Turbulence and CFD models: Theory and applications

Roadmap to Lecture 6

Part 6

1. On the closure coefficients
2. Galilean invariance
3. URANS and RANS

Roadmap to Lecture 6

Part 6

1. **On the closure coefficients**
2. Galilean invariance
3. URANS and RANS

On the closure coefficients

- The coefficients used in the turbulence models do not come out of thin air.
- They have been calibrated using experiments, numerical simulations, or empirical correlations.
- During this calibration process, many assumptions are taken that sometimes might not be very realistic.
    - Such as, local equilibrium, local isotropy, two-dimensional flow, fully developed flow, and so on.
- Optimization methods, data driven simulations, and machine learning is also being used to calibrate these coefficients.
- Notice that we called then coefficients and not constants.
- They certainly can be adjusted.

On the closure coefficients

- Let us review the $k - \epsilon$ turbulence model,

$$\nabla_t k + \nabla \cdot (\bar{\mathbf{u}} k) = \tau^R : \nabla \bar{\mathbf{u}} - \epsilon + \nabla \cdot \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \nabla k \right]$$

$$\nabla_t \epsilon + \nabla \cdot (\bar{\mathbf{u}} \epsilon) = C_{\epsilon_1} \frac{\epsilon}{k} \tau^R : \nabla \bar{\mathbf{u}} - C_{\epsilon_2} \frac{\epsilon^2}{k} + \nabla \cdot \left[\left(\nu + \frac{\nu_t}{\sigma_\epsilon} \right) \nabla \epsilon \right]$$

- This model uses the following relation for the kinematic eddy viscosity,

$$\nu_t = \frac{C_\mu k^2}{\epsilon}$$

- With the following closure coefficients,

$$C_{\epsilon_1} = 1.44 \qquad C_{\epsilon_2} = 1.92 \qquad C_\mu = 0.09 \qquad \sigma_k = 1.0 \qquad \sigma_\epsilon = 1.3$$

- And auxiliary relationships,

$$\omega = \frac{\epsilon}{C_\mu k} \qquad\qquad l = \frac{C_\mu k^{3/2}}{\epsilon}$$

On the closure coefficients

- Talking about canonical or simplified solutions, the RANS equations for a two-dimensional boundary layer (or pure shear flow) can be written as follows,

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} = 0$$

$$\overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} + \nu\frac{\partial^2 \overline{u}}{\partial y^2} + \frac{\partial}{\partial y}\left(\overline{u'v'}\right)$$

$$\frac{\partial \overline{p}}{\partial y} \approx 0$$

- Where the following assumptions were taken,

$$v << u \qquad \frac{\partial}{\partial x} << \frac{\partial}{\partial y} \qquad \frac{\partial p}{\partial y} << \frac{\partial p}{\partial x}$$

- Under certain conditions, these equations provide high quality solutions of turbulent flows that can be used to validate models and calibrate coefficients.

On the closure coefficients

- Let us address how to calibrate the eddy viscosity coefficient C_μ which is used in the following eddy viscosity relation,

$$\nu_t = C_\mu \frac{k^2}{\epsilon}$$

- As we have seen, this equation is used in two-equation models.
- In the $k - \epsilon$ model, the coefficient C_μ is equal to 0.09.
- This coefficient also appears in the $k - \omega$ model, but it is called β^* instead.
- This coefficient can be calibrated using the approximation of the two-dimensional shear layer flow.
- That is, we are dealing with a simple pure shear flow, a big simplification.
- Additionally, let us assume local equilibrium and local isotropy,

$$P = \epsilon$$

- Which is not entirely true, as this quantities are not entirely in equilibrium, even in shear flows.

References:
S. Pope. Turbulent Flows, Cambridge University Press, 2000.
P. Bernard, J. Wallace. Turbulent Flow. Analysis, Measurement, and Prediction. Wiley. 2002.

