Turbulence and CFD models: Theory and applications

Roadmap to Lecture 6

Part 3

1. The Reynolds stress model
2. Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress
3. Transition models – Review of the $\gamma - Re_\theta$ model

Roadmap to Lecture 6

Part 3

1. The Reynolds stress model
2. Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress
3. Transition models – Review of the $\gamma - Re_\theta$ model

The Reynolds stress model

- The extra term appearing in the RANS/URANS equations is know as the Reynolds stress tensor,

$$\nabla \cdot (\bar{\mathbf{u}}) = 0$$

$$\frac{\partial \bar{\mathbf{u}}}{\partial t} + \nabla \cdot (\bar{\mathbf{u}}\bar{\mathbf{u}}) = -\frac{1}{\rho}(\nabla p) + \nu \nabla^2 \bar{\mathbf{u}} + \frac{1}{\rho} \nabla \cdot \boldsymbol{\tau}^R$$

- Where τ^R is the Reynolds stress tensor, and it can be written as,

$$\tau^R = -\rho \left(\overline{\mathbf{u}'\mathbf{u}'} \right) = - \begin{pmatrix} \rho\overline{u'u'} & \rho\overline{u'v'} & \rho\overline{u'w'} \\ \rho\overline{v'u'} & \rho\overline{v'v'} & \rho\overline{v'w'} \\ \rho\overline{w'u'} & \rho\overline{w'v'} & \rho\overline{w'w'} \end{pmatrix}$$

- So far, we have modeled this term using the Boussinesq approximation.

The Reynolds stress model

- The Reynolds stress tensor τ^R, is the responsible for the increased mixing and larger wall shear stresses.
- Remember, increased mixing and larger wall shear stresses are properties of turbulent flows.
- The RANS/URANS approach to turbulence modeling requires the Reynolds stresses to be appropriately modeled, for example, by using eddy viscosity models (EVM).
- However, it is possible to derive its own governing equations (six new equations as the tensor is symmetric).
- This approach is known as Reynolds stress models (RSM).
- Probably, the RSM is the most physically sounded RANS/URANS approach as it avoids the use of hypothesis/assumptions to model the Reynolds stress tensor.
- However, it is computationally expensive, and less robust than EVM.
    - It can be unstable if proper boundary conditions and initial conditions are not used.
    - And, as you may guess, it is heavily modeled.

The Reynolds stress model

- The RSM models are more general than EVM models.
- They potentially have better accuracy than the EVM model.
- However, this does not mean that they are better than EVM models.
- RSM models perform better in situations where the EVM models have poor performance,
    - Flows with strong curvature or swirl (cyclone separators and flows with concentrated vortices).
    - Flows in corners with secondary motions.
    - Very complex 3D interacting flows.
    - Highly anisotropic flows.
- In general, RSM models can be considered in non-equilibrium conditions (production not equal to dissipation),

$$P \neq D$$

The Reynolds stress model

- Let us recall the exact Reynolds stress transport equations,

$$\frac{\partial \tau_{ij}}{\partial t} + \bar{u}_k \frac{\partial \tau_{ij}}{\partial x_k} = -\tau_{ik} \frac{\partial \bar{u}_j}{\partial x_k} - \tau_{jk} \frac{\partial \bar{u}_i}{\partial x_k} + \epsilon_{ij} - \Pi_{ij} + \frac{\partial}{\partial x_k} \left[\nu \frac{\partial \tau_{ij}}{\partial x_k} + C_{ijk} \right]$$

- Where the following terms require modeling,

$$\epsilon_{ij} = 2\nu \overline{\frac{\partial u_i'}{\partial x_k} \frac{\partial u_j'}{\partial x_k}}$$

Dissipation tensor

$$\Pi_{ij} = \overline{\frac{p'}{\rho} \left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i} \right)}$$

Pressure-strain correlation tensor

$$\rho C_{ijk} = \rho \overline{u_i' u_j' u_k'} + \overline{p' u_i'} \delta_{jk} + \overline{p' u_j'} \delta_{ik}$$

Turbulent transport tensor

- The most critical term is the pressure-strain term.
- RSM models differ by how this term is modeled.

The Reynolds stress model

- The dissipation tensor of the Reynolds stress equations is also a tensor and can be modeled as follows,

$$\epsilon_{ij} = \frac{2}{3}\epsilon\delta_{ij}$$

- Where ϵ denotes the scalar dissipation rate of turbulence kinetic energy,

$$\epsilon = \nu \overline{\frac{\partial u'_i}{\partial x_k}\frac{\partial u'_i}{\partial x_k}}$$

- The use of this assumption avoids the need for employing a dissipation transport equation for each component of the Reynolds stress tensor.
- Which results in a reduction in the number of transport equation to be solved and thus the computational cost.
- It is clear that ϵ needs to be modeled.
- For this we use a similar approach to the one used in the two-equations models presented in the previous lectures.
- Most of the time, the turbulent dissipation rate transport equation is solved.

The Reynolds stress model

- The turbulent transport tensor of the Reynolds stress equations is also a tensor and can be modeled as follows,

$$C_{ijk} = \frac{\partial}{\partial x_k}\left[\frac{\mu_t}{\sigma}\frac{\partial}{\partial x_k}\left(\overline{u_i' u_j'}\right)\right]$$

- Using this approach [1], the turbulent transport tensor is modeled using a gradient-diffusion model (this is the easiest and most robust approach).
- And alternative approach is the one proposed by Daly and Harlow [2],

$$C_{ijk} = C_s \frac{2}{3}\frac{k^2}{\epsilon}\left[\frac{\partial \tau_{jk}}{\partial x_i} + \frac{\partial \tau_{ik}}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_k}\right]$$

- Have in mind that there are more complex forms to model the turbulent transport tensor, but they are not very robust for industrial applications.

[1] F. S. Lien, M. A. Leschziner. Assessment of Turbulent Transport Models Including Non-Linear RNG Eddy-Viscosity Formulation and Second-Moment Closure. 1994.
[2] B. J. Daly, F. H. Harlow. Transport Equations in Turbulence. 1970.

The Reynolds stress model

- The modeling of the pressure-strain term is critical. It contains complex correlations that are difficult to measure.
- Major difference between RSM models is due to the approach taken to model this term.
- The pressure-strain tensor can be decomposed as follows,

$$\Pi_{ij} = \underbrace{\Pi_{ij,1}}_{\text{Slow pressure strain term}} + \underbrace{\Pi_{ij,2}}_{\text{Fast pressure strain term}}$$

- To most widely used approach to model this term is the LRR [1] and is given as follows,

$$\Pi_{ij,1} = -C_1 \frac{\epsilon}{k}\left(\tau_{ij} - \frac{2}{3}\delta_{ij} k\right) \qquad \Pi_{ij,2} = -C_2\left(P_{ij} - \frac{2}{3}\delta_{ij} P\right)$$

$$P_{ij} = -\tau_{ki}\frac{\partial \overline{u}_j}{\partial x_k} - \tau_{kj}\frac{\partial \overline{u}_i}{\partial x_k} \qquad P = \frac{1}{2}P_{kk}$$

Capital P stand for production not pressure

[1] B. E. Launder, G. J. Reece, W. Rodi. Progress in the Development of a Reynolds-Stress Turbulence Closure. 1975.

The Reynolds stress model

- The solvable equations of the LRR model are given as follows,

$$\frac{\partial \tau_{ij}}{\partial t} + \bar{u}_k \frac{\partial \tau_{ij}}{\partial x_k} = -\tau_{ik} \frac{\partial \bar{u}_j}{\partial x_k} - \tau_{jk} \frac{\partial \bar{u}_i}{\partial x_k} + \epsilon_{ij} - \Pi_{ij} + \frac{\partial}{\partial x_k} \left[\nu \frac{\partial \tau_{ij}}{\partial x_k} + C_{ijk} \right]$$

- With the following auxiliary relationships,

$$C_{ijk} = \frac{\partial}{\partial x_k} \left[\frac{\mu_t}{\sigma} \frac{\partial}{\partial x_k} \left(\overline{u_i' u_j'} \right) \right]$$

$$\Pi_{ij} = -C_1 \frac{\epsilon}{k} \left(\tau_{ij} - \frac{2}{3} \delta_{ij} k \right) - C_2 \left(P_{ij} - \frac{2}{3} \delta_{ij} P_k \right)$$

If you compare this term with the original formulation of the LRR method, you will notice that this term has been further simplified

$$P_{ij} = -\tau_{ki} \frac{\partial \overline{u}_j}{\partial x_k} - \tau_{kj} \frac{\partial \overline{u}_i}{\partial x_k} \qquad P = \frac{1}{2} P_{kk} \qquad \epsilon_{ij} = \frac{2}{3} \epsilon \delta_{ij}$$

- And closure coefficients,

$$\sigma = 0.82 \qquad C_1 = 1.8 \qquad C_2 = 0.6$$

- With the following relation for the kinematic eddy viscosity (if it is based on the dissipation rate equation),

$$\mu_t = \rho C_\mu \frac{k^2}{\epsilon}$$

The Reynolds stress model

- The Reynolds stress model (RSM) [1, 2, 3, 4] is the most elaborate type of RANS turbulence model. It abandons the isotropic eddy-viscosity hypothesis.
- The RSM closes the RANS equations by solving transport equations for the Reynolds stresses, together with an equation for the turbulent dissipation rate or the specific dissipation rate.
- This means that five additional transport equations are required in 2D flows, and seven additional transport equations are solved in 3D.
- Then, the Reynolds stresses are inserted directly into the momentum equations.
- If additional scalars are present (temperature, passive scalars, and so on), three additional equations need to be added.
- If the turbulent kinetic energy equation is needed for specific terms, it is obtained by taking the trace of the Reynolds tress tensor.
- The most used versions of the RSM are the LRR [3] and the SSG [5].
- The RSM might not always yield results that are clearly superior to EVM models. However, use of the RSM is a must when the flow features of interest are the result of anisotropy in the Reynolds stresses.
- Among the examples are cyclone flows, highly swirling flows in combustors, rotating flow passages, and the stress-induced secondary flows in ducts.
- Despite its apparent superiority over EVM models, the RSM is not widely used.
- Also, the RSM is not widely validated as other EVM models.
- There are also algebraic version of the RSM models that solve two equations.
  - Explicit Algebraic Reynolds Stress Model [6, 7].
  - They are usually an extension of the $k - \epsilon$ and $k - \omega$ family models.

[1] M. M. Gibson, B. E. Launder. Ground Effects on Pressure Fluctuations in the Atmospheric Boundary Layer. 1978.
[2] B. E. Launder. Second-Moment Closure: Present... and Future?. 1989.
[3] B. E. Launder, G. J. Reece, W. Rodi. Progress in the Development of a Reynolds-Stress Turbulence Closure. 1975.
[4] B. J. Daly, F. H. Harlow. Transport Equations in Turbulence. 1970.
[5] C. G. Speziale, S. Sarkar, T. B. Gatski. Modelling the Pressure-Strain Correlation of Turbulence: An Invariant Dynamical Systems Approach. 1991.
[6] W. Rodi. A New Algebraic Relation for Calculating Reynolds Stress. 1976.
[7] S. Girimaji. Fully Explicit and Self-Consistent Algebraic Reynolds Stress Model. 1996.

