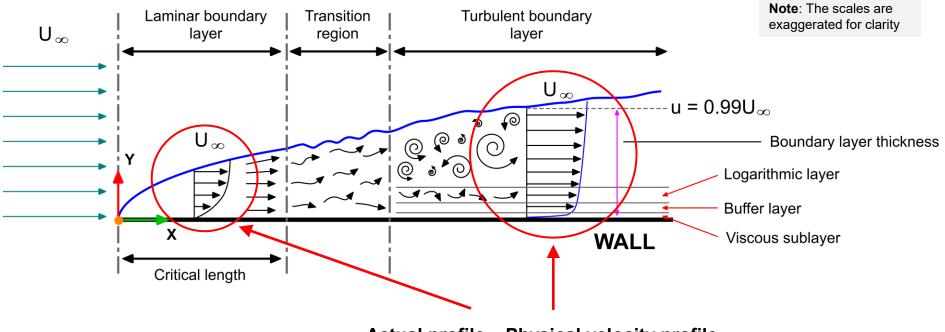
Turbulence and CFD models: Theory and applications

Roadmap to Lecture 6

Part 4

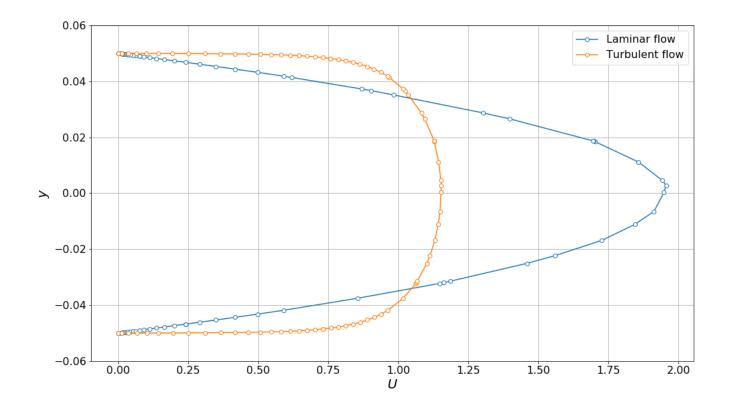
Turbulence near the wall – Boundary layer



Actual profile – Physical velocity profile

- Walls are the main source of turbulence generation in flows.
- The presence of walls imply the existence of boundary layers.
 - In the boundary layer, large gradients exist (velocity, temperature, and so on).
 - To properly resolve these gradients, we need to use very fine meshes close to the walls.

- Comparison of laminar and turbulent velocity profiles in a pipe.
- As it can be observed, close to the walls the velocity gradient is larger in the turbulent case.
- Therefore, fine meshes are required in order to properly resolve the steep gradients (velocity, temperature, etc.) close to the walls.



- To resolve the flow close to the walls, two approaches can be used:
 - Wall resolving approach.
 - Wall modeling approach.
- In the **wall resolving approach**, the equations are integrated down to the viscous sublayer.
 - This approach allows for the accurate computation of steep gradients of velocity and transported quantities near the walls.
 - However, it is computationally expensive as it requires very fine meshes close to the walls.
- In the **wall modeling approach**, the equations are solved a distance away from the wall, in the log-law region.
 - In this approach, we apply boundary conditions based on log-law relations some distance away from the wall, so we do not need to resolve the viscous sublayer.
 - The reduction of the computational overhead, because we are using a coarser mesh, is a compelling reason to support the use of wall functions in CFD.
 - However, under many flow conditions, such as, strong adverse pressure gradient, separated and impinging flows, strongly anisotropic flows, and so on; their physical basis is uncertain, and their accuracy is questionable.

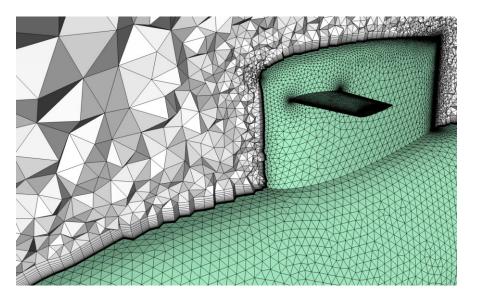
Wall resolving mesh

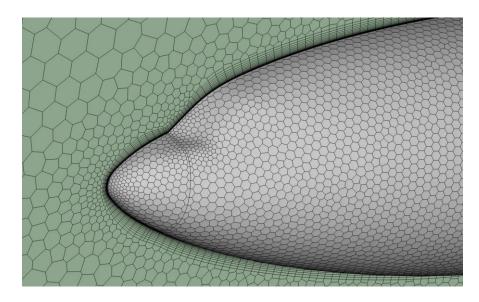
- Wall resolving meshes resolve the boundary layer down to the viscous sublayer.
- These meshes allow for the accurate computation of steep gradients of velocity and transported quantities near the walls.
- The only drawback is that you will require a lot of cells close to the walls.

Wall modeling mesh

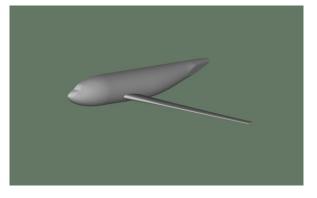
- In wall modeling meshes we apply boundary conditions some distance away from the wall, so we do not need to resolve the viscous sublayer.
- The goal is to use coarser meshes without losing accuracy.
- In the cell next to the wall, the field quantities and wall shear stresses are approximated using correlations (*e.g.*, log-law layer).

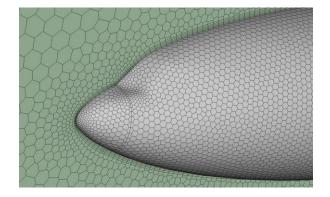
- The easiest way to resolve the steep gradients near the walls is by resolving the viscous sublayer.
 - To resolve the viscous sublayer, we need to cluster a lot of cells in the region where y⁺ is less than 5.
 - Usually, we need to cluster more than 10 layers in order to accurately resolve the profiles of velocity and the transported quantities.
 - This can significantly increase the cell count.
 - And in the case of unsteady simulations, it can have a significant impact in the time-step.
 - In unsteady simulations, a CFL number in the order of one is usually required for stability and accuracy reasons.



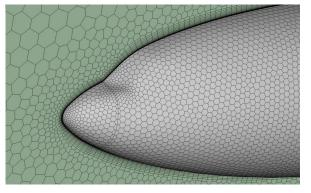


- In the wall resolving mesh, we only modified the inflation layers parameters (the prismatic cells close to the walls).
- The parameters of the surface mesh and volume mesh remained the same.
- By taking a mesh resolving approach, we almost doubled the cell count.





