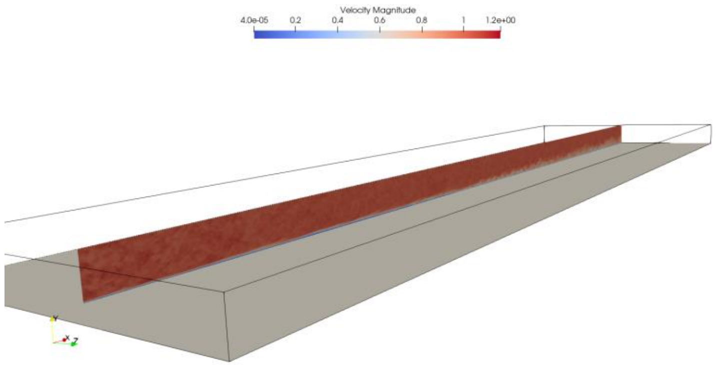
Spanwise direction

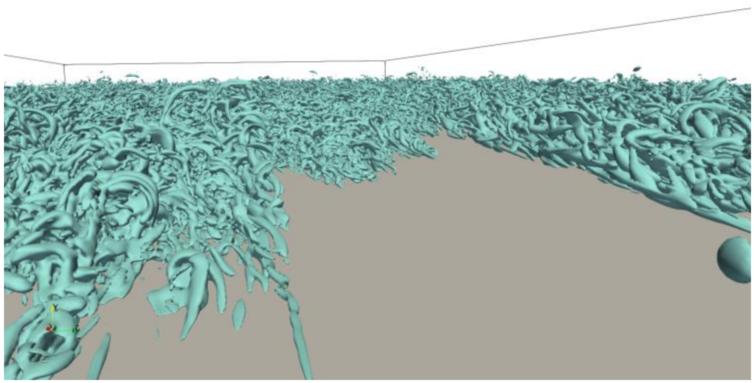
Streamwise direction

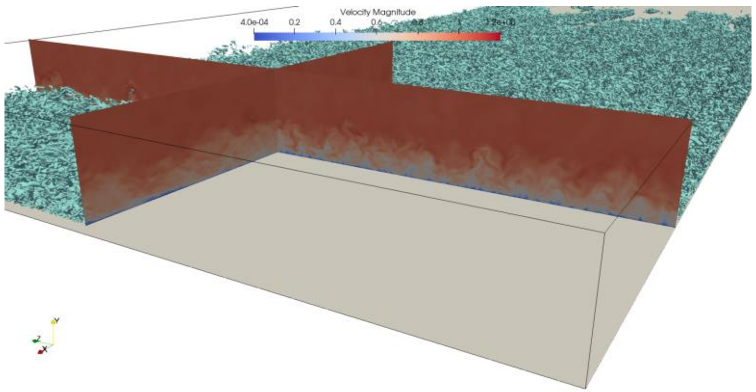
Wall normal direction

Wall distance units x^+ – y^+ – z^+ and some rough estimates





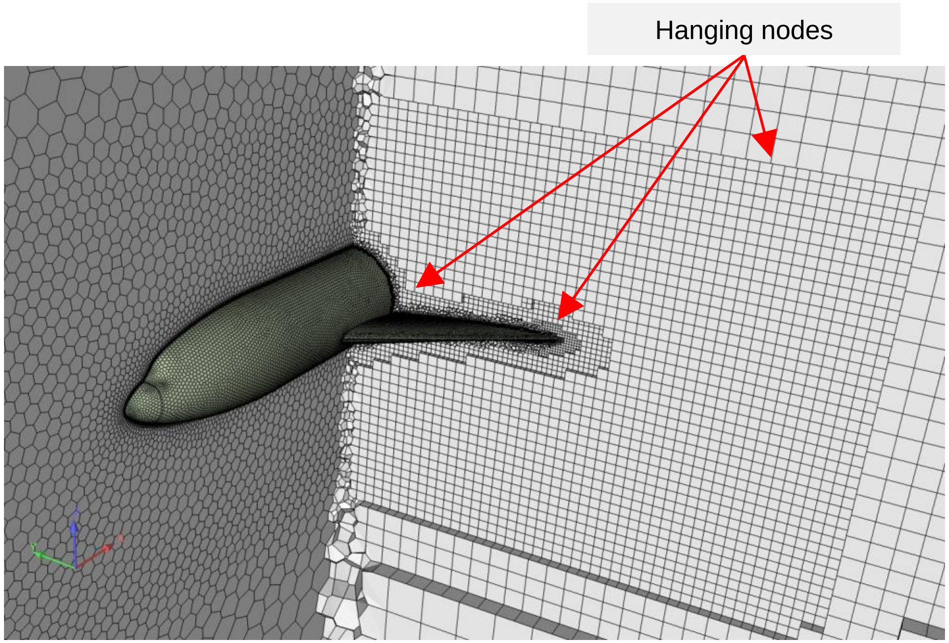




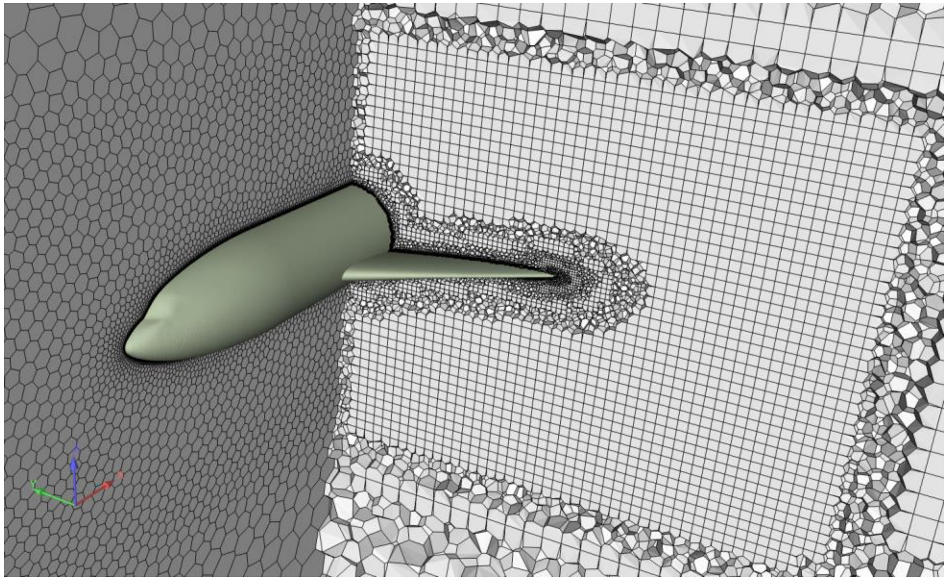
- In proper resolved DNS, the wall distance units requirements in all directions (spanwise, streamwise and normal) are close or lower than one.

On meshes with hanging nodes

- Hanging nodes are often used by meshers and supported by solvers to ease the mesh generation process.
- In SRS simulations, it is extremely recommended to avoid hanging nodes.
- Hanging nodes are source of fast transition between small and large cells and can add a lot numerical diffusion to the solution.
- In particular, avoid hanging nodes close to the walls.



Hexahedral dominant mesh with polyhedral surface mesh
Hanging nodes allowed – Octree subdivision



Hexahedral dominant mesh with tetrahedral surface mesh
Hanging nodes not allowed – Octree subdivision filled with polyhedral

LES wall functions

- It is highly recommended to use wall functions with LES simulations.
- Otherwise, the meshing requirements will be close to those of DNS.
- Conservatively speaking, the upper limit of y^+ for LES and DES simulations should be less than $y^+ < 200$.
- And you should cluster enough points in the boundary layer to accurately resolve the velocity profile normal to the wall.
- Usually, at least 15 inflation layers are required to meet good resolution requirements.
- It is also imperative to use y^+ insensitive wall functions.
- That is, formulations that cover viscous sublayer, buffer region, and log-law region.

LES wall functions

- Remember, you also need to resolve the boundary layer in the spanwise and streamwise directions, and this is what mainly imposes stringent meshing requirements in LES simulations.
- Recommended wall distance units values are (these are indicatives values based on different references),

$$\Delta x^+ < 50, \, \Delta z^+ < 50 \quad \text{for} \quad y^+ < 5$$

Wall resolving

$$\Delta x^+ < 4 \Delta y^+, \, \Delta z^+ < 4 \Delta y^+ \quad \text{for} \quad y^+ \leq 200$$

Wall modeling

Note: it is recommended to use a surface mesh aspect ratio of 10 or less.