On the closure coefficients

- If production is equal to dissipation, *i.e.*, $P = \epsilon$, we obtain,

$$\underbrace{\tau_{uv} \frac{\partial \overline{u}}{\partial y}}_{P} = \epsilon$$

- Using the Boussinesq hypothesis to derive τ_{uv},

$$\tau_{uv} = -\overline{u'v'} = \nu_t \frac{\partial \overline{u}}{\partial y}$$

- Combining the two previous equations, so we drop the derivative, we obtain,

$$\frac{\tau_{uv}^2}{\nu_t} = \epsilon$$

On the closure coefficients

- Finally, using the following relation of eddy viscosity,

$$\nu_t = C_\mu \frac{k^2}{\epsilon}$$

- Together with the following relationship (that we obtained in the previous slide),

$$\frac{\tau_{uv}^2}{\nu_t} = \epsilon$$

- We obtain the following relation to estimate the eddy viscosity coefficient C_μ,

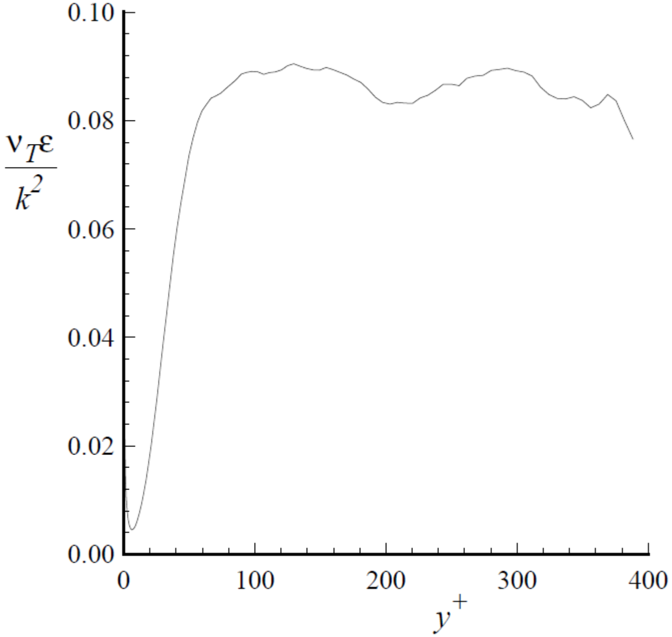
$$C_\mu = \frac{\tau_{uv}^2}{k^2}$$

This quantity can be measured using DNS or experiments
It represents the ratio of the shear Reynolds stress to turbulent kinetic energy

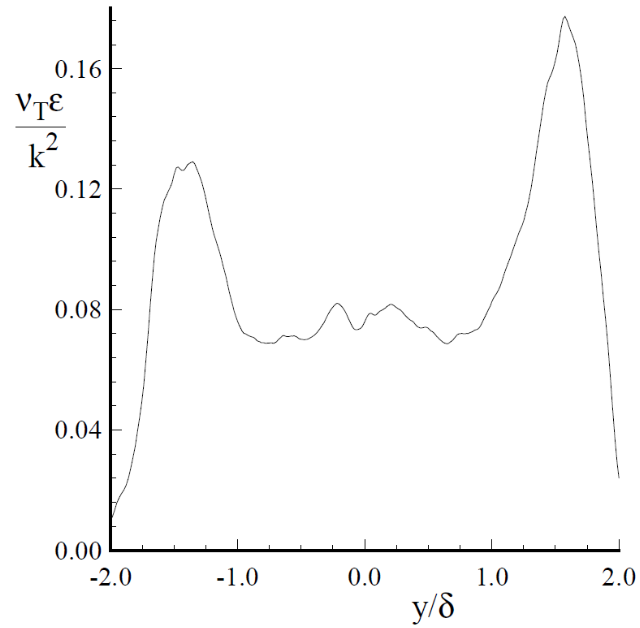
On the closure coefficients

- For simple shear flows the quantities k, ϵ, $\nu_t = -\overline{u'v'}/(\partial\overline{u}/\partial y)$, can be measured.

$$\nu_t \epsilon = \tau_{uv}^2$$



J. Kim, P. Moin, R. Moser. Turbulence statistics in fully developed channel flow at low Reynolds number. 1987.



M. Rogers, R. Moser. Direct simulation of a self-similar turbulent mixing layer. 1994.

- From these results, it can be seen that the eddy viscosity coefficient C_μ is approximately equal to 0.09
- This is where DNS and canonical flows come in handy.