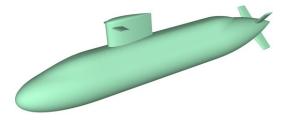
Wall modeling mesh Average y⁺ approximately 60

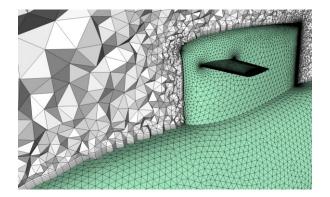


Wall resolving mesh Average y⁺ approximately 1

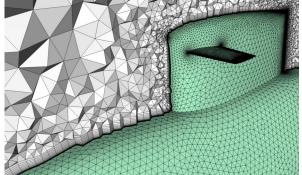
	Wall modeling mesh	Wall resolving mesh
Number of cells	649 619	1 160 135

- In the wall resolving mesh, we only modified the inflation layers parameters (the prismatic cells close to the walls).
- The parameters of the surface mesh and volume mesh remained the same.
- By taking a mesh resolving approach, we almost doubled the cell count.





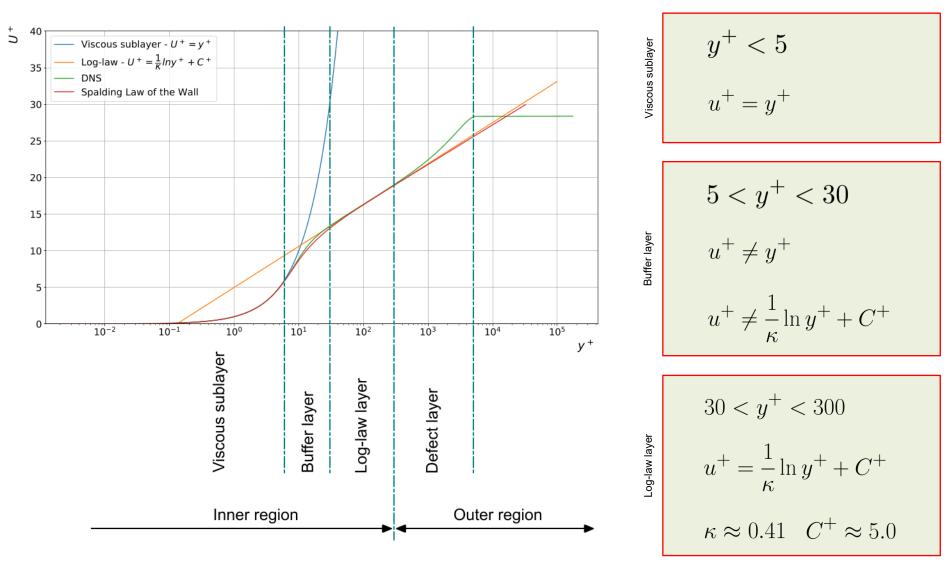
Wall modeling mesh Average y⁺ approximately 60



Wall resolving mesh Average y⁺ approximately 1

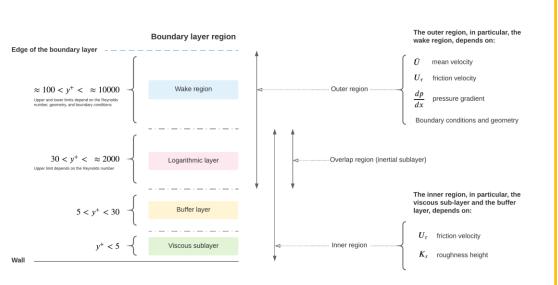
	Wall modeling mesh	Wall resolving mesh
Number of cells	6 613 049	11 149 266

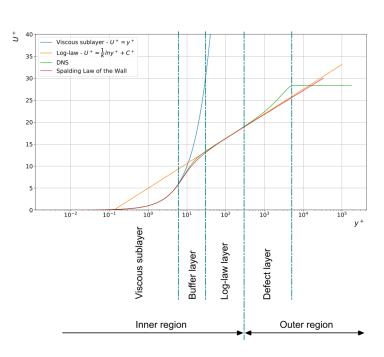
Turbulence near the wall – Relations according to the y⁺ value



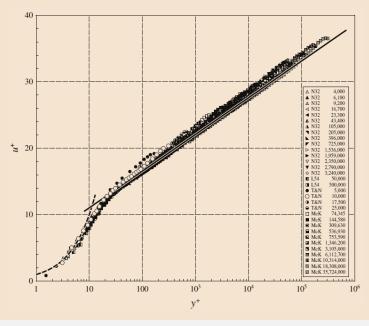
Note: the range of y⁺ values might change from reference to reference but roughly speaking they are all close to these values.

- The velocity profile near the wall can be represented by using the previous non-dimensional quantities and correlations.
- By using non-dimensional quantities, the flow behavior near the wall is independent of the Reynolds number, geometry, or relevant physics (to some extent).
- The correlations take a very predictable behavior close to the walls for a wide variety of flows.
- The outer or mean flow, depends on the geometry, boundary conditions, physics, and so on.

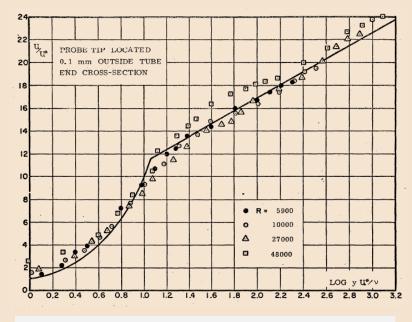




- The law of the wall, is one of the cornerstones of fluid dynamics and turbulence modeling.
- The logarithmic law, refers to the region of the inner-region of the boundary layer that can be described using a simple analytic function in the form of a logarithmic equation.
- This is one of the most famous empirically determined relationships in turbulent flows near solid boundaries.
- Measurements show that, for both internal and external flows, the streamwise velocity in the flow near the wall varies logarithmically with distance from the surface.



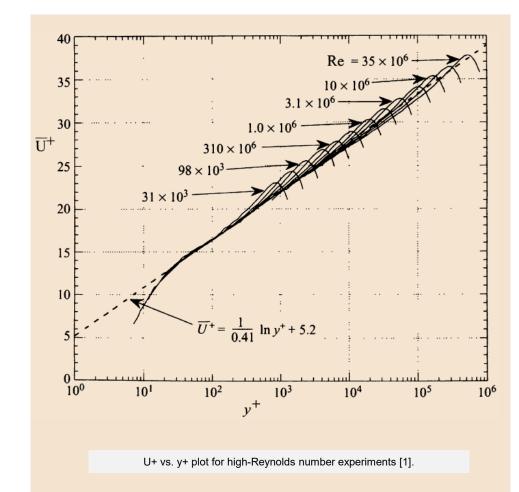
Dimensionless mean velocity profile u^+ as a function of the dimensionless wall distance y^+ for turbulent pipe flow with Reynolds numbers between 4000 and 3600000 [1].



