Roadmap to Lecture 5

1. ~~Governing equations of fluid dynamics~~
2. ~~RANS equations – Reynolds averaging~~
3. ~~The Boussinesq hypothesis~~
4. ~~The gradient diffusion hypothesis~~
5. **Sample turbulence models**
6. Quick review of solution methods for the governing equations of fluid dynamics
7. What is Ansys Fluent? – Executive summary

Sample turbulence models

RANS equations – EVM equations

- By using the Boussinesq approximation in the incompressible RANS equations, we obtain the following set of equations using vector notation (**the solvable equations**),

$$\nabla \cdot (\bar{\mathbf{u}}) = 0$$

$$\frac{\partial \bar{\mathbf{u}}}{\partial t} + \nabla \cdot (\bar{\mathbf{u}}\bar{\mathbf{u}}) = -\frac{1}{\rho}\left(\nabla \bar{p} + \frac{2}{3}\rho \nabla k\right) + \nabla \cdot \left[\frac{1}{\rho}\left(\mu + \mu_t\right)\nabla \bar{\mathbf{u}}\right]$$

- The problem now reduces to computing the turbulent eddy viscosity in the momentum equation.
- Let us explore two closure models, the $k - \epsilon$ model and $k - \omega$ the model.

Sample turbulence models

RANS equations – EVM equations

- By using the Boussinesq approximation in the incompressible RANS equations, we obtain the following set of equations using index notation (**the solvable equations**),

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho}\left[\frac{\left(\partial \bar{p} + \partial \frac{2}{3}\rho k\right)}{\partial x_i}\right] + \frac{\partial}{\partial x_j}\left[\frac{1}{\rho}\left(\mu + \mu_t\right)\frac{\partial \bar{u}_i}{\partial x_j}\right]$$

- The problem now reduces to computing the turbulent eddy viscosity in the momentum equation.
- Let us explore two closure models, the $k-\epsilon$ model and $k-\omega$ the model.

Sample turbulence models

$k - \epsilon$ Turbulence model governing equations

- It is called $k - \epsilon$ because it solves two additional equations for modeling the turbulent eddy viscosity, namely, the turbulent kinetic energy k and the turbulence dissipation rate ϵ.

$$\nabla_t k + \nabla \cdot (\bar{\mathbf{u}} k) = \tau^R : \nabla \bar{\mathbf{u}} - \epsilon + \nabla \cdot \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \nabla k \right]$$

$$\nabla_t \epsilon + \nabla \cdot (\bar{\mathbf{u}} \epsilon) = C_{\epsilon_1} \frac{\epsilon}{k} \tau^R : \nabla \bar{\mathbf{u}} - C_{\epsilon_2} \frac{\epsilon^2}{k} + \nabla \cdot \left[\left(\nu + \frac{\nu_t}{\sigma_\epsilon} \right) \nabla \epsilon \right]$$

- With the following closure coefficients,

$$C_{\epsilon_1} = 1.44 \qquad C_{\epsilon_2} = 1.92 \qquad C_\mu = 0.09 \qquad \sigma_k = 1.0 \qquad \sigma_\epsilon = 1.3$$

- And the following closure relationships,

$$\nu_t = \frac{C_\mu k^2}{\epsilon} \qquad \omega = \frac{\epsilon}{C_\mu k} \qquad l = \frac{C_\mu k^{3/2}}{\epsilon}$$

Sample turbulence models

$k - \epsilon$ Turbulence model governing equations

- The closure equations correspond to the standard $k - \epsilon$ model.
- They have been manipulated so there are no terms including fluctuating quantities (*i.e.*, velocity and pressure), and double or triple correlations of the fluctuating quantities.
- The Reynolds stress tensor is modeled using the Boussinesq approximation.
- The turbulence dissipation rate is modeled using a second transport equation.

$$\nabla_t k + \nabla \cdot (\bar{\mathbf{u}} k) = \tau^R : \nabla \bar{\mathbf{u}} - \epsilon + \nabla \cdot \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \nabla k \right]$$

Production, Dissipation, Diffusion

$$\nabla_t \epsilon + \nabla \cdot (\bar{\mathbf{u}} \epsilon) = C_{\epsilon_1} \frac{\epsilon}{k} \tau^R : \nabla \bar{\mathbf{u}} - C_{\epsilon_2} \frac{\epsilon^2}{k} + \nabla \cdot \left[\left(\nu + \frac{\nu_t}{\sigma_\epsilon} \right) \nabla \epsilon \right]$$

Production, Dissipation, Diffusion

$k-\omega$ Turbulence model governing equations

- It is called $k-\omega$ because it solves two additional equations for modeling the turbulent viscosity, namely, the turbulent kinetic energy k and the turbulence specific dissipation rate ω.

$$\nabla_t k + \nabla \cdot (\bar{\mathbf{u}} k) = \tau^R : \nabla \bar{\mathbf{u}} - \beta^* k\omega + \nabla \cdot [(\nu + \sigma^* \nu_t) \nabla k]$$

$$\nabla_t \omega + \nabla \cdot (\bar{\mathbf{u}} \omega) = \alpha \frac{\omega}{k} \tau^R : \nabla \bar{\mathbf{u}} - \beta \omega^2 + \nabla \cdot [(\nu + \sigma \nu_t) \nabla \omega]$$

- With the following closure coefficients,

$$\alpha = 5/9 \qquad \beta = 3/40 \qquad \beta^* = 9/100 \qquad \sigma = 1/2 \qquad \sigma^* = 1/2$$

- And the following closure relationships,

$$\nu_t = \frac{k}{\omega} \qquad \epsilon = \beta^* \omega k \qquad l = \frac{k^{1/2}}{\omega}$$

Sample turbulence models

$k-\omega$ Turbulence model governing equations

- The closure equations correspond to the Wilcox (1988) $k-\omega$ model.
- They have been manipulated so there are no terms including fluctuating quantities (*i.e.*, velocity and pressure), and doble or triple correlations of the fluctuating quantities.
- The Reynolds stress tensor is modeled using the Boussinesq approximation.