On the closure coefficients

- The calibrated eddy viscosity coefficient C_μ can be expressed as,

$$C_\mu = \frac{\tau_{uv}^2}{k^2} = 0.09$$

- Sometimes in the literature you will find,

$$\sqrt{C_\mu} = \beta_r = \frac{\tau_{uv}}{k} = 0.3$$

- Where the ratio of the Reynolds stress to turbulent kinetic energy, *i.e.*, β_r, is often referred to as Bradshaw's constant [1], and sometimes as to Townsend's constant [2].

References:
[1] P. Bradshaw, D. Ferriss, N. Atwell. Calculation of Boundary Layer Development Using the Turbulent Energy Equation. 1967.
[2] A. Townsend. The Structure of Turbulent Shear Flow. 1976.

On the closure coefficients

- Calibration of the coefficient $C_{\epsilon 2}$.
- This constant can be calibrated using the hypothesis of decaying homogenous turbulence.
- In this type of flow the mean velocity and mean velocity gradients are equal to zero (huge simplification).
- Therefore, the governing equations can be expressed as follows,

$$\frac{\partial k}{\partial t} = -\epsilon \qquad \qquad \frac{\partial \epsilon}{\partial t} = -C_{\epsilon 2}\frac{\epsilon^2}{k}$$

- Using experimental data and looking for power law solutions, an expression for $C_{\epsilon 2}$ is obtained as a function of a decay exponent.

$$k = \frac{k_0}{(t/t_0 + 1)^n} \qquad \qquad C_{\epsilon 2} = \frac{n+1}{n}$$

- Fitting experimental data, the decay coefficient n can be obtained.
- As can be seen, depending on the data, the value of the coefficient $C_{\epsilon 2}$ will vary.
- Values ranging from 1.5 to 2.2 are often found in the literature.

References:
S. Pope. Turbulent Flows, Cambridge University Press, 2000.
P. Durbin, B. Petterson-Reif. Statistical Theory and Modeling for Turbulent Flow. Wiley. 2011.

On the closure coefficients

- Let us address the coefficient $C_{\epsilon 1}$, its calibration is also related to experimental data.
- In this case we use the assumption of homogenous shear flow.
- Therefore, the governing equations can be expressed as follows,

$$\frac{\partial k}{\partial t} = P_k - \epsilon \qquad \frac{\partial \epsilon}{\partial t} = \frac{C_{\epsilon 1} P_k - C_{\epsilon 2} \epsilon}{k/\epsilon}$$

- Combining these equation we can find the following relationship,

$$C_{\epsilon 1} = \frac{C_{\epsilon 2} - 1}{P_k/\epsilon} + 1$$

- By using growth rates of homogenous sheared turbulence (for the spreading rate in a plane mixing layer), we can find the coefficient $C_{\epsilon 1}$.
- Values ranging from 1.2 to 2.0 are often found in the literature.

References:
S. Pope. Turbulent Flows, Cambridge University Press, 2000.
P. Durbin, B. Petterson-Reif. Statistical Theory and Modeling for Turbulent Flow. Wiley. 2011.

On the closure coefficients

- Let us address the coefficient σ_ϵ.
- This closure coefficient acts like an effective Prandtl number for dissipation diffusion and is specified to ensure the correct log-law slope of κ^{-1}.
- Using the previous coefficients, the value of σ_ϵ can be found using the following relationship,

$$\sigma_\epsilon = \frac{\kappa^2}{(C_{\epsilon 1} - C_{\epsilon 2})\sqrt{C_\mu}}$$

- This is the equation used by Jones and Launder (see references) to determine the value of the coefficient σ_ϵ.
- The original value obtained by Jones and Launder is equal to 1.3.
- Values ranging from 1.2 to 1.5 are often found in the literature.
- Notice that this relation can also be used to derive the value of the Karman constant κ by adjusting the value of the coefficients to produce a particular value of κ.
- This implies the use of data fitting or optimization methods.

References:
W. Jones, B. Launder. The prediction of laminarization with a two equation model of turbulence. Int. J. Heat Mass Transfer 15, 301–314. 1972.
P. Durbin, B. Petterson-Reif. Statistical Theory and Modeling for Turbulent Flow. Wiley. 2011.

