
Turbulence and CFD models: Theory and applications

Roadmap to Lecture 2

- 1. The turbulent world around us**
- 2. Turbulence, does it matter?**
- 3. Introduction to turbulence modeling**
- 4. Wall bounded flows and shear flows**

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Leonardo da Vinci pioneered the flow visualization genre about 500 years ago. The illustrations to the left (*Studies of water passing obstacles and falling* c. 1508-1509) represents perhaps the world's first use of visualization as a scientific tool to study a turbulent flow.

The following da Vinci's observation is close to the Reynold's decomposition.

“Observe the motion of the surface of the water; which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to the random and reverse motion.”



Leonardo da Vinci pioneered the flow visualization genre about 500 years ago. The illustrations to the left (*Studies of water passing obstacles and falling* c. 1508-1509) represents perhaps the world's first use of visualization as a scientific tool to study a turbulent flow.

The following da Vinci's description is maybe the earliest reference to the importance of vortices in fluid motion.

“So moving water strives to maintain the course pursuant to the power which occasions it and, if it finds an obstacle in its path, completes the span of the course it has commenced by a circular and revolving movement.”



Leonardo da Vinci pioneered the flow visualization genre about 500 years ago. The illustrations to the left (*Studies of water passing obstacles and falling* c. 1508-1509) represents perhaps the world's first use of visualization as a scientific tool to study a turbulent flow.

The following da Vinci's observation is an analogy to the energy cascade and coherent structures.

“...the smallest eddies are almost numberless, and large things are rotated only by large eddies and not by small ones and small things are turned by small eddies and large”

The turbulent world around us

What is turbulence?

- Due to the extremely complex nature of turbulence and its incomplete understanding, there is not a single accepted definition of turbulence.
 - Leonardo da Vinci was so intrigued by turbulence that he depicted it in many of his sketches (see previous slide). While observing the flow of water, he gave one of the very first definitions of turbulence (if not the first one),

“...the smallest eddies are almost numberless, and large things are rotated only by large eddies and not by small ones and small things are turned by small eddies and large”

- Richardson [1] in 1922 stated that,

*“Big whorls have little whorls,
which feed on their velocity;
And little whorls have lesser whorls,
And so on to viscosity”*

The turbulent world around us

What is turbulence?

- Due to the extremely complex nature of turbulence and its incomplete understanding, there is not a single accepted definition of turbulence.
 - T. von Karman [1] who is known for his studies about Fluid Dynamics, quotes G. I. Taylor with the following definition of turbulence in 1937,

“Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous or liquid, when they flow past solid surfaces or even when neighboring streams same fluid past or over one another.”

- J.O. Hinze [2] offers yet another definition for turbulence in 1959,

“Turbulent fluid motion is an irregular condition of the flow in which quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned.”

[1] T. Von Karman. “Some remarks on the statistical theory of turbulence”. Proc. 5th Int. Congr. Appl. Mech, Cambridge, MA, 347, 1938.

[2] J. O. Hinze. “Turbulence”. McGraw-Hill, New York, 1959.

The turbulent world around us

What is turbulence?

- Due to the extremely complex nature of turbulence and its incomplete understanding, there is not a single accepted definition of turbulence.
 - A more modern and highly specific definition of turbulence is given by G. T. Chapman and M. Tobak [1],

“Turbulence is any chaotic solution to the 3D Navier–Stokes equations that is sensitive to initial data and which occurs as a result of successive instabilities of laminar flows as a bifurcation parameter is increased through a succession of values.”

- S. Rodriguez [2], gives an even more modern definition linked to the use of approximations to deliver solutions,

“Turbulent flows is the dynamic superposition of an extremely large number of eddies with random (irregular) but continuous spectrum of sizes and velocities that are interspersed with small, discrete pockets of laminar flow (as a result of the Kolmogorov eddies that decayed, as well as in the viscous laminar sublayer and in the intermittent boundary). In this sense, turbulent flows are intractable in its fullest manifestation; this is where good, engineering common sense and approximations can deliver reasonable solutions, albeit approximate.”

[1] G. T. Chapman and M. Tobak. “Observations, Theoretical Ideas, and Modeling of Turbulent Flows — Past, Present and Future, in Theoretical Approaches to Turbulence”. Dwoyer et al.(eds), Springer-Verlag, New York, pp. 19–49, 1985.

[2] S. Rodriguez. “Applied Computational Fluid Dynamics and Turbulence modeling”. Springer, 2019.

What is turbulence?

- Due to its complexity, a definition does not work properly for turbulence, instead of it, it's better to explain its characteristics.
- Tennekes and Lumley [1] in their book called "*A First Course in Turbulence*", list the characteristics of turbulence:
 - Irregularity.
 - Diffusivity.
 - Large Reynolds numbers.
 - Three-Dimensional vorticity fluctuations.
 - Dissipation.
 - Continuum.
 - Feature of a flow, not fluid.

Turbulent flows have the following characteristics

- **Irregularity.** One characteristic of turbulent flows is their irregularity (or randomness). A fully deterministic approach to characterize turbulent flows is very difficult, if not impossible. Turbulent flows are usually described statically.



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The turbulent world around us

Turbulent flows have the following characteristics

- **Diffusivity.** The diffusivity of turbulence causes rapid mixing and increased rates of momentum, heat, and mass transfer. A flow that looks random but does not exhibit the spreading of velocity fluctuations through the surrounding fluid is not turbulent.



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The turbulent world around us

Turbulent flows have the following characteristics

- **Large Reynolds number.** Turbulent flows always occur at high Reynolds numbers. They are caused by a complex interaction between the viscous terms and nonlinear terms in the equations of motion. Randomness and nonlinearity combine to make the equations of turbulence nearly intractable.



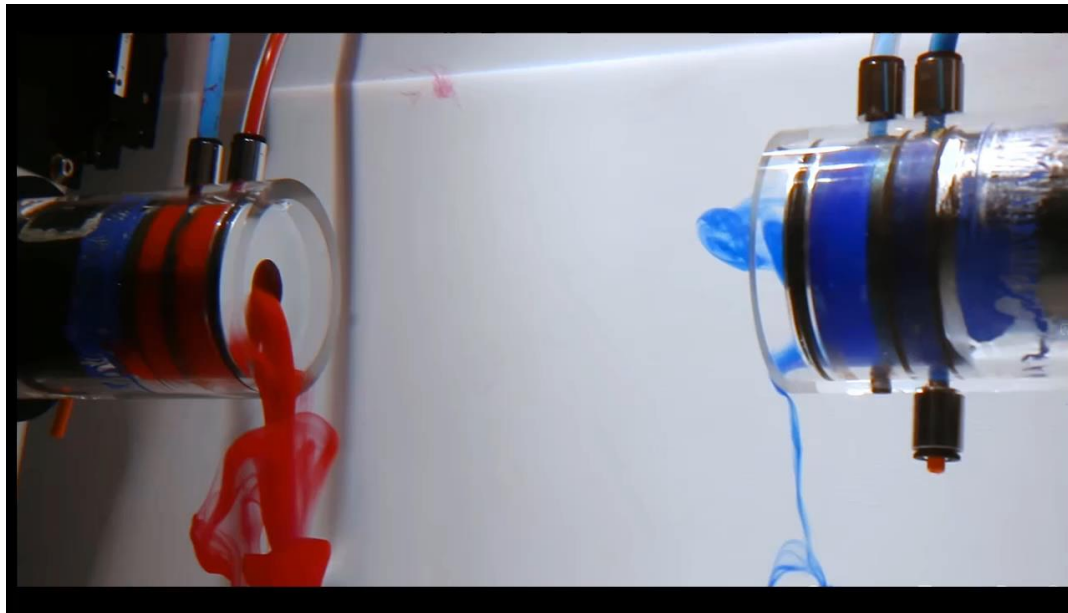
Photo credit: <https://www.pinterest.it/pin/763993524266658708/>

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The turbulent world around us

Turbulent flows have the following characteristics

- **Three-Dimensional vorticity fluctuations.** Turbulence is rotational and three dimensional. Turbulence always exhibit high levels of fluctuating vorticity. The random vorticity fluctuations that characterize turbulence could not maintain themselves if the velocity fluctuations were two dimensional. Mechanisms such as the stretching of three-dimensional vortices play a key role in turbulence.



Video credit: <https://www.youtube.com/watch?v=EVbdbVhzcM4&t>. This video has been edited to fit this presentation. Copyright on the images is held by the contributors. Apart from Fair Use, permission must be sought for any other purpose.

The turbulent world around us

Turbulent flows have the following characteristics

- **Dissipation.** Turbulent flows are dissipative. Kinetic energy gets converted into heat due to viscous shear stresses. Turbulent flows die out quickly when no energy is supplied.



Photo credit: <https://flic.kr/p/5qv1Bs>

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The turbulent world around us

Turbulent flows have the following characteristics

- **Continuum.** Turbulence is a continuum phenomenon. Even the smallest eddies are significantly larger than the molecular scales.



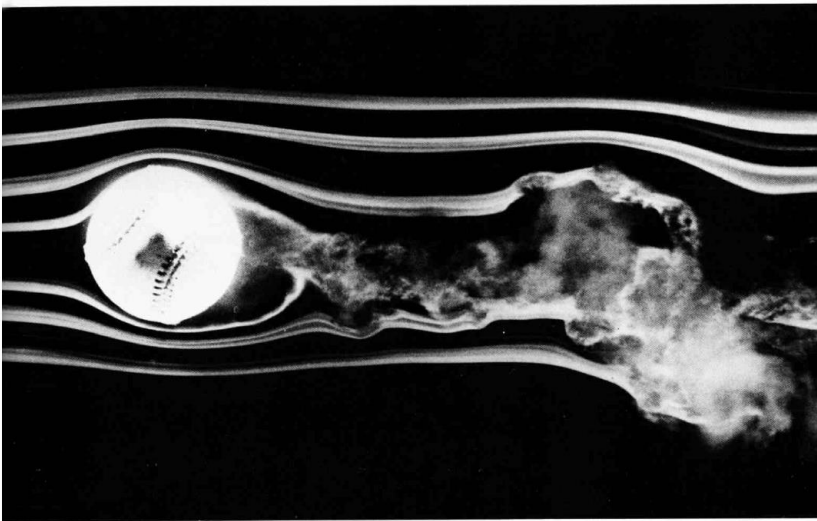
Photo credit: Sid Balachandran on Unsplash

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The turbulent world around us

Turbulent flows have the following characteristics

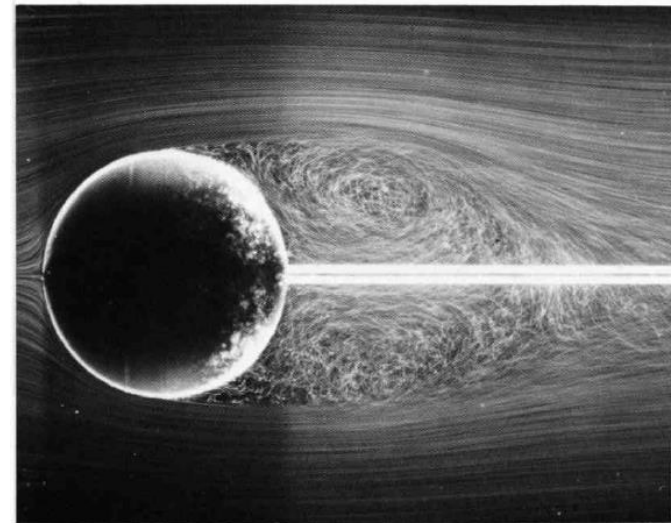
- **Feature of a flow, not fluid.** Turbulence is a feature of fluid flow and is not a property of the flow. A liquid or a gas at high Reynolds number will exhibit the same dynamics.



Air flow over a spinning baseball.

M. Van Dyke. An album of fluid motion.

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Water flow over a sphere.

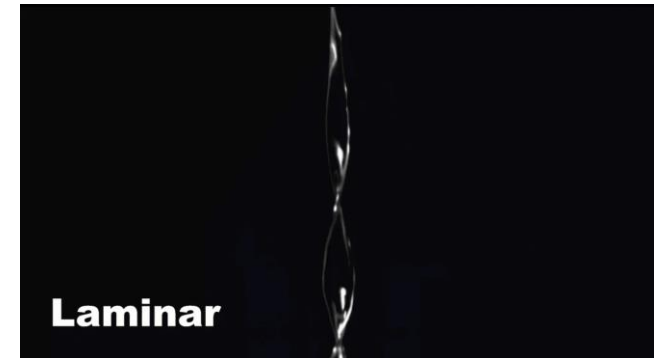
M. Van Dyke. An album of fluid motion.

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The turbulent world around us

Turbulent flows have the following characteristics

- In summary:
 - One characteristic of turbulent flows is their **irregularity** (or randomness). A fully deterministic approach to characterize turbulent flows is very difficult. Turbulent flows are usually described statically. Turbulent flows are always chaotic. But not all chaotic flows are turbulent. Magma flowing can be chaotic but not necessarily turbulent.
 - The **diffusivity** of turbulence causes rapid mixing and increased rates of momentum, heat, and mass transfer. A flow that looks random but does not exhibit the spreading of velocity fluctuations through the surrounding fluid is not turbulent.
 - Turbulent flows are **dissipative**. Kinetic energy gets converted into heat due to viscous shear stresses. Turbulent flows die out quickly when no energy is supplied.
 - Turbulent flows always occur at **high Reynolds numbers**. They are caused by a complex interaction between the viscous forces and convection.
 - Turbulent flows are **rotational**, that is, they have non-zero vorticity. Mechanisms such as the stretching of three-dimensional vortices play a key role in turbulence.
 - Turbulence is a **continuum** phenomenon. Even the smallest eddies are significantly larger than the molecular scales.
 - Turbulence is a **feature of fluid flow** and is not a property of the flow. A liquid or a gas at high Reynolds number will exhibit the same dynamics.

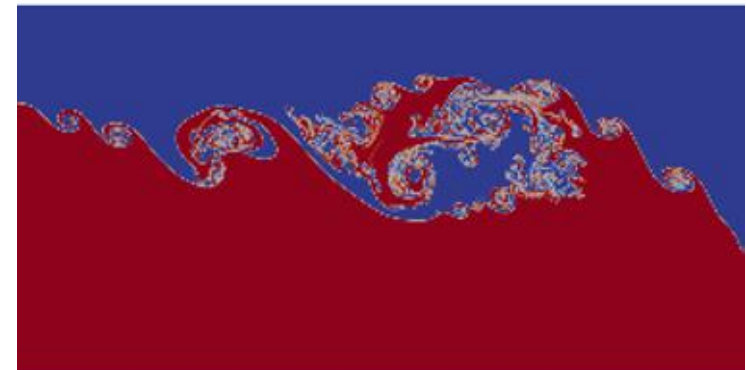
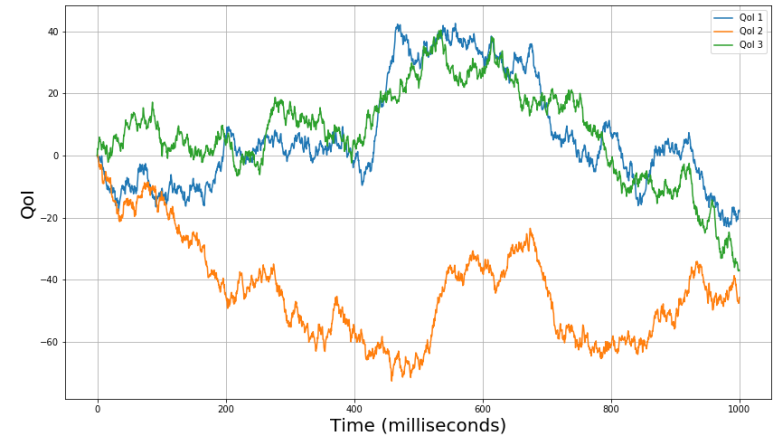


Video credit: <https://www.youtube.com/watch?v=9dz7fUJBFOk>
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The turbulent world around us

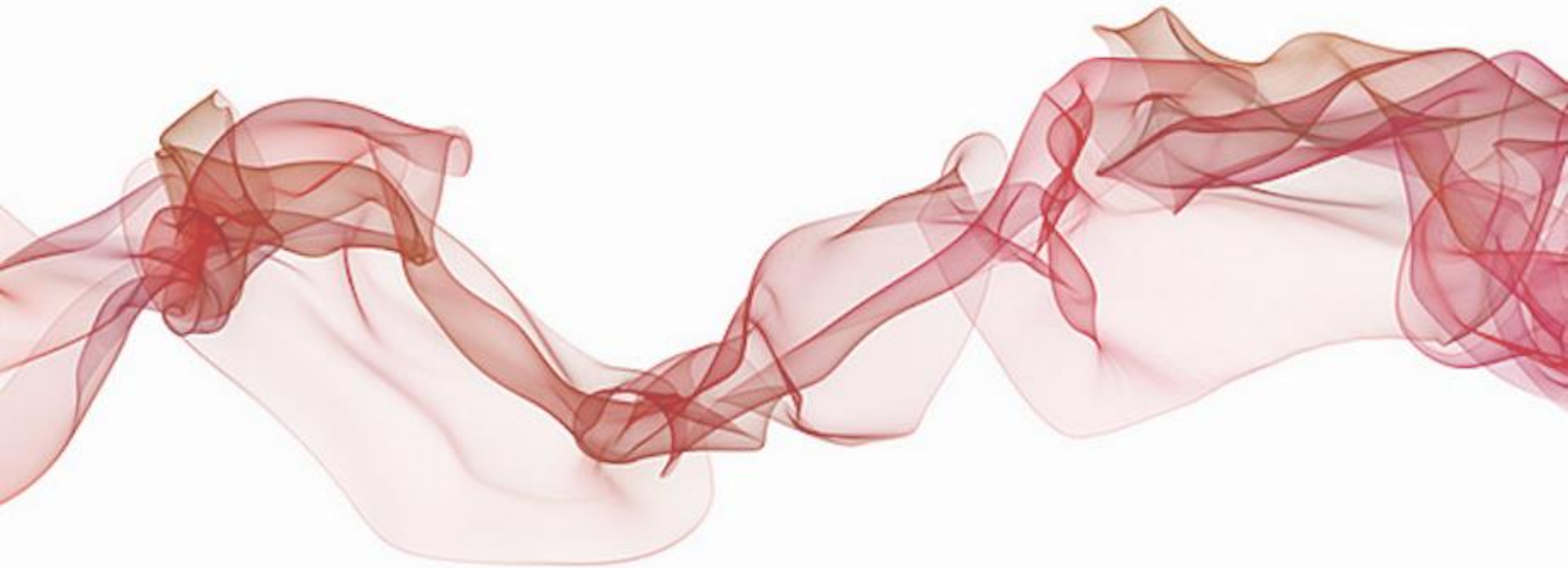
... so, what is turbulence?

- Turbulent flows share all the previous characteristics.
- In addition, let us state the following:
 - Turbulence is an unsteady, aperiodic motion in which all three velocity components fluctuate in space and time.
 - Every transported quantity shows similar fluctuations (pressure, temperature, species, concentration, and so on)
 - Turbulent flows contains a wide range of eddy sizes (scales):
 - Large eddies derives their energy from the mean flow. The size and velocity of large eddies are on the order of the mean flow.
 - Large eddies are unstable and they break-up into smaller eddies.
 - The smallest eddies convert kinetic energy into thermal energy via viscous dissipation.
 - The behavior of small eddies is more universal in nature.

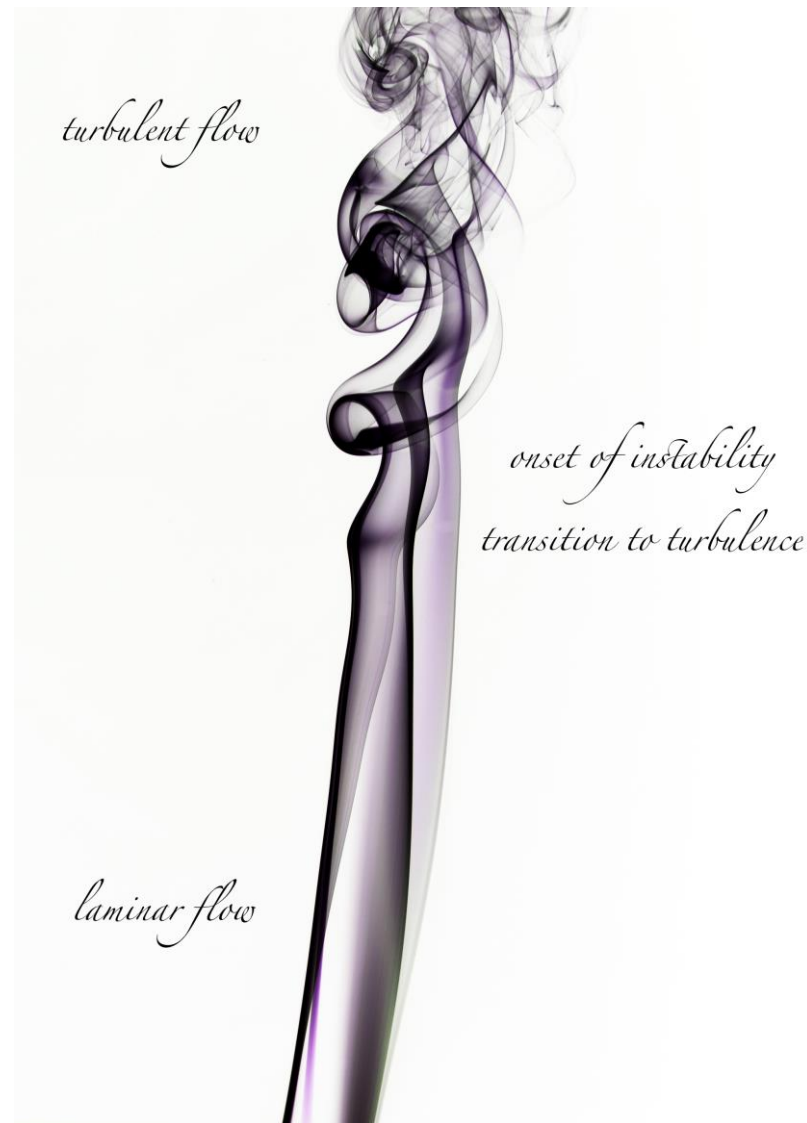


The turbulent world around us

Before continuing, let me share a few more amazing images that show the beauty and complexity of turbulence in nature and engineering applications.