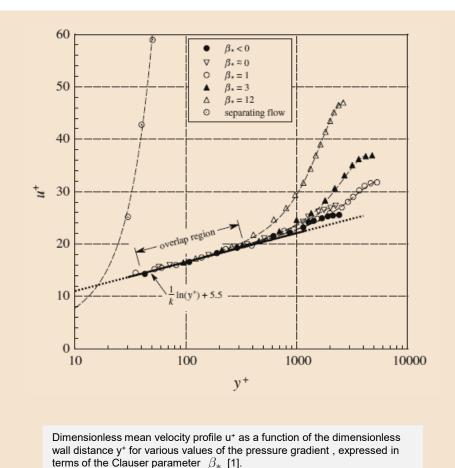
Nondimensional velocity profile. Experimental measurements [2].

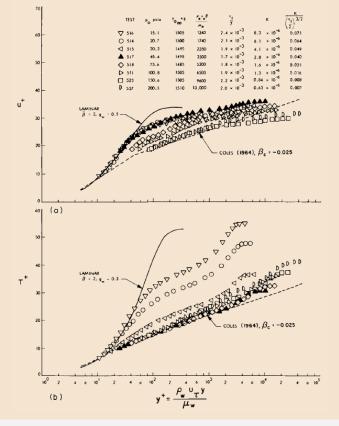
[1] F. Nieuwstadt, B. Boersma, J. Westerweel. Turbulence. Introduction to Theory and Applications of Turbulent Flows. Springer. 2016.
 [2] E. Lindgren. Experimental study on turbulent pipe flows of distilled water. Oklahoma State University, AD621071, Technical Report No. 2, 1965.

- In the literature, you will find that wall functions are valid for 30 < y⁺ < 300.
- This is the most common range that you will find in the literature but have in mind that different authors might define different values.
- However, this range is fine for most applications.
- It is a subject of discussion the upper limit of the loglaw layer.
- Most of the times you will find in the literature a y⁺ upper limit of 300.
- In reality, this upper limit depends on the Reynolds number, as shown in the figure.
- But
- For high Reynolds number, the overlap region is large.
- Whereas, for lower Reynolds number, the overlap region is shorter.
- And this imposes a limit on the usability of wall functions for lower Reynolds numbers, as it becomes very difficult to cluster enough computational cells in the log-law region to resolve the profiles.
- If the y⁺ upper limit is below or close to 100, it is better to use a wall resolving approach.
- The value of the lower limit is generally accepted to be equal to 30.



- As shown in the figures, the pressure gradient has an influence on the velocity profile.
- Therefore, if we are planning to use wall functions, it is strongly recommended to use wall functions that use corrections to take into account this effect.

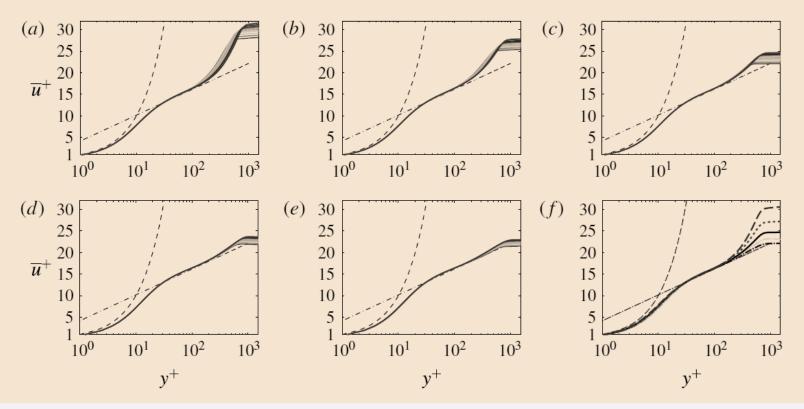




Effect of strong flow acceleration on the (a) velocity profiles and (b) temperature profiles measured in a nozzle. [2].

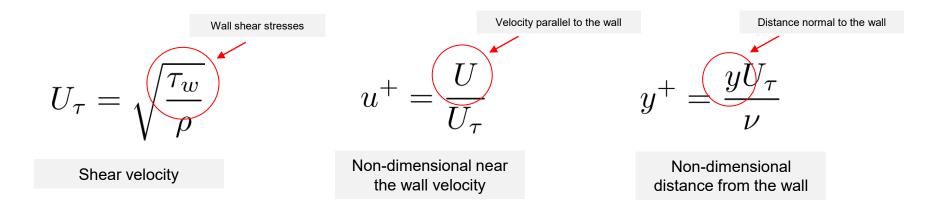
[1] F. Nieuwstadt, B. Boersma, J. Westerweel. Turbulence. Introduction to Theory and Applications of Turbulent Flows. Springer. 2016. [2] T. Cebeci, A. M. O. Smith. Analysis of Turbulent Boundary Layers. Academic Press, 1974.

- As shown in the figures, the pressure gradient has an influence on the velocity profile.
- Therefore, if we are planning to use wall functions, it is strongly recommended to use wall functions that use corrections to take into account this effect.



Mean velocity profiles in wall units u+. Thirteen profiles between xmin and xmax of the domain. (a) APGs (strong adverse pressure gradient). (b) APGw (weak adverse pressure gradient). (c) ZPG (zero pressure gradient). (d) FPGw (weak favorable pressure gradient). (e) FPGs (strong favorable pressure gradient). (f) Comparison of the five cases at similar friction Reynolds number [1].

• Before addressing how to compute the flow close to the walls, let us summarize all the nondimensional variables near the walls.

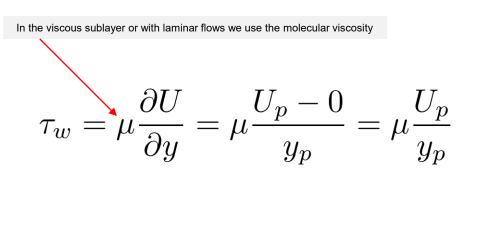


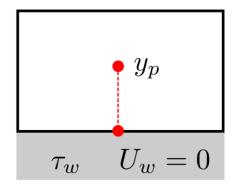
Close to the walls we only know the wall shear stress, viscosity, and distance,

$$U = f(\tau_w, \rho, \mu, y)$$

• Therefore, we use these quantities to create the non-dimensional groups.