On the closure coefficients

- Let us look back at the relationship for σ_ϵ presented in the previous slide.
- That relationship was obtained by manipulating the **solvable** dissipation rate equation and using some additional relations and assumptions.
- By rearranging the relationship, we obtain the following equation,

$$\frac{\sqrt{C_\mu}\sigma_\epsilon}{\kappa^2}(C_{\epsilon 1} - C_{\epsilon 2}) = 1$$

- Using the standard coefficients of the $k - \epsilon$ model,

$$C_{\epsilon_1} = 1.44 \qquad C_{\epsilon_2} = 1.92 \qquad C_\mu = 0.09 \qquad \sigma_k = 1.0 \qquad \sigma_\epsilon = 1.3$$

- In the left-hand side of the previous condition, we obtain a value of 1.11, showing that this constraint is reasonably well satisfied.
- Which suggests that some kind of optimization method or data drive approach could be used to calibrate all these coefficients.
- In fact, machine learning methods are being used to calibrate turbulence models.

References:
S. Pope. Turbulent Flows, Cambridge University Press, 2000.
R. Bernard. Turbulent Fluid Flow. Wiley. 2019.

On the closure coefficients

- Finally, let us address the coefficient σ_k .
- This term affect the effective diffusivity in the TKE equation.
- Generally speaking, there is no consensus within the turbulence community about the value of this coefficient.
- Different values can be used depending on the set of experimental data available.
- In most cases this coefficient is assumed to be one.
- In fact, we have assumed that the value of this coefficient is one.
- Values ranging from 0.8 to 1.2 are often found in the literature.

On the closure coefficients

- Some observations of the effect of changing the closure coefficients in the standard $k - \epsilon$ turbulence model,

Coefficient	Value	Result of Increasing Value
C_μ	0.09	More mixing, more shear, greater change in pressure
$C_{\epsilon 1}$	1.44	Less mixing, lower shear, smaller change in pressure
$C_{\epsilon 2}$	1.92	More mixing, more shear, greater change in pressure
σ_k	1.0	Less mixing (Prandtl-Schmidt number)
σ_ϵ	1.3	Less mixing (Turbulent Prandtl number)

$$\nabla_t k + \nabla \cdot (\bar{\mathbf{u}} k) = \tau^R : \nabla \bar{\mathbf{u}} - \epsilon + \nabla \cdot \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \nabla k \right]$$

$$\nabla_t \epsilon + \nabla \cdot (\bar{\mathbf{u}} \epsilon) = C_{\epsilon_1} \frac{\epsilon}{k} \tau^R : \nabla \bar{\mathbf{u}} - C_{\epsilon_2} \frac{\epsilon^2}{k} + \nabla \cdot \left[\left(\nu + \frac{\nu_t}{\sigma_\epsilon} \right) \nabla \epsilon \right]$$

$$\nu_t = \frac{C_\mu k^2}{\epsilon}$$

- These observation can also be use with other variants of the $k - \epsilon$ turbulence models.
- Use with care as these are rough observations.

On the closure coefficients

- All the coefficients used in all turbulence models, undergo a similar calibration process.
- We briefly discussed how to calibrate the coefficients; the interest reader can refer to the original references of the turbulence models for a detailed description of the calibration of all coefficients.
- If you are interested in the standard $k-\epsilon$ turbulence model, these are the original references,

[1] B. E. Launder, D. B. Spalding. The Numerical Computation of Turbulent Flows. Computer Methods in Applied Mechanics and Engineering. 1974.

[2] B. E. Launder, B. I. Sharma. Application of the Energy-Dissipation Model of Turbulence to the Calculation of Flow Near a Spinning Disc. Letters in Heat and Mass Transfer. 1974.

- If you are interested in the Wilcox 1988 $k-\omega$ turbulence model,

[1] D. C. Wilcox. Reassessment of the Scale-Determining Equation for Advanced Turbulence Models. AIAA Journal, 1988.

[2] D. C. Wilcox. Turbulence Modeling for CFD. DCW Industries, 2010.

- Plus, many additional cross-references.