The turbulent world around us

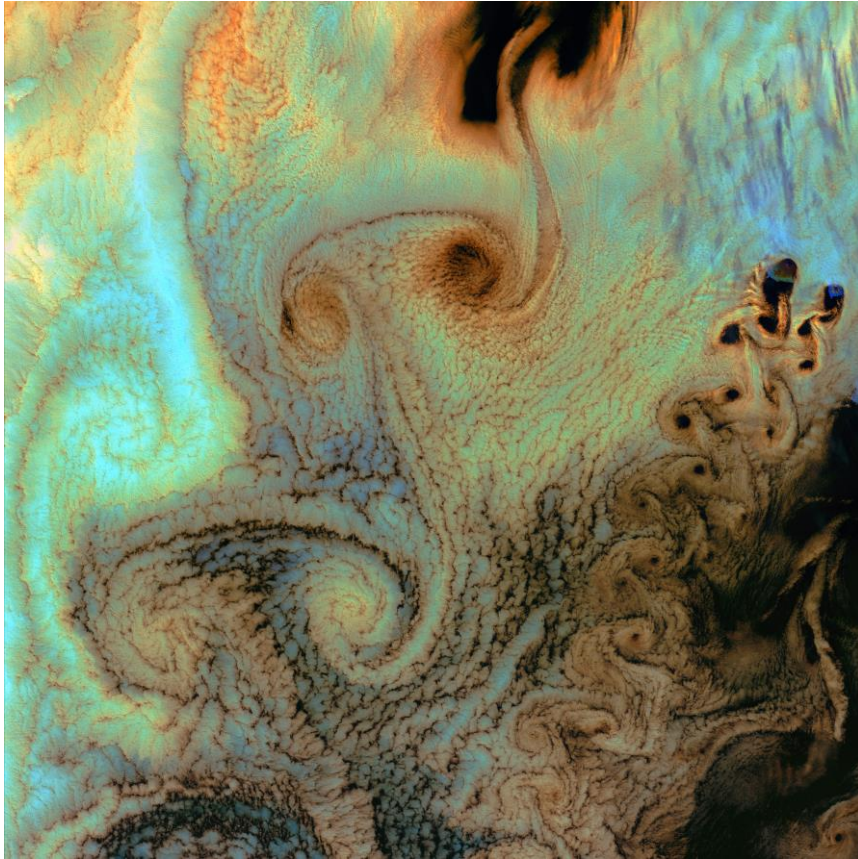


Buoyant plume of smoke rising from a stick of incense

Photo credit: <https://www.flickr.com/photos/jlhpgood/>

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The turbulent world around us



Von Karman vortices created when prevailing winds sweeping east across the northern Pacific Ocean encountered Alaska's Aleutian Islands

Photo credit: USGS EROS Data Center Satellite Systems Branch.
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Von Karman Vortex Streets in the northern Pacific Photographed from the International Space Station

Photo credit: NASA

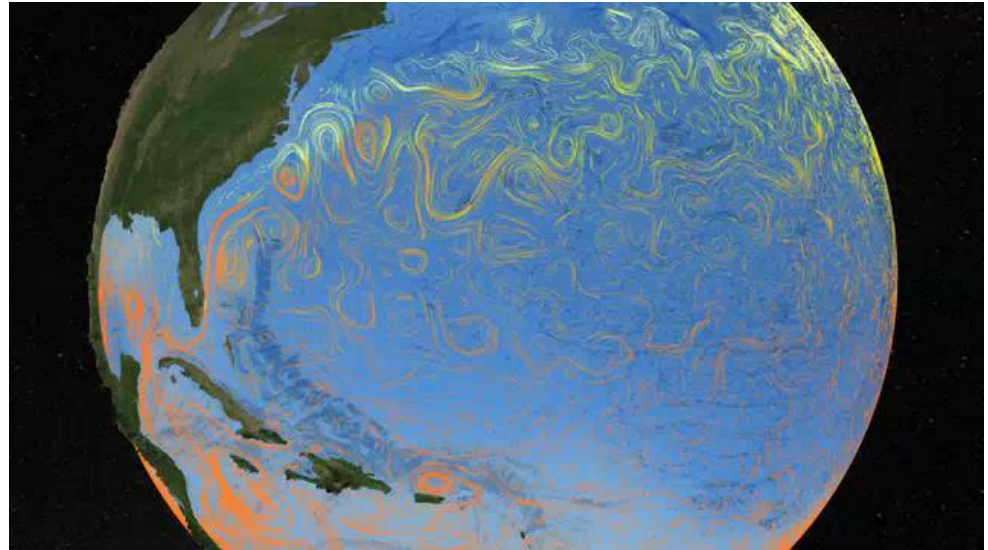
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The turbulent world around us



Cirrus clouds - Kelvin-Helmholtz instability

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NASA Aquarius mission - Studies of ocean and wind flows

The Aquarius mission measured the salinity in the ocean, giving scientists the tools needed to improve predictions of future climate trends and events. Aquarius salinity data, combined with data from other sensors that measure sea level, rainfall, temperature, ocean color, and winds, gave us a much clearer picture of how the ocean works. Will higher temperatures intensify evaporation and alter sea surface salinity patterns? Will changes in salinity affect ocean circulation and how heat is distributed over the globe? Aquarius measurements provide a new perspective on the ocean, how it is linked to climate, and how it will respond to climate change

Video credit: <https://svs.gsfc.nasa.gov/cgi-bin/details.cgi?aid=3829>
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The turbulent world around us



Turbulent waters

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Spring vortex in turbulent waters

Photo credit: <https://www.flickr.com/photos/kenii/>

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The turbulent world around us



Tugboat riding on the turbulent wake of a ship

Photo credit: <https://www.flickr.com/photos/oneeighteen/>

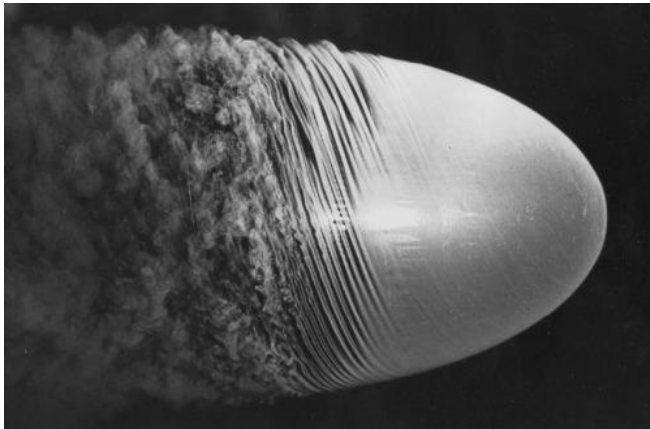
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Trailing vortices

Photo credit: Steve Morris. AirTeamImages.

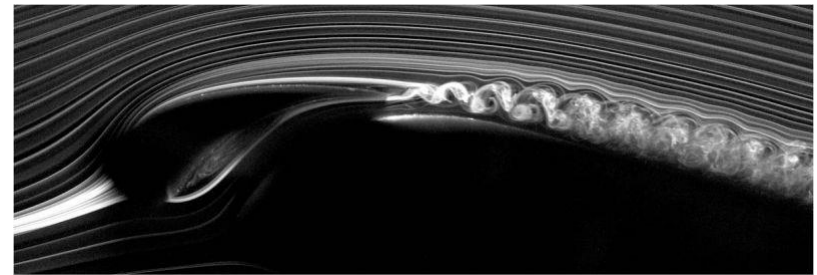
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Flow visualization over a spinning spheroid

Photo credit: Y. Kohama.

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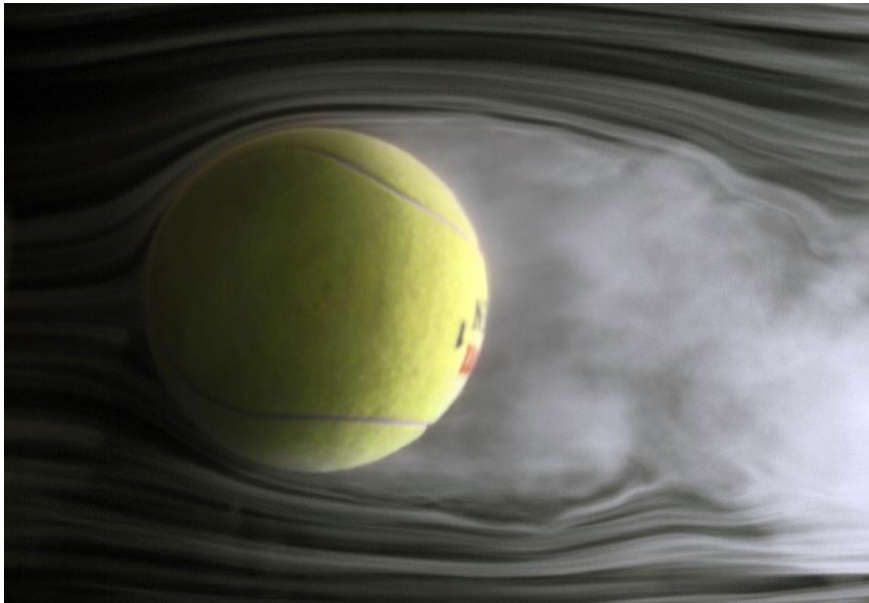


Flow around an airfoil with a leading-edge slat

Photo credit: S. Makiya et al.

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The turbulent world around us



Wind Tunnel Test of New Tennis Ball

Photo credit: NASA

<http://tennisclub.gsfc.nasa.gov/tennis.windtunnelballs.html>

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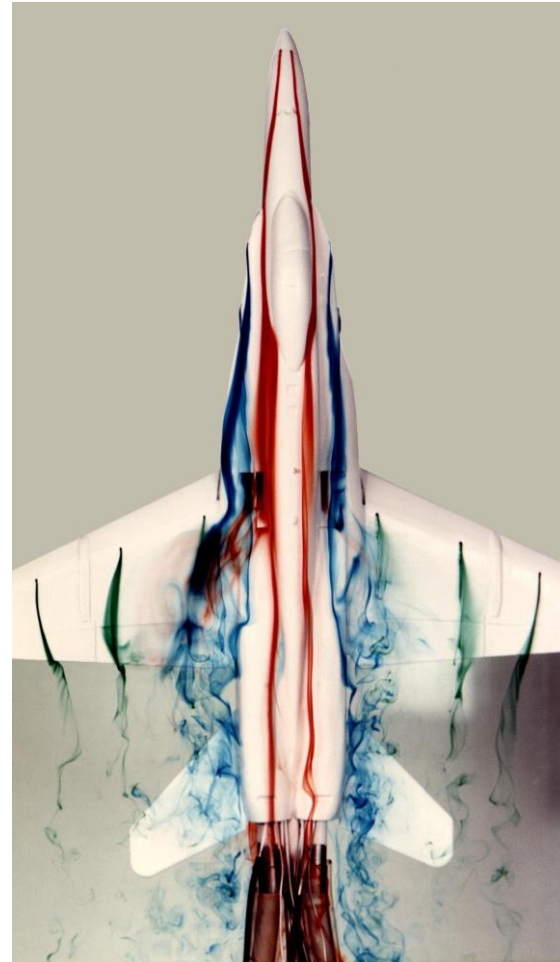
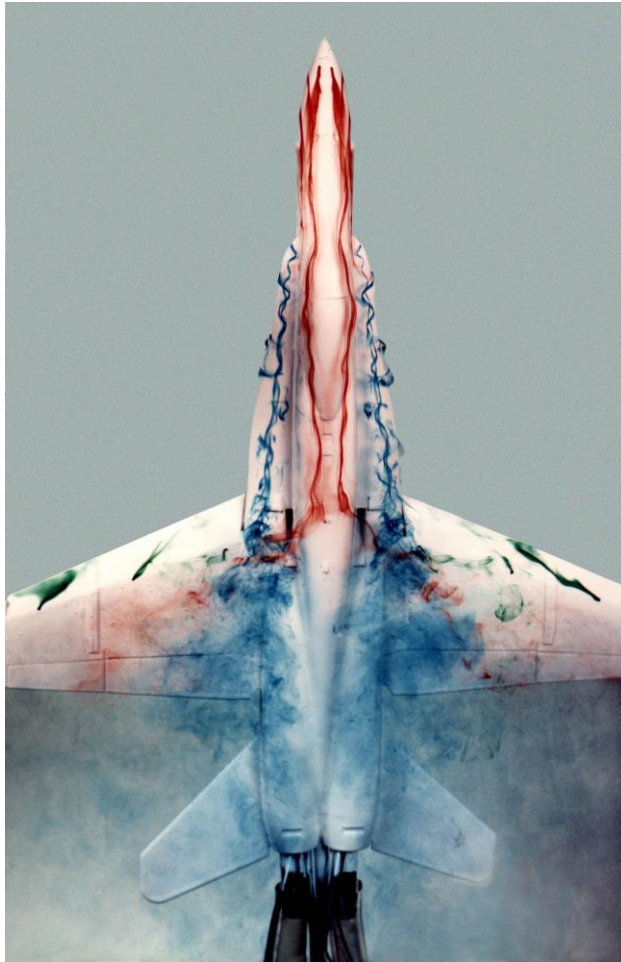


Wake turbulence behind individual wind turbines

Photo credit: NREL's wind energy research group.

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The turbulent world around us



Vortices on a 1/48-scale model of an F/A-18 aircraft inside a Water Tunnel

Photo credit: NASA Dryden Flow Visualization Facility. <http://www.nasa.gov/centers/armstrong/multimedia/imagegallery/FVF>
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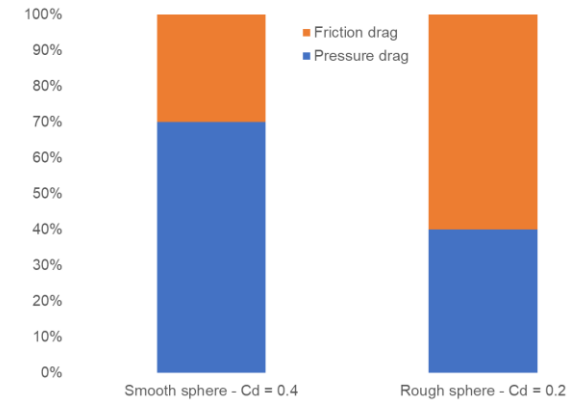
The turbulent world around us



(a) $C_D \approx 0.4$



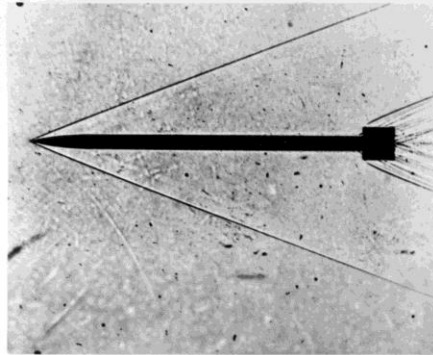
(b) $C_D \approx 0.2$



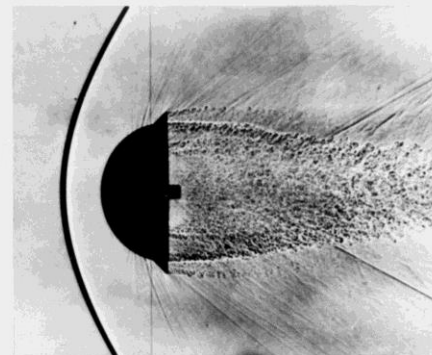
Drag decomposition

The turbulent world around us

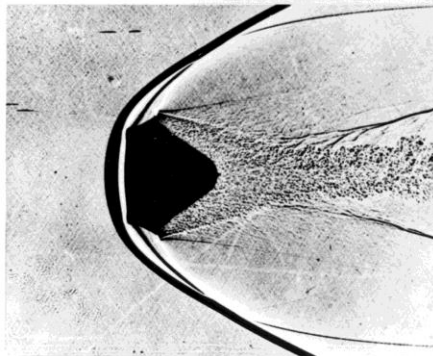
RESEARCH CONTRIBUTING TO PROJECT MERCURY



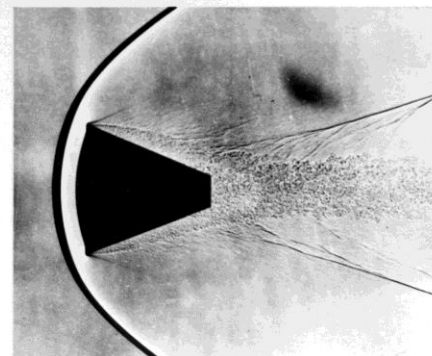
INITIAL CONCEPT



BLUNT BODY CONCEPT 1953



MISSILE NOSE CONES 1953-1957



MANNED CAPSULE CONCEPT 1957

Shadowgraph Images of Re-entry Vehicles

Photo credit: NASA on the Commons. <https://www.flickr.com/photos/nasacommons/>

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The turbulent world around us

Astrophysical, plasma, planetary and quantum turbulence

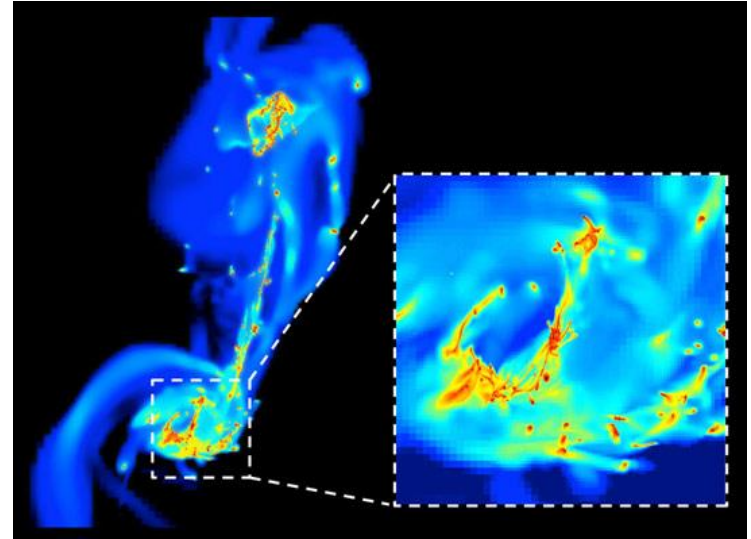


M8: The Lagoon Nebula

Photo credit: Steve Mazlin, Jack Harvey, Rick Gilbert, and Daniel Verschate.

Star Shadows Remote Observatory, PROMPT, CTIO

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A frame from the simulation of the two colliding Antennae galaxies.

Photo credit: F. Renaud / CEA-Sap.

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Jupiter photo taken by Juno's cam.

Photo credit: NASA / JPL / SwRI / MSSS / David Marriott

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Roadmap to Lecture 2

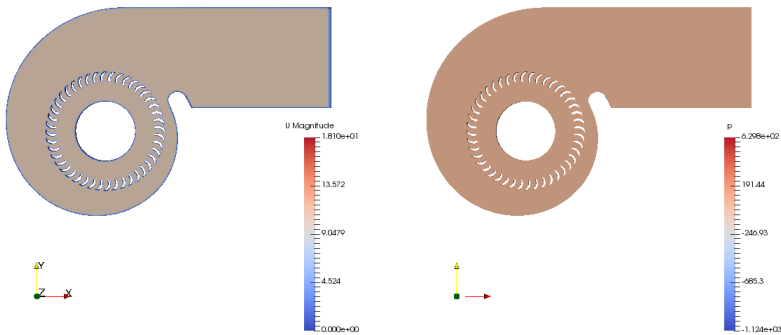
- ~~1. The turbulent world around us~~
- 2. Turbulence, does it matter?**
- ~~3. Introduction to turbulence modeling~~
- ~~4. Wall bounded flows and shear flows~~

Turbulence, does it matter?

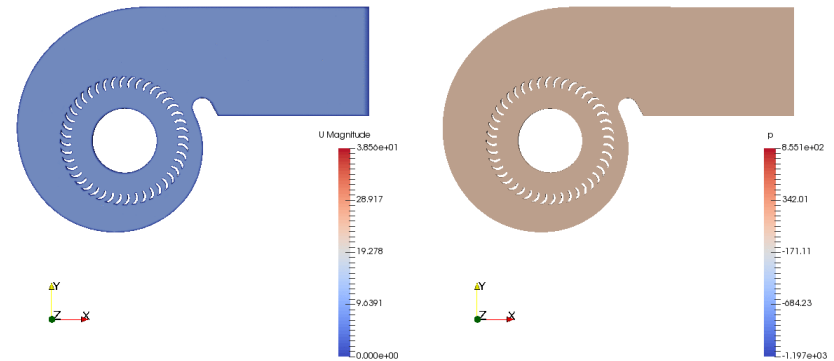
Blower simulation using sliding grids



Time: 0.000000



Time: 0.000000



No turbulence model used (laminar, no turbulence modeling, DNS, unresolved DNS, name it as you want)

<http://www.wolfdynamics.com/training/turbulence/image1.gif>

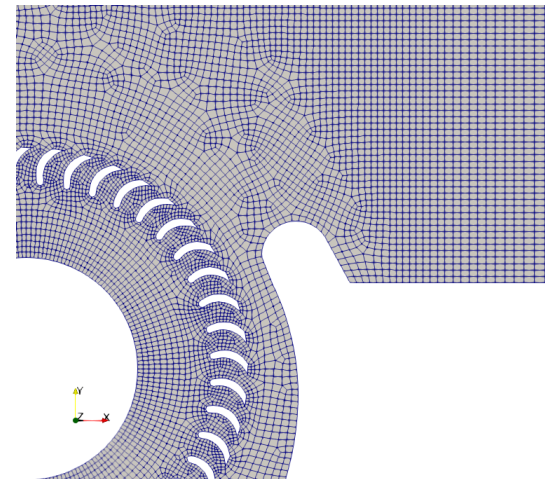
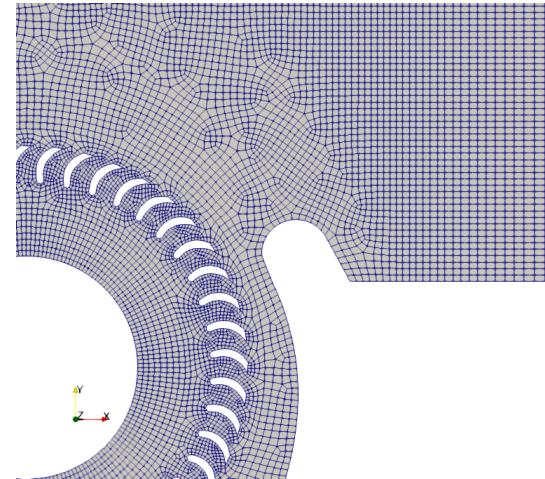
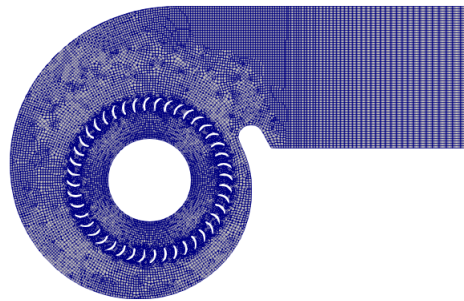
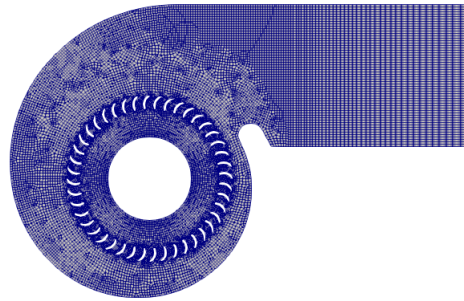
K-epsilon turbulence model

<http://www.wolfdynamics.com/training/turbulence/image2.gif>

Turbulence, does it matter?