The Reynolds stress model

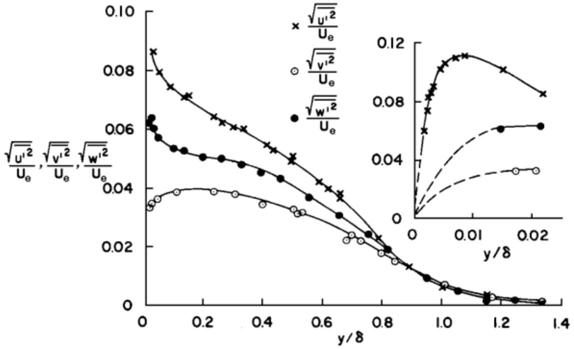
- The RSM model can be used with wall functions.
- The wall boundary conditions for the solution variables are all taken care of by the wall functions implementation.
- Therefore, when using commercial solvers (Fluent in our case) you do not need to be concerned about the boundary conditions at the walls.
- If you are using a wall resolving approach, all Reynolds stresses must approach in an asymptotic way to zero at the wall.
- The freestream values can be computed as follows,

$$\overline{u_1'^2} = k$$

$$\overline{u_2'^2} = \overline{u_3'^2} = \frac{1}{2}k$$

$$\overline{u_i' u_j'} = 0 \qquad (i \neq j)$$

$$\overline{u'^2} : \overline{v'^2} : \overline{w'^2} \approx 4 : 2 : 3$$



- The boundary condition for turbulent dissipation rate or specific dissipation rate are determined in the same manner as for the two-equations turbulence models.

Part 3

1. ~~The Reynolds stress model~~
2. **Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress**
3. Transition models – Review of the $\gamma - Re_\theta$ model

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Now that we have addressed the main EVM, let us talk about the turbulence kinetic energy, dissipation rate, and Reynolds stress budgets.
- We have seen that the transport equations of the turbulent quantities can be expressed in the following way,

$$\underbrace{\nabla_t \phi}_{\text{Transient term}} + \underbrace{\nabla \cdot \bar{\mathbf{u}}\phi}_{\text{Convection}} = \underbrace{P^{\phi}}_{\text{Production}} + \underbrace{\epsilon^{\phi}}_{\text{Dissipation}} + \underbrace{D^{\phi}}_{\text{Diffusion}} + \underbrace{S^{\phi}}_{\text{Source terms}}$$

- Where ϕ represents the transported turbulent quantity.
- At the same time, each term can be decomposed into sub-terms.
- For example, the pressure-strain tensor can be decomposed into the following contributions,

$$\Pi_{ij} = \underbrace{\Pi_{ij,1}}_{\text{Slow pressure strain term}} + \underbrace{\Pi_{ij,2}}_{\text{Fast pressure strain term}}$$

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- The budgets for the turbulence kinetic energy, dissipation rate, and Reynolds stress budgets are computed using DNS data of canonical flows or experimental measurements.
- The budget data represent the contribution of each term to the whole turbulence process (production, dissipation, transport, redistribution, and diffusion).
- This data also reveal that all the terms in the budget become important close to the wall.
- The budget data is used for model development and to test existing closure models.
- The turbulence budgets are usually plotted in normalized units.
- Let us take a look at the data from the following reference:
    - N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

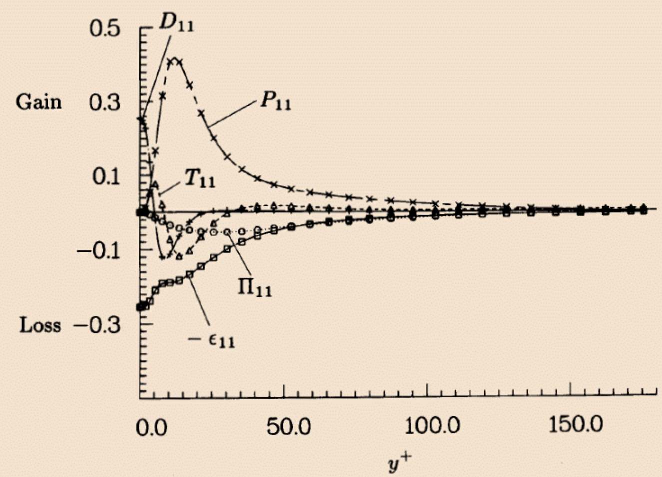


Figure 1. Terms in the budget of $\overline{u_1'u_1'}$ in wall coordinates. P_{11} = Production; T_{11} = Turbulent transport; D_{11} = Viscous diffusion; ϵ_{11} = Dissipation rate; Π_{11} = Velocity pressure-gradient term.

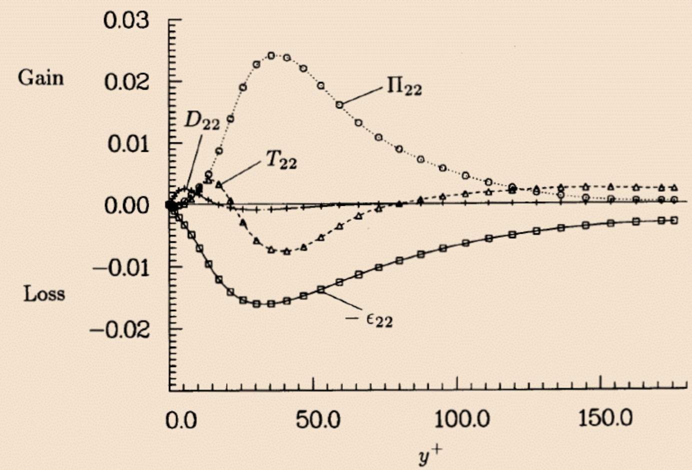


Figure 2. Terms in the budget of $\overline{u_2'u_2'}$ in wall coordinates. T_{22} = Turbulent transport; D_{22} = Viscous diffusion; ϵ_{22} = Dissipation rate; Π_{22} = Velocity pressure-gradient term.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

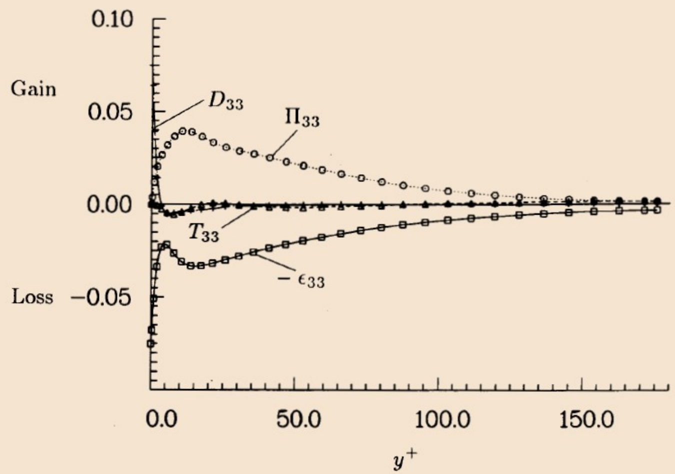


Figure 3. Terms in the budget of $\overline{u_3' u_3'}$ in wall coordinates. T_{33} = Turbulent transport; D_{33} = Viscous diffusion; ϵ_{33} = Dissipation rate; Π_{33} = Velocity pressure-gradient term.

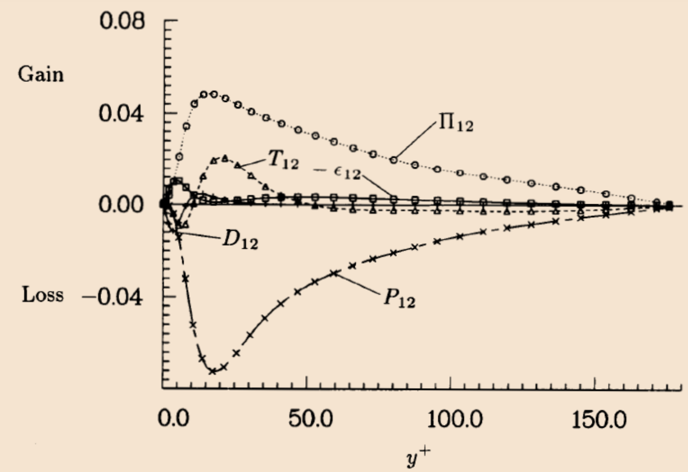


Figure 4. Terms in the budget of $\overline{u_1' u_2'}$ in wall coordinates. P_{12} = Production; T_{12} = Turbulent transport; D_{12} = Viscous diffusion; ϵ_{12} = Dissipation rate; Π_{12} = Velocity pressure-gradient rate.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

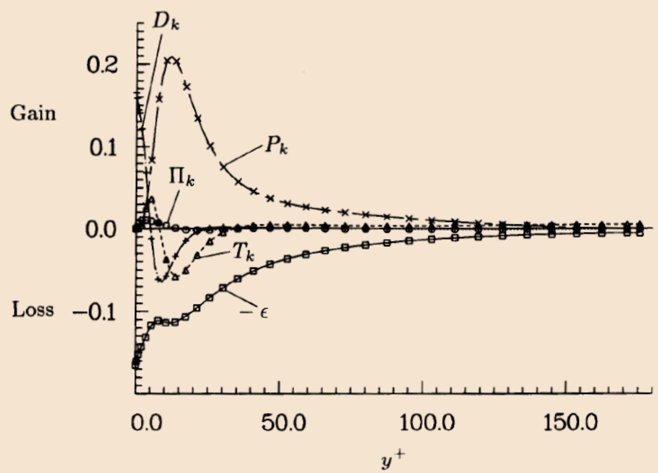


Figure 5. Terms in the budget of the turbulence kinetic energy, k, in wall coordinates. P_k = Production; T_k = Turbulent transport; D_k = Viscous diffusion; ϵ_k = Dissipation rate; Π_k = Velocity pressure-gradient term.

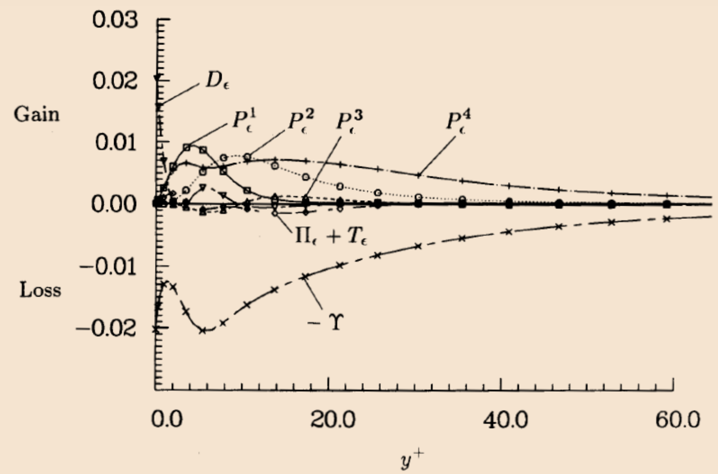


Figure 6. Terms in the budget of the dissipation rate of the turbulence kinetic energy, ϵ, in wall coordinates. P_ϵ^1 = Production by mean velocity gradient; P_ϵ^2 = Mixed production; P_ϵ^3 = Gradient production; P_ϵ^4 = Turbulent production; T_ϵ = Turbulent pransport; D_ϵ = Viscous diffusion; Υ = Dissipation rate; Π_ϵ = Pressure transport.