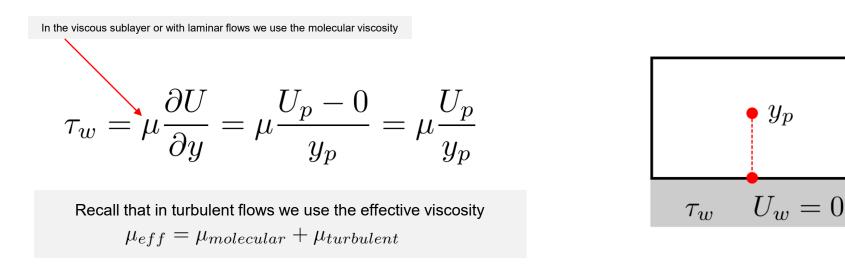
• If we are dealing with globally laminar flows, we can compute the wall shear stress as follows,





- In our notation, the subscript p indicates values at the cell center and the subscripts w indicates values at the walls
- Remember, some solvers use cell-centered quantities, and some solvers use node-centered quantities.
- From now on, we are going to assume that all quantities are computed at the cell center.
- Sometimes in this approach, damping functions are added to gain robustness.

- If the same way, if we are dealing with turbulent flows, and the mesh fine enough to resolve the viscous sublayer, we can compute the wall shear stress in the same way as for laminar flows.
- After all, we are resolving the viscous sublayer, which is laminar.



- In our notation, the subscript p indicates values at the cell center and the subscripts w indicates values at the walls
- Remember, some solvers use cell-centered quantities, and some solvers use node-centered quantities.
- From now on, we are going to assume that all quantities are computed at the cell center.
- Sometimes in this approach, damping functions are added to gain robustness.

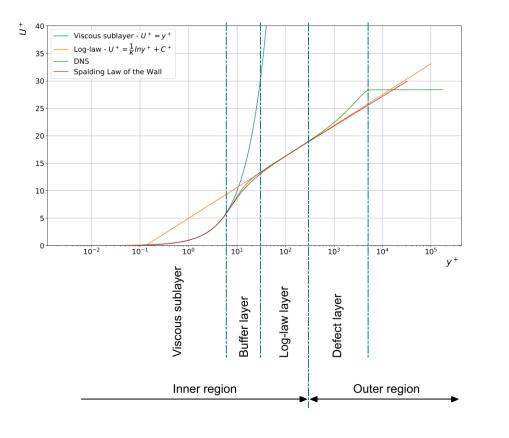
- We just described how to compute the wall shear stresses in laminar flows and in turbulent flows using the wall resolving approach.
- As you can see, this approach is straight forward to implement.
- Let us address how to compute the wall shear stresses using the wall modeling approach.
- If we are dealing with turbulent flows and if we are using a coarse mesh such that y⁺ > 30, using the previous relations to compute the wall shear stresses is not accurate anymore.
- We are missing a lot of gradient information if we use the previous approach, namely,

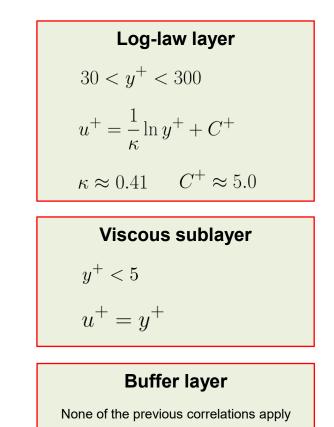
$$\tau_w = \mu \frac{\partial U}{\partial y} = \mu \frac{U_p - 0}{y_p} = \mu \frac{U_p}{y_p}$$

This is not accurate if $y^+ > 10$. This value roughly corresponds to the intersection of the viscous sublayer law and the log-law

- In the wall modeling approach, we need to somehow correct this computation.
- We do this by constructing a bridge between the wall values and the log-law correlations.
- The question is?
 - How do we transfer information from the empirical correlations to the walls and to the flow?
 - How do we compute the wall shear stresses?

- If the first cell center is in log-law layer, we cannot use the viscous sublayer relationship because it is too inaccurate. Therefore, we need to use wall functions.
- By using wall functions, we can use empirical correlations to bridge wall conditions to the log-law layer.
- The correlations provide a link between U and U_{τ} (or τ_w).





- By using wall functions, we bridge the wall conditions and cell centered values with the log-law empirical correlations.
- The wall functions reduce the computational effort significantly because we do not need to resolve the viscous sublayer.
- Let us explain the standard wall functions using the method proposed by Launder and Spalding [1], which is probably the most widely used method.
- In this approach,

$$u^* = \begin{cases} y^* & \text{In the viscous sublayer} \\ \frac{1}{\kappa} \ln (Ey^*) & \text{In the log-law layer} \end{cases}$$

- Notice that we are using u^* and y^* instead of u^+ and y^+ .
- Also, the log-law layer correlation is slightly different from what we have seen so far.
- Let us address these two issues.

- The idea of introducing the new quantity u*, is to avoid the singularity that occurs when the wall shear stress is equal to zero in u⁺ (*i.e.*, in a separation point).
- Recall that the shear velocity is equal to,

$$U_{\tau} = \sqrt{\frac{\tau_w}{\rho}}$$

• In a separation point, the wall shear stress is equal to zero, therefore,

$$U_{\tau} = 0 \quad \text{if} \quad \tau_w = 0$$

 If the shear velocity is equal to zero, then we have a singularity when computing u⁺, and this might pose numerical problems.

$$u^+ = \frac{U}{U_\tau}$$

The new quantities u* and y⁺ are defined as follows [1],

$$U_{\tau}^{*} = C_{\mu}^{1/4} k_{p}^{1/2} \qquad \qquad y^{*} = \frac{C_{\mu}^{1/4} k_{p}^{1/2} y_{p}}{\nu} \qquad \qquad u^{*} = \frac{1}{\kappa} \ln \left(Ey^{*}\right)$$

- Notice that these new quantities do not depend anymore on the shear velocity.
- Therefore, in the case of wall shear stress equal to zero, we do not risk anymore a singularity when computing u⁺.
- It is worth noting that y⁺ is equal to y^{*} in the ideal situation of equilibrium conditions (production equal to dissipation).
- We will address the concept of equilibrium when addressing the origins of the C_{μ} coefficient.