On the closure coefficients

At this point cynics might protest:

"This is pure empiricism! How can an entirely fictional equation, plucked out of thin air, and forced, through the judicious choice of some arbitrary coefficients, to reproduce one or two laboratory results, possibly hope to anticipate the evolution of a wide range of flows?"

The extraordinary thing, however, is that, by and large, it works reasonably well, at least much better than it ought to. So perhaps there is more to

$$\nabla_t \epsilon + \nabla \cdot (\bar{\mathbf{u}}\epsilon) = C_{\epsilon_1} \frac{\epsilon}{k} \tau^R : \nabla \bar{\mathbf{u}} - C_{\epsilon_2} \frac{\epsilon^2}{k} + \nabla \cdot \left[\left(\nu + \frac{\nu_t}{\sigma_\epsilon} \right) \nabla \epsilon \right]$$

than meets the eye. Perhaps there is some underlying rationale for this equation. It turns out that there is.

P. Davidson [1]

References:
[1] P. Davidson. Turbulence. An Introduction for Scientists and Engineers. Oxford University Press. 2015.

Roadmap to Lecture 6

Part 6

1. ~~On the closure coefficients~~
2. **Galilean invariance**
3. URANS and RANS

Galilean invariance

- Something we all take for granted as part of our Newtonian worldview is that the phenomena of fluid mechanics are physically the same, regardless of what reference frame we choose to view them in.
- Any fluid flow may be described with equal validity whether the reference frame is inertial (not accelerating or rotating) or is accelerating and/or rotating.
- The Navier-Stokes equations and the general boundary conditions used with them are invariant under Galilean transformation, which means that the forms of the equations and the boundary conditions are unchanged by a transformation from one inertial frame to another.
- The fact that we can look at the equations of fluid dynamics and their related physical interpretation in different reference frames and know that they are effectively the same comes in handy in many ways.
  - For example, it is basic to the similarity between wind-tunnel testing and flight.
- Just like all phenomena described by classical mechanics, the behavior of fluid flows is the same in all inertial frames.
- The idea of independence of reference frame worth a cursory look.

Reference:
D. McLean. Understanding aerodynamics. Arguing from the real physics. Wiley. 2013.

Galilean invariance

- The Galilean transformation between two coordinated systems can be defined as follows,

$$x' = x - Vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

- The moving coordinate system (the primed notation) is moving with respect to the fix reference system (unprimed) with a velocity V (as measured in the fix system) in the x direction.

$$U'(\mathbf{x}', t') = U(\mathbf{x}, t) - V$$

$$u' = u - V$$

- A quantity that is the same in different inertial frames is said to be Galilean invariant
- Let us transform one coordinate system to the other coordinate system

Galilean invariance

- Let us differentiate the variable ϕ (which is in function of space **x** and time t).
- By applying the chain rule, we obtain,

$$\frac{\partial t'}{\partial x} = 0$$

$$\frac{\partial \phi}{\partial x} = \frac{\partial x'}{\partial x}\frac{\partial \phi}{\partial x'} + \frac{\partial t'}{\partial x}\frac{\partial \phi}{\partial t'} = \frac{\partial \phi}{\partial x'}$$

$$\frac{\partial x'}{\partial x} = 1$$

- Therefore, the gradient of ϕ is the same in both coordinate systems.
- The gradient is Galilean invariant.
- A similar result can be obtained for the Laplacian.

Galilean invariance

- Let us differentiate the variable ϕ (which is in function of space **x** and time t).
- By applying the chain rule, we obtain,

$$\frac{\partial t'}{\partial t} = 1$$

$$\frac{\partial \phi}{\partial t} = \frac{\partial x'}{\partial t}\frac{\partial \phi}{\partial x'} + \frac{\partial t'}{\partial t}\frac{\partial \phi}{\partial t'} = -V\frac{\partial \phi}{\partial x'} + \frac{\partial \phi}{\partial t'}$$

$$\frac{\partial x'}{\partial t} = -V$$

- Therefore, the partial time derivative of ϕ is not Galilean invariant.
- For example, the partial time derivative of velocity is not Galilean invariant.
- Also, the velocity is not Galilean invariant.

