GMA 2008

AIMETA
ASSOCIAZIONE ITALIANA
DI MECCANICA TEORICA
ED APPLICATA

Weight functions and asymptotics of elastic fields for interfacial cracks loaded by asymmetric forces

UNIVERSITÀ DEGLI STUDI DI TRENTO

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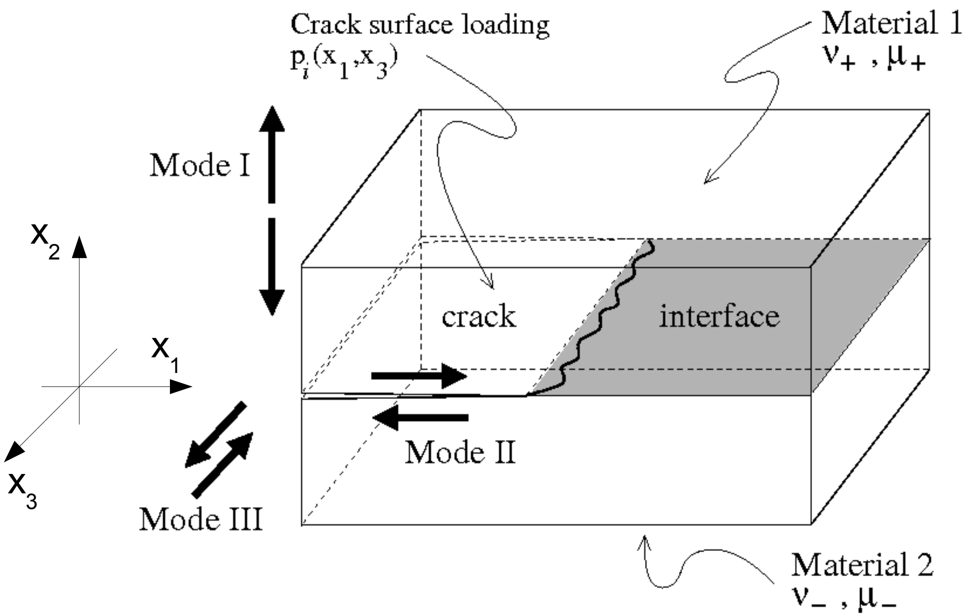
PROBLEM FORMULATION

Semi-infinite plane crack along the interface between two different elastic isotropic half-spaces



Two characteristic features:

- coupling of symmetric and skew-symmetric opening modes
- oscillatory behaviour of the solution near the crack edge

Complex SIF: $K(x_3) = K_{\mathrm{I}}(x_3) + iK_{\mathrm{II}}(x_3)$

Bimaterial constant: $\epsilon = \frac{1}{2\pi} \log \frac{\mu_+ + (3-4\nu_+)\mu_-}{\mu_- + (3-4\nu_-)\mu_+}$

Asymptotics:

$$\begin{cases} \sigma_{22}(x_1,0,x_3) + i\sigma_{12}(x_1,0,x_3) \sim \dfrac{K(x_3)}{\sqrt{2\pi x_1}} x_1^{i\epsilon}, \\ \sigma_{23}(x_1,0,x_3) \sim \dfrac{K_{\mathrm{III}}(x_3)}{\sqrt{2\pi x_1}}, \end{cases}$$

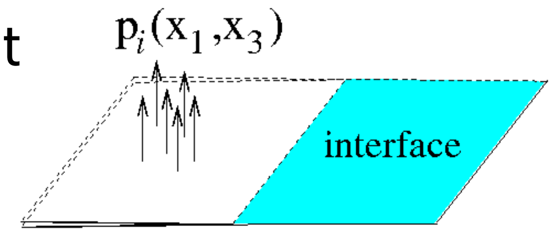
$$\begin{cases} [u_2 + iu_1](x_1,x_3) \sim \dfrac{(1-\nu_+)/\mu_+ + (1-\nu_-)/\mu_-}{(1/2+i\epsilon)\cosh(\pi\epsilon)} K(x_3) \sqrt{\dfrac{-x_1}{2\pi}} (-x_1)^{i\epsilon}, \\ [u_3](x_1,x_3) \sim 2\left(\dfrac{1}{\mu_+} + \dfrac{1}{\mu_-}\right) K_{\mathrm{III}}(x_3) \sqrt{\dfrac{-x_1}{2\pi}}, \end{cases}$$