• All the relations of the standard wall functions formulation of Launder and Spalding [1], can be summarized as follows,

$$\begin{split} u^* &= \frac{U_p C_\mu^{1/4} k_p^{1/2}}{\tau_w / \rho} & \quad \text{The only unknown quantity is the wall shear stress} \\ P_k &\approx \tau_w \frac{\partial \overline{u}}{\partial y} = \tau_w \frac{\tau_w}{\kappa \rho C_\mu^{1/4} k_p^{1/2} y_p} \\ \epsilon_p &= \frac{C_\mu^{3/4} k_p^{3/2}}{\kappa y_p} \end{split}$$

• Recall that the subscript p indicates values at the cell center and the subscripts w indicates values at the walls

• The boundary condition for TKE at the walls is,

$$\frac{\partial k}{\partial n} = 0$$

• And recall that,

$$u^{*} = \frac{1}{\kappa} \ln (Ey^{*}) \qquad y^{*} = \frac{\rho C_{\mu}^{1/4} k_{p}^{1/2} y_{p}}{\mu} \qquad \underbrace{\kappa = 0.4187 \qquad E = 9.793}_{\text{These are the values used in Fluent}}$$

• These relations apply only to the cells adjacent to the walls.

• We can also use an automatic wall treatment just by simply adding a conditional clause,

$$u^* = \begin{cases} y^* & y^* < 11.225\\ \frac{1}{\kappa} \ln(Ey^*) & y^* > 11.225 \end{cases}$$

- The value of 11.225 (which is the one used in Fluent), comes from the intersection of the two correlations.
- This value might change depending on the constant used.
- If you use the relations that we defined for the law of the wall in the previous lectures, you will find a value of approximately 10.8, of course, we used different values for the constants.

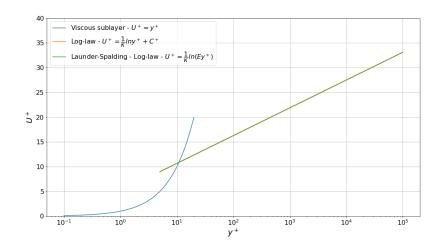
• In the standard wall functions formulation of Launder and Spalding [1], the correlation for the log-law layer is given as follows,

$$u^* = \frac{1}{\kappa} \ln\left(Ey^*\right)$$

• Whereas the traditional correlation is given as follows (apply a logarithmic rule and you will get similar formulations),

$$u^{+} = \frac{1}{\kappa} \ln\left(y^{+}\right) + C^{+}$$

- These two correlations are approximately the same, as shown in the figure.
- Any difference is due to the values of the constants used.



- We just presented the wall functions for the momentum and turbulence variables.
- Similarly, wall functions can be derived for temperature, species, and so on.
- We will briefly address temperature wall functions in Lecture 9.
- The approach we just presented, is also known as a log-law based approach.
 - Specifically, we presented the Launder-Spalding methodology [1].
 - Which is probably the most popular approach.
 - However, this does not mean that it is the most robust approach.
- Remember, you need to assign boundary conditions to the wall functions.
- In Fluent, the boundary conditions for the wall functions are all taken care by the solver. You do not need to be concerned about the numerical values.
- Finally, this is not the only approach when using wall functions.
- In the literature, you will find many approaches.

- There are many wall functions implementations, just to name a few,
 - Standard wall functions.
 - Standard wall functions Launder-Spalding methodology (the approach we just presented).
 - Generalized wall functions.
 - Scalable wall functions.
 - Non-equilibrium wall functions.
 - Two-layer approach.
 - y⁺ insensitive wall treatment.
 - Menter-Lechner wall functions.
 - Werner and Wengle wall functions.
 - Craft wall functions.
 - Chien-Launder wall functions.
 - Eddy viscosity-based wall functions.
 - Spalding continuous wall function.
 - Roughness wall functions.
 - And so on.

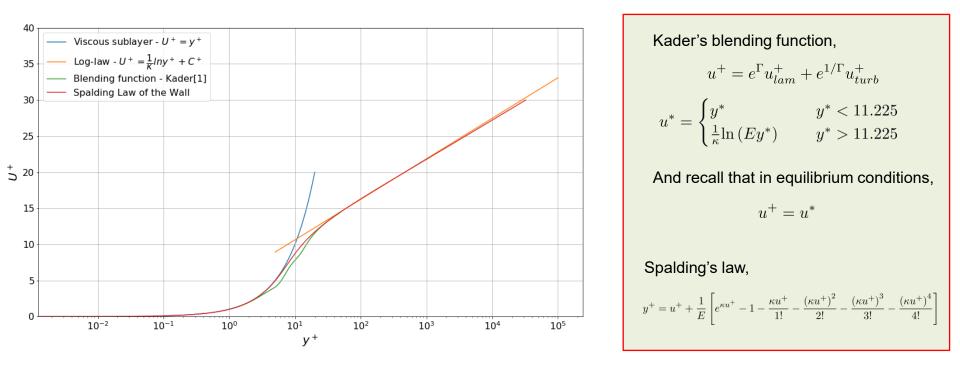
- It is also possible to formulate y⁺ insensitive wall functions.
- That is, formulations that cover viscous sublayer, buffer region, and log-law region.
- This can be achieved by using a blending function between the viscous sublayer and the loglaw layer [1].
- To use this approach, you need to use turbulence models able to deal with wall resolving meshes and wall modeling meshes.
- The $k \omega$ family of turbulence models are y⁺ insensitive.
- Kader [1] proposed the following blending function to obtain a y⁺ insensitive formulation,

$$u^{+} = e^{\Gamma} u_{lam}^{+} + e^{1/\Gamma} u_{turb}^{+}$$

$$\Gamma = -\frac{a (y^{+})^{4}}{1 + by^{+}} \qquad a = 0.01 \qquad b = 5$$

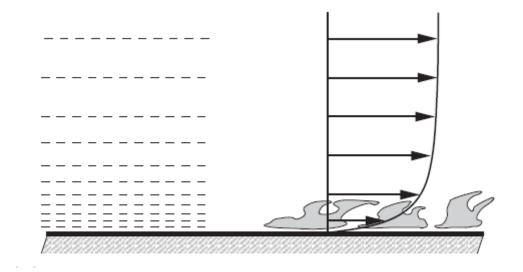
 This formula guarantees the correct asymptotic behavior for large and small values of y⁺ and a reasonable representation of velocity profiles in the cases where y⁺ falls inside the buffer region.

- Plot of Kader's [1] blending function.
- In the plot, the Spalding function [2] is also represented.
- The Spalding function is another alternative to obtain a y^+ insensitive treatment.
 - It is essentially a fit of the laminar, buffer and logarithmic regions of the boundary layer.



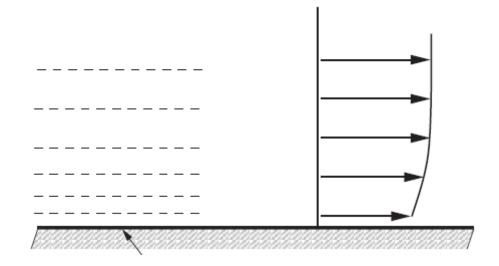
Near-wall treatment and wall functions

- When dealing with wall turbulence, we need to choose a near-wall treatment.
- If you want to resolve the boundary layer, all the way down to the viscous sub-layer, you need very fine meshes close to the wall.
- In terms of y^+ , you need to cluster at least 5 to 10 layers at $y^+ < 5$.
 - You need to properly resolve the profiles (U, k, epsilon, Reynolds stresses and so on).
- Usually, this kind of meshes will cluster from 15 to 30 layers (or even more) close to the walls.
- This is the most accurate approach, but it is computationally expensive.