Blower simulation using sliding grids

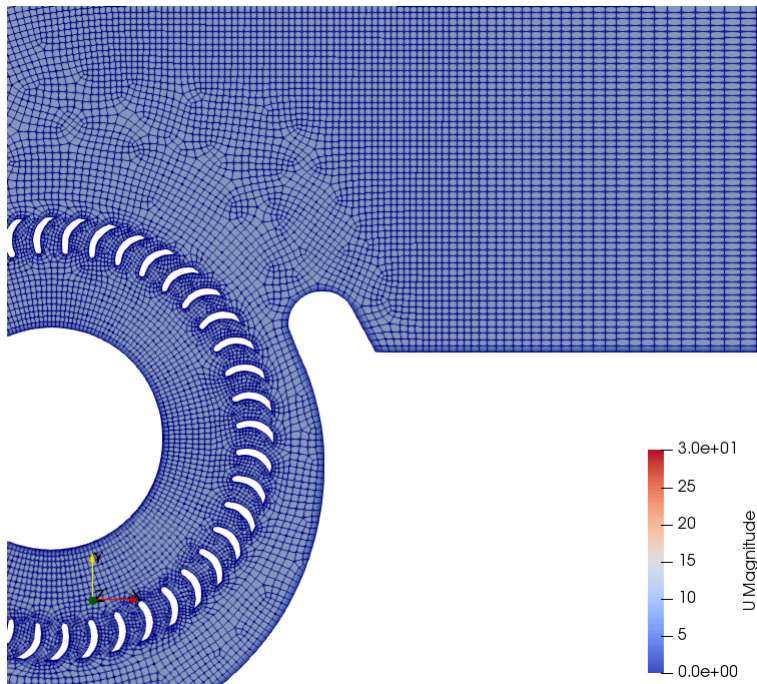
- Even if the mesh is coarse, thanks to the help of the turbulence model, we managed to capture the right physics.



Turbulence, does it matter?

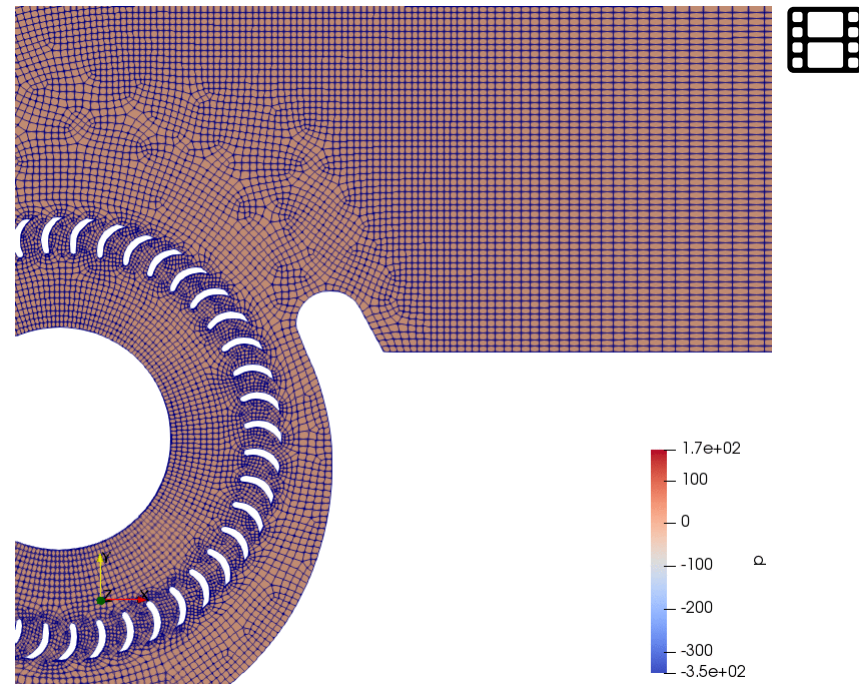
Blower simulation using sliding grids

- Even if the mesh is coarse, thanks to the help of the turbulence model, we managed to capture the right physics.



K-epsilon turbulence model – Velocity magnitude field (m/s)

<http://www.wolfdynamics.com/training/turbulence/blower3.gif>

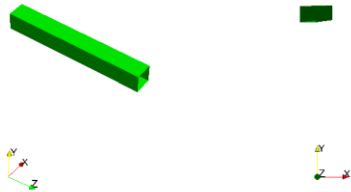


K-epsilon turbulence model – Relative pressure field (Pa)

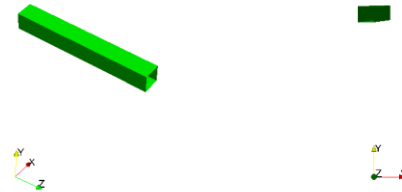
<http://www.wolfdynamics.com/training/turbulence/blower4.gif>

Turbulence, does it matter?

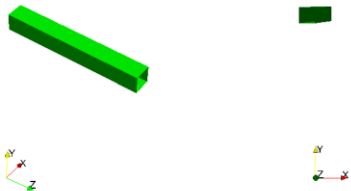
Vortex shedding past square cylinder



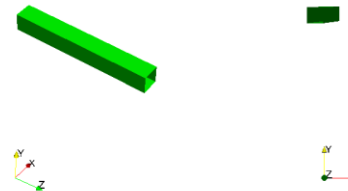
URANS (K-Omega SST with no wall functions) – Vortices visualized by Q-criterion
www.wolfdynamics.com/wiki/squarecil/urans2.gif



LES (Smagorinsky) – Vortices visualized by Q-criterion
www.wolfdynamics.com/wiki/squarecil/les.gif



Laminar (no turbulence model) – Vortices visualized by Q-criterion
www.wolfdynamics.com/wiki/squarecil/laminar.gif



DES (SpalartAllmarasDDES) – Vortices visualized by Q-criterion
www.wolfdynamics.com/wiki/squarecil/des.gif

Turbulence, does it matter?

Vortex shedding past square cylinder

Turbulence model	Drag coefficient	Strouhal number	Computing time (s)
Laminar	2.81	0.179	93489
LES	2.32	0.124	77465
DES	2.08	0.124	70754
SAS	2.40	0.164	57690
URANS (WF)	2.31	0.130	67830
URANS (No WF)	2.28	0.135	64492
RANS	2.20	-	28246 (10000 iter)
Experimental values	2.05-2.25	0.132	-

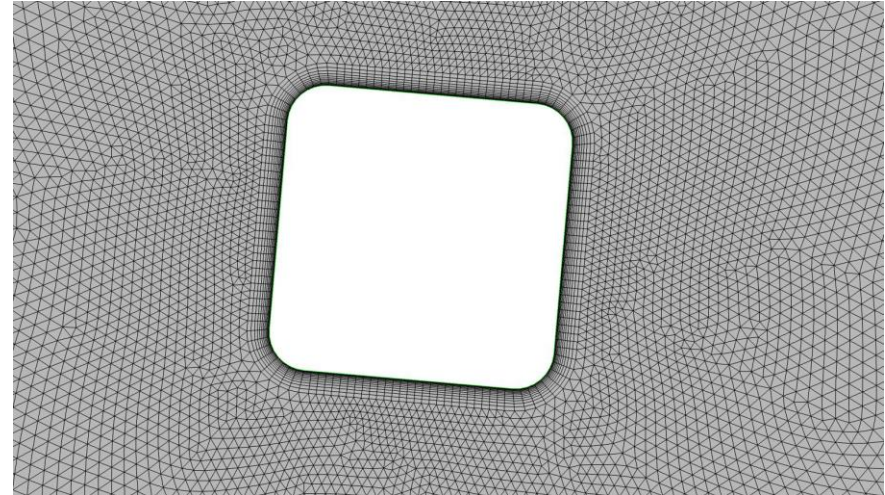
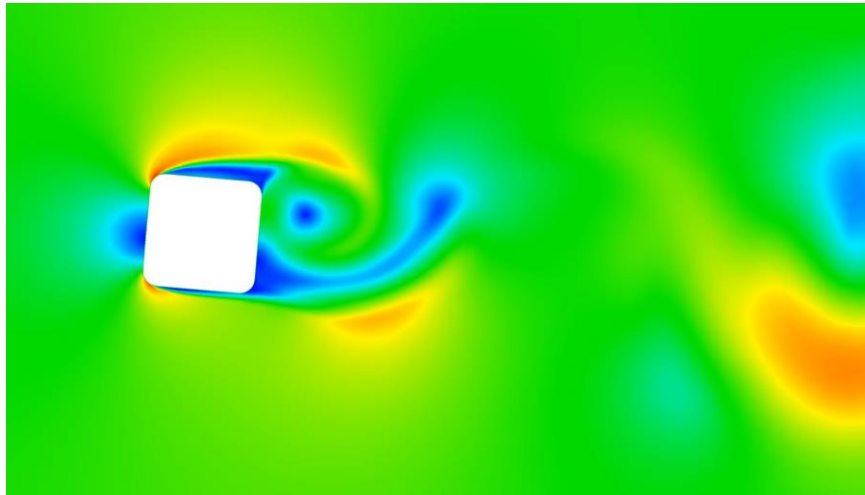
Note: all simulations were run using 4 cores.

References:

D. A. Lyn and W. Rodi. "The flapping shear layer formed by flow separation from the forward corner of a square cylinder". *J. Fluid Mech.*, 267, 353, 1994.
D. A. Lyn, S. Einav, W. Rodi and J. H. Park. "A laser-Doppler velocimetry study of ensemble-averaged characteristics of the turbulent near wake of a square cylinder". *Report. SFB 210 /E/100*.

Turbulence, does it matter?

Transitional flow past square cylinder with rounded corners – $Re = 54000$



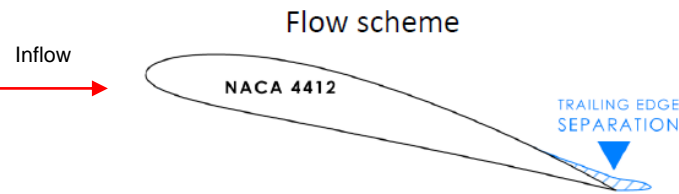
Velocity magnitude

www.wolfdynamics.com/wiki/turb/media1.mp4

Turbulence model	Drag coefficient	Lift coefficient
DNS	0.06295	0.07524
LES	0.1146	0.03269
SAS	0.1058	0.0258
URANS (No WF)	0.1107	0.00725
Transition K-KL-Omega	0.059	-0.0104
Transition K-Omega SST	0.0987	-0.0143
Experimental values	0.045 to 0.075	-0.011 to -0.015

Turbulence, does it matter?

Separated flow around a NACA-4412 airfoil



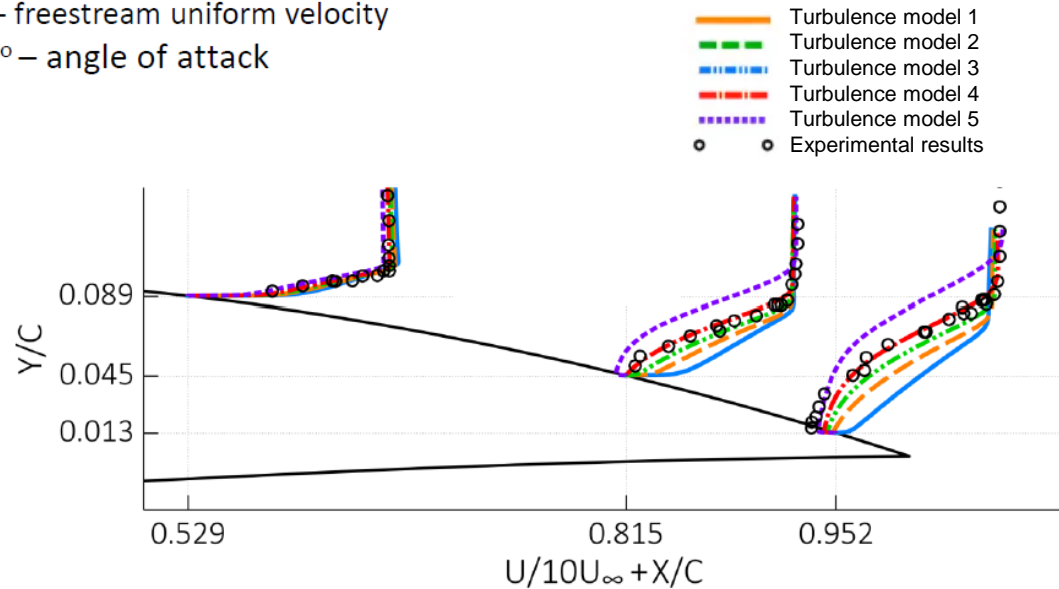
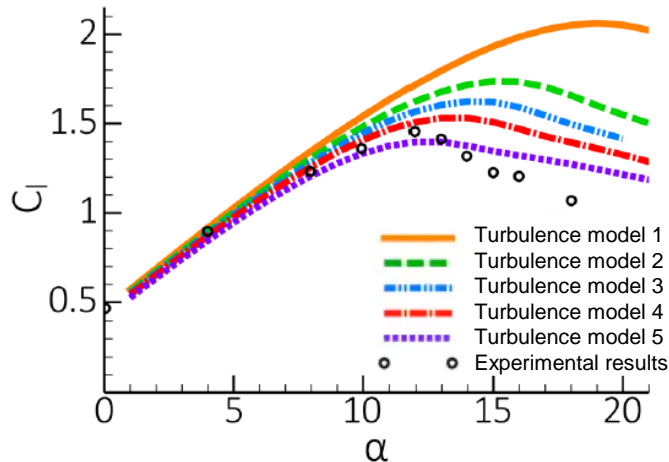
Incompressible flow

$$Re = U_{\infty} \cdot C / \nu = 1.64 \cdot 10^6$$

C - airfoil chord

U_{∞} - freestream uniform velocity

$\alpha = 12^\circ$ - angle of attack



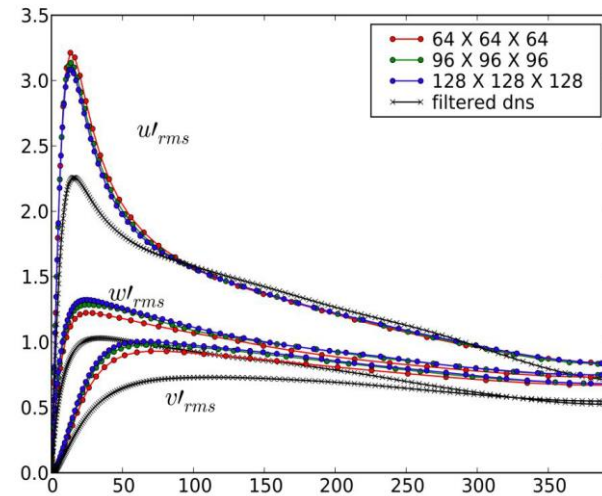
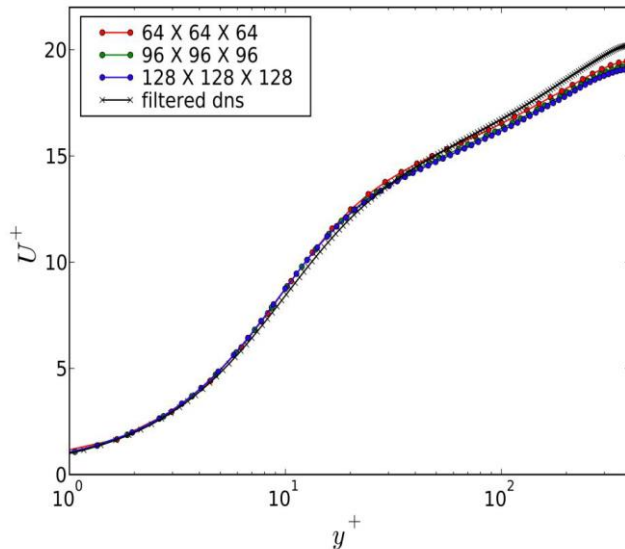
- CFD has been around since the late 1970s, and after all these years is not that easy to compute the flow around 2D airfoils.
- In particular, predicting the maximum lift and stall characteristics is not trivial.

References:

- F. Menter. "A New Generalized k-omega model. Putting flexibility into Turbulence models (GEKO)", Ansys Germany
- A. J. Wadcock. "Investigation of Low-Speed Turbulent Separated Flow Around Airfoils", NASA Contractor Report 177450

Turbulence, does it matter?

Grid independent solutions and modeling errors



- CFD has been around since the late 1970s. Since then, a lot of progress has been done in hardware, software, algorithms and turbulence models.
- But many times, even if we get converged, grid-independent solutions we fail to get a good match with the experiments (not that the experiments are always right) or a reference solution due to modeling errors.

*“The multiscale nature of turbulence creates unique challenges for numerical simulations. Discretization methods must preserve the physical processes, reducing or eliminating artificial dissipation and dispersion ...
... How do you establish confidence in the numerical simulations of turbulent flows?”*

References:

P. Moin. “Turbulence: V&V and UQ Analysis of a Multi-scale Complex System”. Center for Turbulence Research Stanford University

Turbulence, does it matter?

Turbulence is not a trivial problem

“Turbulence is the most important unresolved problem of classical physics”

Richard Feynman

“Turbulence was probably invented by the devil on the seventh day of creation when the good lord was not looking”

Peter Bradshaw (1994)

“Turbulence is the graveyard of theories”

Hans W. Liepmann (1997)

Turbulence, does it matter?

Turbulence is not a trivial problem

- Probably my favorite quote, as it covers the largest elephants in CFD, mesh and turbulence.

“Geometry modeling is to meshing what turbulence modeling is to computational fluid dynamics (CFD) – a mathematically complex model of something important that we try to treat as the proverbial black box.”

John Chawner - Pointwise

Roadmap to Lecture 2

- ~~1. The turbulent world around us~~
- ~~2. Turbulence, does it matter?~~
- 3. Introduction to turbulence modeling**
- ~~4. Wall bounded flows and shear flows~~

Introduction to turbulence modeling

*“Essentially, all models are wrong,
but some are useful”*

G. E. P. Box



George Edward Pelham Box

18 October 1919 – 28 March 2013. Statistician, who worked in the areas of quality control, time-series analysis, design of experiments, and Bayesian inference. He has been called *“one of the great statistical minds of the 20th century”*.

Turbulence modeling in engineering

- Most natural and engineering flows are turbulent, hence the necessity of modeling turbulence.
- The goal of turbulence modeling is to develop equations that predict the time averaged velocity, pressure, temperature fields without calculating the complete turbulent flow pattern as a function of time.
- Turbulence can be wall bounded or free shear. Depending of what you want to simulate, you will need to choose an appropriate turbulence model.
- There is no universal turbulence model, hence you need to know the capabilities and limitations of the turbulence models.
- Due to the multi-scale and unsteady nature of turbulence, modeling it is not an easy task.
- Simulating turbulent flows in any general CFD solver (e.g., OpenFOAM®, SU2, Fluent, CFX, Star-CCM+) requires selecting a turbulence model, providing initial conditions and boundary conditions for the closure equations of the turbulent model, selecting a near-wall modeling, and choosing runtime parameters and numerics.

Why turbulent flows are challenging?

- Unsteady aperiodic motion.
- All fluid properties and transported quantities exhibit random spatial and temporal variations.
- They are intrinsically three-dimensional due to vortex stretching.
- Strong dependence from initial conditions.
- Contains a wide range of scales (eddies).
- Therefore, in order to accurately model/resolve turbulent flows, the simulations must be three-dimensional, time-accurate, and with fine enough meshes such that all spatial scales are properly captured.
- Additional physics that makes turbulence modeling even harder:
 - Buoyancy, compressibility effects, heat transfer, multiphase flows, transition to turbulence, surface finish, combustion, and so on.

Introduction to turbulence modeling

Reynolds number and Rayleigh number

- It is well known that the Reynolds number characterizes if the flow is laminar or turbulent.
- So before doing a simulation or experiment, check if the flow is turbulent.
- The Reynolds number is defined as follows,

$$\text{Convective effects} \longrightarrow Re_L = \frac{\rho U L}{\mu} \longleftarrow \text{Viscous effects}$$

- Where U is a characteristic velocity, e.g., free-stream velocity.
- And L is representative length scale, e.g., length, height, diameter, etc.

Introduction to turbulence modeling

Reynolds number and Rayleigh number

- If you are dealing with natural convection, you can use the Rayleigh number, Grashof number, and Prandtl number to characterize the flow.

$$\begin{array}{lcl}
 \text{Buoyancy effects} & \longrightarrow & \\
 & Ra = \frac{g\beta L^3 \Delta T}{\nu\alpha} = \frac{\rho^2 \overset{\text{Specific heat}}{c_p} \overset{\text{Thermal expansion coefficient}}{\beta} g L^3 \Delta T}{\mu \overset{\text{Thermal conductivity}}{k}} = Gr \times Pr \\
 \text{Viscous effects} & \longrightarrow &
 \end{array}$$

$$\begin{array}{lcl}
 \text{Momentum diffusivity} & \longrightarrow & \\
 & Pr = \frac{\nu}{\alpha} = \frac{\mu c_p}{k} \\
 \text{Thermal diffusivity} & \longrightarrow &
 \end{array}$$

$$Gr = \frac{g\beta(T_S - T_\infty)L^3}{\nu^2}$$

Introduction to turbulence modeling

Reynolds number and Rayleigh number

- Turbulent flow occurs at large Reynolds number.

- For external flows,

$$Re_x \geq 500000 \quad \text{Around slender/streamlined bodies (surfaces)}$$

$$Re_d \geq 20000 \quad \text{Around an obstacle (bluff bodies)}$$

- For internal flows,

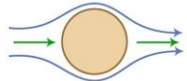
$$Re_{d_h} \geq 2300$$

- Notice that other factors such as free-stream turbulence, surface conditions, blowing, suction, roughness and other disturbances, may cause transition to turbulence at lower Reynolds number.
- If you are dealing with natural convection and buoyancy, turbulent flows occurs when

$$\frac{Ra}{Pr} \geq 10^9$$

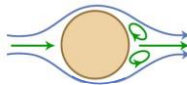
Introduction to turbulence modeling

What happens when we increase the Reynolds number?