- From figure 5,
  - TKE peaks in the buffer layer (about y^+ 15).
  - Dissipation peaks in the viscous sublayer.
  - In the core of the flow, production and dissipation are in balance

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

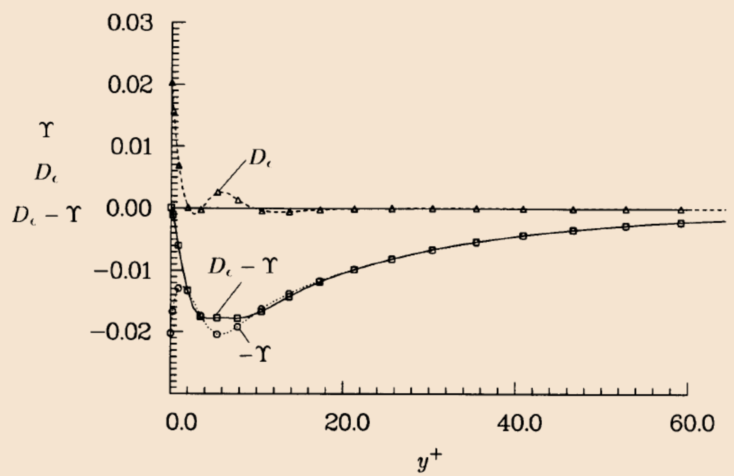


Figure 7. Split of the viscous term into a dissipation rate term, Υ, and a viscous diffusion term, D_ϵ.

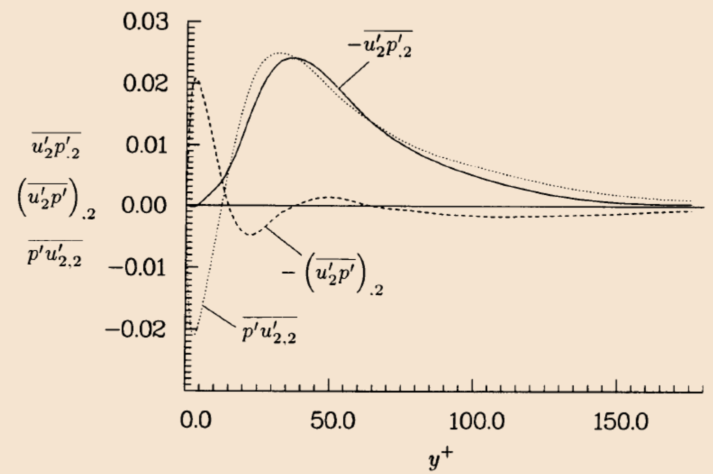


Figure 8. Split of the velocity pressure-gradient term into a pressure transport term, $(\overline{pu'_2})_{,y^+}$, and a pressure-strain term, $\overline{pu'_{2,y^+}}$.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

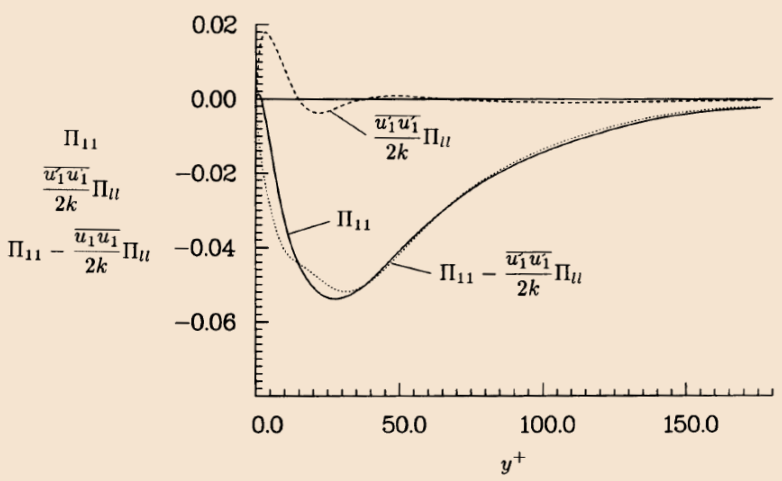


Figure 9. Split of the velocity pressure-gradient term, Π_{11}, into a pressure transport term, $\overline{u_1' u_1'}/2k\ \Pi_{ll}$, and a redistributive term, $\Pi_{11} - \overline{u_1' u_1'}/2k\ \Pi_{ll}$.

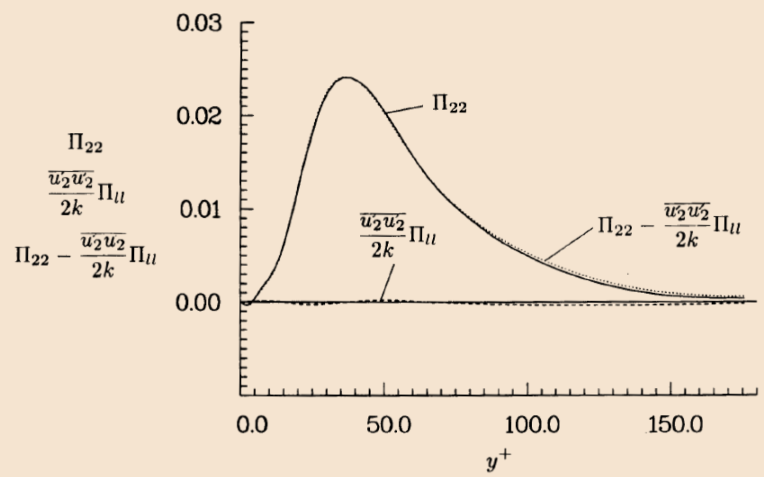


Figure 10. Split of the velocity pressure-gradient term, Π_{22}, into a pressure transport term, $\overline{u_2' u_2'}/2k\ \Pi_{ll}$, and a redistributive term, $\Pi_{22} - \overline{u_2' u_2'}/2k\ \Pi_{ll}$.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

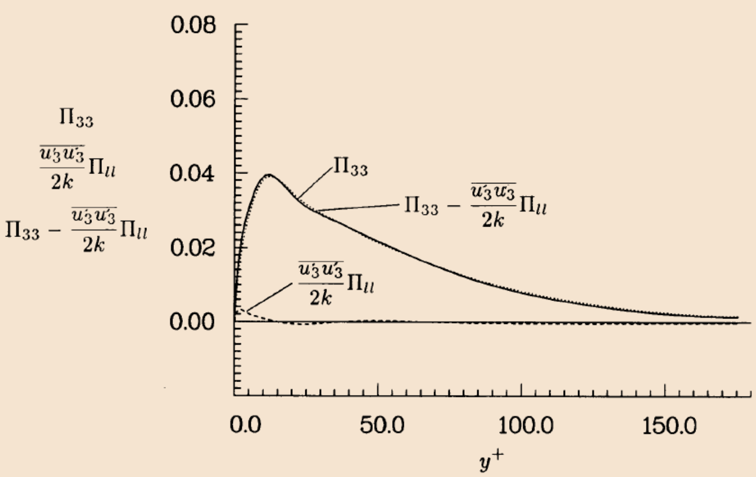


Figure 11. ____ Split of the velocity pressure-gradient term, Π_{33}, into a pressure transport term, $\overline{u'_3 u'_3}/2k\ \Pi_{ll}$, and a redistributive term, $\Pi_{33} - \overline{u'_3 u'_3}/2k\ \Pi_{ll}$.

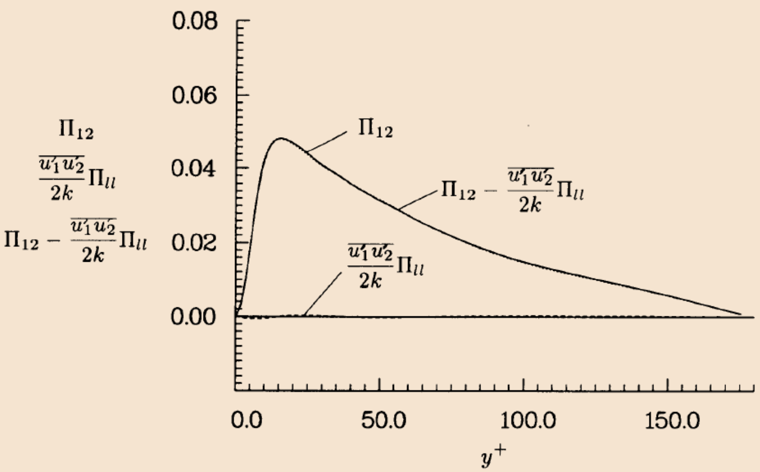


Figure 12. ____ Split of the velocity pressure-gradient term, Π_{12}, into a pressure transport term, $\overline{u'_1 u'_2}/2k\ \Pi_{ll}$, and a redistributive term, $\Pi_{12} - \overline{u'_1 u'_2}/2k\ \Pi_{ll}$.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

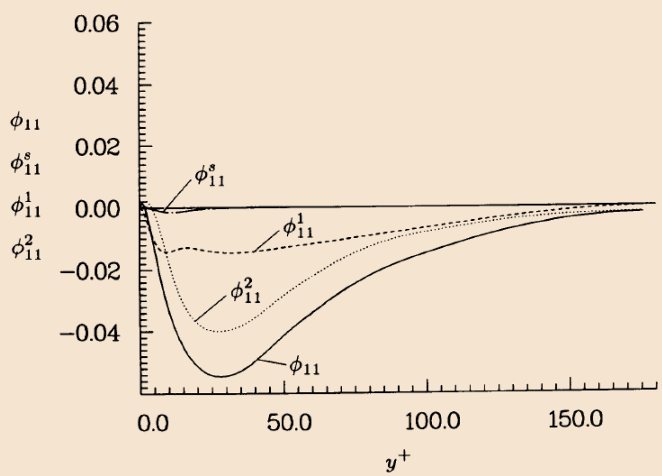


Figure 13. Split of pressure-strain term, ϕ_{11}, into a rapid term, ϕ_{11}^1, a return term, ϕ_{11}^2, and a Stokes term, ϕ_{11}^S.