- As you can see, these requirements translate in much finer surface meshes, low surface aspect ratios, and clustered meshes towards the walls.
- You also need to use low growth rate factors, in the order of 1.1 or less.

LES wall functions

- The wall functions treatment is similar to that of RANS/URANS models.
- It is extremely recommended (if not imperative) to use y^+ insensitive wall functions.
- This can be achieved by using a blending function between the viscous sublayer and the log-law layer [1].
- Kader [1] proposed the following blending function to obtain a y^+ insensitive formulation,

$$u^+ = e^{\Gamma} u^+_{lam} + e^{1/\Gamma} u^+_{turb}$$

$$\Gamma = -\frac{a\left(y^+\right)^4}{1 + b y^+} \qquad a = 0.01 \qquad b = 5$$

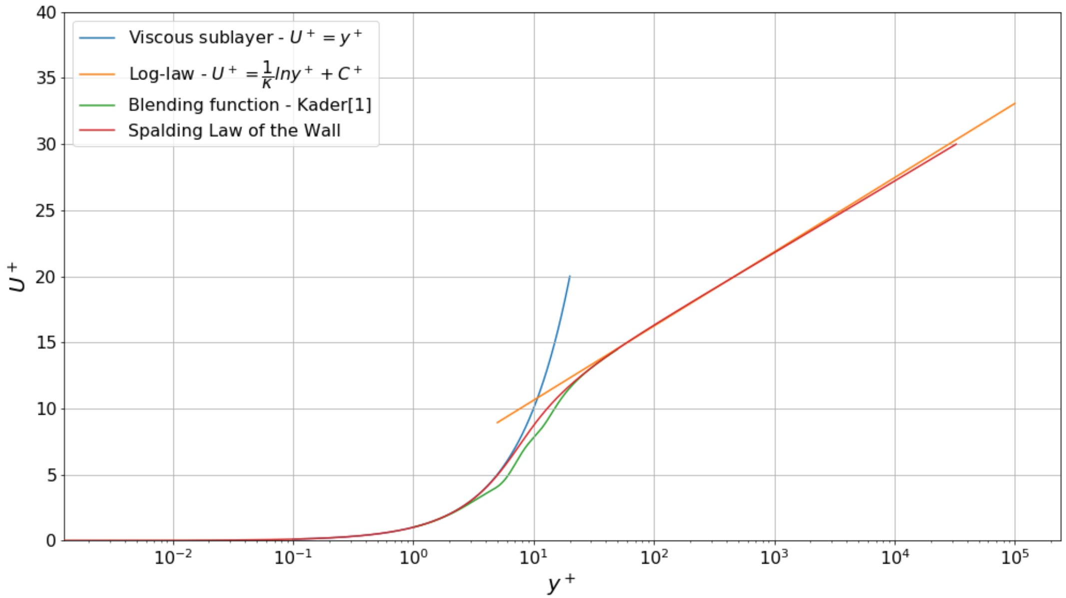
$$u^+_{lam} = y^+ \qquad u^+_{turb} = \frac{1}{\kappa} \ln y^+ + C^+$$

- This formula guarantees the correct asymptotic behavior for large and small values of y^+ and a reasonable representation of velocity profiles in the cases where y^+ falls inside the buffer region.

References:
[1] B. Kader. Temperature and Concentration Profiles in Fully Turbulent Boundary Layers. 1981.

LES wall functions

- Plot of Kader's [1] blending function.
- In the plot, the Spalding function [2] is also represented.
- The Spalding function is another alternative to obtain a y^+ insensitive treatment.
- The Spalding function is essentially a fit of the laminar, buffer and logarithmic regions of the boundary layer.