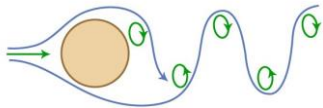
Creeping flow (no separation)
Steady flow

$$Re < 5$$



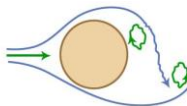
**A pair of stable vortices
in the wake**
Steady flow

$$5 < Re < 40 - 46$$



**Laminar vortex street
(Von Karman street)**
Unsteady flow

$$40 - 46 < Re < 150$$

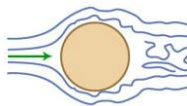


**Laminar boundary layer up to
the separation point, turbulent
wake**
Unsteady flow

$$150 < Re < 300$$

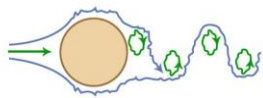
Transition to turbulence

$$300 < Re < 3 \times 10^5$$



**Boundary layer transition to
turbulent**
Unsteady flow

$$3 \times 10^5 < Re < 3 \times 10^6$$



**Turbulent vortex street, but the
wake is narrower than in the
laminar case**
Unsteady flow

$$3 \times 10^6 > Re$$

- Easy to simulate
- Steady

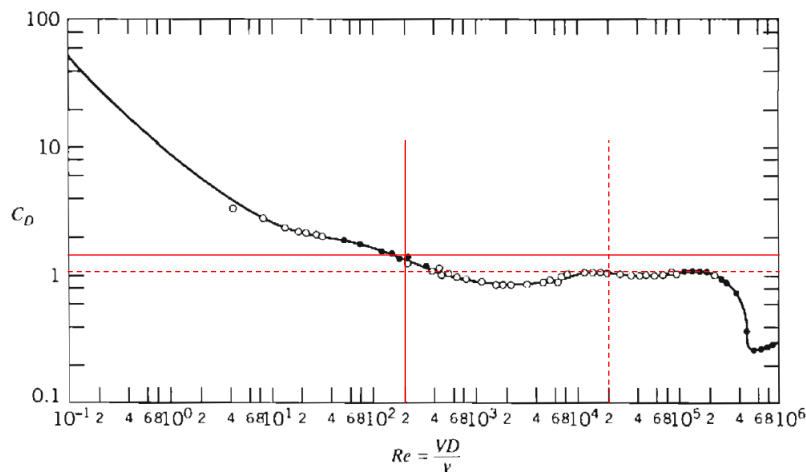
- Relatively easy to simulate.
- It becomes more challenging when the boundary layer transition to turbulent
- Unsteady

- Challenging to simulate
- Unsteady

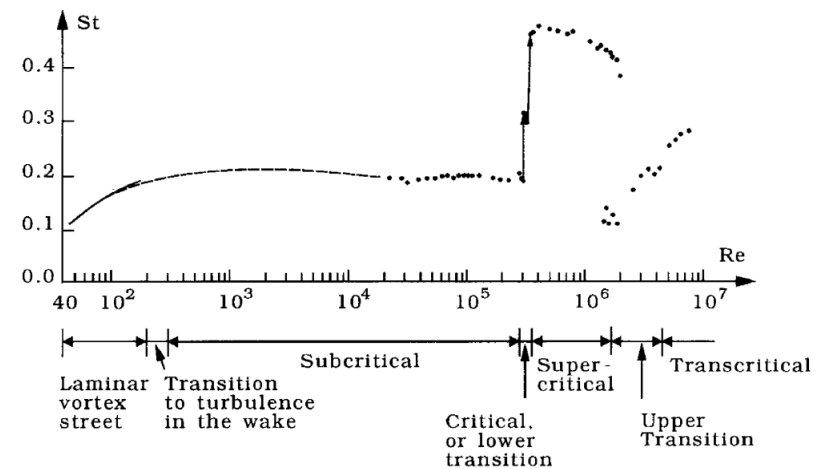
Vortex shedding behind a cylinder and Reynolds number

Introduction to turbulence modeling

What happens when we increase the Reynolds number?



Drag coefficient as a function of Reynolds number for a smooth cylinder [1]



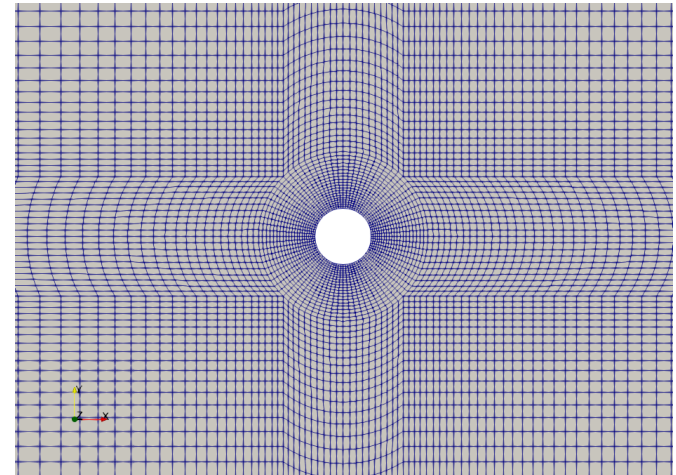
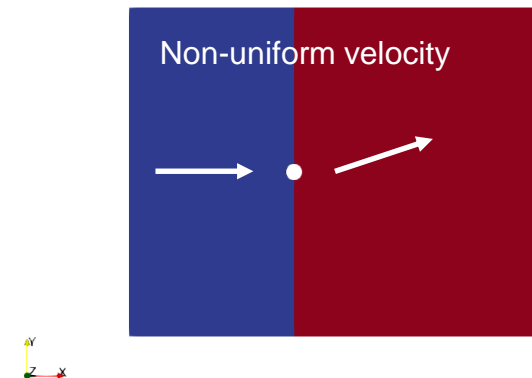
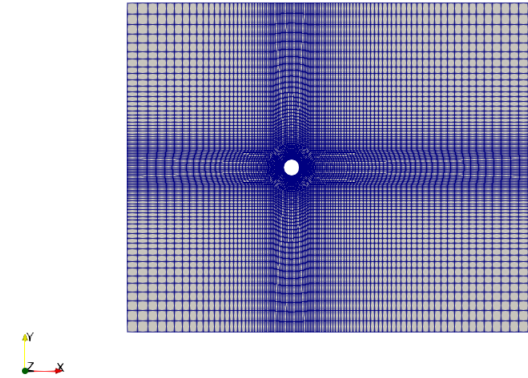
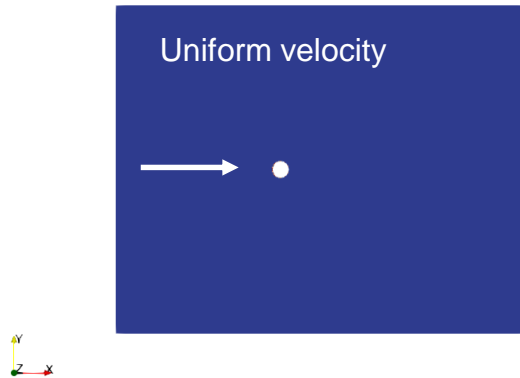
Strouhal number $St = \frac{fL}{U}$ for a smooth cylinder [2]

References:

1. Fox, Robert W., et al. Introduction to Fluid Mechanics. Hoboken, NJ, Wiley, 2010
2. Sumer, B. Mutlu, et al. Hydrodynamics Around Cylindrical Structures. Singapore, World Scientific, 2006

Introduction to turbulence modeling

What happens when we increase the Reynolds number?

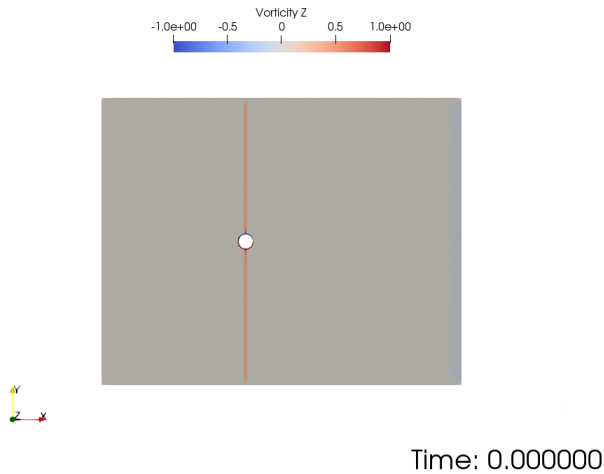


Field initialization - Velocity

Mesh

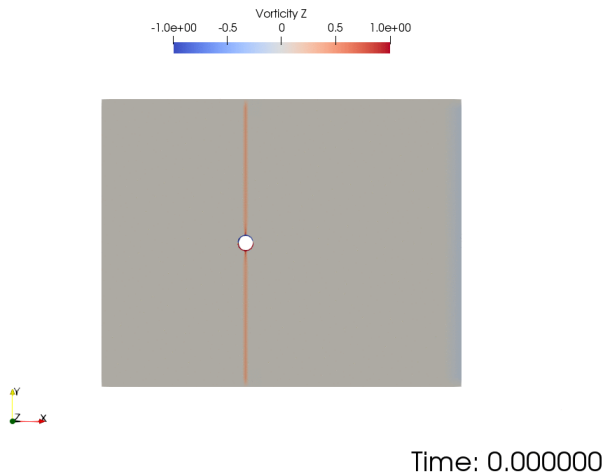
Introduction to turbulence modeling

What happens when we increase the Reynolds number?



Reynolds = 20 – Non-uniform initialization
Laminar flow with separation

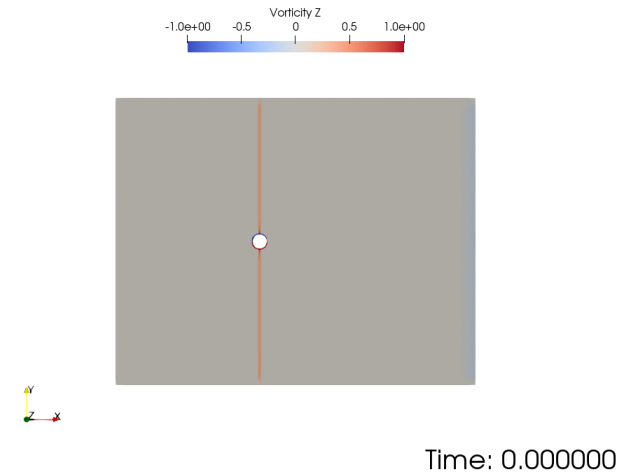
<http://www.wolfdynamics.com/training/turbulence/unscyl1.gif>



Reynolds = 50 – Non-uniform initialization

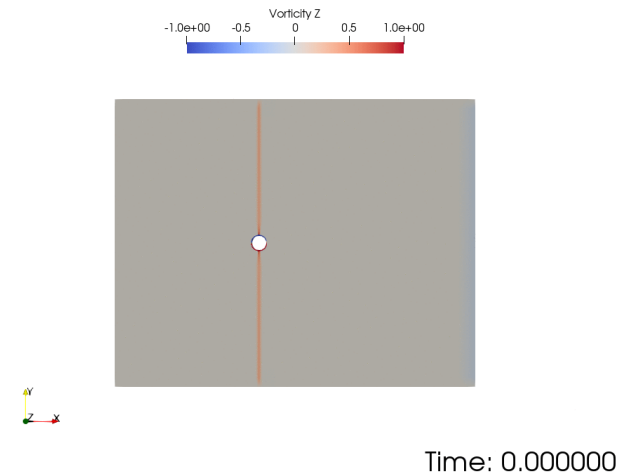
Laminar flow with vortex shedding (maybe on the limit of the onset of the Von Karman street.

<http://www.wolfdynamics.com/training/turbulence/unscyl2.gif>



Reynolds = 200 – Non-uniform initialization
Laminar flow with vortex shedding

<http://www.wolfdynamics.com/training/turbulence/unscyl3.gif>



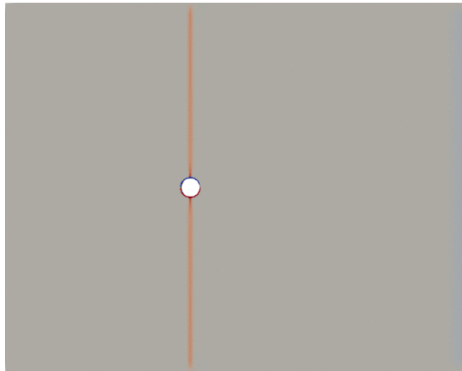
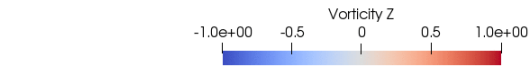
Reynolds = 20000 – Non-uniform initialization

Turbulent flow with vortex shedding (turbulence model enable)

<http://www.wolfdynamics.com/training/turbulence/unscyl4.gif>

Introduction to turbulence modeling

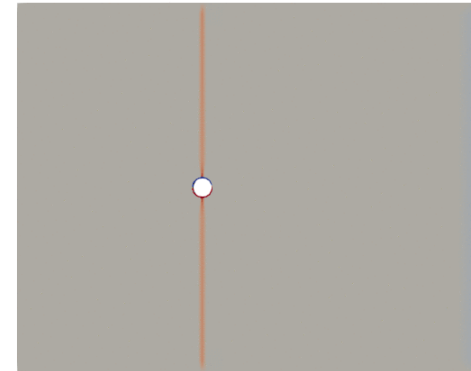
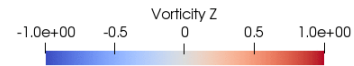
What happens when we increase the Reynolds number?



Time: 0.000000

Reynolds = 20000 – Non-uniform initialization
No turbulence model enable

<http://www.wolfdynamics.com/training/turbulence/unscyl5.gif>



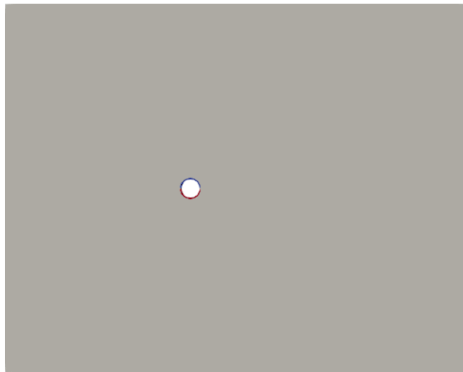
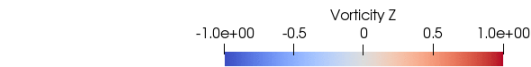
Time: 0.000000

Reynolds = 20000 – Non-uniform initialization
Turbulence model enable (k-omega SST)

<http://www.wolfdynamics.com/training/turbulence/unscyl4.gif>

Introduction to turbulence modeling

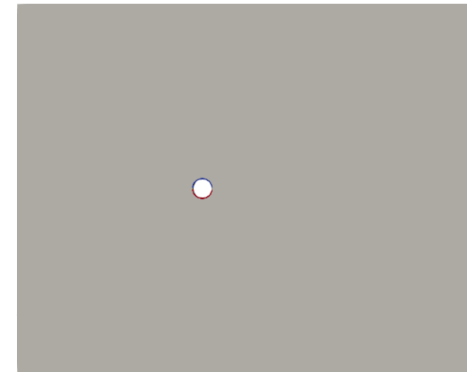
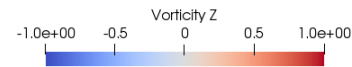
What happens when we increase the Reynolds number?



Time: 0.000000

Reynolds = 20000 – Uniform initialization
Turbulence model enable (k-omega SST)

<http://www.wolfdynamics.com/training/turbulence/unscyl6.gif>



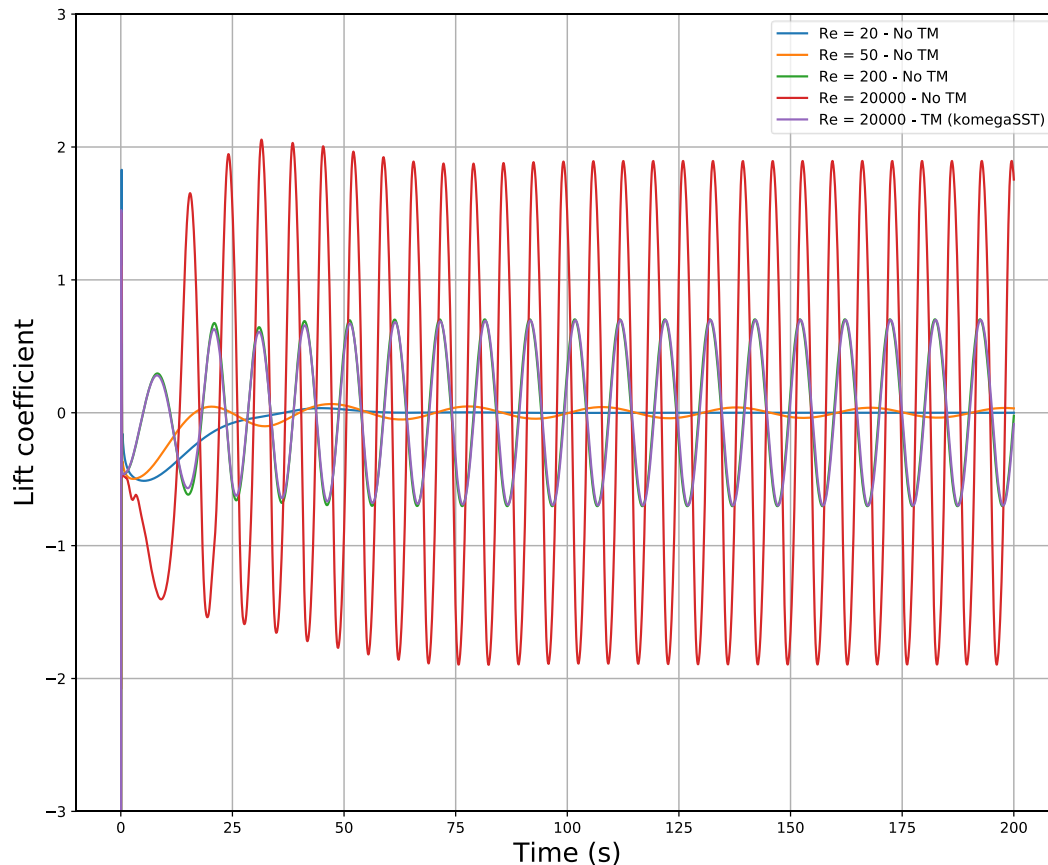
Time: 0.000000

Reynolds = 20000 – Uniform initialization
Turbulence model enable (LES Smagorisky)

<http://www.wolfdynamics.com/training/turbulence/unscyl7.gif>

Introduction to turbulence modeling

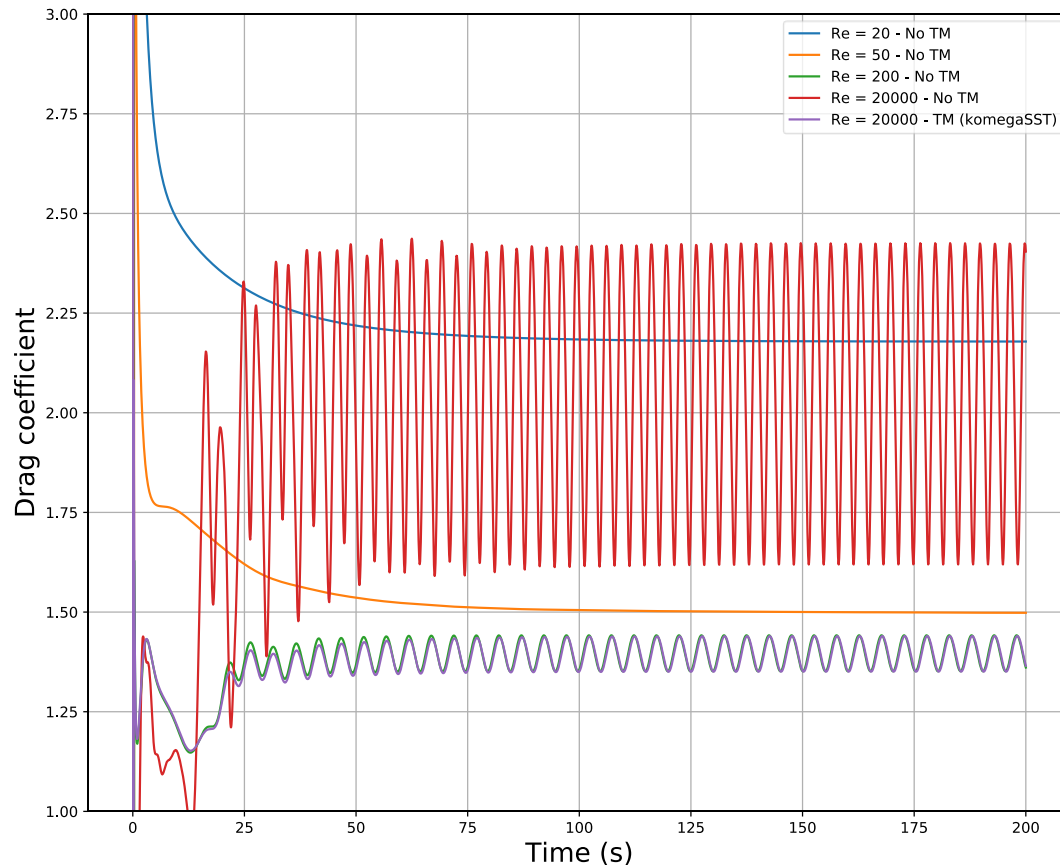
What happens when we increase the Reynolds number?



