Homework

Turbulence and CFD models: Theory and applications
Lecture 5

Question 1

Derive the incompressible Reynolds-Averaged Navier-Stokes (RANS) equations,

$$\begin{aligned} \nabla \cdot (\bar{\mathbf{u}}) &= 0, \\ \frac{\partial \bar{\mathbf{u}}}{\partial t} + \nabla \cdot (\bar{\mathbf{u}}\bar{\mathbf{u}}) &= \frac{-\nabla \bar{p}}{\rho} + \nu \nabla^2 \bar{\mathbf{u}} + \frac{1}{\rho} \nabla \cdot \boldsymbol{\tau}^R \end{aligned} \tag{1}$$

where $\boldsymbol{\tau}^R = -\rho \left(\overline{\mathbf{u}'\mathbf{u}'} \right)$. Write down all steps, averaging rules, and vector identities used.

Question 2

Using the Boussinesq hypothesis,

$$\boldsymbol{\tau}^R = -\rho \left(\overline{\mathbf{u}'\mathbf{u}'} \right) = 2\mu_T \bar{\mathbf{D}}^R - \frac{2}{3}\rho k \mathbf{I} = \mu_T \left[\nabla \bar{\mathbf{u}} + (\nabla \bar{\mathbf{u}})^{\mathrm{T}} \right] - \frac{2}{3}\rho k \mathbf{I} \tag{2}$$

Derive the solvable incompressible Reynolds-Averaged Navier-Stokes (RANS) equations,

$$\begin{aligned} \nabla \cdot (\bar{\mathbf{u}}) &= 0 \\ \frac{\partial \bar{\mathbf{u}}}{\partial t} + \nabla \cdot (\bar{\mathbf{u}}\bar{\mathbf{u}}) &= -\frac{1}{\rho} \left(\nabla \bar{p} + \frac{2}{3}\rho \nabla k \right) + \nabla \cdot \left[\frac{1}{\rho} (\mu + \mu_t) \nabla \bar{\mathbf{u}} \right] \end{aligned} \tag{3}$$

What is the definition of effective viscosity μ_{eff}?

Write down all steps, averaging rules, and vector identities used.

Question 3 - Optional

Using index notation, we can write the viscous stress tensor as follows,

$$\tau_{ij} = 2\mu S_{ij} \tag{4}$$

where S_{ij} is the strain-rate tensor,

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{5}$$

Verify that the divergence of the viscous stress tensor is equivalent to,

$$\mu \frac{\partial^2 u_i}{\partial x_j x_j} \tag{6}$$

Remember the following constraints and properties,

- Divergence-free constraint.
- Symmetry of the second derivatives.

General guidelines

- Write down all the steps followed to derive the equations.
- You can use vector notation or index notation.
- You can also write the equations in their expanded Cartesian form.
- Write down all the averaging rules and vector identities used.
- The notation used is the same one used in Lecture 5.
- You can write your report in English or Italian.
- Do not hesitate to contact me if you have any questions.

Deadline

The deadline to submit your homework is 29 April 2020. You can send it to my email: joel.guerrero@unige.it