- Kader's [1] blending function,

$$u^+ = e^{\Gamma} u^+_{lam} + e^{1/\Gamma} u^+_{turb}$$

$$u^+ = \begin{cases} y^+ & y^+ < 11.225 \\ \frac{1}{\kappa} \ln(y^+) + C^+ & y^+ > 11.225 \end{cases}$$

- Or using u* wall units

$$u^* = \begin{cases} y^* & y^* < 11.225 \\ \frac{1}{\kappa} \ln(E y^*) & y^* > 11.225 \end{cases}$$

- And recall that in equilibrium conditions,

$$u^+ = u^* \qquad y^* = \frac{C_\mu^{1/4} k_p^{1/2} y_p}{\nu} \qquad y^+ = \frac{\rho \times U_\tau \times y}{\mu}$$

- However, in the context of LES simulations the use of u^+ wall units is preferred because k^{sgs} is not explicitly computed.
- The Spalding's law [2] is written as follows,

$$y^+ = u^+ + \frac{1}{E}\left[e^{\kappa u^+} - 1 - \frac{\kappa u^+}{1!} - \frac{(\kappa u^+)^2}{2!} - \frac{(\kappa u^+)^3}{3!} - \frac{(\kappa u^+)^4}{4!}\right]$$

References:
[1] B. Kader. Temperature and Concentration Profiles in Fully Turbulent Boundary Layers. 1981.
[2] D. Spalding. A single formula for the law of the wall. J. of Applied Mechanics. 1961.

LES wall functions

- Finally, this is not the only approach when using wall functions in LES simulations.
- Just to name a few other approaches,
    - The Shumann model.
    - The Grotzbach model.
    - The shifted corrections model.
    - Election model.
    - The optimized ejection model by Marusic.
    - The model of Werner and Wengle [1] (this model is available in Ansys Fluent).
    - The model of Murakami.
    - The suboptimal control-based model.
    - The Deardorff model.
    - Two-layer model by Balaras.
    - Das-Moser embedded wall model
    - Hybrid LES/RANS models.
- For a general review of these models please refer to references [2,3,4].

References:
[1] H. Werner, H. Wengle. Large-Eddy Simulation of Turbulent Flow Over and Around a Cube in a Plate Channel. Eighth Symposium on Turbulent Shear Flows, Munich, Germany. 1991.
[2] P. Sagaut. Large eddy simulation for incompressible flows. An introduction. Springer, 2006.
[3] S. Pope. Turbulent Flows. Cambridge University Press. 2014.
[4] U. Piomelli, E. Balaras. Wall-layer models for large-eddy simulations. Annual Review of Fluid Mechanics. Vol. 35:349-374, 2002

Roadmap to Lecture 9

1. ~~SRS simulations~~
2. ~~LES equations – Filtered Navier-Stokes equations~~
3. ~~Sub-grid scale models for LES~~
4. ~~DES brief review~~
5. ~~A few mesh resolution guidelines and rough estimates for LES/DES simulations~~
6. **Final remarks on LES/DES turbulence models**

Final remarks on LES/DES turbulence models

A few acronyms that you will find while working with DES and LES models

- **LES**: large eddy simulation (it resolves 80% of the energy spectrum).
- **VLES**: very large eddy simulation (it resolves 50% of the energy spectrum).
- **LES-NWR**: LES with near wall resolution (it resolves the boundary layer).
- **LES-NWM**: LES with near wall modelling (it uses wall functions).
- **WMLES**: wall modeled LES (same as LES-NWM).
- **DES**: detached eddy simulation (hybrid RANS-LES).
- **DDES**: delayed DES (DES with shielding functions).
- **DDES-SA**: DDES or DES based on the Spalart-Allmaras RANS model.
- **DDES-SST**: DDES or DES based on the $k-\omega$ SST RANS model.
- **IDDES**: improved delayed DES (DES with shielding functions and WMLES).
- **ALES**: adaptive LES (LES with adaptive mesh refinement).
- **GS**: grid scales
- **SGS**: sub-grid scales
- **HRM**: high Reynolds part of the boundary layer (logarithmic layer).
- **LRN**: low Reynolds part of the boundary layer (viscous layer).