Dissipation

Production

Diffusion

$$\nabla_t k + \nabla \cdot (\bar{\mathbf{u}} k) = \underbrace{\tau^R : \nabla \bar{\mathbf{u}}}_{\text{Production}} - \underbrace{\beta^* k \omega}_{\text{Dissipation}} + \underbrace{\nabla \cdot \left[\left(\nu + \sigma^* \nu_t \right) \nabla k \right]}_{\text{Diffusion}}$$

$$\nabla_t \omega + \nabla \cdot (\bar{\mathbf{u}} \omega) = \alpha \frac{\omega}{k} \underbrace{\tau^R : \nabla \bar{\mathbf{u}}}_{\text{Production}} - \underbrace{\beta \omega^2}_{\text{Dissipation}} + \underbrace{\nabla \cdot \left[\left(\nu + \sigma \nu_t \right) \nabla \omega \right]}_{\text{Diffusion}}$$

Production

Diffusion

Dissipation

Final remarks

- The previous EVM models, are probably the most widely used ones.
- The standard $k - \epsilon$ is a wall modeling model (high Reynolds number model), and $k - \omega$ the (Wilcox 1998) is wall resolving model (low Reynolds number).
- By inspecting the closure equations of the $k - \epsilon$ model, we can evidence that the turbulent kinetic energy k and the turbulent dissipation rate ϵ must go to zero at the correct rate in order to avoid turbulent viscosity production close to walls.

$$\nabla_t k + \nabla \cdot (\bar{\mathbf{u}} k) = \tau^R : \nabla \bar{\mathbf{u}} - \epsilon + \nabla \cdot \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \nabla k \right]$$

$$\nabla_t \epsilon + \nabla \cdot (\bar{\mathbf{u}} \epsilon) = C_{\epsilon_1} \frac{\epsilon}{k} \tau^R : \nabla \bar{\mathbf{u}} - C_{\epsilon_2} \frac{\epsilon^2}{k} + \nabla \cdot \left[\left(\nu + \frac{\nu_t}{\sigma_\epsilon} \right) \nabla \epsilon \right]$$

$$\nu_t = \frac{C_\mu k^2}{\epsilon}$$

Sample turbulence models

Final remarks

- Instead, the $k-\omega$ model does not suffer of this problem as the turbulence specific dissipation rate ω is proportional to $\omega \propto y^{-2}$.
- Therefore, the specific dissipation rate ω close to the walls is usually a large value.

$$\nabla_t k + \nabla \cdot (\bar{\mathbf{u}} k) = \tau^R : \nabla \bar{\mathbf{u}} - \beta^* k \omega + \nabla \cdot \left[(\nu + \sigma^* \nu_t) \nabla k \right]$$

$$\nabla_t \omega + \nabla \cdot (\bar{\mathbf{u}} \omega) = \alpha \frac{\omega}{k} \tau^R : \nabla \bar{\mathbf{u}} - \beta \omega^2 + \nabla \cdot \left[(\nu + \sigma \nu_t) \nabla \omega \right]$$

$$\nu_t = \frac{k}{\omega}$$

$$\omega = \frac{6\nu}{\beta_0 d^2} \qquad \beta_0 \approx 0.075$$

d is the distance to the first cell center normal to the wall (y)

- We will talk more about closure models in Lecture 6.

Final remarks

- Remember, you need to define initial conditions and boundary conditions for all transported quantities when using turbulence models.
    - Including the turbulent variables.
- No need to say that the values need to be physically realistic.
- For example, if you set the turbulent dissipation rate ϵ or the specific dissipation rate ω to zero, you will have convergence problems.
    - Recall that,

$$\nu_t = \frac{k}{\omega} \qquad \qquad \nu_t = \frac{C_\mu k^2}{\epsilon}$$

- By the way, some models can be very sensitive to initial conditions.
- We addressed turbulence estimates in Lecture 4.
- We will revisit this subject in the next lectures.

Sample turbulence models

Short description of some RANS turbulence models

Model	Short description
Spalart-Allmaras	This is a one equation model. Suitable for external aerodynamics, tubomachinery and high-speed flows. Good for mildly complex external/internal flows and boundary layer flows under pressure gradient (e.g., airfoils, wings, airplane fuselages, ship hulls). Performs poorly with flows with strong separation.
Standard k–epsilon	This is a two-equation model. Very robust and widely used despite the known limitations of the model. Performs poorly for complex flows involving severe pressure gradient, separation, strong streamline curvature. Suitable for initial iterations, initial screening of alternative designs, and parametric studies. Can only be used with wall functions.
Realizable k–epsilon	This is a two-equation model. Suitable for complex shear flows involving rapid strain, moderate swirl, vortices, and locally transitional flows (e.g., boundary layer separation, massive separation, and vortex shedding behind bluff bodies, stall in wide-angle diffusers, room ventilation). It overcomes the limitations of the standard k-epsilon model.
Standard k–omega	This is a two-equation model. Superior performance for wall-bounded boundary layer, free shear, and low Reynolds number flows compared to models from the k-epsilon family. Suitable for complex boundary layer flows under adverse pressure gradient and separation (external aerodynamics and turbomachinery).
SST k–omega	This is a two-equation model. Offers similar benefits as the standard k–omega. Not overly sensitive to inlet boundary conditions like the standard k–omega. Provides more accurate prediction of flow separation than other RANS models. Can be used with and without wall functions. Probably the most widely used RANS model.