Galilean invariance

- Let us apply the previous results to the material derivative,

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x}$$

- Substituting and doing some algebra, we obtain,

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial t'} - V\frac{\partial \phi}{\partial x'} + u\frac{\partial \phi}{\partial x'} = \frac{\partial \phi}{\partial t'} + u'\frac{\partial \phi}{\partial x'}$$

$$u = u' + V$$

- Notice that the terms where the velocity V appears cancel out.
- Therefore, the material derivative is Galilean invariant.

Galilean invariance

- We just showed that the velocity gradients and the material derivative are Galilean invariant.
- Whereas the velocity and its partial time derivative are not Galilean invariant.
- Other quantities that are Galilean invariant include transported scalars (such as temperature, species, concentration, and pressure), and quantities related to velocity gradients, e.g., strain rate, vorticity, and so on.
- All these operators appear in the Navier-Stokes equations; therefore, they are Galilean invariant.
- The Reynolds stress tensor modeled using the Boussinesg hypothesis is also Galilean invariant.
- Therefore, the RANS/URANS equations are Galilean invariant.
- However, the closure models are not all born Galilean invariant.
- The concept of Galilean invariance is an important tool when developing closure models.
- Models must not be sensitive to system translation, must not depend on the orientation of the system axes, and must be applicable to an accelerating or rating system.
- The fact that a turbulence model is not Galilean invariant does not mean that it is useless. It only limits the model to fixed coordinate systems.
- Recall that the $\gamma - Re_\theta$ transition model is not Galilean invariant.

Roadmap to Lecture 6

Part 6

1. ~~On the closure coefficients~~
2. ~~Galilean invariance~~
3. **URANS and RANS**

URANS and RANS

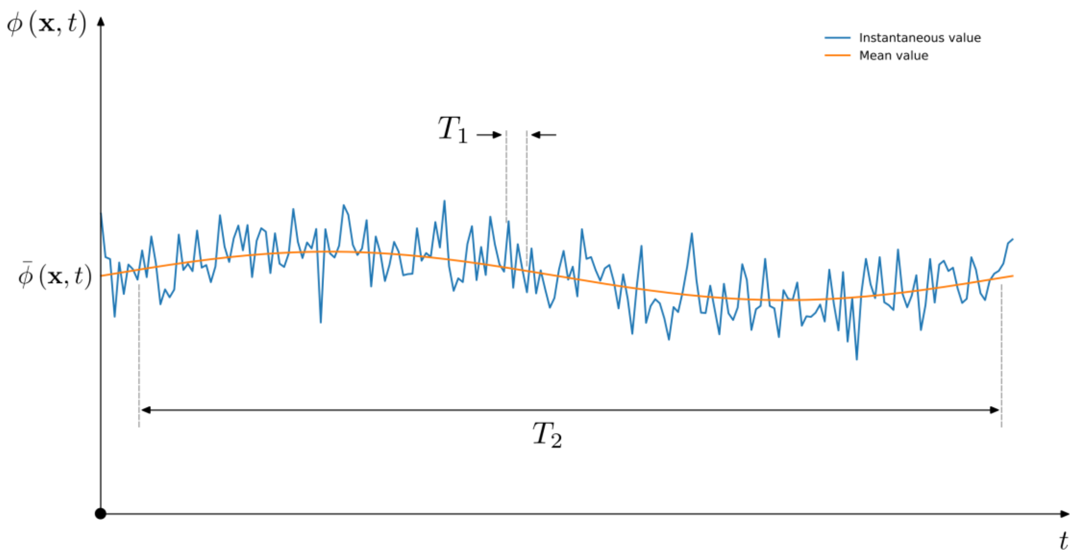
- So far, we have said that in RANS simulations we neglect the time derivative and in URANS simulations we retain the time derivative.
- It is a little bit more complicated than that, but it is a good interpretation.
- We should also take into consideration what we know about the flow physics, such as, shedding frequency or flow separation.
- The choice of the approach, RANS or URANS, strongly depends on the unsteadiness of the flow physics.
  - The unsteadiness can be due the coherent vortical structures, large scale eddies, multiphase flows, external forces, boundary conditions, moving bodies, and so on.
- Have in mind that it is possible to use RANS solvers with unsteady physics.
  - However, the results will be very questionable.
- Ultimately, it is up to the user.
- You must understand your physics, know your resources, and get a compromise between accuracy and computing speed.