Reynolds	Turbulence model	Mean c_l
20	No	-0.00012
50	No	0.00274
200	No	-0.00149
20000	No	0.02176
20000	K-Omega SST	-0.00214

Introduction to turbulence modeling

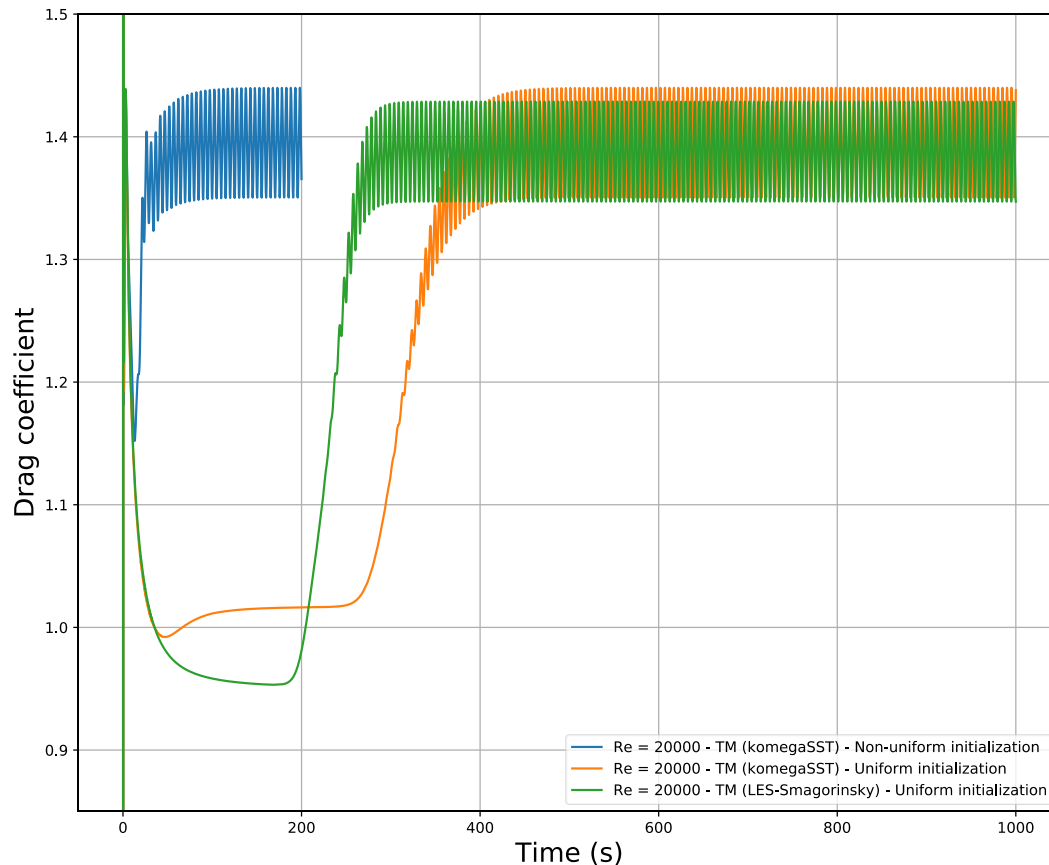
What happens when we increase the Reynolds number?



Reynolds	Turbulence model	Mean c_d
20	No	2.17987
50	No	1.50056
200	No	1.39786
20000	No	2.05043
20000	K-Omega SST	1.39459

Introduction to turbulence modeling

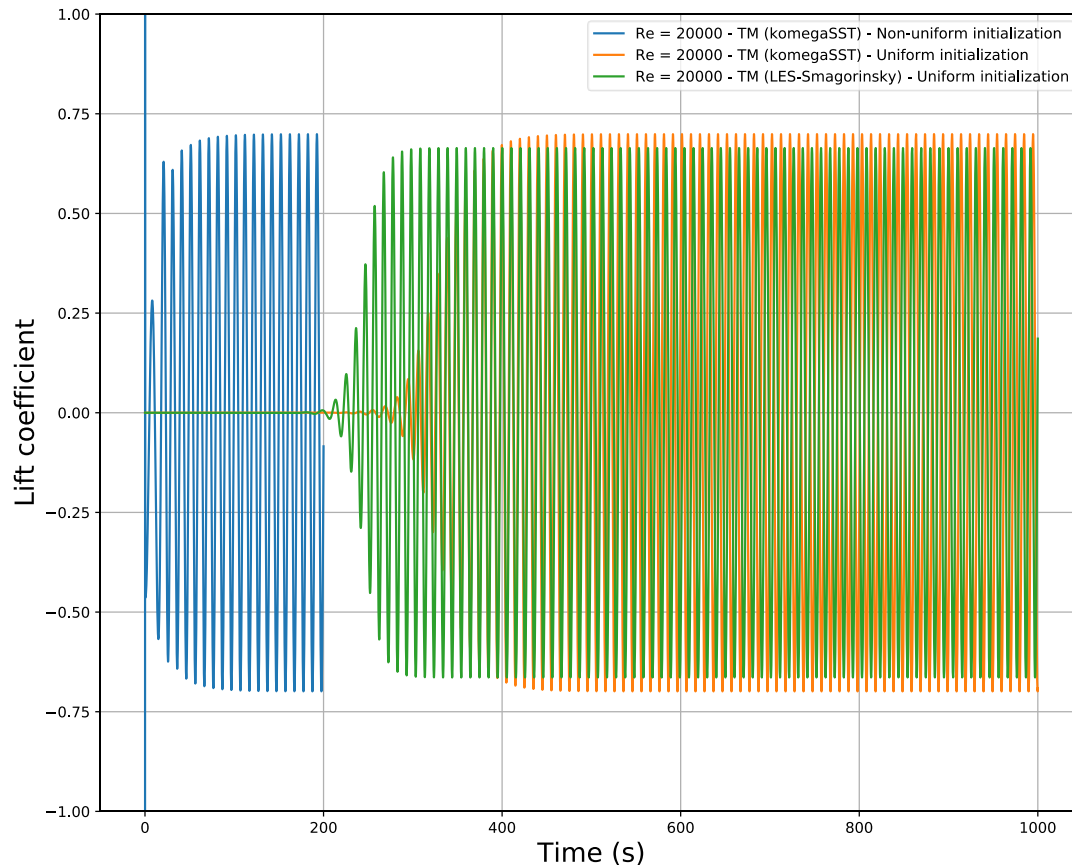
What happens when we increase the Reynolds number?



Reynolds	Turbulence model	Mean c_d
20000	K-Omega SST (NUI)	1.39786
20000	K-Omega SST (UI)	1.39617
20000	LES-Smagorinsky (UI)	1.38865

Introduction to turbulence modeling

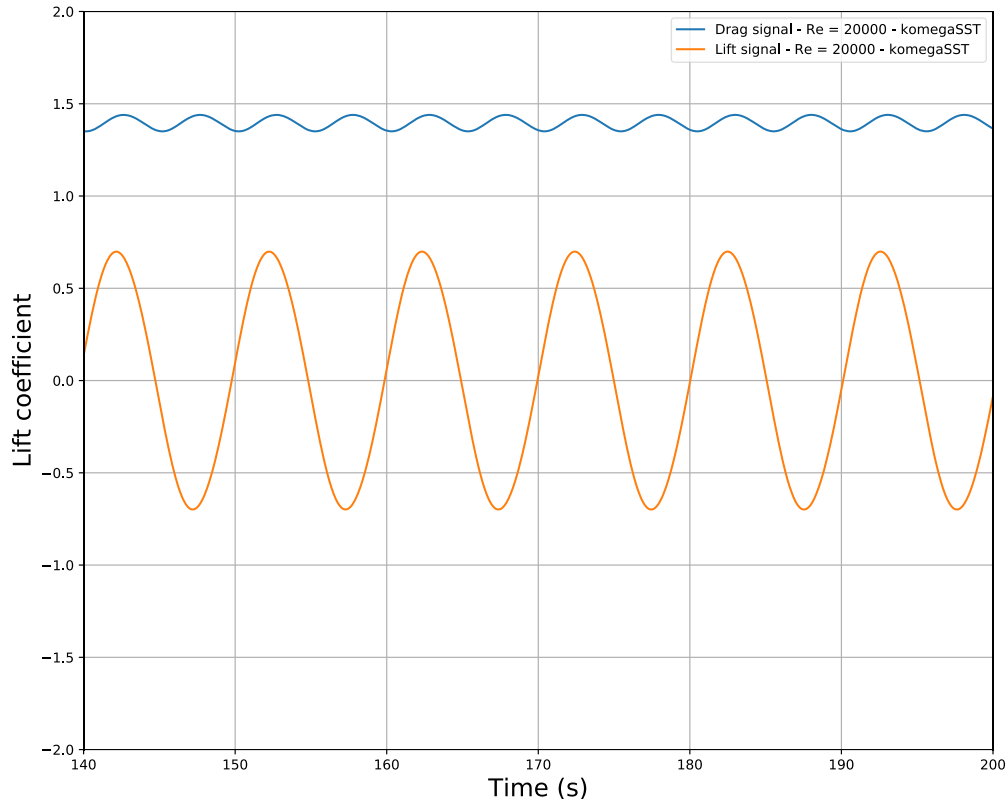
What happens when we increase the Reynolds number?



Reynolds	Turbulence model	Mean c_l
20000	K-Omega SST (NUI)	-0.00214
20000	K-Omega SST (UI)	0.00190
20000	LES-Smagorinsky (UI)	-0.00118

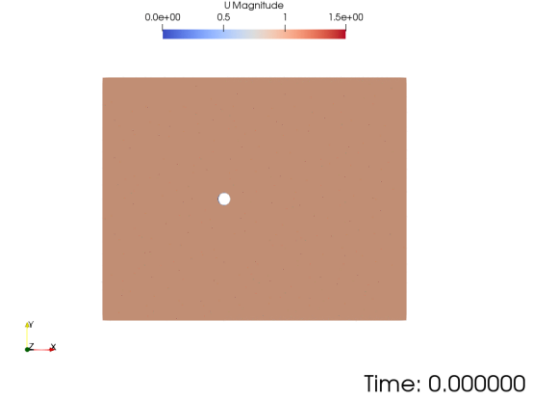
Introduction to turbulence modeling

What happens when we increase the Reynolds number?

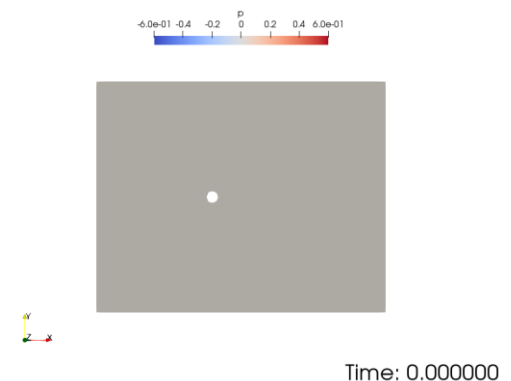


- Do you notice anything peculiar in the force coefficient signals?
- Look at the frequencies?
- Try different Reynolds number, do you see the same behavior?

<http://www.wolfdynamics.com/training/turbulence/unscyl8.gif>



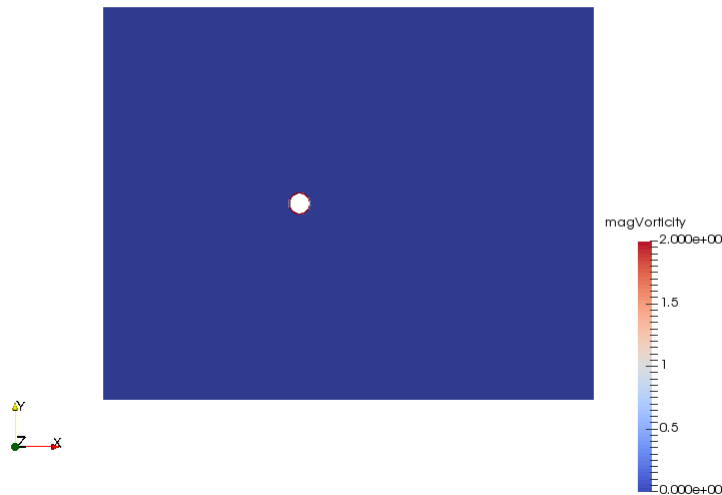
<http://www.wolfdynamics.com/training/turbulence/unscyl9.gif>



Introduction to turbulence modeling

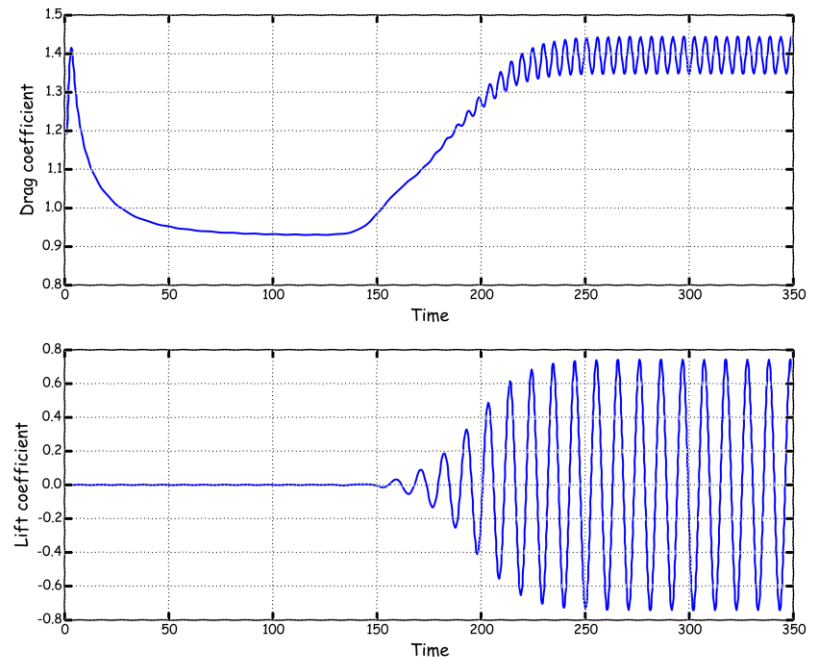
Vorticity does not always mean turbulence

Time: 0.000000



Instantaneous vorticity magnitude field

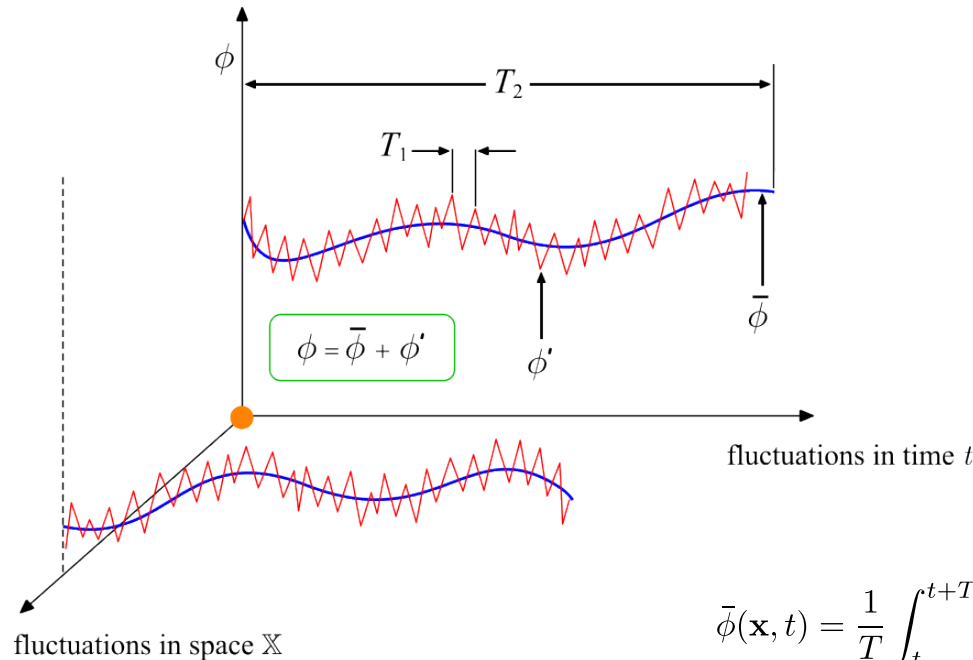
<http://www.wolfdynamics.com/training/turbulence/image6.gif>



- The Reynolds number in this case is 100, for these conditions, the flow still is laminar.
- We are in the presence of the Von Karman vortex street, which is the periodic shedding of vortices caused by the unsteady separation of the fluid around blunt bodies.
- Vorticity is not a direct indication of turbulence.
- However, turbulent flows are rotational, they exhibit vortical structures.

Introduction to turbulence modeling

Turbulence modeling – Fluctuations of transported quantities

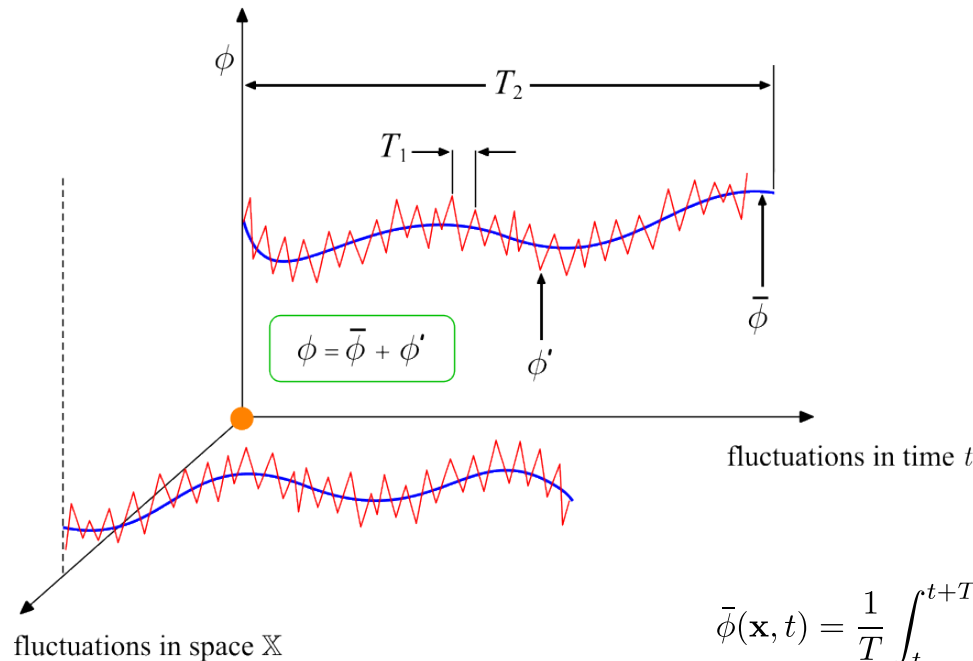


$$\bar{\phi}(\mathbf{x}, t) = \frac{1}{T} \int_t^{t+T} \phi(\mathbf{x}, t) dt, \quad T_1 \ll T \ll T_2$$

- We have defined turbulence as an unsteady, aperiodic motion in which velocity components and every transported quantity fluctuate in space and time.
- For most engineering applications it is impractical to account for all these instantaneous fluctuations.
- Therefore, we need to somehow remove, avoid, or filter those small scales by using models.
- To remove, avoid, or filter the instantaneous fluctuations (or small scales), two methods can be used: Reynolds averaging and filtering the governing equations.
- Both methods introduce additional terms that must be modeled for closure, Turbulence Modeling.
- We are going to talk about closure methods later.

Introduction to turbulence modeling

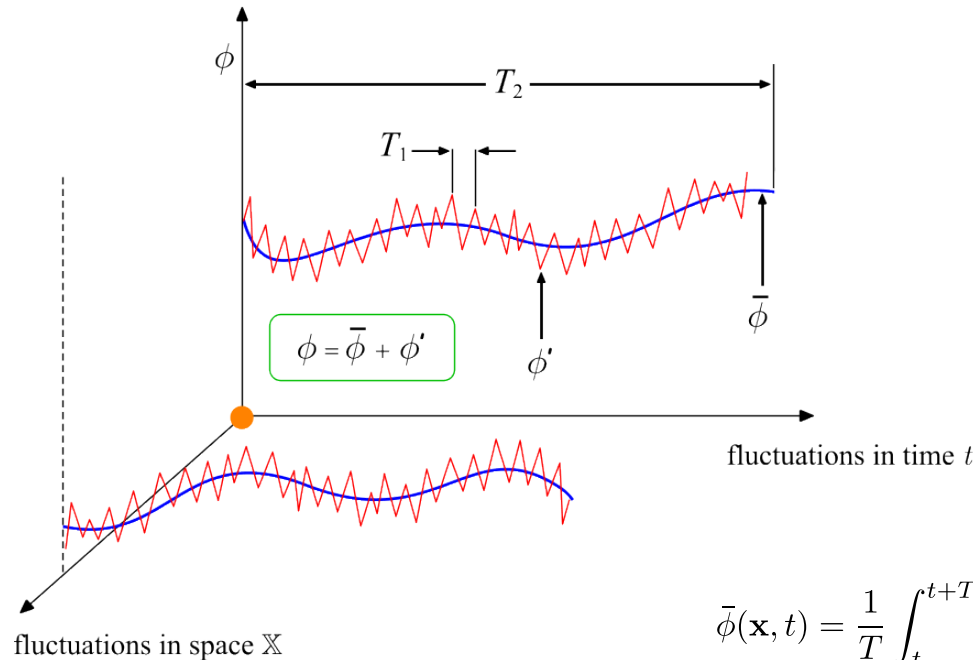
Turbulence modeling – Fluctuations of transported quantities



- The objective of turbulence modeling is to develop equations that will predict the time averaged primitive fields (velocity, pressure, temperature, concentration, and so on), without calculating the complete turbulent flow pattern as a function of time.
- In other words, we do not want to resolve all the space and time scales.
 - This reduces the computational time and resources.
 - However, you can also compute instantaneous values and calculate other statistical properties (e.g., RMS, two point-correlations, PDF, and so on).
 - Turbulence modeling is very accurate, if you know what are you doing.

Introduction to turbulence modeling

Turbulence modeling – Fluctuations of transported quantities

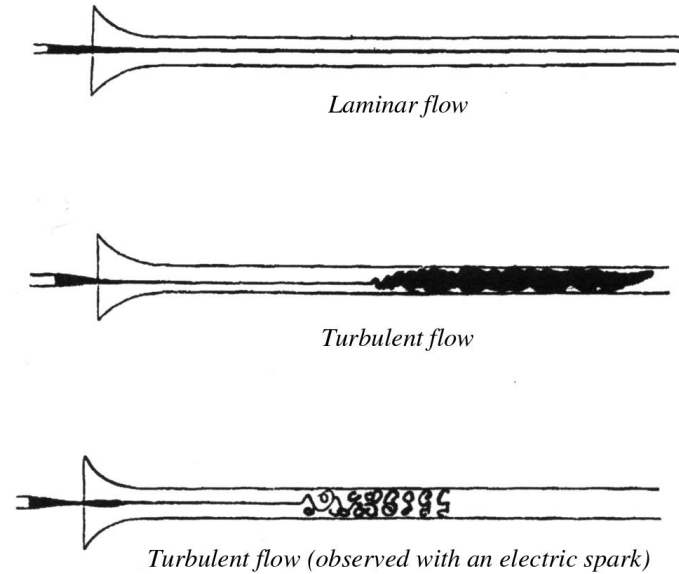
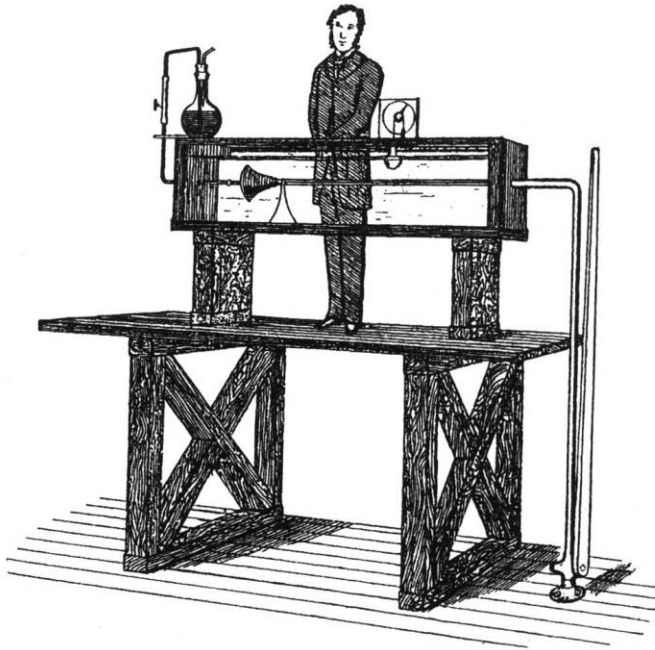


$$\bar{\phi}(\mathbf{x}, t) = \frac{1}{T} \int_t^{t+T} \phi(\mathbf{x}, t) dt, \quad T_1 \ll T \ll T_2$$

- Important to understand:
 - The time averaged flow pattern is a statistical property of the flow.
 - It is not the actual flow pattern.
 - The flow pattern changes from instant to instant.
 - In engineering applications, most of the time it is enough to know the average value.
- Later we are going to make a distinction between averaging stationary turbulence, averaging nonstationary turbulence, and statistically steady turbulence.