Final remarks on LES/DES turbulence models

Short description of some LES turbulence models

Model	Short description
Smagorinsky	Simple algebraic model (0-equations). Because of its simplicity and low computational cost it is widely used. It is not a particularly good choice for wall-bounded flows, but for flows far from solid boundaries it can be quite adequate. The model constants can depend on the flow conditions. This model is a good starting point for complex simulations.
Smagorinsky-Lilly	Simple algebraic model (0-equations). Because of its simplicity and low computational cost it is widely used. It overcomes some of the limitations of the Smagorinsky model by using damping functions in near-wall regions, therefore, it works better with wall-bounded flows. The model constants can depend on the flow conditions.
Wall-Addaptive Local Eddy Viscosity model (WALE)	Simple algebraic model (0-equations). Retains the simplicity and low computational cost of of the Smagorinsky model. Wall damping effects are accounted for without using the damping function explicitly. It predicts accurately the flow near the walls and transition. The model constants can depend on the flow conditions.
Dynamic Smagorinsky	This model is based on the similarity concept and Germano's identity. It is more universal because the constants are computed dynamically. The model predicts accurately the wall behavior, transition, and allow energy backscatter. The computation of dynamic constants requires additional computational power and fluctuations while computing the constant can cause stability issues.
Dynamic Kinetic Energy Transport	This model overcomes some of the limitations of the Smagorinsky-Lilly and dynamic Smagorinsky models. It solves an additional transport equation for the sub-grid scale kinetic energy. The model predicts accurately the wall behavior, transition, and allow energy backscatter, it also allows for history of the kinetic energy. It is more computational expensive as it solve an additional equation and it performs explicit filtering.

LES/DES general meshing requirements

- Summary of the most important requirements of SRS simulations.

	LES	DES	SAS	URANS
Wall resolution y^+	< 200	1*	1*	1-1000**
Wall resolution x^+	40	No strict requirements		
Wall resolution z^+	20	No strict requirements		
CFL number ***	1 <	5 <	5 <	10 <
Growth rate	1.1 <	1.1 < ****	1.1 < ****	1.2 <
Internal flows	YES	NO	NO	YES
External flows	YES	YES	YES	YES

* Recommend value. It is strongly advised to resolve the boundary layer (LRN approach). The wall modeling approach can also be used. However, the benefits of using a HRN approach are questionable.

** The upper limit depends on the Reynolds number.

*** Recommended values for good accuracy but you can use higher values at your own risk.

**** Recommended values in the boundary layer region.

Final remarks on LES/DES turbulence models

A few mesh resolution guidelines and rough estimates

- The mesh is everything in CFD, and when it comes to turbulence modeling it is extremely important to have meshes with good quality and acceptable resolution.
- Some general guidelines for meshes to be used with RANS/DES/LES:
    - Resolve well the curvature.
    - Allow a smooth transition between cell of different sizes (at least 3 cells).
    - Identify the integral scales and try to cluster at least 5 cells in the domain regions where you expect to find the integral scales.
    - If you have no restrictions, use a wall resolving approach, that is, fine meshes in the direction normal to the walls.
    - It is recommended to avoid hanging nodes.

Final remarks on LES/DES turbulence models

A few mesh resolution guidelines and rough estimates

- The mesh is everything in CFD, and when it comes to turbulence modeling it is extremely important to have meshes with good quality and acceptable resolution.
- Some guidelines specific to RANS meshes:
  - When it comes to RANS, the most important metric for mesh resolution is the y^+ value.
  - Identify your wall treatment a-priory and mesh your domain according to this requirement.
  - If you are doing 3D simulations, there are no strict requirements when it comes to the span-wise and stream-wise directions,
  - But as a general rule, you can use Δx^+ and Δz^+ values as high as 300 the value of Δy^+ and less than a 1000 wall distance units.

Final remarks on LES/DES turbulence models

A few mesh resolution guidelines and rough estimates

- The mesh is everything in CFD, and when it comes to turbulence modeling it is extremely important to have meshes with good quality and acceptable resolution.
- Some guidelines specific to DES meshes:
  - Usually in DES simulations wall resolving meshes are used with RANS requirements similar to RANS.
  - That is, there are no specific requirements on the streamwise and spanwise wall units values.
  - However, it is extremely important to resolve well the integral length scales, as in LES simulations.
  - What is very tricky in DES, is how to control the transition from the RANS mesh close to the walls to the fine-medium mesh far from the walls.
  - You might encounter a mismatch between the RANS mesh and the LES mesh, because the RANS mesh tends to be coarser in the streamwise and spanwise directions.
  - This transition, if not well controlled can cause grid induced separation and stability problems.
  - Hexahedral meshes are extremely recommended.