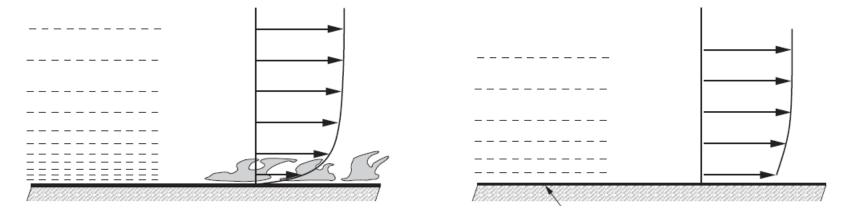
Near-wall treatment and wall functions

- When dealing with wall turbulence, we need to choose a near-wall treatment.
- If you are not interested in resolving the boundary layer up to the viscous sub-layer, you can use wall functions.
- In terms of y^+ , wall functions will model everything below $y^+ < 30$ or the target y^+ value.
- This approach use coarser meshes, but you should be aware of the limitations of the wall functions.
- You will need to cluster at least 5 to 10 layers close to the walls in order to resolve the profiles (U, k, epsilon, Reynolds stresses and so on).
- As a general rule, when using wall functions, the first cell center should be located above y+ > 40-50 and below $y \approx 0.2\delta_{99}$ (boundary layer thickness).



Near-wall treatment and wall functions

- When dealing with wall turbulence, we need to choose a near-wall treatment.
- You can also use the y⁺ insensitive wall treatment (sometimes known as continuous wall functions or scalable wall functions).
- This kind of wall functions are valid in the whole boundary layer.
- In terms of y⁺, you can use this approach for values between 1 < y⁺ < 300-500 (the upper limit depends on the Reynolds number).
- This approach is very flexible as it is independent of the y⁺ value, but is not available in all turbulence models
- Again, you should cluster enough cells close to the walls to resolve the profiles (at least 8-10 layers).



This approach automatically switch between one or the other approach.

Final remarks

- If you want good accuracy, use a wall resolving approach.
- This approach is relatively affordable if you are running steady simulations.
- If you have flow separation, have in mind that wall functions are not very accurate.
- Heat transfer and non-equilibrium applications requires high accuracy (wall resolving treatment).
 - This requirement is not compulsory; however, it is strongly recommended.
- Using wall functions is not about putting one single cell in the log-law layer.
 - You need to put enough cells in the log-law region to resolve the velocity, temperature, and turbulence variables profiles.
- When using the wall resolving approach, try to get an average y⁺ value close to 1 or lower.
- Values of y⁺ lower that 0.1 will not give large improvement.
- Pushing the mesh to values of y⁺ below 0.1 can results in low quality meshes for industrial applications.
- It is a common agreement that the upper limit of the viscous sublayer is five. When placing the first cell center, you can go as high as six, without loosing too much accuracy.
 - But ideally, you should aim for a y⁺ value of 1-2.

Final remarks

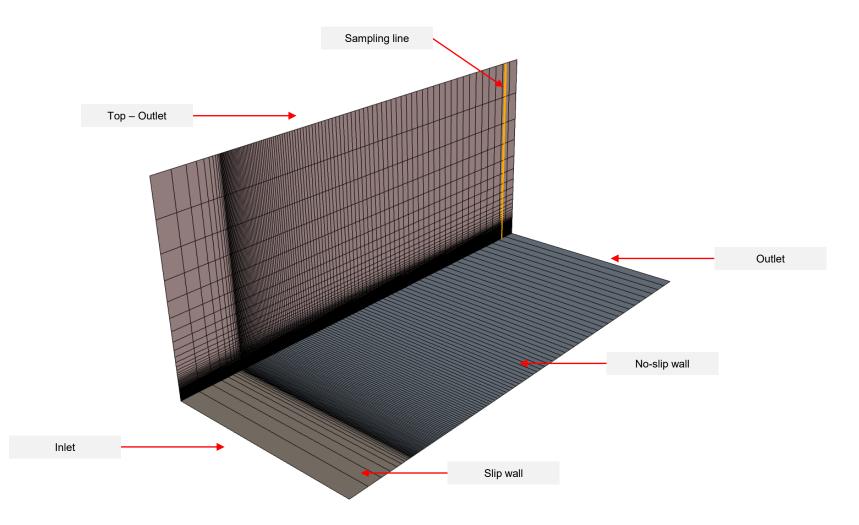
- As for the wall modeling approach, in the wall resolving treatment you need to cluster enough cells to resolve the viscous sublayer profiles.
 - Namely, velocity, temperature, turbulence quantities, and so on.
- It is recommended to use at least 15 inflation layers with a low expansion ratio (1.2 or less) to properly resolve the profiles.
- No need to mention it, but hexahedral or prismatic cells are preferred over any other type of cells in the boundary layer region.
- Do not use mesh refinement with standard wall functions as the solution tends to deteriorate.
 - Generally speaking, standard wall functions perform bad with wall resolving meshes.
- The absolute minimum number of inflation layers when using wall functions is five.
- It is recommended to use y⁺ insensitive wall functions instead of standard wall functions.
 - However, not all turbulence models support the use of y⁺ insensitive wall functions .
- As a general rule, when using wall functions, the first cell center should be located above y+ > 40-50 and below $y \approx 0.2\delta_{99}$ (boundary layer thickness).

