A practical application of the variational approach to fracture mechanics

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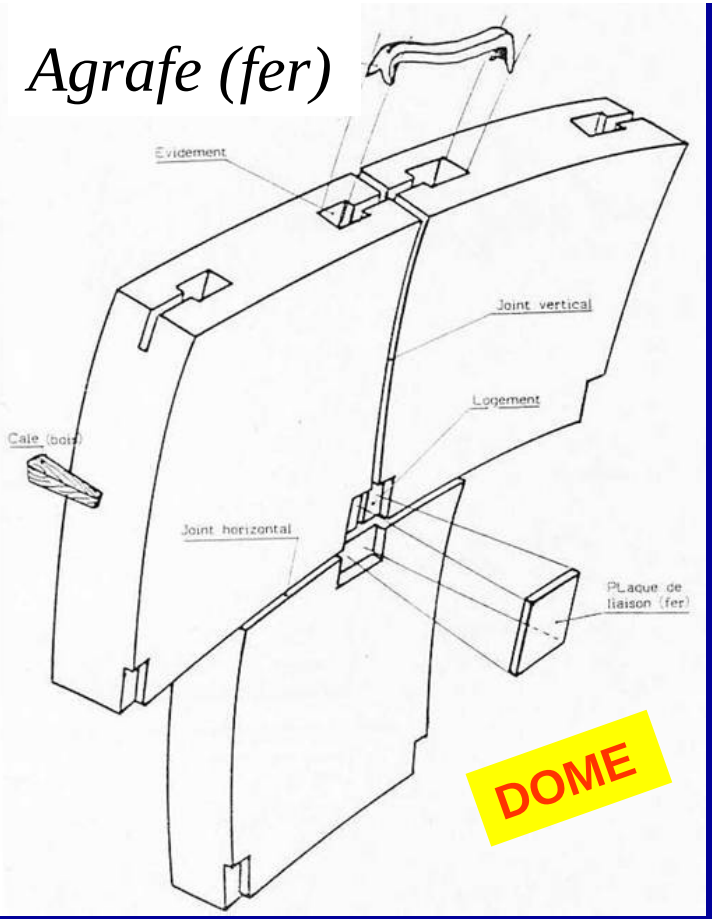
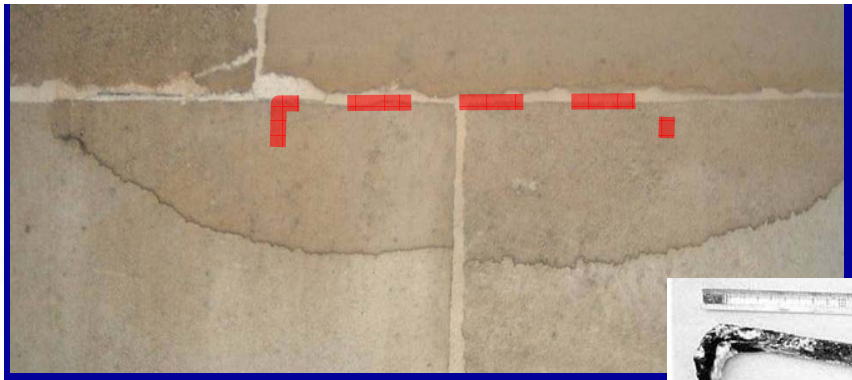
Agrafe (fer)