Final remarks on LES/DES turbulence models

A few mesh resolution guidelines and rough estimates

- The mesh is everything in CFD, and when it comes to turbulence modeling it is extremely important to have meshes with good quality and acceptable resolution.
- Some guidelines specific to LES meshes:
  - When it comes to LES meshes, it is recommended to use wall functions.
  - Otherwise, the meshing requirements are similar to those of DNS.
  - It is recommended to use values of y^+ in the range of $1 < y^+ < 200$.
  - You can also use values between $10 < y^+ < 30$.
    - LES uses wall functions that can deal with the buffer layer.
  - In LES, it is extremely important to resolve well the stream-wise and span-wise directions. Recommended values,

$$\Delta x^+ < 50, \Delta z^+ < 50 \quad \text{for} \quad y^+ < 5$$ Wall resolving

$$\Delta x^+ < 4\Delta y^+, \Delta z^+ < 4\Delta y^+ \quad \text{for} \quad y^+ \leq 200$$ Wall modeling

  - Use hexahedral meshes.

Wall distance units and some rough estimates

- DES and RANS simulations do not have stream-wise and span-wise wall distance units requirements as in LES simulations. Therefore, they are more affordable.
- If you are conducting DES simulations, it is highly recommended to resolved the boundary layer.
- In DES simulations you can also use wall functions.
- The upper limit of y^+ for LES and DES simulations should be less than $y^+ < 200$-300 .
- Remember, it is strongly recommended to use wall functions with LES simulations.
    - Otherwise, your meshing requirements will be close to those of DNS.
- If you are doing DNS, y^+ should be close or less than 1.
- The spanwise and streamwise values should be less than 10, but ideally close to 1.

Additional remarks on DES/LES turbulence models

- The mesh requirements of LES-NWR are close to those of DNS; therefore, it is highly recommended the use of wall functions.
- LES wall functions are valid in the whole boundary layer (including the buffer region).
- Remember, DES/LES methods are intrinsically 3D and unsteady.
- LES simulations are very sensitive to mesh element type; it is highly recommended to use hexahedral meshes.
- For good accuracy, try to keep the mesh growth rate below 1.1.
- The WALE and dynamic methods are the best LES choices. However, you can use the Smagorinsky method for simple flows or getting an initial solution.
- The DDES Spalart-Allmaras is the best choice when it comes to DES simulations.
- If you are dealing with external aerodynamics and detached flows, DES simulations are very affordable.
- In DES, as it is not necessary to resolve eddies located in the wall region, you can use coarser meshes in stream-wise and span-wise directions (close to the walls).
- For LES simulations, keep the CFL below 1.

Additional remarks on DES/LES turbulence models

- DES simulations have more relaxed time-stepping requirements, but in general you should not go above 5 (CFL number).
- Use RANS simulations as starting point for LES/DES simulations.
- When it comes to post-processing SRS turbulence simulations it can be quite time consuming, especially if we are dealing with unsteady data and large meshes.
- Most of the times we are interested in computing averaged quantities, so do not forget to compute the unsteady statistics.
- Not all discretization schemes are born with LES in mind. In LES simulation we must use low dissipation and non-dispersive discretization methods.
- Same applies for element type. Tetrahedral elements are not very desirable when conducting LES simulations, even if we use high-accuracy and non-dispersive methods.
- Hexes are preferred over the rest of element types.
- Remember, many LES filters are designed with hexes in mind.
- Low dissipation methods translate in energy preserving methods, that is, the energy spectrum should not increase (or accumulate) with large wave number (small scales).
- It is not recommended to use meshes with hanging nodes.