Final remarks

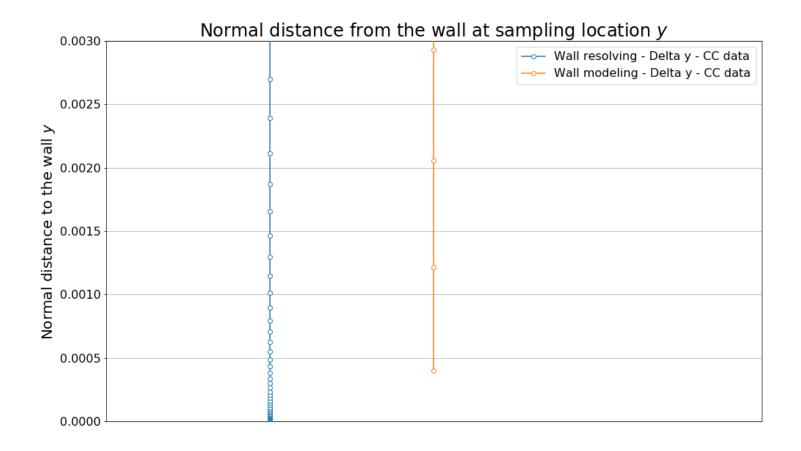
- When using wall functions, it is extremely advised to avoid placing the first cell center in the buffer layer, as errors are large in this region.
 - None of the correlation found in literature will give you accurate results in the buffer region.
- Remember, it is very difficult (if not impossible) to have a uniform y⁺ value.
- Therefore, you should monitor the average y⁺ value at the walls.
- It is also recommended to monitor the maximum and minimum values of y⁺ and verify that they
 do not cover more that 10% of the surface or are located in critical areas.
- Generally speaking, wall functions is the approach to use if you are more interested in the mixing in the outer region, rather than the forces on the wall.
- If accurate prediction of forces, heat transfer, and species concentration on the walls are key to your simulation (aerodynamic drag, turbomachinery blade performance, heat transfer, combustion) it is better to use a wall resolving approach.
- By following good standard practices, both approaches can give similar results.

At this point you are invited to revisit Lecture 3 Turbulence near the wall - Law of the wall

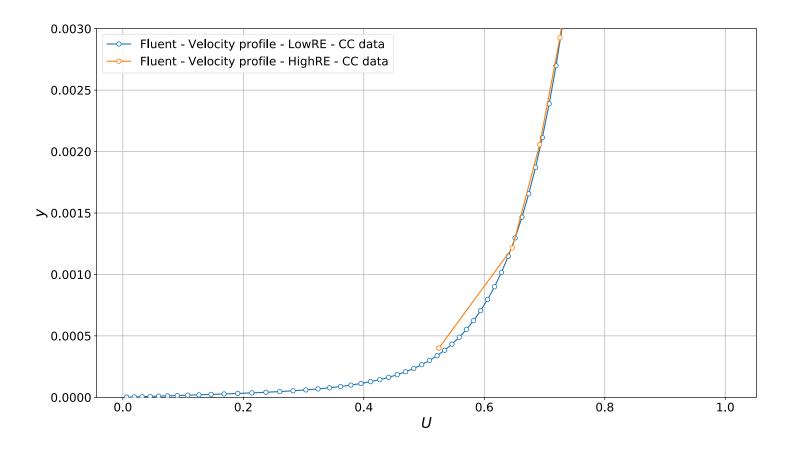
Comparison of the wall resolving approach and wall modeling approach for a sample application



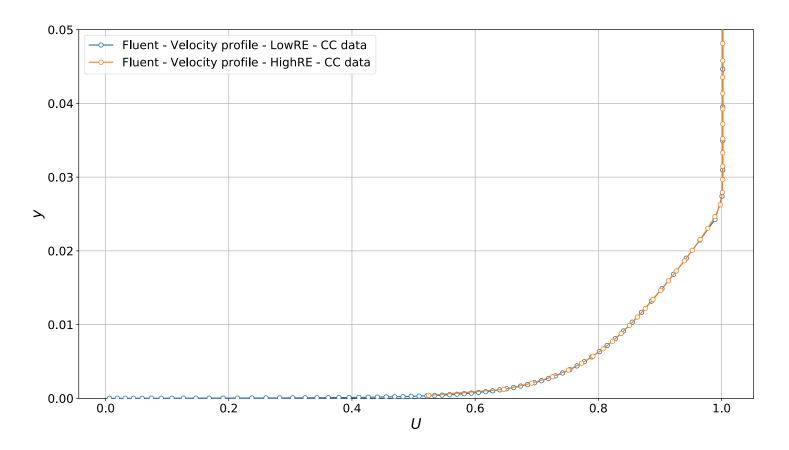
- Distance normal to wall y (m).
- Each circle represents a cell center.



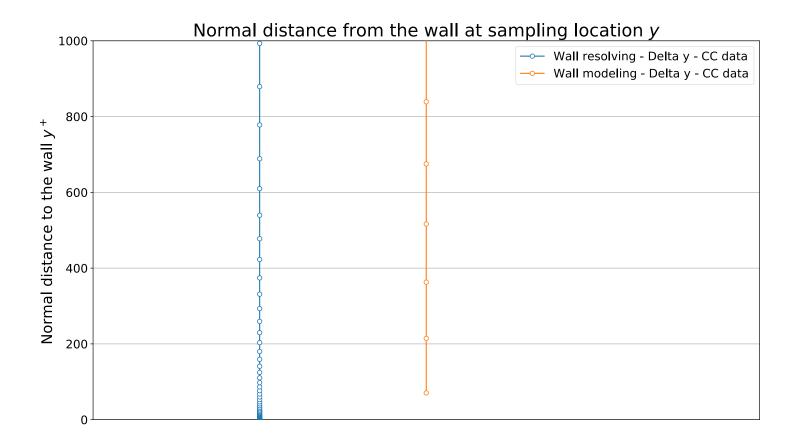
- Velocity profile close to the wall y vs. U
- Each circle represents a cell center.



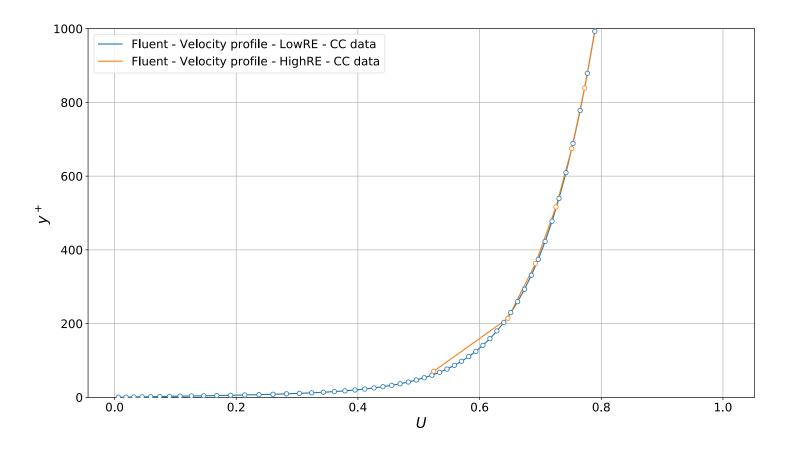
- Velocity profile close to the wall y vs. U
- Each circle represents a cell center.