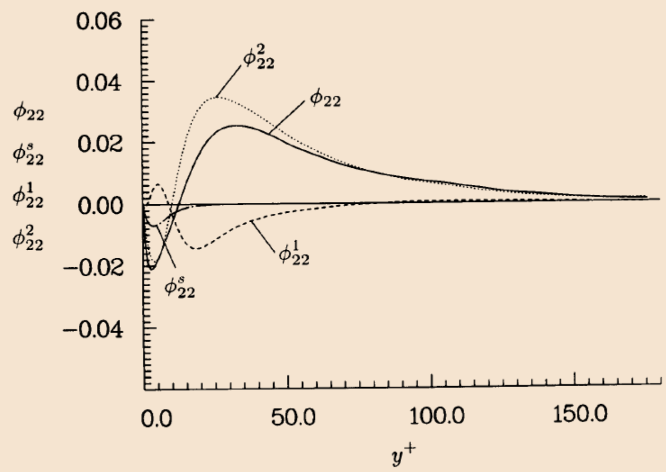


Figure 14. Split of pressure-strain term, ϕ_{22}, into a rapid term, ϕ_{22}^1, a return term, ϕ_{22}^2, and a Stokes term, ϕ_{22}^S.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

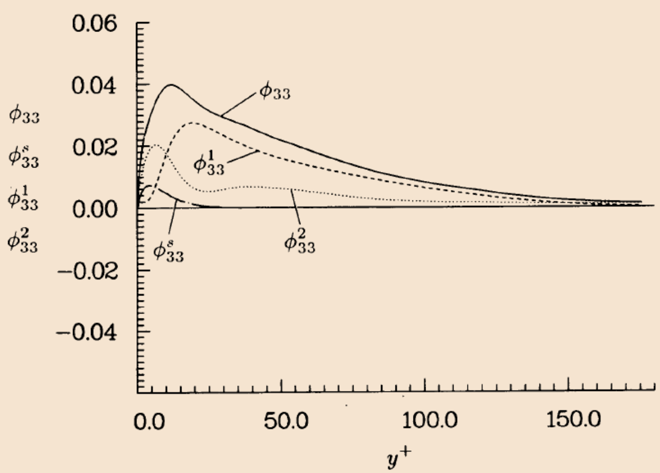


Figure 15. Split of pressure-strain term, ϕ_{33}, into a rapid term, ϕ^1_{33}, a return term, ϕ^2_{33}, and a Stokes term, ϕ^S_{33}.

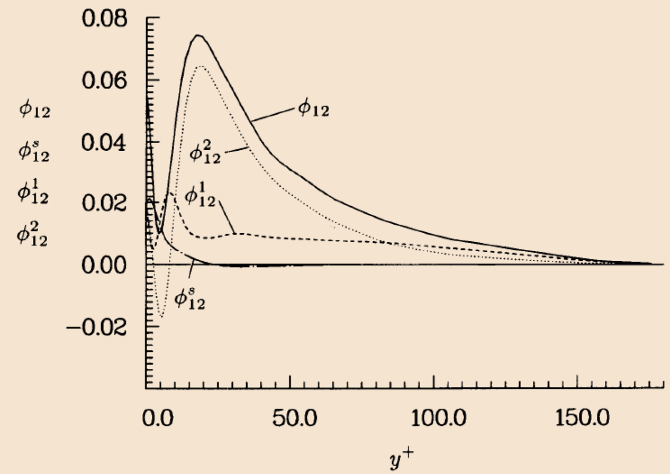


Figure 16. Split of pressure-strain term, ϕ_{12}, into a rapid term, ϕ^1_{12}, a return term, ϕ^2_{12}, and a Stokes term, ϕ^S_{12}.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

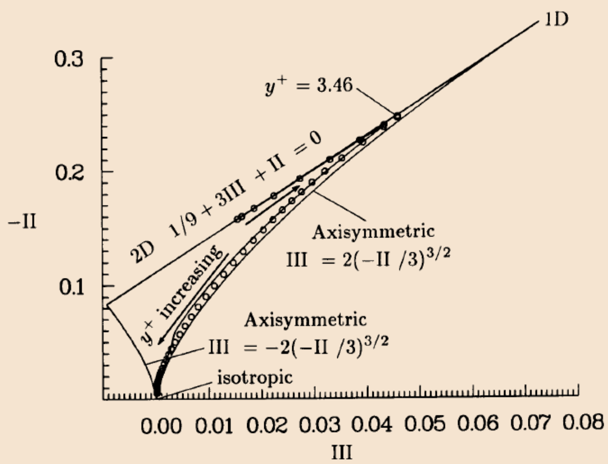


Figure 17. Anisotropy invariant map. ○ ○ ○ ○ d_{ij} at various y^+ in the channel; ——— b_{ij} at various y^+ in the channel.

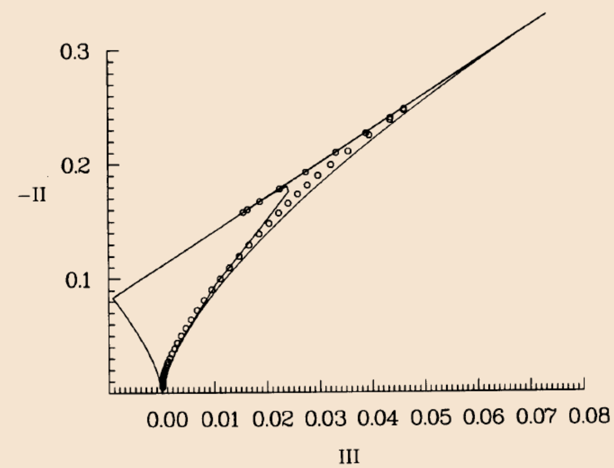


Figure 18. Anisotropy invariant map. ○ ○ ○ ○ d_{ij} at various y^+ in the channel; ——— model, equation (40).

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

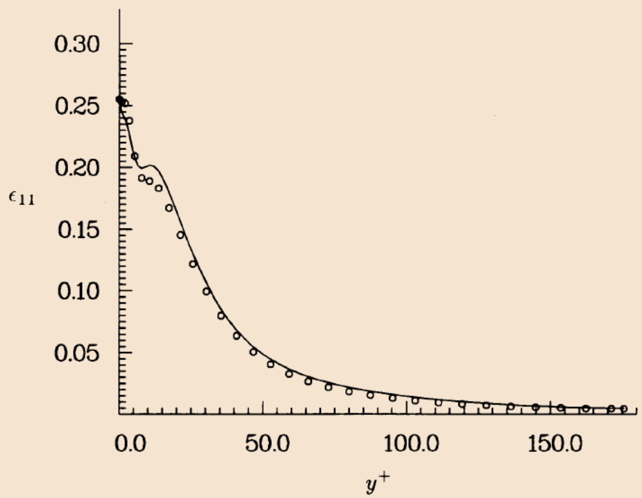


Figure 19. Distribution of ϵ_{11} across the channel. ○ ○ ○ ○ ϵ_{11} term computed from the channel data; —— model, $\epsilon/k\ \overline{u_1'u_1'}$.

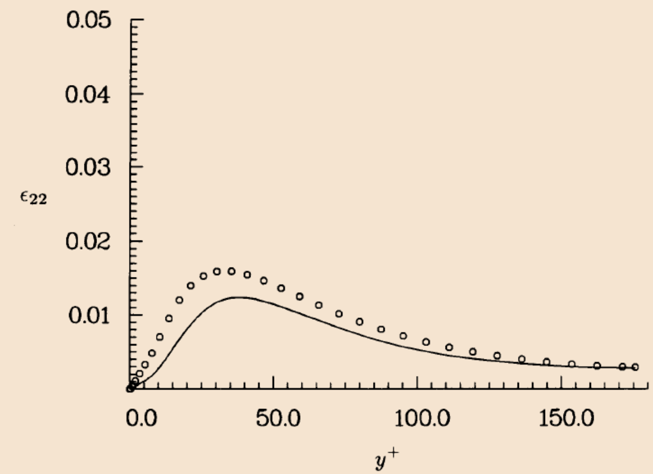


Figure 20. Distribution of ϵ_{22} across the channel. ○ ○ ○ ○ ϵ_{22} term computed from the channel data; —— model, $\epsilon/k\ \overline{u_2'u_2'}$.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

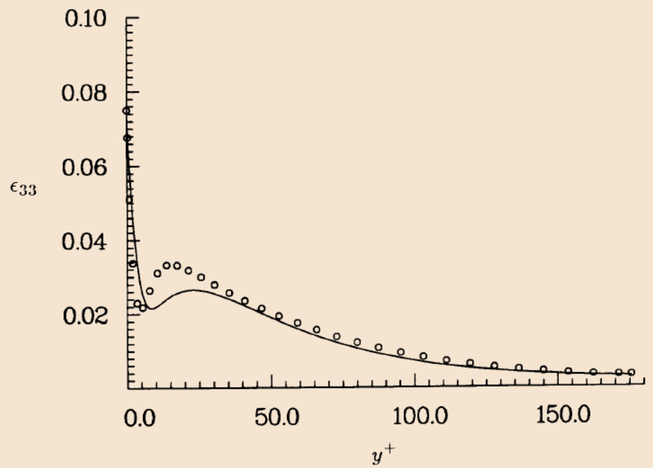


Figure 21. Distribution of ϵ_{33} across the channel. ○ ○ ○ ○ ϵ_{33} term computed from the channel data; —— model, $\epsilon/k\ \overline{u_3'u_3'}$.

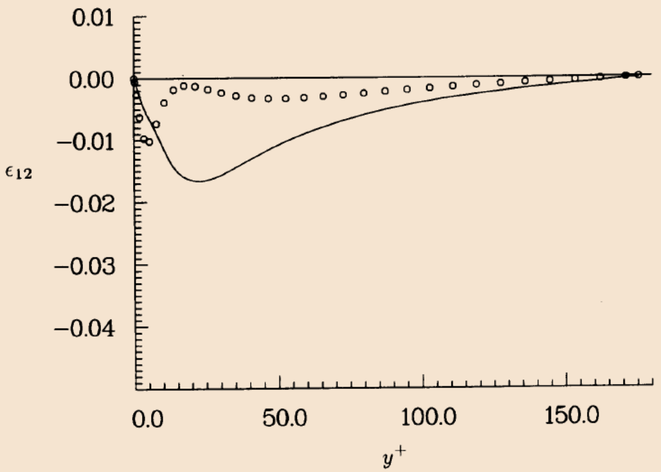


Figure 22. Distribution of ϵ_{12} across the channel. ○ ○ ○ ○ ϵ_{12} term computed from the channel data; —— model, $\epsilon/k\ \overline{u_1'u_2'}$.

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

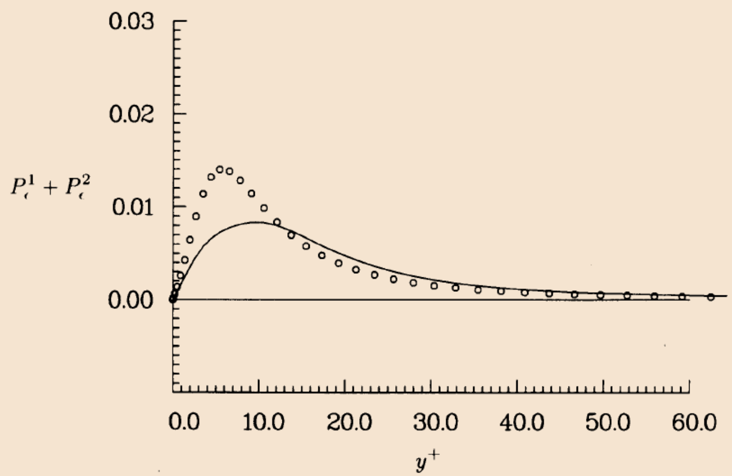


Figure 23. Distribution of the production term, $P_\epsilon^1 + P_\epsilon^2$, in the budget of ϵ across the channel; ○ ○ ○ ○ term computed from the channel data; —— model, equation (42).