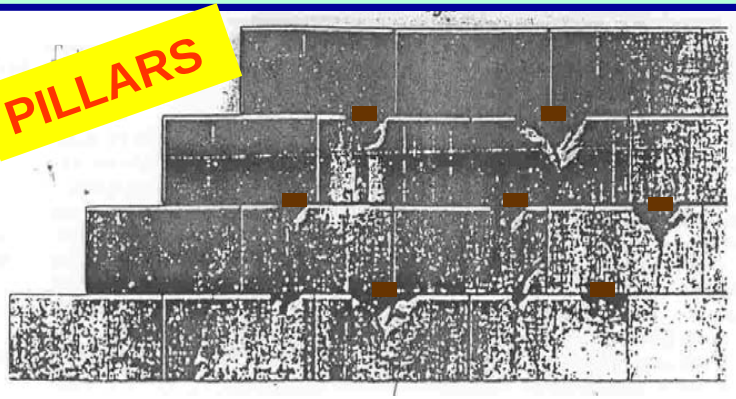
DOME



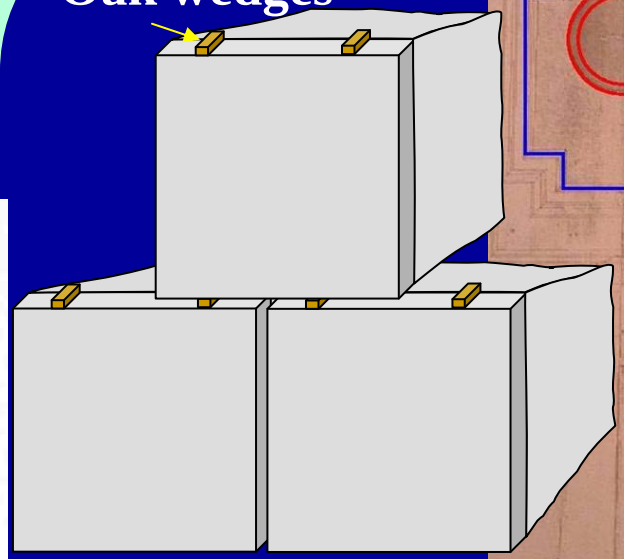
Agrafe

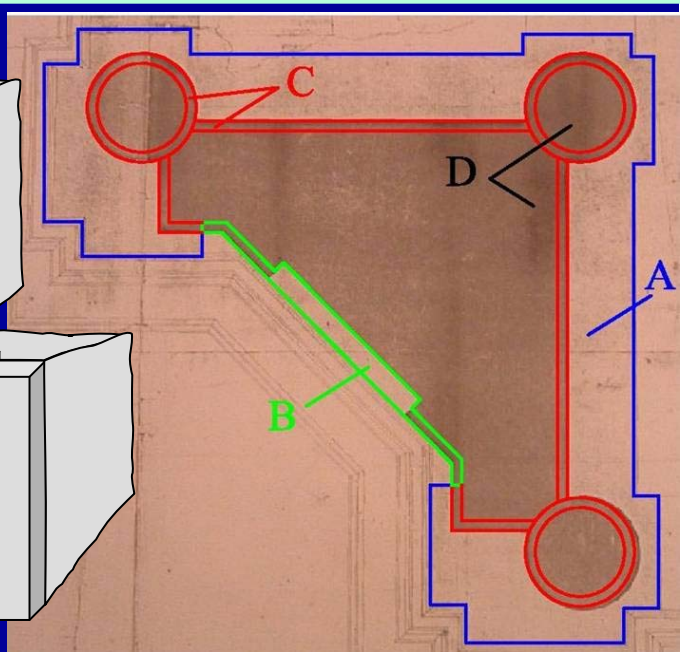
Observed damage

PILLARS



Oak wedges



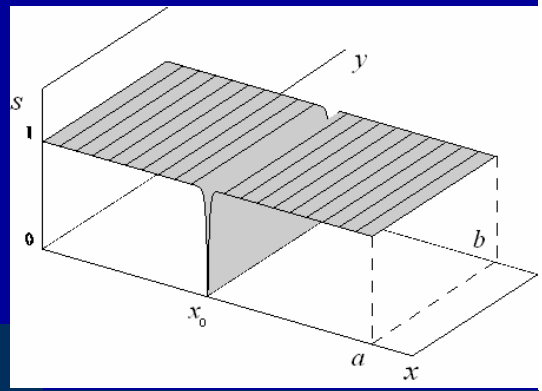


Variational approach to fracture mechanics

$$\min_{\mathbf{u}\in\mathcal{A}_t^{SBV}} \pi[\mathbf{u}] := \int_\Omega \frac{1}{2}\mathbb{C}\mathbf{e}[\mathbf{u}]\cdot\mathbf{e}[\mathbf{u}]\,d\mathbf{x} + \gamma\,\mathcal{H}^1(S_{\mathbf{u}}\setminus\mathcal{N})$$

$$\mathcal{A}_t^{SBV} = \left\{\mathbf{u}\in SBV(\tilde{\Omega}) : \mathbf{u} = t\cdot\mathbf{U} \text{ on } \tilde{\Omega}\setminus\bar{\Omega}\right\}$$

Francfort & Marigo, *JMPS*, 2002



Boudin, Francfort & Marigo, *JMPS*, 2004

$$\min_{(\mathbf{u},s)\in\mathcal{A}_t} \Pi_\varepsilon[\mathbf{u},s] = \int_{\Omega\setminus\mathcal{N}} (s^2+k_\varepsilon)\frac{1}{2}\mathbb{C}\mathbf{e}[\mathbf{u}]\cdot\mathbf{e}[\mathbf{u}]d\mathbf{x} + \gamma\int_{\tilde{\Omega}\setminus\mathcal{N}}\left[\frac{\varepsilon}{2}|\nabla s|^2 + \frac{(1-s)^2}{2\varepsilon}\right]d\mathbf{x}$$

$$\tilde{\mathcal{A}}_t = \left\{(\mathbf{u},s)\in W^{1,2}(\tilde{\Omega}\setminus\mathcal{N},\mathbb{R}^2)\times W^{1,2}(\tilde{\Omega}\setminus\mathcal{N};[0,1]) : \mathbf{u} = t\cdot\mathbf{U} \text{ on } \tilde{\Omega}\setminus\bar{\Omega}\right\}$$