MAIN RESULTS

- Willis-Movchan weight functions
  - Special singular solutions of the homogeneous BVP
  - Fourier transform and the Wiener-Hopf equation $$[\overline{\boldsymbol{U}}]^{+}(\beta,\lambda) = \frac{1}{\rho}\boldsymbol{G}(\beta,\lambda)\overline{\boldsymbol{\Sigma}}^{-}(\beta,\lambda)$$
  - Reciprocal theorem (Betti identity)

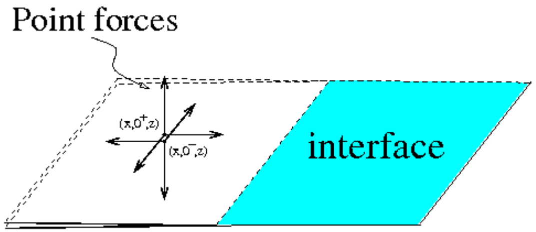
$$\{\sigma_{i2}^{(+)} * R_{ih}[U_h] - R_{ih}\Sigma_{h2} * [u_i]\}(x_1', x_3') = -\{p_i * R_{ih}[U_h]\}(x_1', x_3')$$

- Analytical formula for the SIFs for the straight crack front



$$\tilde{K}_k(\lambda) = -i \lim_{x_1' \to 0} \mathcal{A}_{kj}^{-1}(\lambda) \int_{-\infty}^{0} \tilde{p}_i(x_1,\lambda) R_{ih}[\tilde{U}_h^j]^{+}(x_1' - x_1, \lambda) dx_1$$

- Bueckner weight functions
  - SIFs corresponding to point forces applied on the crack surfaces
  - Related to the Green's function for the crack



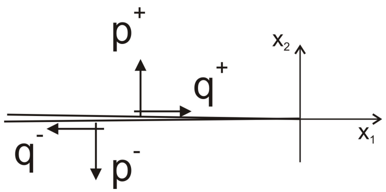
$$h_{kp}(x,z;x_3) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} i\tilde{\mathcal{B}}_{kj}(\lambda)\delta_{ip}R_{ih}[\overline{U}_h^j]^{+}(\beta,\lambda)e^{-i(-x)\beta}e^{-i(x_3-z)\lambda}d\lambda d\beta$$

MAIN RESULTS

- Asymmetric loading (2D case)
  - **Interfacial cracks**: both the symmetric and antisymmetric parts of the loading contribute to the stress singularity at the crack edge and thus both affect the resulting SIFs

$$\sigma_{\theta\theta}(r,0) + i\sigma_{r\theta}(r,0) = \frac{K}{\sqrt{2\pi}} r^{-1/2+i\epsilon} \qquad [u_\theta](r) + i[u_r](r) = -\frac{(1-\nu_+)/\mu_+ + (1-\nu_-)/\mu_-}{(1/2+i\epsilon)\cosh(\pi\epsilon)} K \frac{1}{\sqrt{2\pi}} r^{1/2+i\epsilon}$$

$$\boxed{K = -\sqrt{\frac{2}{\pi}}\cosh(\pi\epsilon)\left\{\int_0^\infty [p^{SYM}(r) - \tanh(\pi\epsilon)p^{SKW}(r)]r^{-1/2-i\epsilon}dr + i\int_0^\infty [q^{SYM}(r) - \tanh(\pi\epsilon)q^{SKW}(r)]r^{-1/2-i\epsilon}dr\right\}}$$



$$p^{SYM} = \frac{p^+ + p^-}{2} \qquad q^{SYM} = \frac{q^+ + q^-}{2}$$

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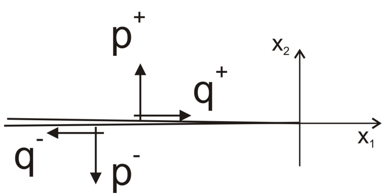

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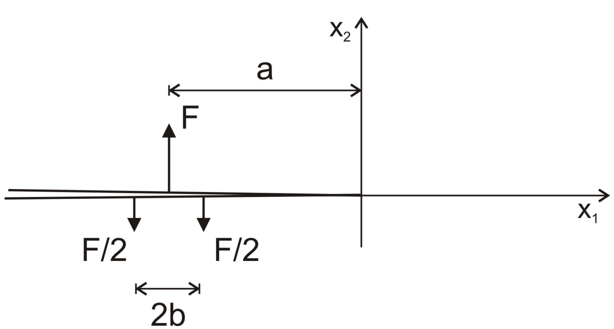
Example:



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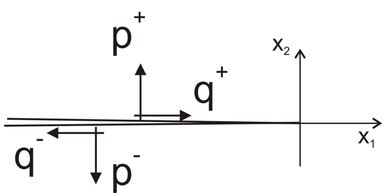

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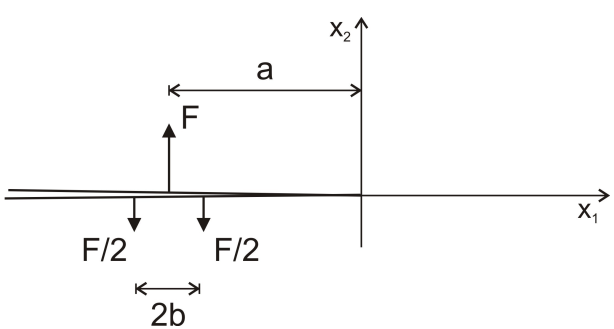
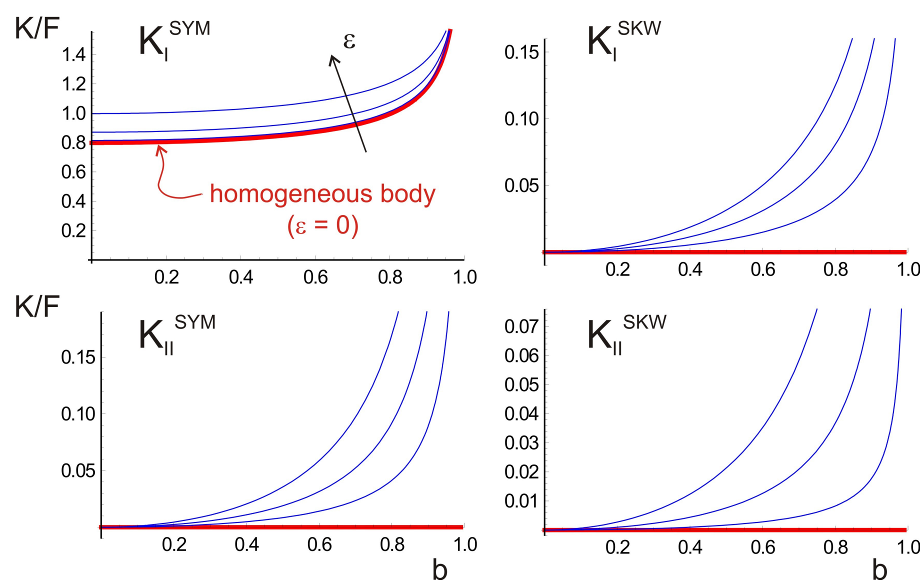
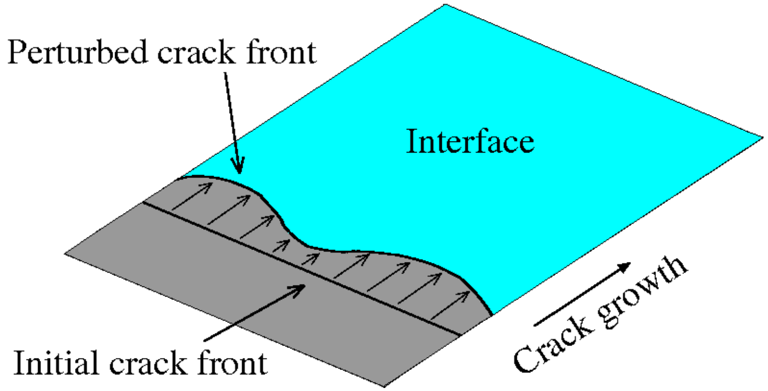




FUTURE DEVELOPMENTS

- Willis-Movchan perturbative approach
  - find the perturbation formula to compute the SIFs for the interfacial wavy crack
  - effective evaluation of the weight functions (from the integral transform space)

Application:
study the perturbation from straightness of the crack front!



- Anisotropic materials
  - Study of crack propagation in laminate composite materials
- Dynamical case
  - Steady-state (constant propagation speed)
  - Essentially dynamical case