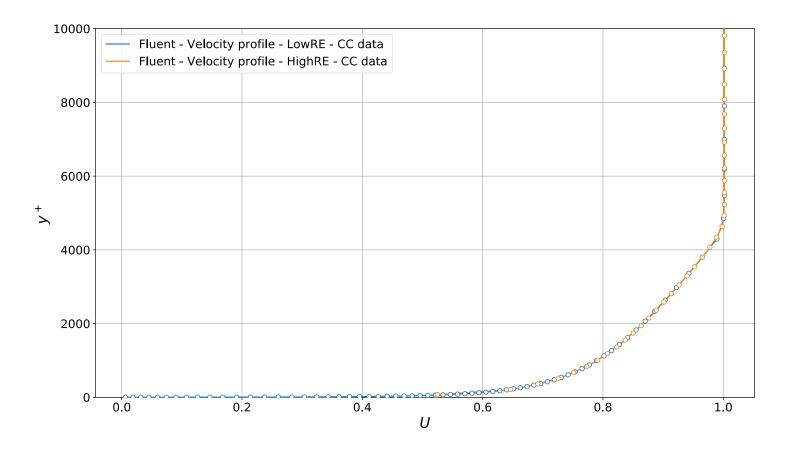
- Non-dimensional distance normal to wall y⁺.
- Each circle represents a cell center.



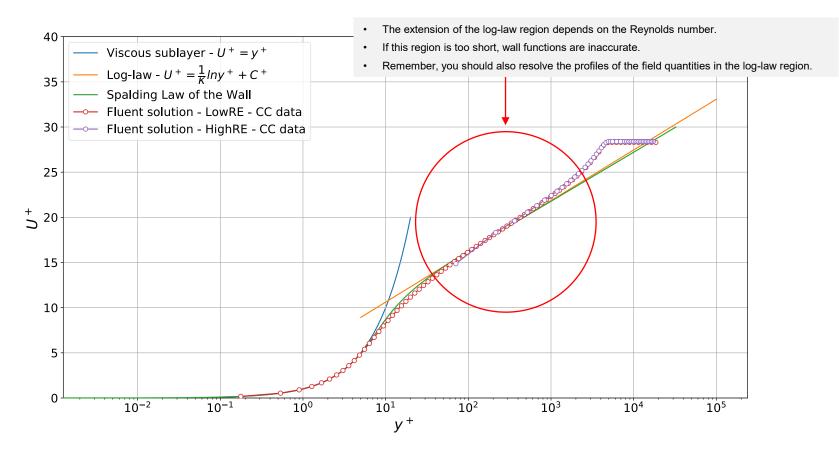
- Velocity profile close to the wall y⁺ vs. U
- Each circle represents a cell center.



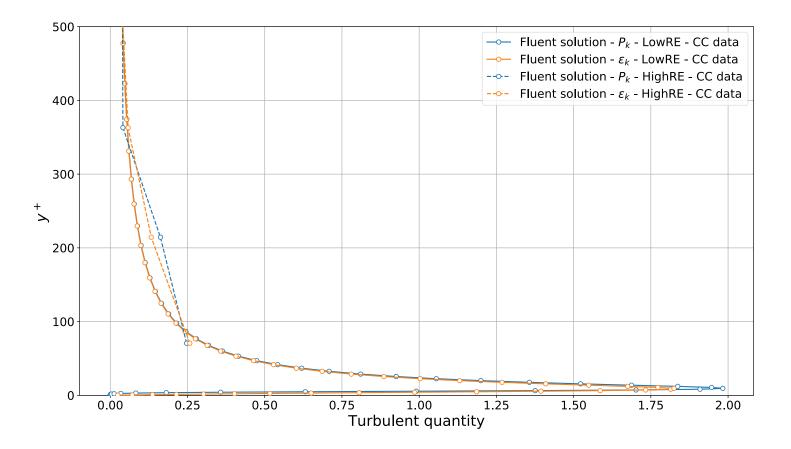
- Velocity profile close to the wall y⁺ vs. U
- Each circle represents a cell center.



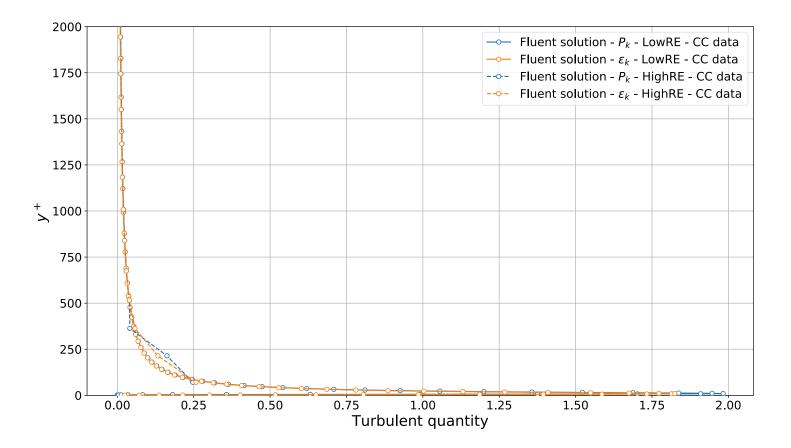
- Normalized velocity profile close to the wall U⁺ vs. y⁺
- Each circle represents a cell center.



- Budget of turbulent kinetic energy production and dissipation close to the walls.
- Each circle represents a cell center.



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- Mesh comparison Wall resolving mesh (left) and wall modeling mesh (right).
- It is important to mention that resolving the streamwise direction is also important.

2D Zero pressure gradient flat plate

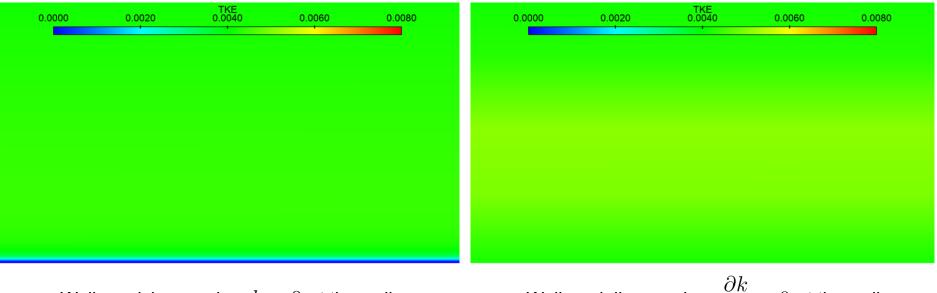
0.00	0.25	√elocity magnitude (m/s) 0.50	0.75	1.00

Wall resolving mesh.

Wall modeling mesh.

• Plot of velocity magnitude contours.

2D Zero pressure gradient flat plate



Wall resolving mesh – k = 0 at the wall

Wall modeling mesh – $\frac{\partial k}{\partial n} = 0$ at the wall

• Plot of turbulent kinetic energy contours.