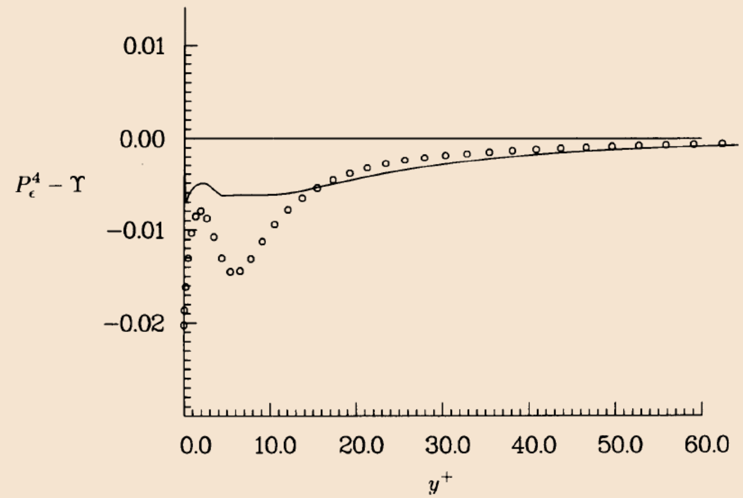


Figure 24. Distribution of the dissipation term, $P_\epsilon^4 - \Upsilon$, in the budget of ϵ across the channel. ○ ○ ○ ○ term computed from the channel data; —— model, equation (44).

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

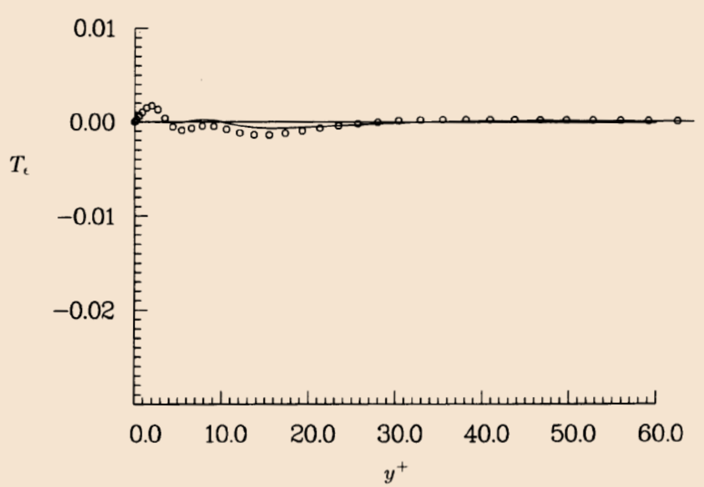


Figure 25. Distribution of the turbulent transport term, T_ϵ, in the budget of ϵ across the channel. ○ ○ ○ ○ term computed from the channel data; —— model, equation (45).

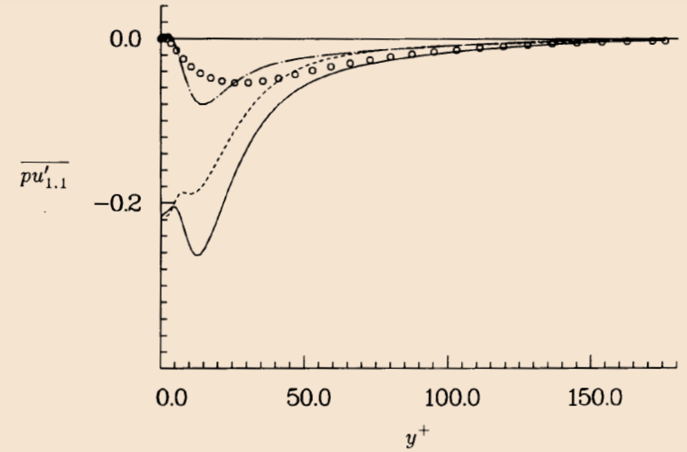


Figure 26. Pressure-strain term, $\overline{pu'_{1,1}}$, in the budget equation for $\overline{u'_1u'_1}$ across the channel. ○ ○ ○ ○ term computed from the channel data; —— model, (eq. (46)+eq. (48)+eq. (49)); ---- model, equation (46); —·— model, (eq. (48)+eq. (49)).

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

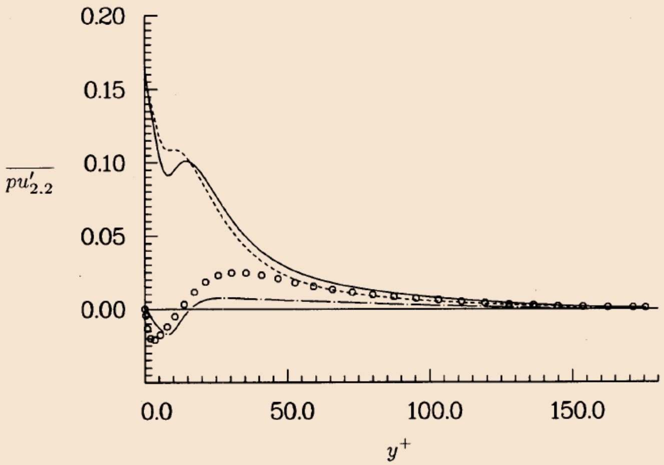


Figure 27. Pressure strain term, $\overline{pu'_{2,2}}$, in the budget equation for $\overline{u'_2u'_2}$ across the channel. ○ ○ ○ ○ term computed from the channel data; —— model, (eq. (46)+eq. (48)+eq. (49)); - - - - model, equation (46). —·— model, (eq. (48)+eq. (49)).

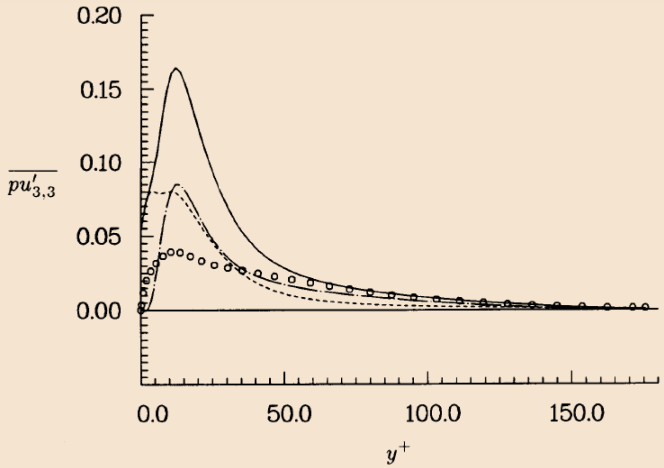


Figure 28. Pressure strain term, $\overline{pu'_{3,3}}$, in the budget equation for $\overline{u'_3u'_3}$ across the channel. ○ ○ ○ ○ term computed from the channel data; —— model, (eq. (46)+eq. (48)+eq. (49)); - - - - model, equation (46). —·— model, (eq. (48)+eq. (49)).

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

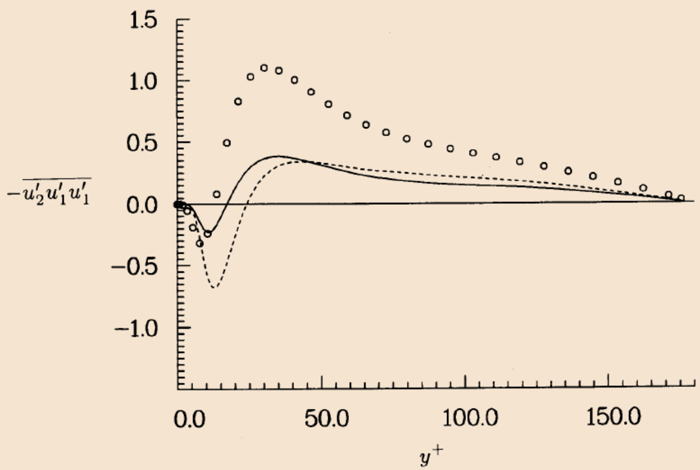


Figure 29. Triple correlation term $-\overline{u_2'u_1'u_1'}$ across the channel. ○ ○ ○ ○ term computed from the channel data; —— model, equation (53); ---- model, equation (54).

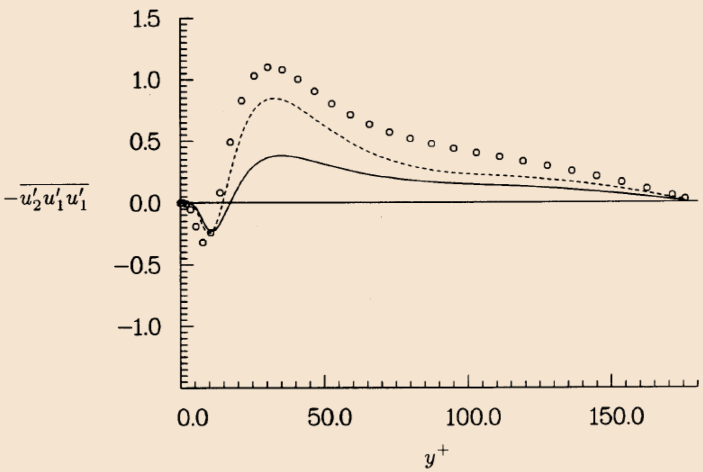


Figure 30. Triple correlation term $-\overline{u_2'u_1'u_1'}$ across the channel. ○ ○ ○ ○ term computed from the channel data; —— model, equation (54); ---- model, equation (55).

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- Turbulence kinetic energy, dissipation rate, and Reynolds stress budgets [1].

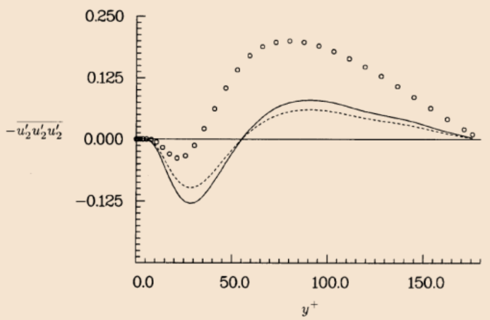


Figure 31. Triple correlation term $-\overline{u_2'u_2'u_2'}$ across the channel. ○ ○ ○ ○ term computed from the channel data; —— model, equation (54); - - - - model, equation (55).

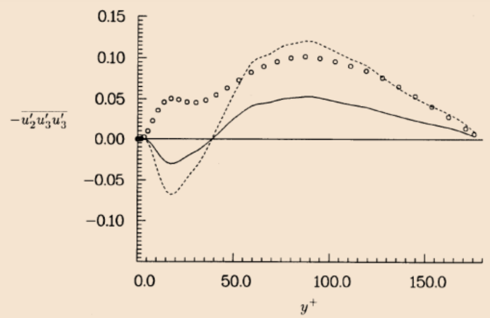


Figure 32. Triple correlation term $-\overline{u_2'u_3'u_3'}$ across the channel. ○ ○ ○ ○ term computed from the channel data; —— model, equation (54); - - - - model, equation (55).