URANS and RANS

- The mean flow computed in RANS simulations can be viewed as an ensemble or time averaged field.
- Any unsteadiness inherent to the turbulent eddies is removed by the averaging process.
  - Yes, the unsteadiness can be present in the steady solver, but the solution is not time accurate.
  - RANS simulations implies that the resolved quantities depends only on space.
- It is important to stress that the hypothesis of steadiness is very questionable, with results that might be even more questionable.
- But if the flow is truly steady (this is the exception rather than the rule) or mildly unsteady, the steady hypothesis is a good one.
- In many engineering applications steady solutions are acceptable and highly desirable.
- Two questions should always be asked when using RANS models,
  - How well does the model capture the flow physics?
  - How accurate are the quantitative predictions obtained in the analysis?
- If you cannot answer positively the previous questions or if you know that the flow physics is strongly unsteady, it is better to use unsteady RANS or URANS.

URANS and RANS

- In unsteady RANS or URANS, the mean flow is viewed as a results of the ensemble average or time averaging over the time period must smaller that T_2 (long period oscillations) and larger than T_1 (we do not want to solve the instantaneous fluctuations).



$$\phi(\mathbf{x}, t) = \underbrace{\bar{\phi}(\mathbf{x}, t)}_{mean\,value} + \underbrace{\phi'(\mathbf{x}, t)}_{fluctuating\,part}$$

- In the figure, T_2 is a time scale characteristic of a slow variation in the flow that we do not wish to regard as belonging to the turbulence fluctuations.

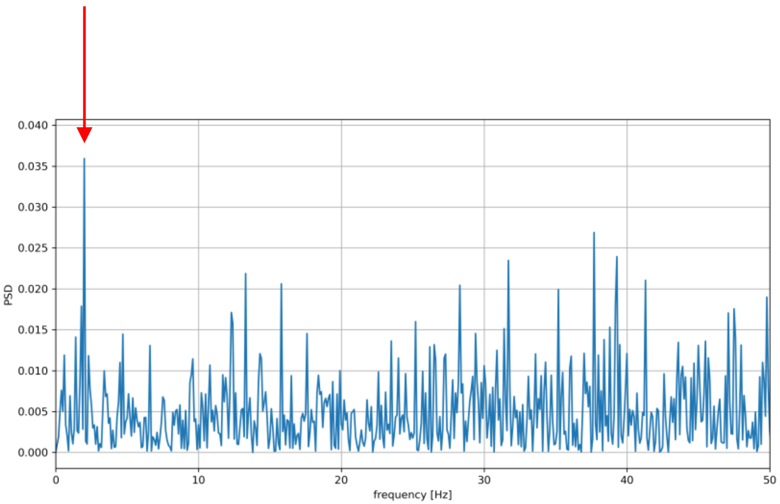
URANS and RANS

- In the previous situation, we were assuming that the time scales T_1 and T_2 exist and differ by several order of magnitude.
- But it might happen that in some situations this requirement is not satisfied.
- In such cases, the time averaging applied to the equations will remove or average these time scales.
- And this is due to the fact that there is no distinct separation between the flow unsteadiness and the turbulent fluctuations.
    - This separation between the flow scales and turbulent fluctuations is known as spectral gap.
- In cases where the spectral gap is not large enough, so it is not possible to distinct between the turbulent fluctuation and the flow scales, the time averaging used in standard URANS methods will add numerical diffusion to solution, and it will underestimate the physics of interest.
- In this situation the mean flow and the fluctuations are correlated.