Introduction to turbulence modeling

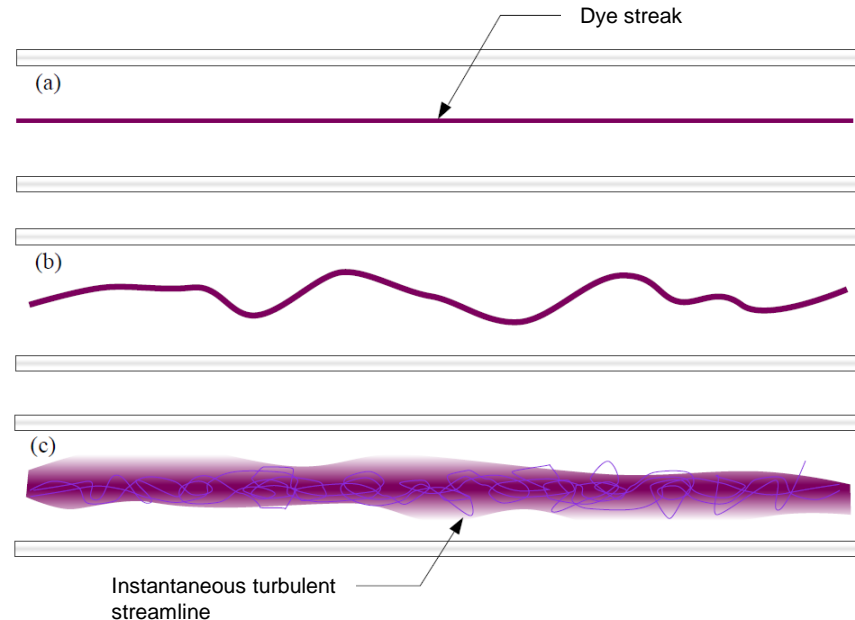
Turbulence modeling – Fluctuations of transported quantities



- Illustration taken from Osborne Reynolds' 1883 influential paper "*An experimental investigation of the circumstances which determine whether the motion of water in parallel channels shall be direct or sinuous and of the law of resistance in parallel channels*".
- Water flows from the tank near the experimenter down to below the ground, through a transparent tube; and dye is injected in the middle of the flow.
- The turbulent or laminar nature of the flow can therefore be observed precisely.

Introduction to turbulence modeling

Turbulence modeling – Fluctuations of transported quantities

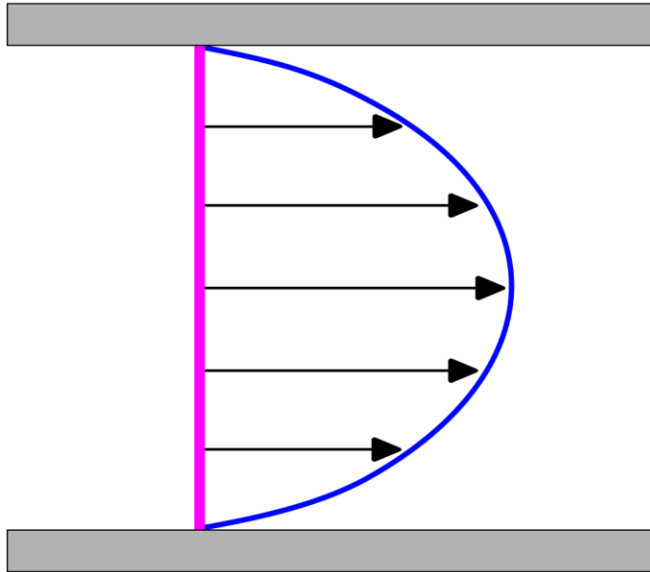


Flow in a pipe. (a) Laminar, (b) Transitional, (c) Turbulent

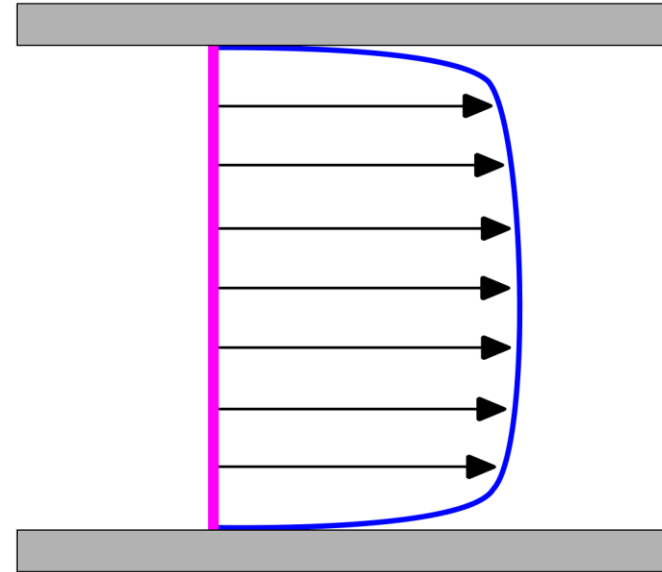
- Turbulence has a direct effect on the velocity profiles and mixing of transported quantities.
- Case (a) corresponds to a laminar flow, where the dye can mix with the main flow only via molecular diffusion, this kind of mixing can take very long times.
- Case (b) shows a transitional state where the dye streak becomes wavy, but the main flow still is laminar.
- Case (c) shows the turbulent state, where the dye streak changes direction erratically, and the dye has mixed significantly with the main flow due to the velocity fluctuations.
- This image gives us an idea of what happens at the core of the flow, but what about the walls?

Introduction to turbulence modeling

Turbulence modeling – Fluctuations of transported quantities



Laminar flow in a pipe

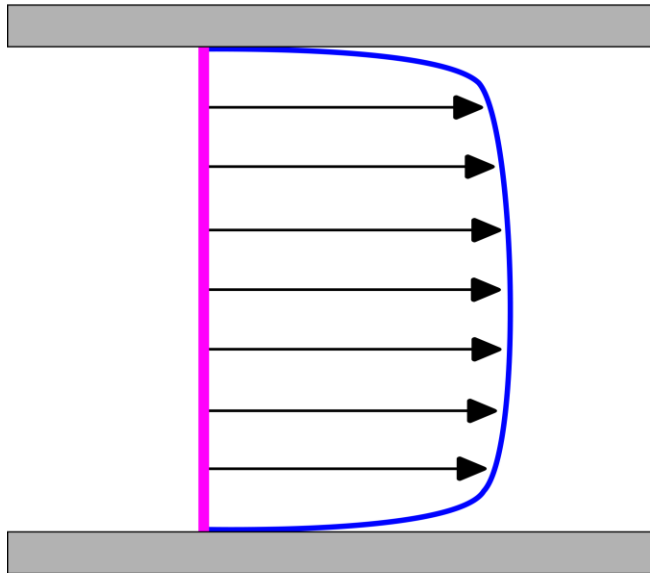


Turbulent flow in a pipe

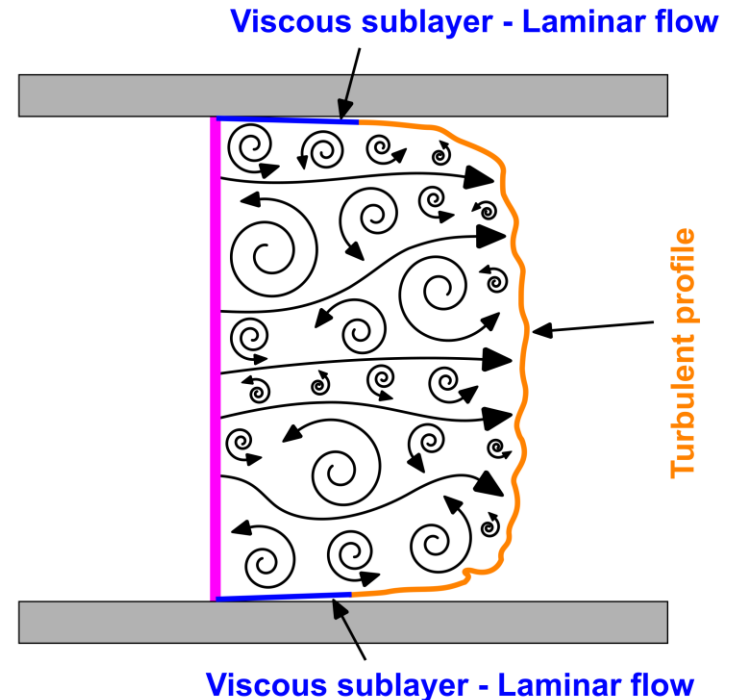
- Turbulence has a direct effect on the velocity profiles and mixing of transported quantities.
- In the laminar case, the velocity gradient close to the walls is small (therefore the shear stresses are lower).
- The turbulent case shows two regions. One thin region close to the walls with very large velocity gradients (hence large shear stresses), and a region far from the wall where the velocity profile is nearly uniform.
- In the illustration, the velocity profile of the turbulent case has been averaged (in reality, there are fluctuations).

Introduction to turbulence modeling

Turbulence modeling – Fluctuations of transported quantities



Averaged turbulent flow

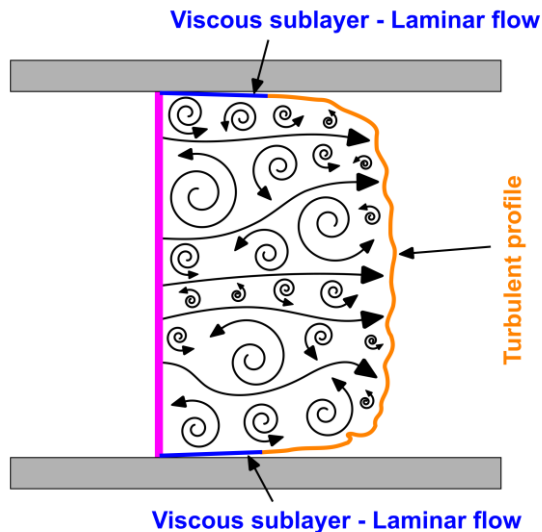


Instantaneous turbulent flow

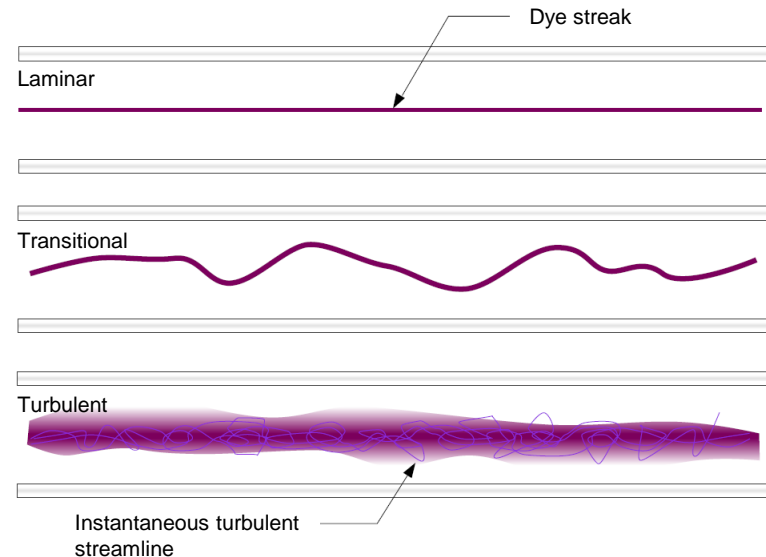
- In the left figure, the velocity profile has been averaged.
- In reality, the velocity profile fluctuates in time (right figure).
- The thin region close to the walls has very large velocity gradients and is laminar.
- Far from the flows, the flow becomes turbulent.

Introduction to turbulence modeling

Turbulence modeling – Fluctuations of transported quantities



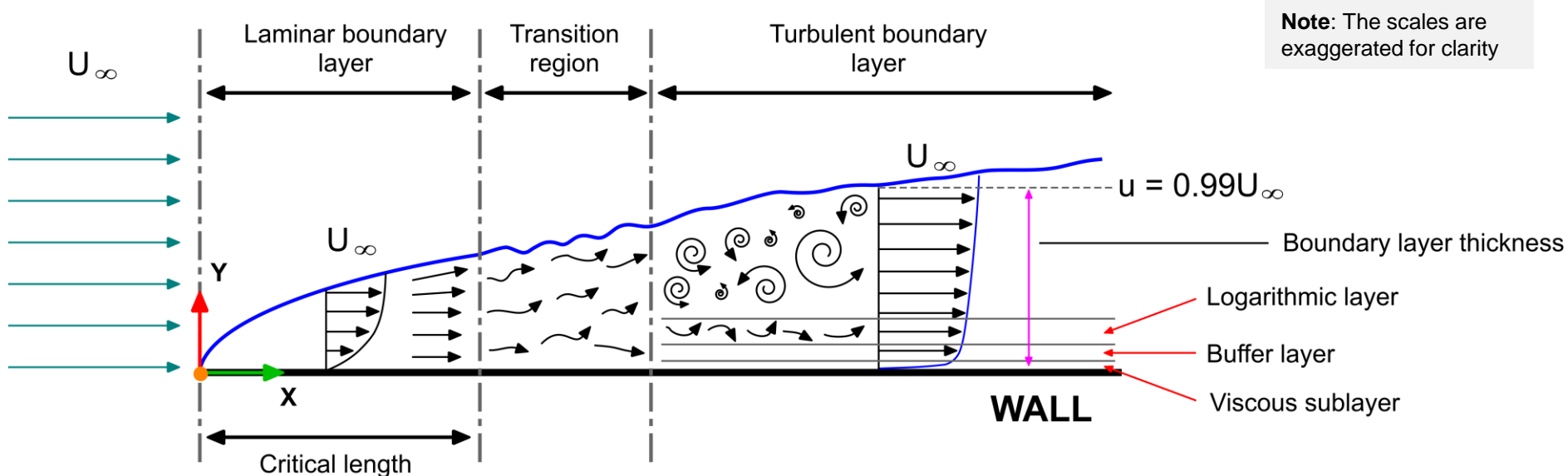
Wall bounded turbulence



Shear free turbulence

- Turbulent flows can originate at the walls. When this is the case, we talk about wall bounded turbulence.
- Turbulent flows can also originate in the absence of walls (or far from walls). When this is the case, we talk about shear free turbulence (usually jets, heated walls, atmospheric flows).

Turbulence modeling – Boundary layer

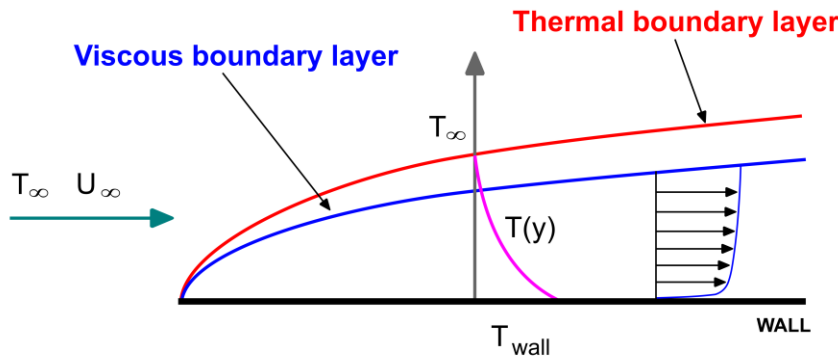


Boundary layer (Laminar-Transitional-Turbulent flow)

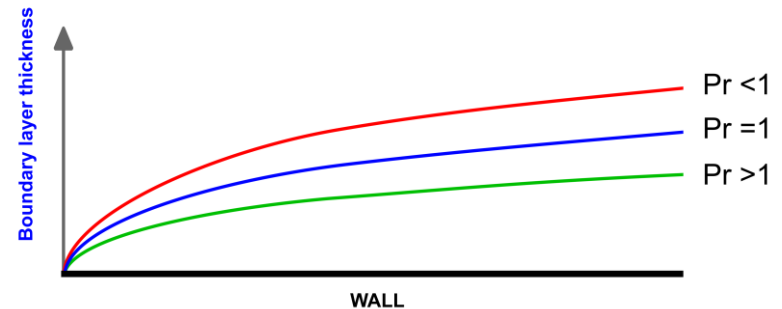
- In this case, a laminar boundary layer starts to form at the leading edge.
- As the flow proceeds further downstream, large shear stresses and velocity gradient develop within the boundary layer. At one point the flow becomes turbulent.
- The turbulent motion increases the mixing and the boundary layer mixing.
- What is happening in the transition region is not well understood. The flow can become laminar again or can become turbulent.
- As for the pipe flow, the velocity profiles in the laminar and turbulent regions are different.

Introduction to turbulence modeling

Turbulence modeling – Thermal boundary layer



Thermal boundary layer vs. Viscous boundary layer
Forced convection



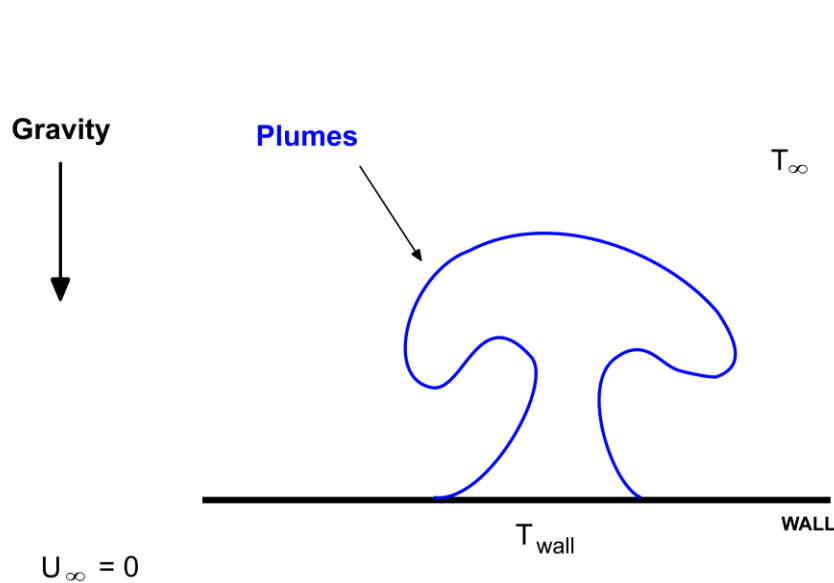
Thermal boundary layer in function of Prandtl number (Pr)

Momentum and thermal boundary layer

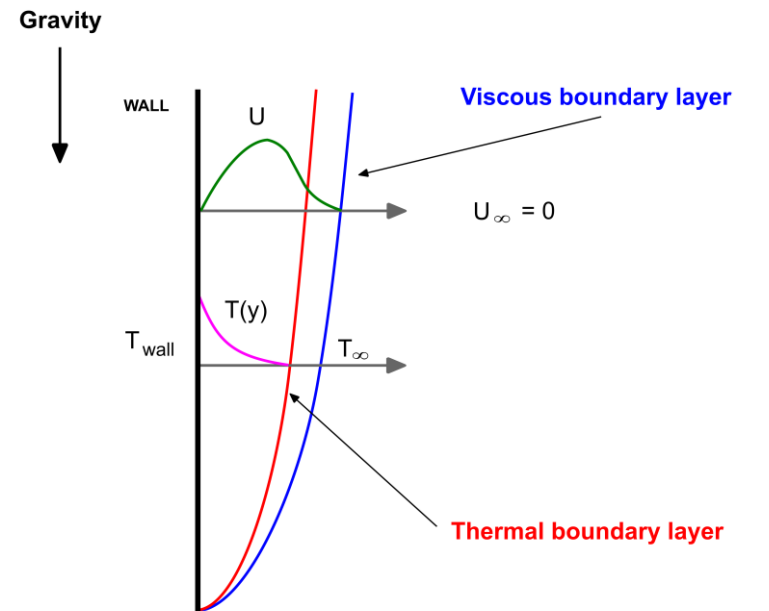
- Just as there is a viscous boundary layer in the velocity distribution (or momentum), there is also a thermal boundary layer.
- Thermal boundary layer thickness is different from the thickness of the viscous sublayer (momentum), and is fluid dependent.
- The thickness of the thermal sublayer for a high Prandtl number fluid (e.g. water) is much less than the momentum sublayer thickness.
- For fluids of low Prandtl numbers (e.g., air), it is much larger than the momentum sublayer thickness.
- For Prandtl number equal 1, the thermal boundary layer is equal to the momentum boundary layer.