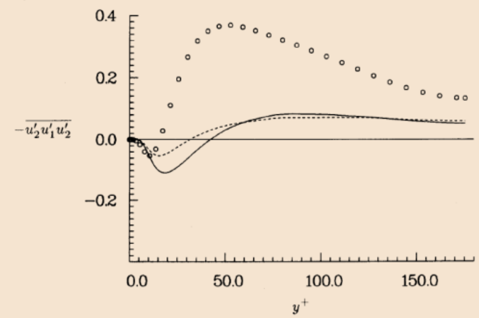


Figure 33. Triple correlation term $-\overline{u_2'u_1'u_2'}$ across the channel. ○ ○ ○ ○ term computed from the channel data; —— model, equation (54); - - - - model, equation (55).

[1] N. Mansour, J. Kim, P. Moin. Reynolds-Stress and Dissipation Rate Budgets in a Turbulent Channel Flow. NASA TM 89451. 1987

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

- These budgets can be obtained from experimental measurements or DNS simulations.
- The turbulence kinetic energy, dissipation rate, and Reynolds stress budgets provide valuable guidelines for model developers, model testing, and model validation.
- This is used for hardcore model development and validation.
- As getting this data can be very time consuming and computationally expensive, there are many well curated databases where this data is already available.

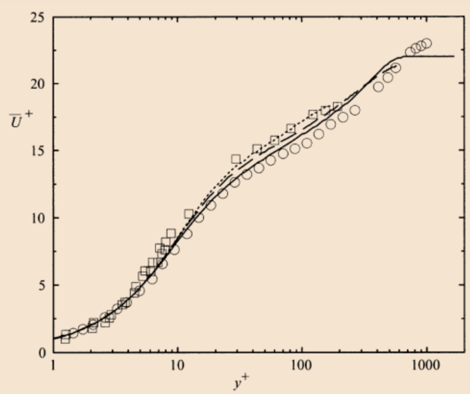


Fig. 4.6 Mean velocity $\overline{U}^+$. □, channel flow measurements [66], $R_\tau = 187$; ○, boundary layer measurements [33], $R_\tau = 1050$; —, DNS of boundary layer [60], $R_\tau = 650$; ···, DNS of channel flow [37], $R_\tau = 180$; – –[49], $R_\tau = 590$.

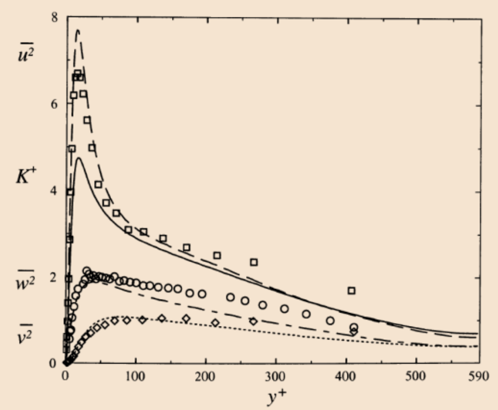


Fig. 4.7 Kinetic energy and normal stresses. Channel flow DNS [49], boundary layer measurements [33]. – –and □, $\overline{u^2}^+$; ··· and ◇, $\overline{v^2}^+$; — · —and ○, $\overline{w^2}^+$; —, K^+.

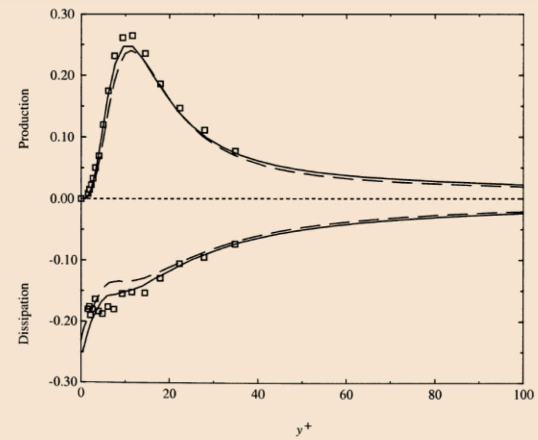


Fig. 4.11 Comparison of turbulent boundary layer and channel flow production and dissipation rates scaled with ν and u_τ. □, boundary layer measurements at $R_\tau = 1050$ [33]; —, DNS boundary layer at $R_\tau = 650$ [60]; – –, DNS channel flow $R_\tau = 590$ [49].

Comparison of budgets using experimental and numerical data. Images reproduced from reference [1].

[1] P. Bernard, J. Wallace. Turbulent Flow. Analysis, Measurement and Prediction. 2002.

Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress

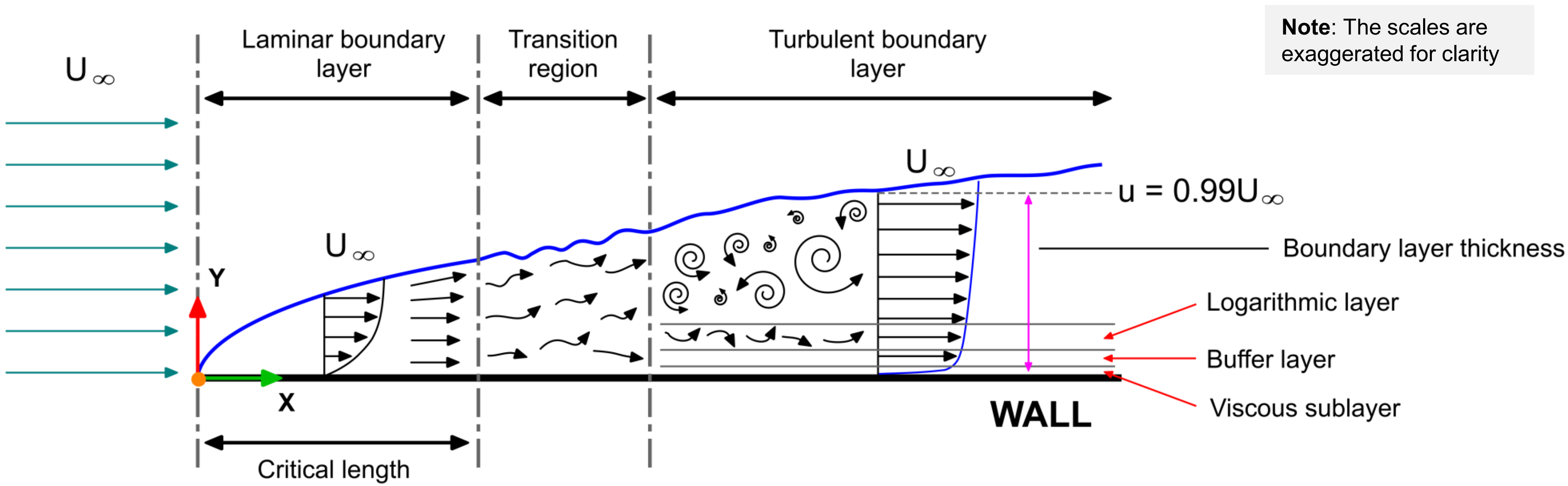
- Incomplete list of turbulence databases:
    - http://turbulence.pha.jhu.edu/
    - http://cfd.mace.manchester.ac.uk/
    - http://turbulence.oden.utexas.edu/
    - https://torroja.dmt.upm.es/turbdata/
    - https://www.rs.tus.ac.jp/t2lab/db/index.html
    - https://warwick.ac.uk/fac/sci/eng/staff/ymc/research/dns_database/
    - http://thtlab.jp/DNS/dns_database.html
    - http://www.tfd.chalmers.se/~lada/projects/databases/proright.html
    - https://ctr.stanford.edu/research-data
    - https://turbase.cineca.it/init/routes/#/logging/welcome
    - https://www.mech.kth.se/~pschlatt/DATA/

Roadmap to Lecture 6

Part 3

1. ~~The Reynolds stress model~~
2. ~~Budgets of turbulence kinetic energy, dissipation rate, and Reynolds stress~~
3. **Transition models – Review of the $\gamma - Re_\theta$ model**

Boundary layer – Laminar, transitional, and turbulent flow



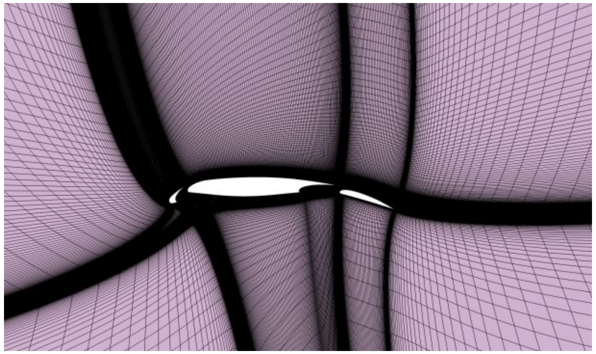
- In this case, a laminar boundary layer starts to form at the leading edge.
- As the flow proceeds further downstream, large shear stresses and velocity gradients develop within the boundary layer.
- At one point, the flow will undergo a transition from laminar to turbulent.
- What is happening in the transition region is not well understood (the flow can become laminar again or can become turbulent).
- Shear stresses, heat transfer rate, mixing rate, and velocities profiles, are very different in each region of the boundary layer (laminar, transitional, or turbulent).

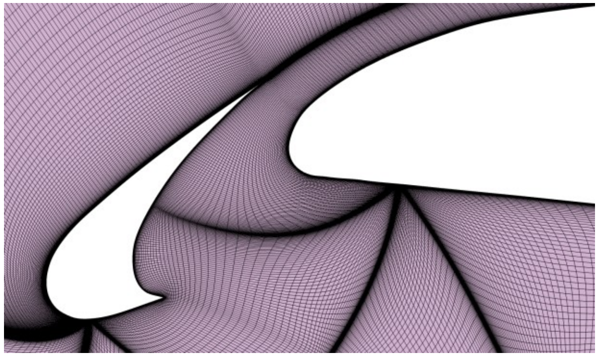
Transition models – Review of the $\gamma - Re_\theta$ model

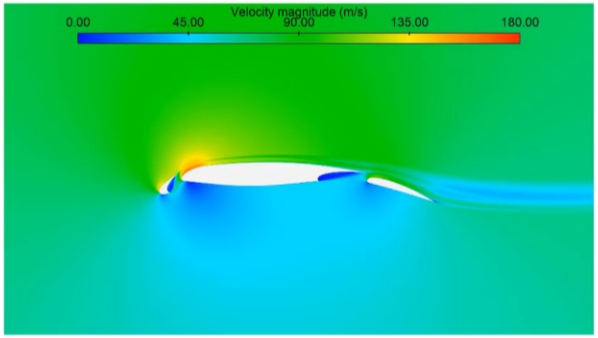
- Maybe, the most challenging topic of turbulence modeling is the prediction of transition to turbulence.
- Trying to predict transition to turbulence in CFD requires very fine meshes and well calibrated models.
- Many traditional turbulence models assume that the boundary layer is turbulent in all its extension.
- But assuming that the boundary layer is entirely turbulent might not be a good assumption, as in some regions the boundary layer might still be laminar, so we may be overpredicting drag forces, overpredicting heat transfer rate, predicting wrong separation points, or predicting wrong mixing rates.
- The main transition to turbulence mechanism are,
    - Natural transition.
    - Bypass transition.
    - Separation induced transition (laminar separation bubbles).
    - Crossflow transition (due to spanwise effects).

