Università degli Studi di Genova

A 3D SMA CONSTITUTIVE MODEL IN THE FRAMEWORK OF FINITE STRAINS

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(INACCURATE) PERFORMANCE OF THE SMALL STRAIN FORMULATION

FINITE STRAIN FORMULATION

multiplicative decomposition

$$\mathbf{F}(\mathbf{X},t)=\mathbf{F}_e(\mathbf{X},t)\mathbf{F}_t(\mathbf{X},t)$$

THERMODYNAMIC APPROACH

control variables: temperature and the right Cauchy-Green tensor $\mathbf{C}=\mathbf{F}^T\mathbf{F}$

internal variable: transformation right Cauchy-Green tensor $\mathbf{C}_t=\mathbf{F}_t^T\mathbf{F}_t$

$$\mathbf{C}_e=\mathbf{F}_e^T\mathbf{F}_e=\mathbf{F}_t^{-T}\mathbf{C}\mathbf{F}_t^{-1}$$

$$\boxed{\Psi(\mathbf{C},\mathbf{C}_t,T)=\Psi_e\left(\mathbf{C}_e\right)+\Psi_t\left(\mathbf{C}_t,T\right)}$$

free energy

$$I_1=tr\mathbf{C}_e \qquad I_2=\frac{1}{2}\left(tr\left(\mathbf{C}_e\right)^2-I_1^2\right)$$

$$\boxed{\Psi_e=\frac{1}{2}\left[\left(\frac{\lambda+2\mu}{4}\right)I_1^2-\left(\frac{3\lambda+2\mu}{2}\right)I_1+\mu I_2+\left(\frac{9\lambda+6\mu}{4}\right)\right]}$$

elastic energy

$$\boxed{\Psi_t=\beta\left\langle T-T_f^{AM}\right\rangle\left\|\mathbf{E}_t\right\|+\frac{1}{2}h\left\|\mathbf{E}_t\right\|^2+\mathcal{I}_{\varepsilon_L}\left(\mathbf{E}_t\right)} \rightarrow \mathbf{E}_t=\frac{1}{2}(\mathbf{C}_t-\mathbf{1})$$

transformation energy

Clausius-Duhem inequality

$$\boxed{-\left(\dot{\Psi}+\eta\dot{T}\right)+\mathbf{S}\cdot\frac{1}{2}\dot{\mathbf{C}}-\mathbf{q}\cdot\frac{\mathbf{grad}\,T}{T}\geq 0}$$

$$\mathbf{S}=2\mathbf{F}_t^{-1}\frac{\partial\Psi_e}{\partial\mathbf{C}_e}\mathbf{F}_t^{-T}$$ **Second Piola-Kirchhoff stress tensor**

STATE LAWS

$$\eta=-\frac{\partial\Psi}{\partial T}$$ **Entropy**

$$\mathbf{T}=2\underbrace{\mathbf{C}_e\frac{\partial\Psi_e}{\partial\mathbf{C}_e}}_{\mathbf{M}}-2\underbrace{\mathbf{F}_t\frac{\partial\Psi_t}{\partial\mathbf{C}_t}\mathbf{F}_t^T}_{\boldsymbol{\alpha}}$$ **Thermodynamic force**

Mandel tensor $\mathbf{M}$ $\boldsymbol{\alpha}$ **Back-stress**

Symmetric deformation rate tensor $\dot{\mathbf{C}}_t=2\mathbf{F}_t^T\mathbf{d}_t\mathbf{F}_t$

