Homework

Turbulence and CFD models: Theory and applications
Lecture 3

Question 1

Using dimensional analysis, deduce the Kolmogorov length, time and velocity scales defined in equations 1-3,

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4} \Longrightarrow \text{Length scale} \tag{1}$$

$$\tau_\eta = \left(\frac{\nu}{\epsilon}\right)^{1/2} \Longrightarrow \text{Time scale} \tag{2}$$

$$\upsilon_\eta = (\nu\epsilon)^{1/4} \Longrightarrow \text{Velocity scale} \tag{3}$$

Question 2

Using dimensional analysis, deduce the Kolmogorov $-5/3$ law (equation 4).

$$E(\kappa) = C_K \epsilon^{2/3} \kappa^{-5/3} \tag{4}$$

Start with the assumption that the energy spectral density per wavenumber, $E(\kappa)$, depends only upon the wavenumber, κ, and the dissipation rate, ϵ.

Recall that by definition the energy spectral density per wavenumber is equal to,

$$k = \int_0^\infty E(\kappa) dk \tag{5}$$

Therefore, the dimensions of $E(\kappa)$ are,

$$E(\kappa) = \frac{k}{\kappa} = \frac{L^2/T^2}{1/L} = \frac{L^3}{T^2} \tag{6}$$

General guidelines

- Write down all the steps.
- If you make assumptions, remember to justify them.
- You can write your report in English or Italian.

The deadline to submit your homework is 8 April 2020. You can send it to my email: joel.guerrero@unige.it