Transition models – Review of the $\gamma - Re_\theta$ model

- In some applications, transition to turbulence is preceded by laminar separation bubbles (LSB).
- LSB are laminar recirculation areas that separate from the wall and reattach in a very short distance and are very sensitive to disturbances.
- After the LSB, the flow becomes turbulent.
- LSB are very difficult to predict. Specialized turbulence models are needed.





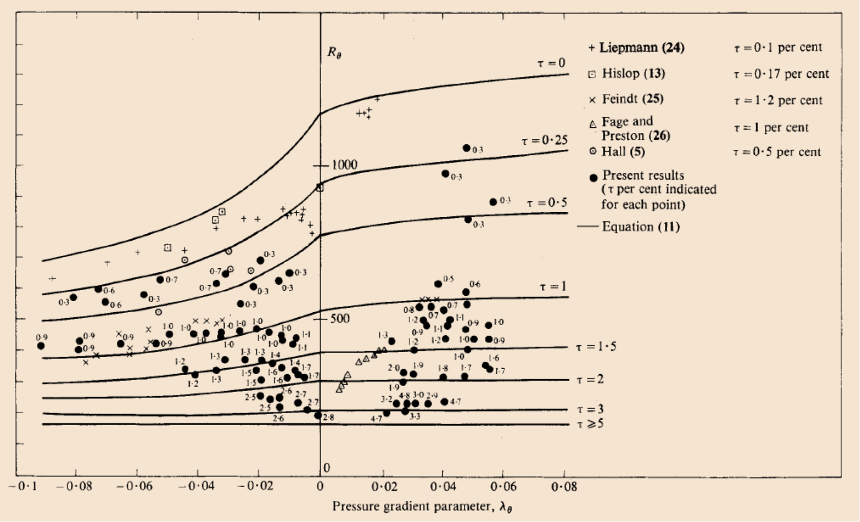


http://www.wolfdynamics.com/images/airfoil.mp4

Transition models – Review of the $\gamma - Re_\theta$ model

- Transition to turbulence is elusive and difficult to solve.
- It requires very fine meshes, and additional computational resources as it solves additional transport equations.
- Transition models are based on correlations to model the mechanism of transition.
- These correlations are then connected to transport equations.
- Transition onset can be affected by:
  - Free-stream turbulence intensity.
  - Pressure gradient λ_θ.
  - Separation.
  - Mach number.
  - Surface conditions.
- Correlations contain all physics, no need to model individual effects.

$$Re_{\theta t} = f(Tu, \lambda)$$



Abu-Ghannam and Shaw transition criterion [1].
Critical Reynolds Re_θ is a function of turbulence intensity τ and pressure gradient λ_θ.

$Re_{\theta t} = f(Tu, \lambda)$	Correlation for the transition Reynolds number
$Re_{\theta c} = f(\tilde{Re}_{\theta t})$	Correlation for the critical Reynolds number
$F_{length} = f(\tilde{Re}_{\theta t})$	Correlation for the length of the transition region

[1] B. J. Abu-Ghannam, R. Shaw. Natural Transition of Boundary Layers—The Effects of Turbulence, Pressure Gradient, and Flow History. 1980.

Transition models – Review of the $\gamma - Re_\theta$ model

- These correlations-based methods basically work in the following way:
  - Transition is triggered as soon as a criterion is met, for example,

$$Re_\theta \geq Re_{\theta t}$$

  - Therefore, Re_θ must be computed in the boundary layer.
  - To compute Re_θ , the velocity at the edge of the boundary layer and the integral of the momentum thickness are needed.
- The transition is then triggered using a ramp function.
- The method requires the computation of local quantities that are not readably available in the solution.
- Namely, boundary layer momentum thickness and the velocity at the edge of the boundary layer).

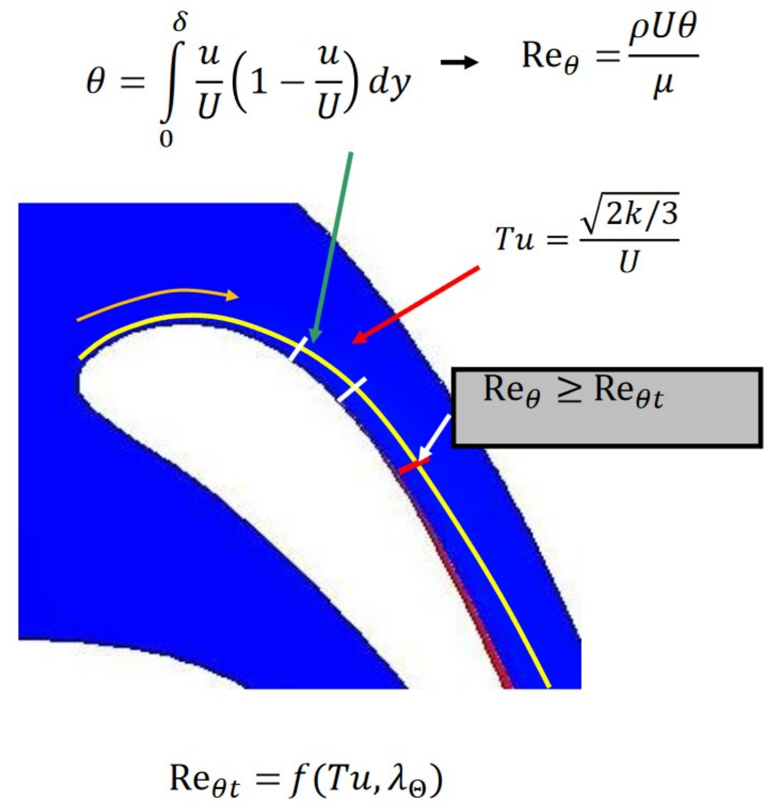


Image taken from:
F. Menter. RANS Transition Modelling Using Transport Equations I. ERCOFTAC Course on Transition Modelling III. May 2015.

Transition models – Review of the $\gamma - Re_\theta$ model

- As we stated previously, transition to turbulence is elusive and difficult to solve; it requires very fine meshes and additional transport equations.
- There a few transitional models around.
- Let us briefly address the $\gamma - Re_\theta$.
- This model is also known as local correction-based transition model or LCTM [1,2].
- And it is based on the $k - \omega$ SST model [3,4].
- It solves two additional equations to model the transition to turbulence.
- One equation for the intermittency γ and one equation for the momentum thickness Reynolds number $\tilde{Re}_{\theta t}$.
- Then, the model couples the intermittency and the momentum thickness Reynolds number with the $k - \omega$ SST model, plus additional corrections.

[1] R. Langtry, F. Menter. Transition Modeling for General CFD Applications in Aeronautics. 2005.
[2] F. Menter, R. Langtry, S. Likki, Y. Suzen, P. Huang, S. Volker. A Correlation-Based Transition Model Using Local Variables Part I – Model Formulation. 2006.
[3] F. Menter. Two-Equation Eddy-Viscosity Turbulence Models for Engineering Applications. 1994.
[4] F. Menter, M. Kuntz, R. Langtry. Ten Years of Industrial Experience with the SST Turbulence Model. 2003.

Transition models – Review of the $\gamma - Re_\theta$ model

- The intermittency transport equation is given as follows,

$$\frac{\partial \rho \gamma}{\partial t} + \frac{\partial(\rho \overline{u}_j \gamma)}{\partial x_j} = P_{\gamma 1} - E_{\gamma 1} + P_{\gamma 2} - E_{\gamma 2} + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\gamma}\right)\frac{\partial \gamma}{\partial x_j}\right]$$

- Where,

$$P_{\gamma 1} = C_{a1} F_{length} \rho S \left(\gamma F_{onset}\right)^{c_{\gamma 3}} \qquad E_{\gamma 1} = C_{e1} P_{\gamma 1} \gamma$$

$$P_{\gamma 2} = C_{a2} \rho \Omega \gamma F_{turb} \qquad E_{\gamma 2} = C_{e2} P_{\gamma 2} \gamma$$

- In the previous relations, F_{length} is an empirical correlation that controls the length of the transition region, F_{onset} controls the onset of the transition when $Re_\theta \geq Re_{\theta t}$, and *S* is the strain rate magnitude.
- As you can see, there are many additional coefficients and auxiliary equations to close this equation.
- The interested reader should refer to the original references for a complete description of the model.

Transition models – Review of the $\gamma - Re_\theta$ model

- The intermittency transport equation is given as follows,

$$\frac{\partial \rho\gamma}{\partial t} + \frac{\partial(\rho \overline{u}_j \gamma)}{\partial x_j} = P_{\gamma 1} - E_{\gamma 1} + P_{\gamma 2} - E_{\gamma 2} + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\gamma}\right)\frac{\partial \gamma}{\partial x_j}\right]$$

- This equation is entirely based on dimensional arguments.
- If the intermittency value is equal to 0, the flow is laminar.
- And when the intermittency is equal to 1, the flow is fully turbulent.
- All values in between 0 and 1 correspond to transition.
- The term F_{onset} triggers the transition onset.
- However, F_{onset} requires as input the critical Reynolds number $Re_{\theta c}$ for the correlation.
- Therefore, another equation needs to be solved for the critical Reynolds number.

$$F_{onset} = \max\left[F_{onset2} - F_{onset3}, 0\right]$$

$$F_{onset1} = \frac{Re_V}{2193 Re_{\theta c}} \qquad F_{onset2} = \min\left[\max\left(F_{onset1}, F_{onset1}^4\right), 2.0\right] \qquad F_{onset3} = \max\left[1 - \left(\frac{Re_T}{25}\right)^3, 0\right]$$

Transition models – Review of the $\gamma - Re_\theta$ model

- The momentum thickness Reynolds number transport equation is given as follows,

$$\frac{\partial \rho \tilde{R}e_{\theta t}}{\partial t} + \frac{\partial \left(\rho \overline{u}_j \tilde{R}e_{\theta t} \right)}{\partial x_j} = P_{\theta t} + \frac{\partial}{\partial x_j} \left[\sigma_{\theta t} \left(\mu + \mu_t \right) \frac{\partial \tilde{R}e_{\theta t}}{\partial x_j} \right]$$

- Where,

$$P_{\theta t} = c_{\theta t} \frac{\rho}{t} \left(Re_{\theta t} - \tilde{R}e_{\theta t} \right) (1.0 - F_{\theta t}) \qquad t = \frac{500\mu}{\rho U^2}$$

- The equation for the transition Reynolds number $\tilde{R}e_{\theta t}$ provides the critical Reynolds number for the intermittency equation.
- Since the correlation is based on freestream conditions, the production term is only active outside the boundary layer.
- This behavior is enabled by means of the blending function $F_{\theta t}$.
- Again, there are many additional coefficients and auxiliary equations to close this equation.
- The interested reader should refer to the original references for a complete description of the model.