URANS and RANS

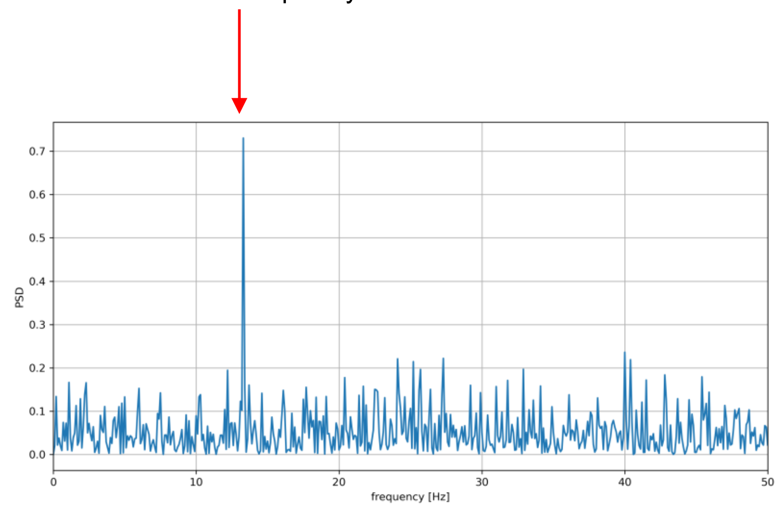
- In recent years, variations of URANS models have been developed that allow for the inclusion of transient, random large-scale structures.
- Such schemes generally are based on lowering the local eddy viscosity of the method to the point that diffusion no longer suppresses instabilities.
- Such effect can also be achieved by recalibrating the coefficients of the URANS model.
- In particular, the value of the coefficient C_μ is often decreased (and as a results μ_t) [1, 2].
- In the figure below, we show the effect of adjusting the coefficient C_μ (the specific application is not important). In this case, a lower value of the coefficient adds less dissipation.

No apparent dominant frequency – Maybe this one?



URANS with standard coefficients

Dominant frequency

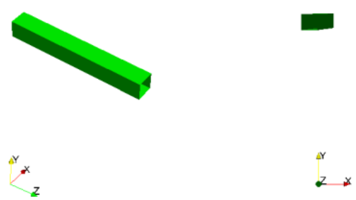


URANS with lower C_μ

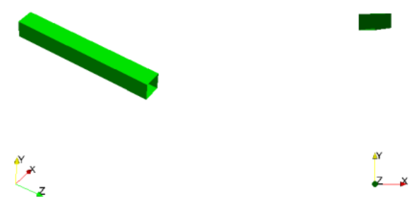
[1] F. Bastin, P. Lafon, S. Candel. Computation of jet mixing noise due to coherent structures: the plane jet case. J. Fluid Mech., 335, 261–304. 1997.
[2] P. Spalart. Strategies for turbulence modelling and simulations. Int. J. Heat and Fluid Flow, 21, 252–263. 2000.

URANS and RANS

Vortex shedding past square cylinder



URANS (K-Omega SST with no wall functions) – Vortices visualized by Q-criterion
www.wolfdynamics.com/wiki/squarecil/urans2.gif



LES (Smagorinsky) – Vortices visualized by Q-criterion
www.wolfdynamics.com/wiki/squarecil/les.gif

- The quantitative results are very similar in both cases.
- But qualitatively speaking, the solutions are very different.
- URANS simulations add a lot of dissipation to the solution.
- They do not resolve well the vortical structures in all directions, in particular in the spanwise direction (vortex stretching).
- And if there is no spectral gap between the turbulent fluctuations and the mean flow fluctuations (the vortex shedding frequency in this case), URANS will remove the mean flow fluctuations.

URANS and RANS

- Adjusting the coefficients of the URANS model, might not always be the best solution when dealing with strong unsteadiness and small spectral gaps.
- In the situations where we are interested in resolving the turbulent spectrum instead of modeling it (as in RANS/URANS), we can resort to another type of approach, namely, scale-resolving simulations or SRS.
- SRS refers to all turbulence models which resolve at least a portion of the turbulence spectrum in at least a part of the domain.
- SRS is a field of intense research, and many new formulations continue to emerge.
- Just to name a few of the approaches found in SRS:
  - Large eddy simulations (LES), detached eddy simulations (DES), scale adaptive simulation (SAS), wall-modeled LES (WMLES), hybrid URANS-LES, embedded LES (ELES), stress-blended eddy simulations (SBES).
- We will explore LES and DES in lecture 10.
- Have in mind that these approaches are intrinsically 3D and unsteady; therefore, computationally expensive.
  - The workhorse of turbulence modeling in CFD, RANS/URANS.