Introduction to turbulence modeling

Turbulence modeling – Thermal boundary layer



Horizontal heated plate immersed in a quiescent fluid.
Natural convection



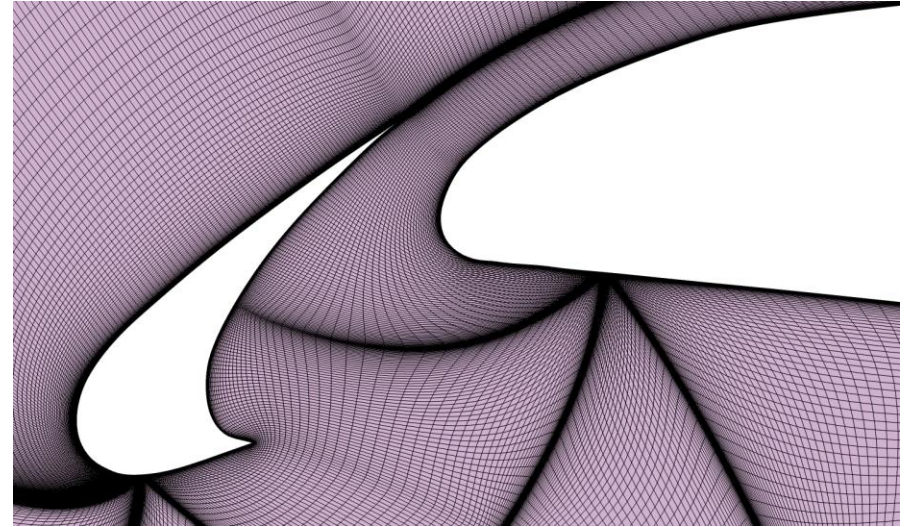
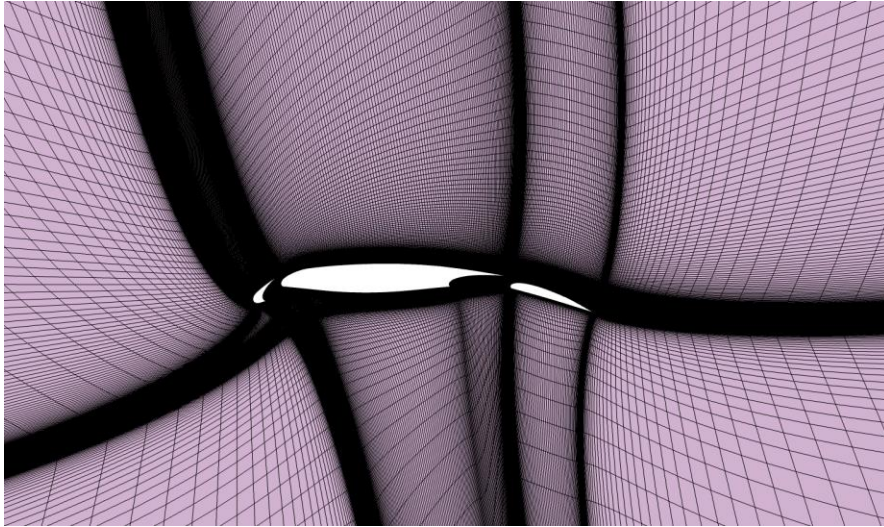
Vertical heated plate immersed in a quiescent fluid.
Natural convection.

Natural convection in a heated plate

- As the fluid is warmed by the plate, its density decreases, and a buoyant force arises which induces flow motion in the vertical or horizontal direction.
- The force is proportional to $(\rho - \rho_{\infty}) \times g$, therefore gravity must be considered.

Introduction to turbulence modeling

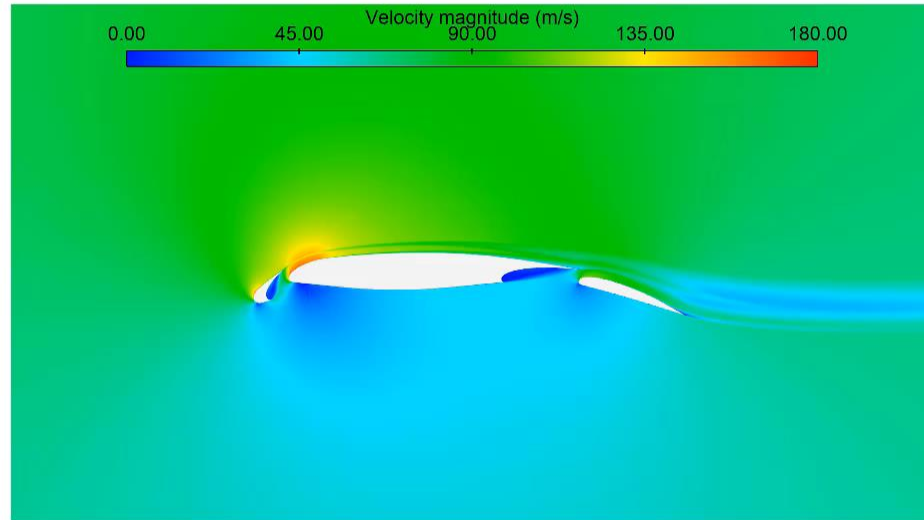
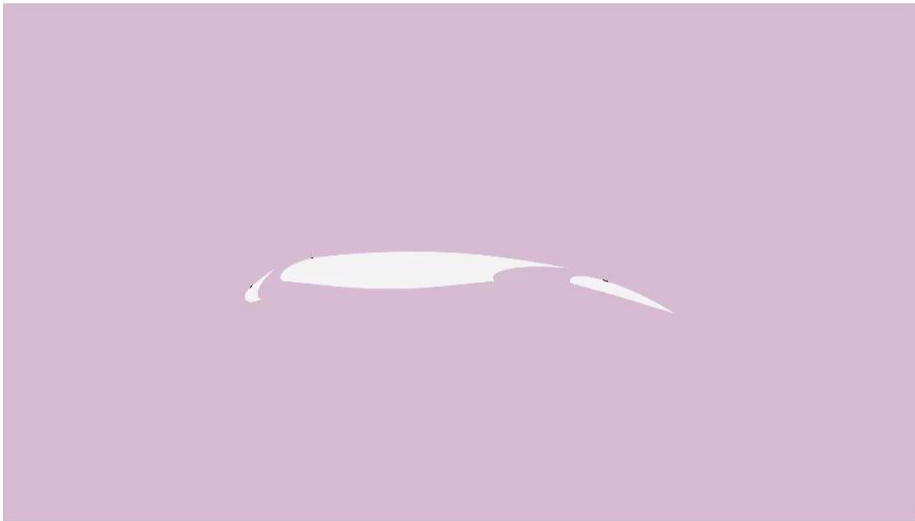
Turbulence modeling – Laminar separation bubbles and transition to turbulence



- Maybe, the most challenging topic of turbulence modeling is the prediction of transition to turbulence.
- Trying to predict transition to turbulence in CFD requires very fine meshes and well calibrated models.
- Many traditional turbulence models assume that the boundary layer is turbulent in all its extension.
- But assuming that the boundary layer is entirely turbulent might not be a good assumption, as in some regions the boundary layer might still be laminar, so we may be overpredicting drag forces or predicting wrong separation points.
- In many applications, transition to turbulence is preceded by laminar separation bubbles (LSB), which are laminar recirculation areas that separate from the wall and reattach in a very short distance and are very sensitive to disturbances.
- After the LSB, the flow becomes turbulent.

Introduction to turbulence modeling

Turbulence modeling – Laminar separation bubbles and transition to turbulence

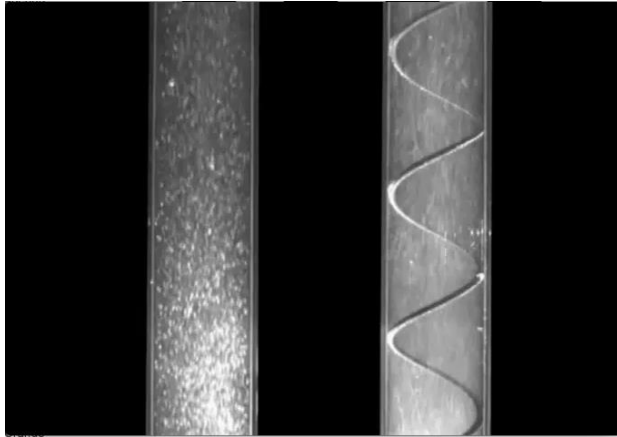


Laminar separation bubbles

<http://www.wolfdynamics.com/images/airfoil.mp4>

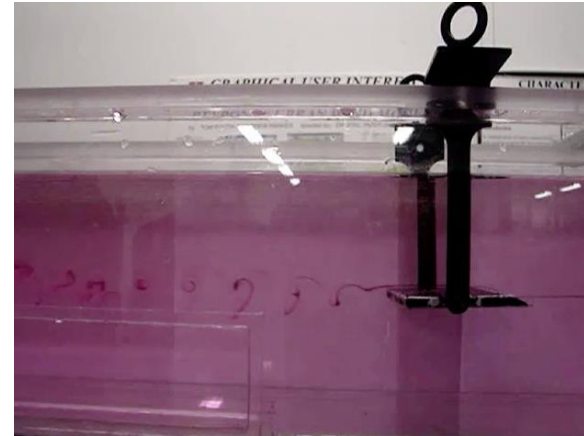
Introduction to turbulence modeling

Turbulence flows videos



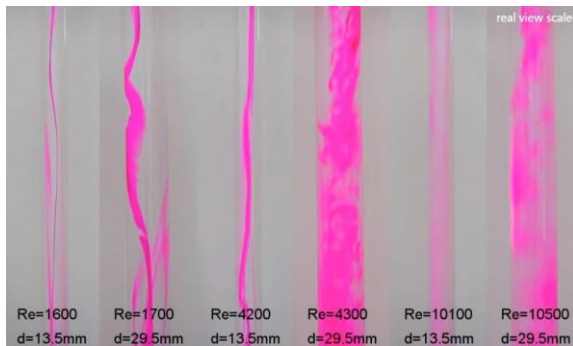
Laminar-Turbulent flow in a pipe
Smooth vs corrugated tube

<https://www.youtube.com/watch?v=WG-YCpAGgQQ>



Flow over a flat plate
Attached and separated boundary layer

https://www.youtube.com/watch?v=zsO5BQA_CZk



Reynolds' dye experiment

Various types of flow - Laminar-Transitional-Turbulent

<https://www.youtube.com/watch?v=ontHCul6eB4>



Laminar-Turbulent vortex shedding

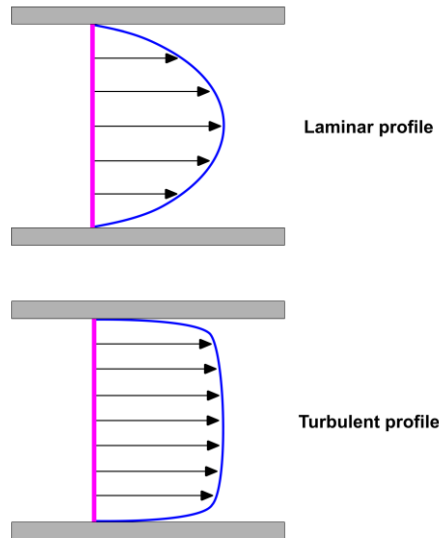
<https://www.youtube.com/watch?v=JI0M1gVNhbw>

Roadmap to Lecture 2

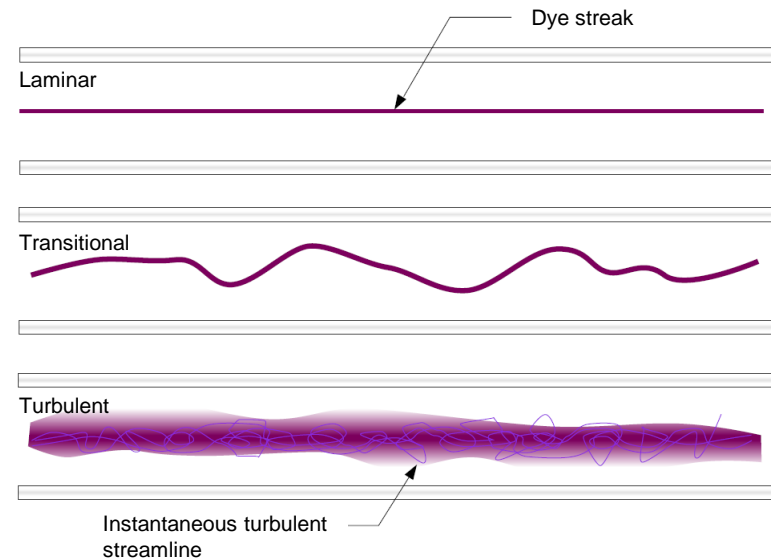
- ~~1. The turbulent world around us~~
- ~~2. Turbulence, does it matter?~~
- ~~3. Introduction to turbulence modeling~~
- 4. Wall bounded flows and shear flows**

Wall bounded flows and shear flows

Wall bounded and shear free flows



Wall bounded turbulence

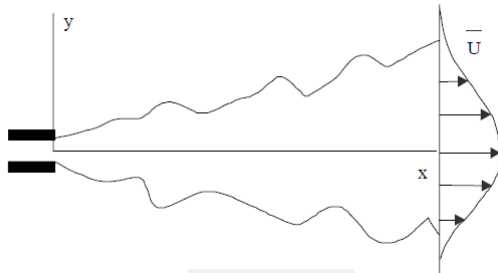


Shear free turbulence

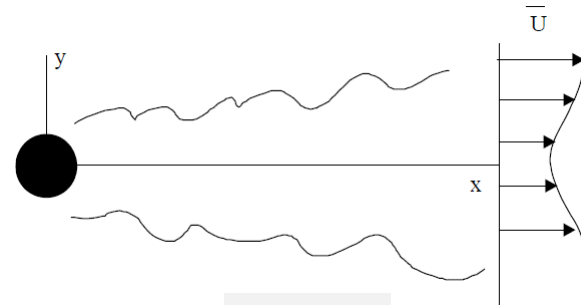
- Flows (laminar or turbulent), can originate at the walls. When this is the case, we talk about wall bounded turbulence (boundary layers).
- They can also originate in the absence of walls (not bounded to walls), or far from walls. When this is the case, we talk about shear free turbulence (jets, har wakes, mixing layers, thermal plumes, atmospheric flows).

Wall bounded flows and shear flows

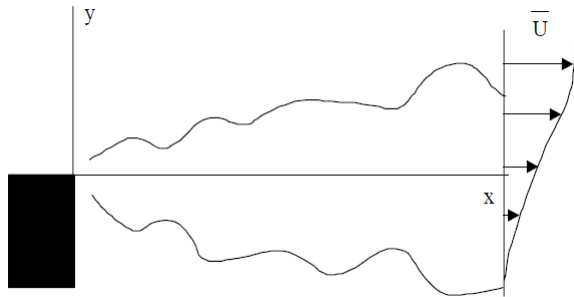
Shear free flows samples



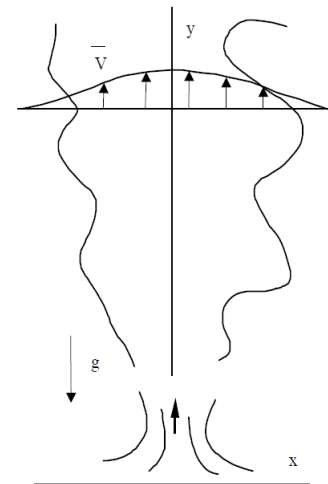
Jet



Far wake



Mixing layer

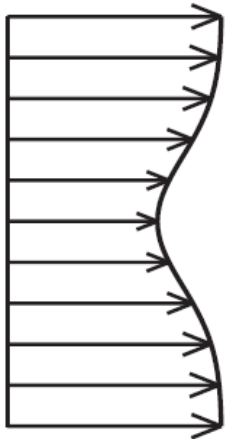


Thermal plume

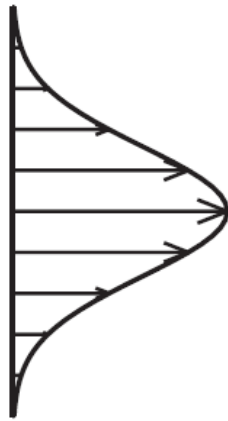
Wall bounded flows and shear flows

Shear free flows samples

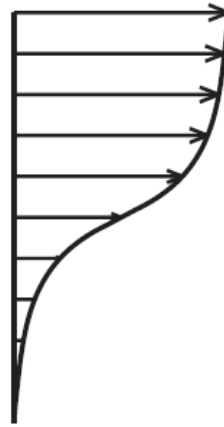
Different velocity profiles of shear free flows



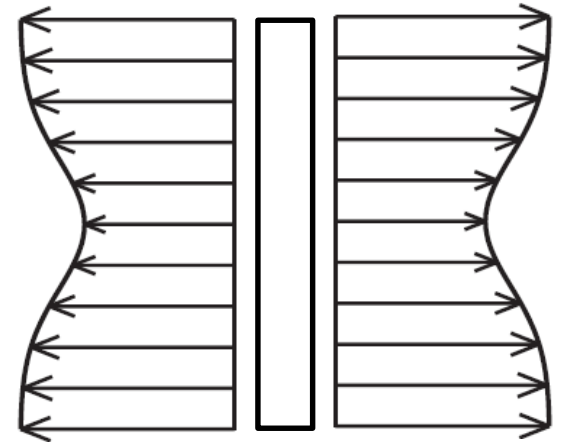
Wake



Plane Jet



Mixing layer

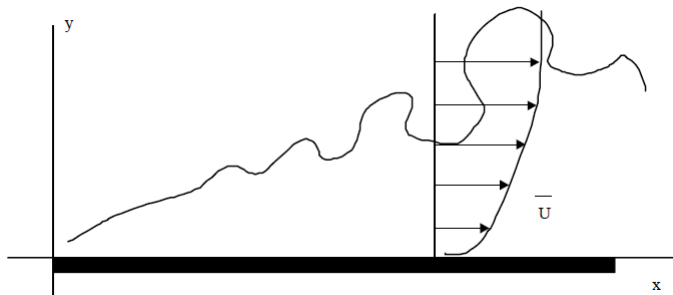


Radial jet

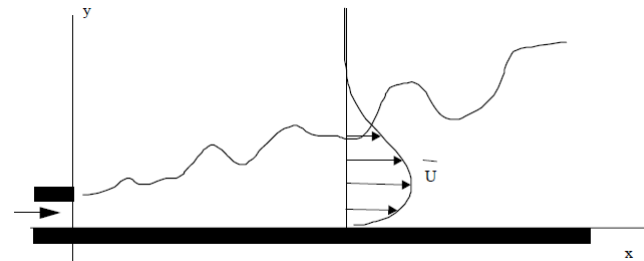
- Under certain conditions, many of these flows have analytical solutions.
- Therefore, turbulence models can be calibrated or validated using these solutions.
- Sample of analytical solutions (self-similarity solutions):
 - Far wake (Schlichting-Gersten, 2017)
 - Mixing layer (Liepmann and Laufer, 1947)
 - Plane jet (Witze and Dwyer, 1976).

Wall bounded flows and shear flows

Wall bounded flows samples



Turbulent boundary layer on a flat plate



Turbulent wall jet

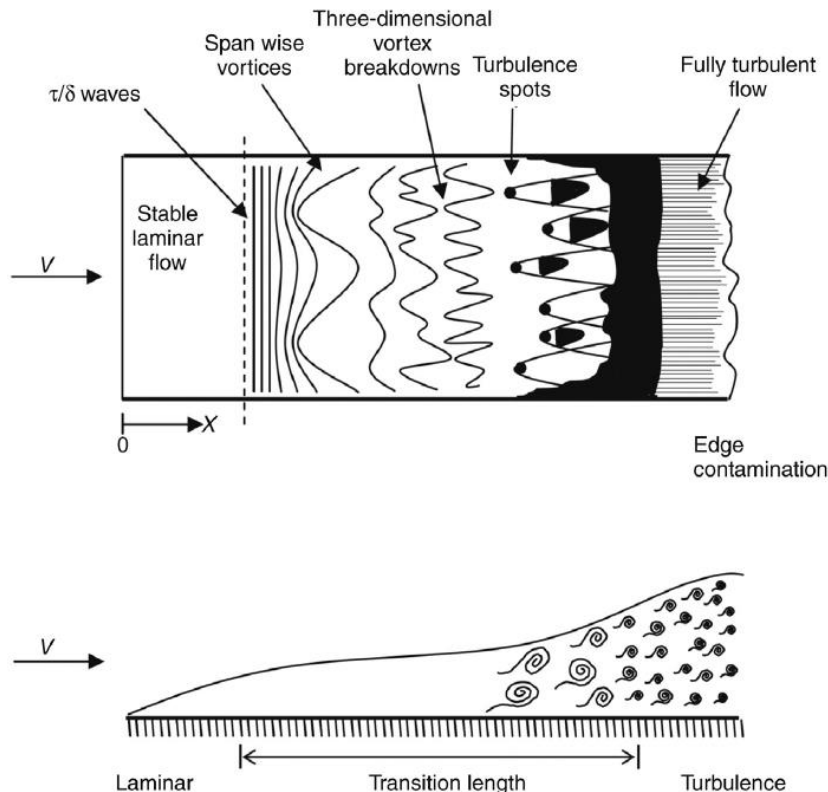


Turbulent flow in a channel or pipe

- Walls can be heated, which adds buoyancy and thermal diffusivity to the physics.
- Walls can also be rough, which will affect the boundary layer.
- Do you think in something else?

Wall bounded flows and shear flows

Wall bounded flows samples

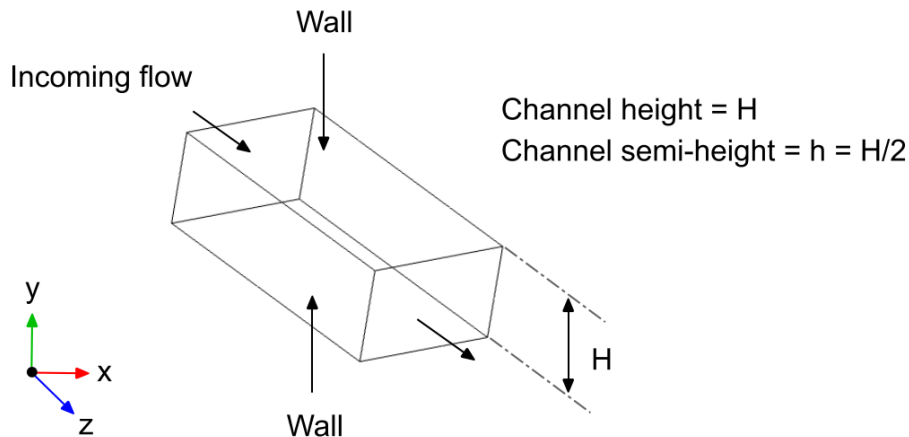


- In these cases, the no-slip boundary condition must be enforced at the walls, and we expect to find a boundary layer similar to the one depicted in the figure.
- The transition to turbulence can follow three different paths:
 - Natural.
 - Bypass.
 - Forced.
- Under certain conditions, these flows have analytical solutions.
- Therefore, turbulence models can be calibrated or validated using these solutions.
- Sample of analytical solutions:
 - Channel flows (Pope, 2000).
 - Boundary layers (Prandtl, 1925; Bradshaw et al. 1995; Pope, 2000).
 - Couette flow (Schlichting-Gersten, 2017)

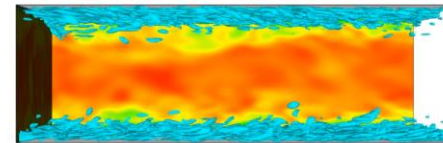
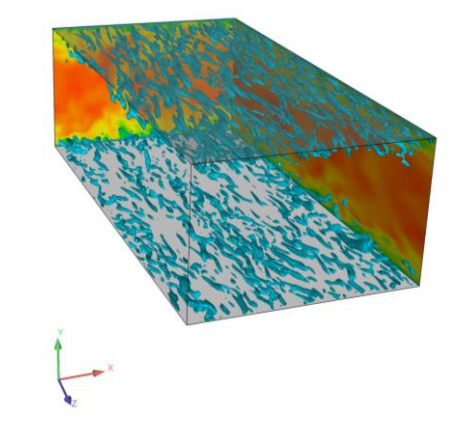
Wall bounded flows and shear flows

Wall bounded flow – Channel flow

- You will find many research on the channel flow as it has an analytical solution under very specific conditions.