Final remarks on LES/DES turbulence models

Turbulence modeling guidelines and tips

- Compute Reynolds number and determine whether the flow is turbulent.
- Try to avoid using turbulent models with laminar flows.
- Choose the near-wall treatment and estimate the normal distance to the wall (y) before generating the mesh.
- Run the simulation for a few time steps and get a better prediction of y^+ and correct your initial prediction of y.
- The realizable $k - \epsilon$ or the $k - \omega$ SST models are good choices for general applications.
- The standard $k - \epsilon$ model is very reliable, you can use it to get initial values for more sophisticated models.
- If you are interested in resolving the large eddies and the inertial range, and modeling the smallest eddies, DES or LES are the right choice.
- If you do not have any restriction in the near wall treatment method, use wall functions.
- Try to avoid the use of symmetry (axial and planar).
- Do not use DES with internal flows.

Final remarks on LES/DES turbulence models

Turbulence modeling guidelines and tips

- Use the default model constants unless you know what are you doing or you are confident that you have better values.
- Set reasonable boundary and initial conditions for the turbulence model variables.
- Always monitor the turbulent variables, most of them are positive bounded.
- Avoid strong oscillations of the turbulent variables.
- If you are doing LES or DES, remember that these models are intrinsically 3D and unsteady.
- In LES you should choose your time-step in such a way to get a CFL of less than 1 and preferably of about 0.5 for LES.
- DES simulations can use larger CFL values (up to 4 for reasonable accuracy).
- If you are doing RANS, it is perfectly fine to use upwind to discretize the turbulence closure equations.
- After all, turbulence is a dissipative process. However, some authors may disagree with this, make your own conclusions.

Final remarks on LES/DES turbulence models

Turbulence modeling guidelines and tips

- On the other hand, if you are doing LES you should keep numerical diffusion to the minimum, so you should use second order methods.
- LES/DES methods can be sensitive to mesh element type, it is highly recommended to use hexahedral meshes.
- Mesh quality if of paramount importance, try to avoid bad elements near the inlets (as they can introduce numerical diffusion) or at the walls (as they can affect the boundary layer or wall functions).
- If you are doing unsteady simulations, always remember to compute the average values (ensemble average).
- Avoid the use of adaptive time-stepping and adaptive save intervals, as they may introduce oscillations in your solution.
- If you are working with combustions and aero-acoustics, you will get best results using LES models but at the cost of higher computational requirements.
- If you are dealing with external aerodynamics and detached flows, DES simulations are very affordable, and surprisingly, they give good results most of the times.

Final remarks on LES/DES turbulence models

Turbulence modeling guidelines and tips

- DNS requires no modeling, but it demands high mesh resolution for the large scales all the way through at least the beginning of the dissipation scales. This requires and incredible amount of mesh cells (in the order of Re^3 or worse).
- LES requires modeling of part of the inertial subrange and into the beginning of the dissipation scales. The amount of required modeling is set by the mesh resolution that can be afforded (at worse in the order of Re^2 which is much less than the mesh resolution for DNS but still is a high requirement).
- In general, LES models are less expensive than DNS, but much more expensive than RANS/URANS.
- RANS/URANS requires modeling of everything from the integral scales into the dissipation range and only mean quantities are computed. Despite this, they perform very well.
- The hybrid method DES, model everything close to the walls and resolves all the scales in the far field (as in LES). DES methods have mesh resolution requirements between RANS and LES.
- **The work-horse of turbulence modeling in CFD: RANS**

Final remarks on LES/DES turbulence models

Future of Turbulence Modelling in Industrial Applications

- Many authors state that the future trends are quite clear: moving from RANS models to LES models.
- However, I politely disagree with this as many industrial applications are quite complex in order to simulate them using LES models.
- LES simulations are unattractive in industry due to the excessive amount of computational resources needed (which cost money), and the amount of time needed to get the outcomes (and time is money).
- RANS models are the work horse of industrial applications and will continue to be until a big leap in computing hardware or solution strategy happens.
- However, academia is moving slowly towards LES and new computing platforms (which hopefully will consume less energy than wind tunnels), so we are the ones responsibly for triggering that big change.
- DES simulations are starting to become more affordable and are slowly replacing URANS.
- Also, ASM and EARSM are becoming more reliable.
- DNS remains out of reach for all engineering use.
- However, it provides a very good base for model development and testing.

These statements are the author's personal opinion (J. Guerrero).