Transition models – Review of the $\gamma - Re_\theta$ model

- The coupling of the intermittency equation to the turbulence model is achieved by modifying the production and dissipation terms of the $k-\omega$ SST turbulence model.
- In the $k-\omega$ SST the transport equation of the turbulence kinetic energy is written as follows,

$$\frac{\partial \rho k}{\partial t} + \frac{\partial \left(\rho k u_i\right)}{\partial x_i} = P_k^* - D_k^* + \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma_k \mu_t\right)\frac{\partial k}{\partial x_j}\right]$$

- Where,

$$P_k^* = \gamma_{eff} P_k$$

$$D_k^* = \min\left[\max\left(\gamma_{eff}, 0.1\right), 1.0\right] D_k$$

- P_k and D_k are the production and dissipation terms of the original formulation of the $k-\omega$ SST model.

Transition models – Review of the $\gamma - Re_\theta$ model

- This model has many empirical correlations, closure coefficients, auxiliary relations, limiter functions, and blending functions that we did not covered here.
- The interested reader should refer to references [1,2,3] for a complete description of the model.
- Also, for a complete description of the $k - \omega$ SST, the interested reader should refer to references [4,5].
- It is worth mentioning that the correlations used are often proprietary and are calibrated to very specific experiments.
- Use transition models only when you are sure that the effect of transition is important and relevant to the flow that you are solving, as these models requires very fine meshes and additional equations.

References:
[1] R. Langtry, F. Menter. Transition Modeling for General CFD Applications in Aeronautics. 2005.
[2] F. Menter, R. Langtry, S. Likki, Y. Suzen, P. Huang, S. Volker. A Correlation-Based Transition Model Using Local Variables Part I – Model Formulation. 2006.
[3] ANSYS Fluent Theory Guide, 2020R1
[4] F. Menter. Two-Equation Eddy-Viscosity Turbulence Models for Engineering Applications. 1994.
[5] F. Menter, M. Kuntz, R. Langtry. Ten Years of Industrial Experience with the SST Turbulence Model. 2003.

Transition models – Review of the $\gamma - Re_\theta$ model

- The model is very sensitive to the inlet turbulence intensity Tu value.
- The location of the transition point and its extension strongly depends on the local Tu.
- For external aerodynamics, high inlet values for Tu and addy viscosity ratio (EVR) are required to allow for decay.
- There are a few correlations to compute this decay.
- It is recommended to initialize the intermittency to 1 in the whole domain.
- It is strongly recommended to use production limiters together with transition to turbulence models.
- Good quality meshes are required.
- The use of hexahedral meshes is strongly recommended.
- Also, low expansion ratios normal to the walls are recommended.

References:
[1] ANSYS Fluent Theory Guide, 2020R1

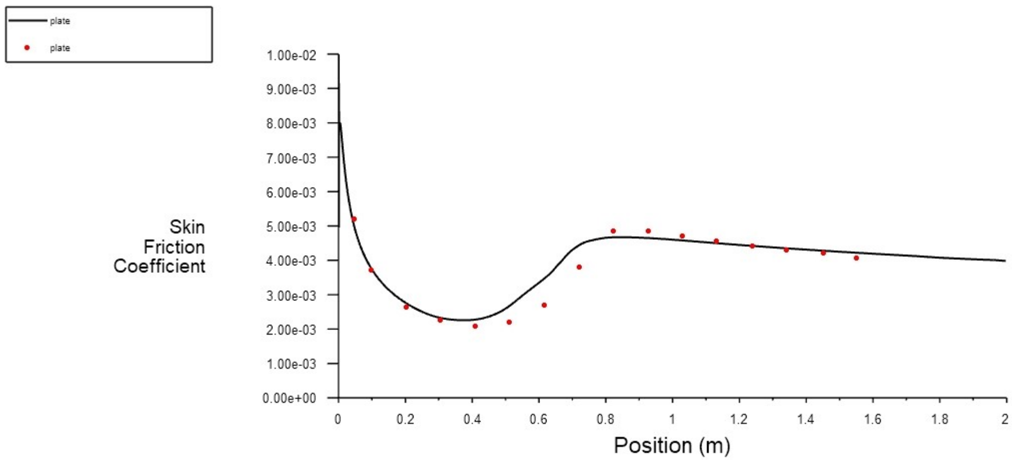
Transition models – Review of the $\gamma - Re_\theta$ model

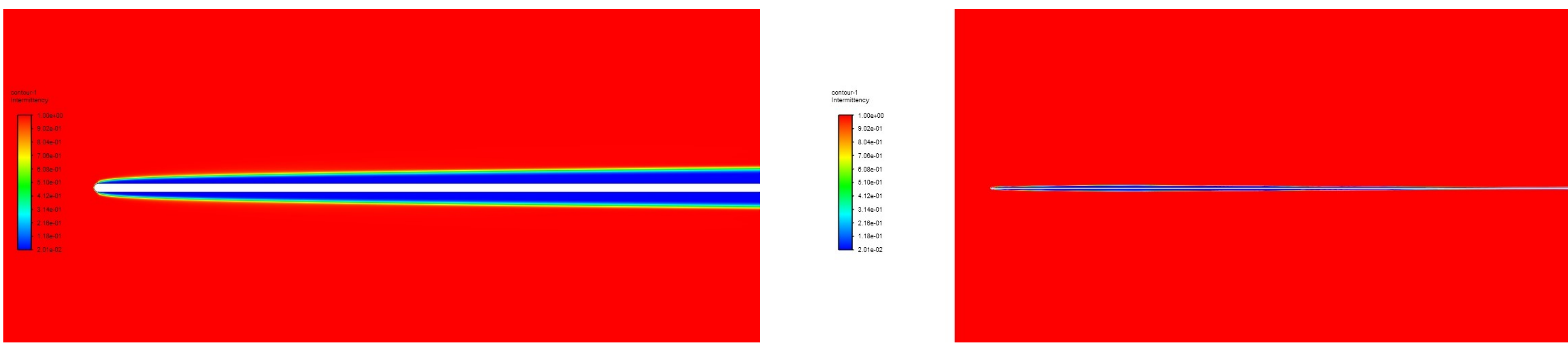
- It is also necessary to use low convergence criterion, in the order of 10e-6 for all variables.
- Transition to turbulence models are wall resolving and require y^+ values lower than 1.
- However, it is not recommended to use y^+ values lower than 0.01.
- You also need to use enough cells in the stream-wise direction so you can properly capture the transition region and eventual LSB.
- It is strongly recommended to follow the standard practices suggested by the model implementation [1].

References:
[1] ANSYS Fluent Theory Guide, 2020R1

Transition models – Review of the $\gamma - Re_\theta$ model

- Laminar to turbulent transition of boundary layer over a flat plate.

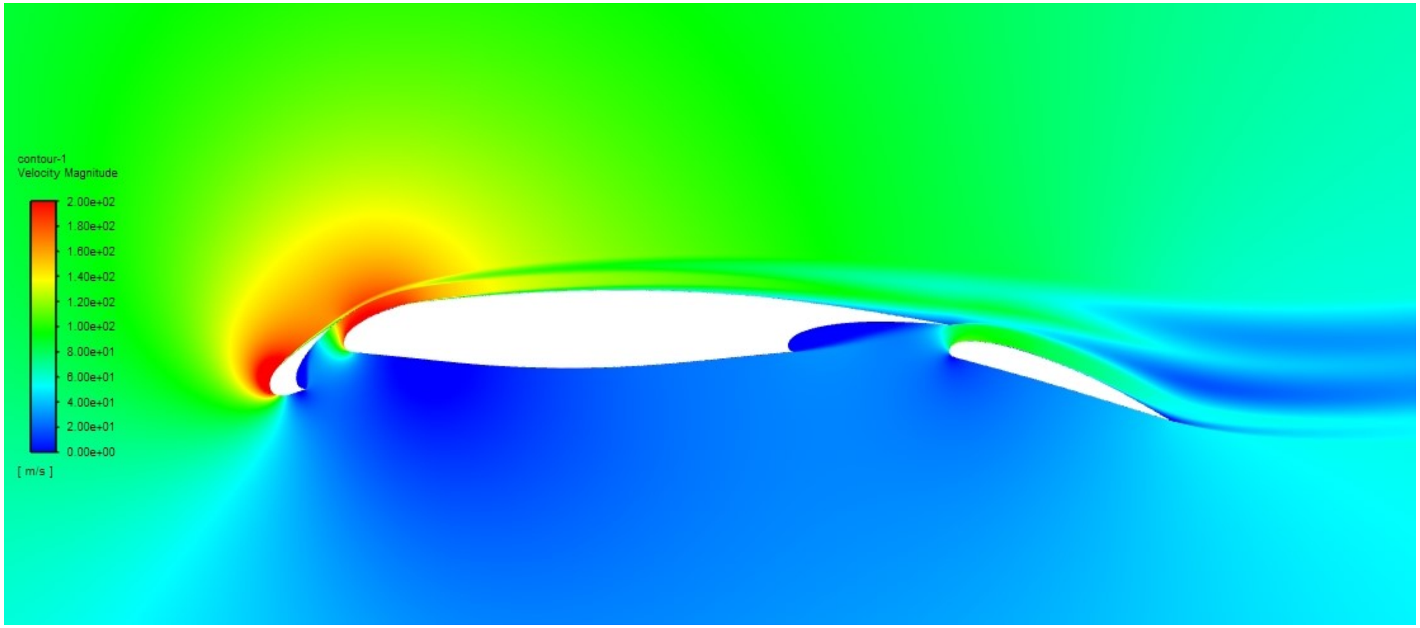


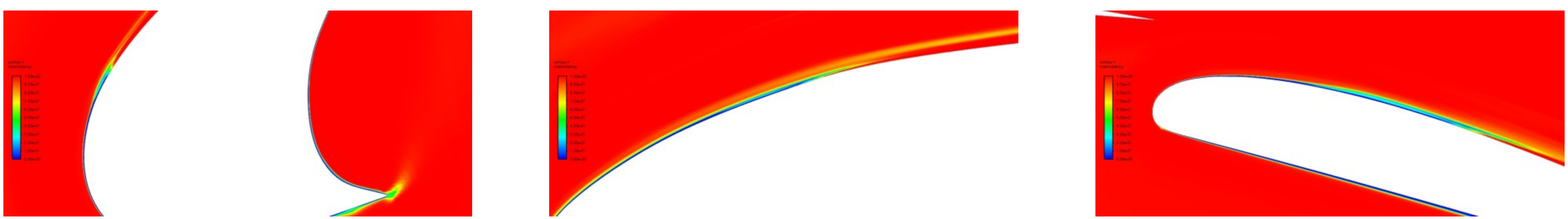


The intermittency field in a transitional flat plate.

Transition models – Review of the $\gamma - Re_\theta$ model

- Transitional flow over a three-element airfoil.





The intermittency field in a three-element airfoil.