Periodic boundary conditions in the streamwise direction (x) and spanwise direction (z)



Wall bounded flows and shear flows

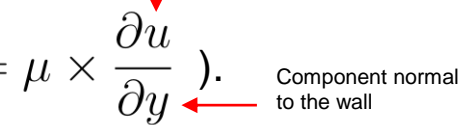
Wall bounded flow – Channel flow

- The channel flow is usually characterized using the Re_τ , which is defined as follows,

$$Re_\tau = \frac{U_\tau \times h}{\nu} \quad Re \approx 20 \times Re_\tau$$

- Where h is the channel semi-height, ν is the kinematic viscosity, and U_τ is the shear velocity and is defined as follows,

$$U_\tau = \left(\frac{\tau_{wall}}{\rho} \right)^{0.5}$$

- In the previous equation, τ_{wall} is the shear stress at the wall ($\tau_{wall} = \mu \times \frac{\partial u}{\partial y}$).
- With these conditions and according to the theory of equilibrium for channels, the equilibrium between the imposed pressure drop and the wall shear stresses is given by,

$$\frac{\partial P}{\partial x} h = -\tau_{wall}$$

- At this point, we only need to set a pressure drop.

Wall bounded flows and shear flows

Wall bounded flow – Channel flow

- The following DNS simulation was conducted at a Re_τ equal to,

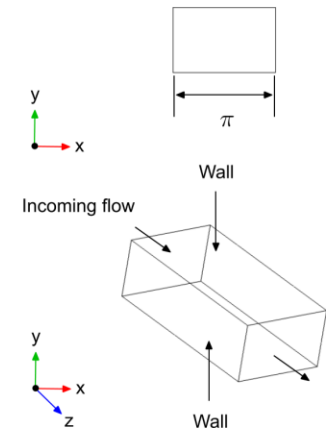
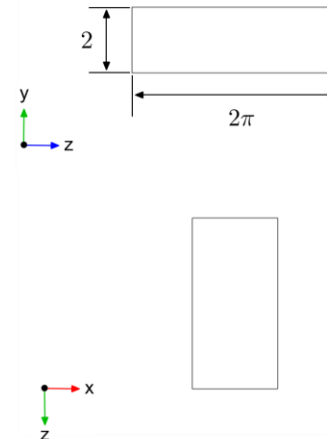
$$Re_\tau = \frac{U_\tau \times h}{\nu} = 590$$

- With the following parameters,

$$\rho = 1 \frac{kg}{m^3}$$

$$\mu = 0.001695 \frac{kg}{m \cdot s}$$

$$h = 1 m$$



Periodic boundary conditions in the streamwise direction (x) and spanwise direction (z)

- Where h is the channel semi-height.
- Notice that this is an incompressible flow.

Wall bounded flows and shear flows

Wall bounded flow – Channel flow

- Periodic boundary conditions in the streamwise (z) and spanwise (x) directions were used.
- The top and bottom walls are no-slip walls.
- To onset the fluid flow we imposed a pressure drop equal to 1 Pa, such as,

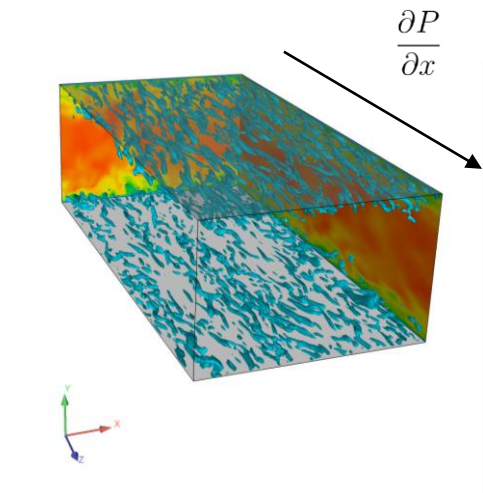
$$\frac{\partial P}{\partial x} h = -1 \frac{Pa}{m}$$

- Therefore the shear stresses at the wall are equal to,

$$\tau_{wall} = 1 Pa$$

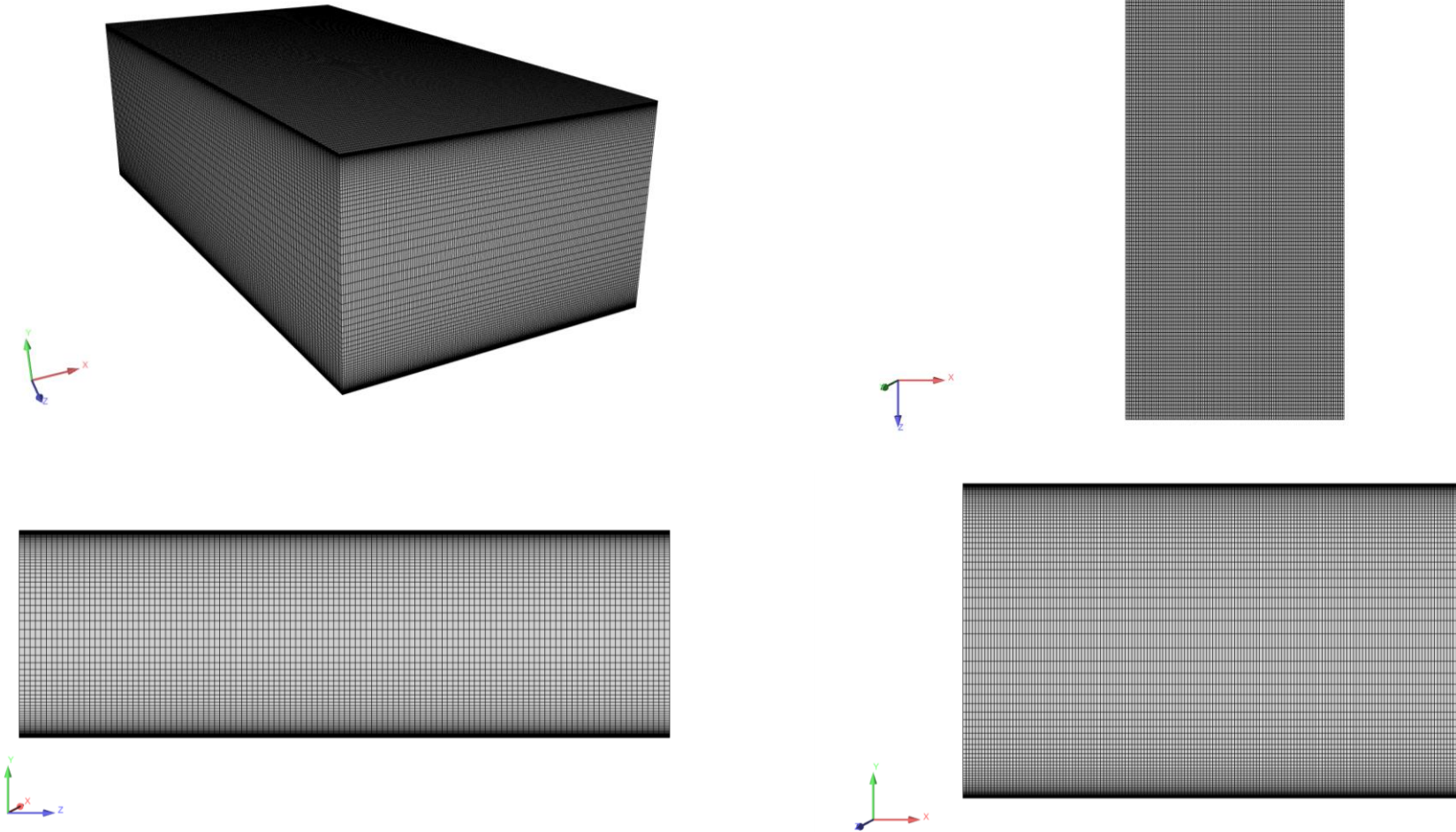
- Finally, the shear velocity is equal to,

$$U_{\tau} = \left(\frac{\tau_{wall}}{\rho} \right)^{0.5} = 1 \frac{m}{s}$$



Wall bounded flows and shear flows

Wall bounded flow – Channel flow

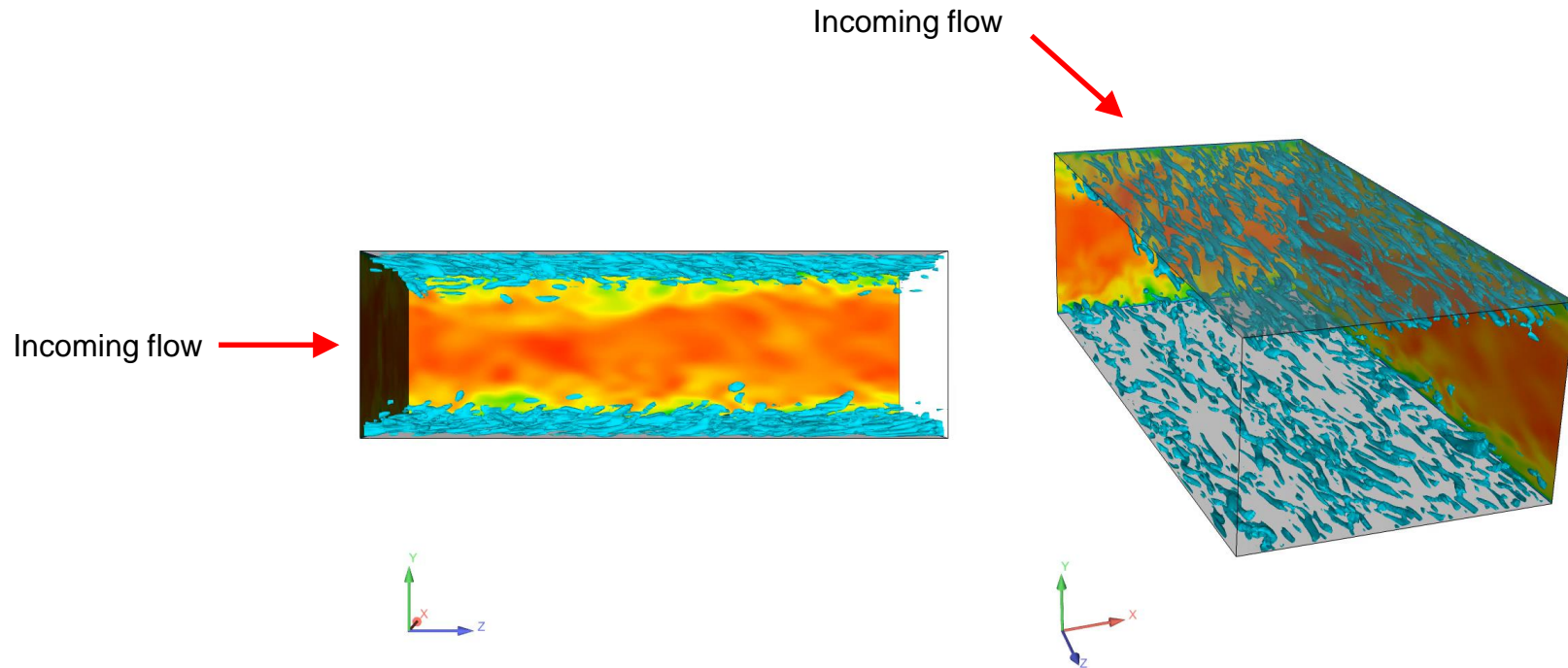


Mesh

Orthogonal hexahedral mesh – Approximately 1.8 million elements

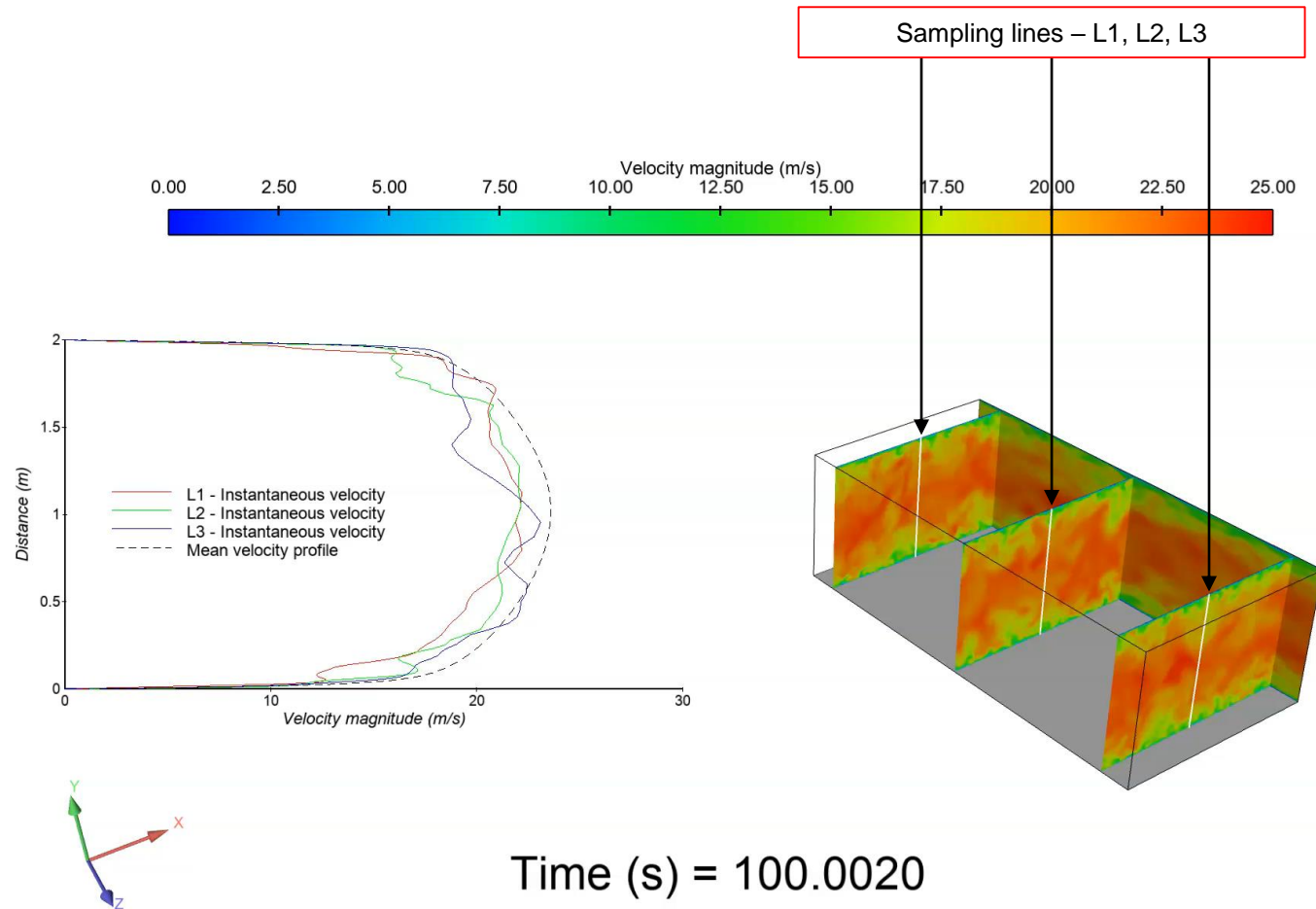
Wall bounded flows and shear flows

Wall bounded flow – Channel flow



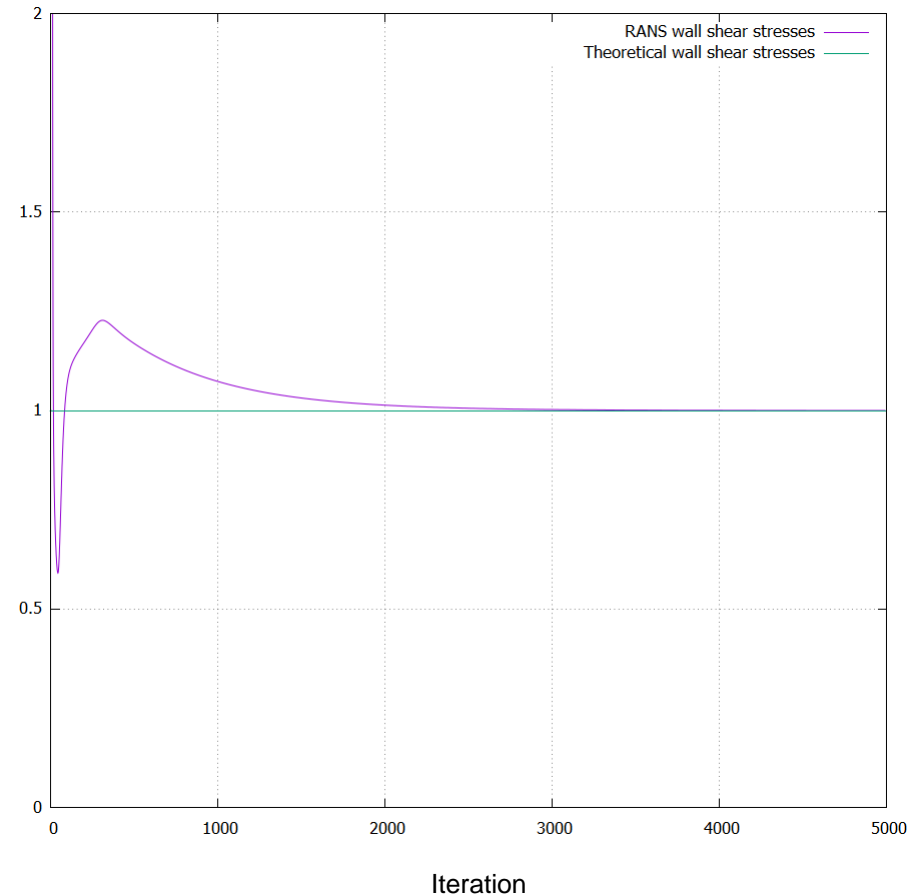
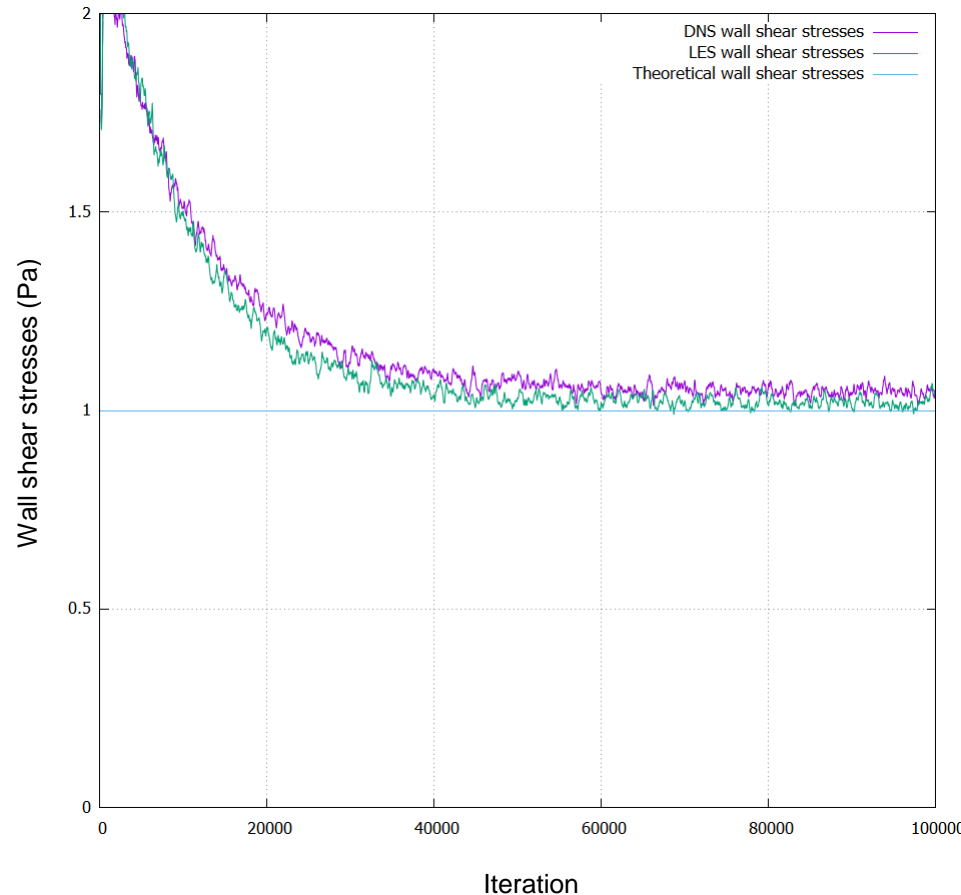
Wall bounded flows and shear flows

Wall bounded flow – Channel flow



Wall bounded flows and shear flows

Wall bounded flow – Channel flow



- The DNS both took approximately 150 hours on 8 cores ($CFL < 1$).
- The RANS simulation took approximately 1 hour on 1 core.

Wall bounded flows and shear flows

Links to a few impressive simulations

- Supercomputer Simulation of NASA's Orion Launch Abort Vehicle
 - https://www.youtube.com/watch?time_continue=1&v=vFgxdD7_LPs&feature=emb_logo
- Aircraft landing gear air flow supercomputer simulation - NASA Ames Research Center
 - https://www.youtube.com/watch?v=-D5N_OnZ_Tg
- Turbulent Boundary Layer (DNS)
 - <https://www.youtube.com/watch?v=W984EOmNaY>
- DNS $Re=400000$ NACA4412
 - <https://www.youtube.com/watch?v=aR-hehP1pTk>
- Exploring Drone Aerodynamics With Computers
 - <https://www.youtube.com/watch?v=hywBEaGiO4k>
- Toward Urban Air Mobility: Air Taxis with Side-By-Side Rotors
 - <https://www.youtube.com/watch?v=eA3SJlzWADQ>
- A computational laboratory for the study of transitional and turbulent boundary layers
 - <https://www.youtube.com/watch?v=wXsl4eyupUY>
- Turning on a Dime – Asymmetric Vortex Formation in Hummingbird Maneuvering Flight
 - <https://www.youtube.com/watch?v=PCj-82oYgUs>