ASSOCIATIVE EVOLUTIONARY LAW

$$\mathbf{d}_t=\dot{\zeta}\frac{\partial F}{\partial\mathbf{T}}$$

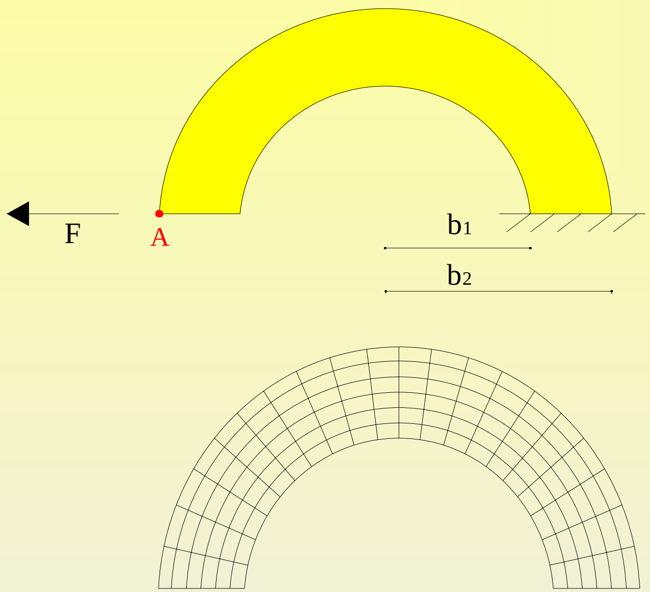
KUHN-TUCKER CONDITIONS

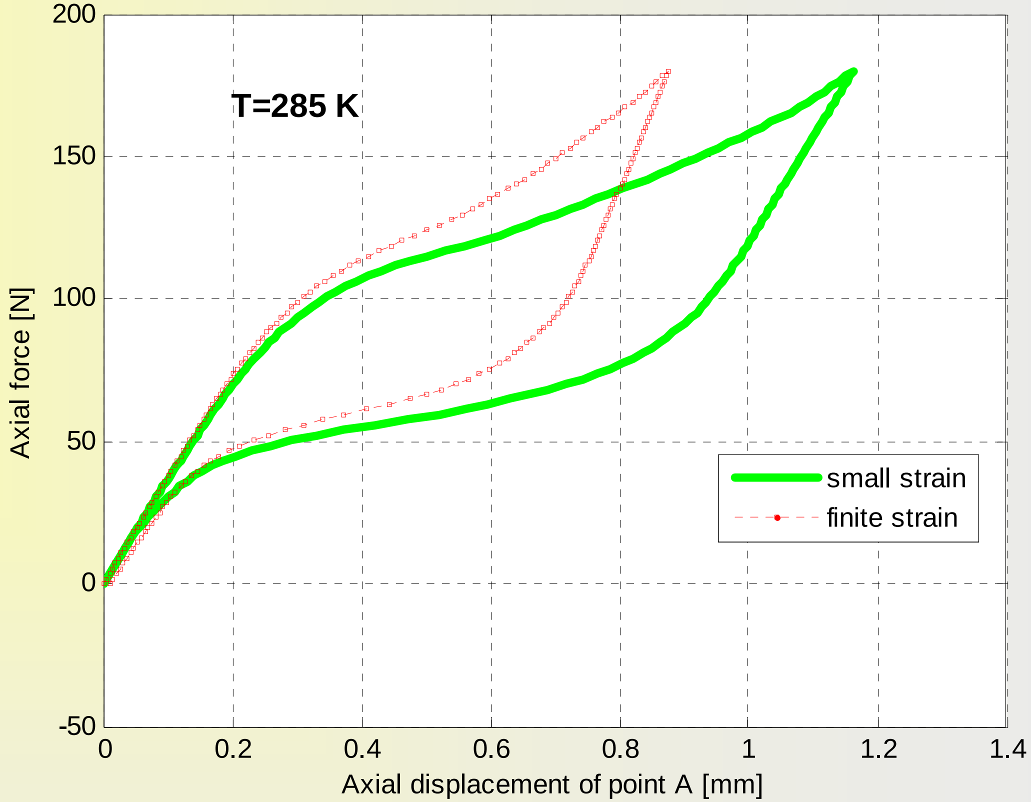
$$\dot{\zeta}\geq 0\qquad F\leq 0\qquad \dot{\zeta}F=0$$

Conclusions and future developments...

✓FEM implementation:

$$b_1 = 5mm \quad b_2 = 8mm$$





✓Experimental testing of the obtained results

✓Modelling of advanced devices: orthodontic wires, stents, microgrips...