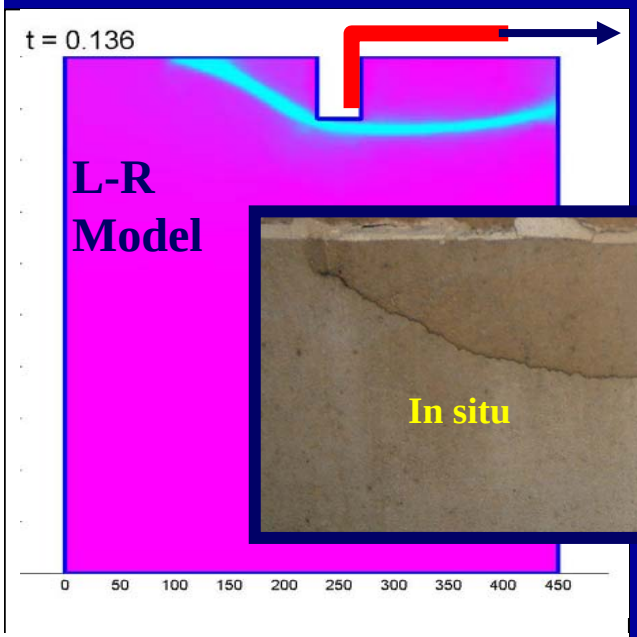
$$\Pi_\varepsilon^D[\mathbf{u},s] = \int_{\Omega\setminus\mathcal{N}} (s^2+k_\varepsilon)\frac{1}{2}\mathbb{C}\mathbf{e}_{dev}[\mathbf{u}]\cdot\mathbf{e}_{dev}[\mathbf{u}]d\mathbf{x} + \int_{\Omega\setminus\mathcal{N}}\frac{1}{2}\mathbb{C}\mathbf{e}_{sph}[\mathbf{u}]\cdot\mathbf{e}_{sph}[\mathbf{u}]d\mathbf{x}$$

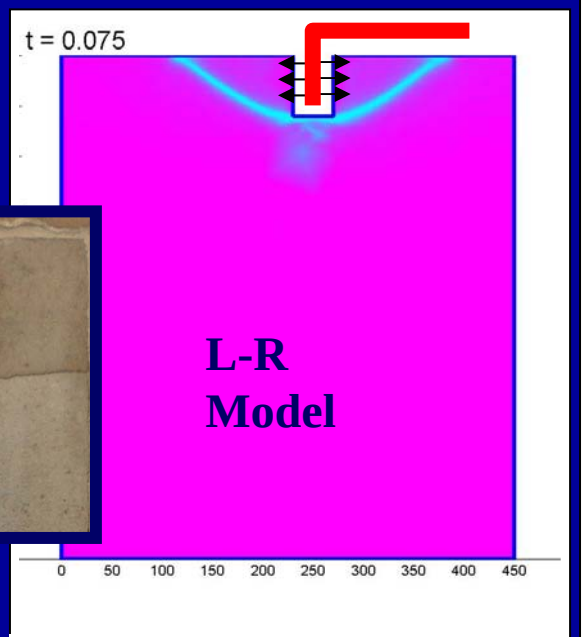
$$+\gamma\int_{\Omega\setminus\mathcal{N}}\left[\frac{\varepsilon}{2}|\nabla s|^2 + \frac{(1-s)^2}{2\varepsilon}\right]d\mathbf{x}$$

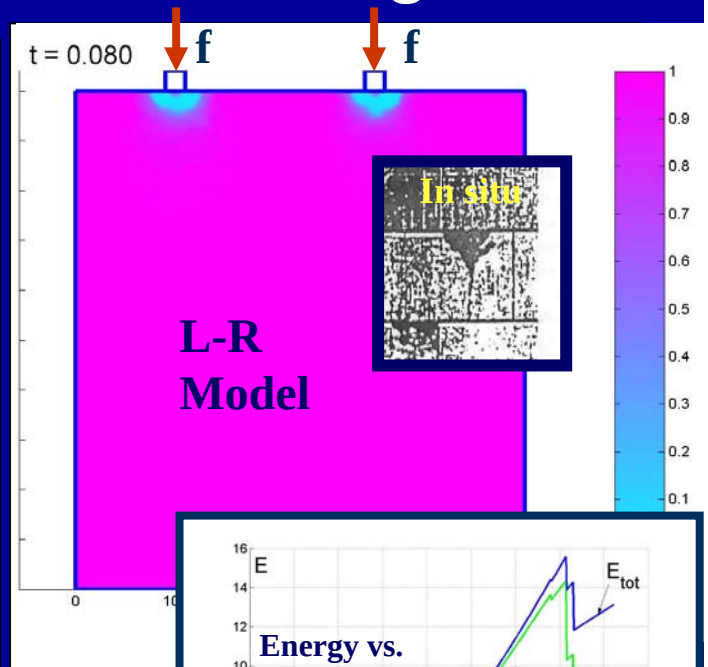
$$\mathcal{A}_t = \left\{(\mathbf{u},s)\in W^{1,2}(\Omega\setminus\mathcal{N},\mathbb{R}^2)\times W^{1,2}(\Omega\setminus\mathcal{N};[0,1]) : \mathbf{u} = t\cdot\mathbf{U} \text{ and } s = 1 \text{ on } \mathcal{D}\right\}$$

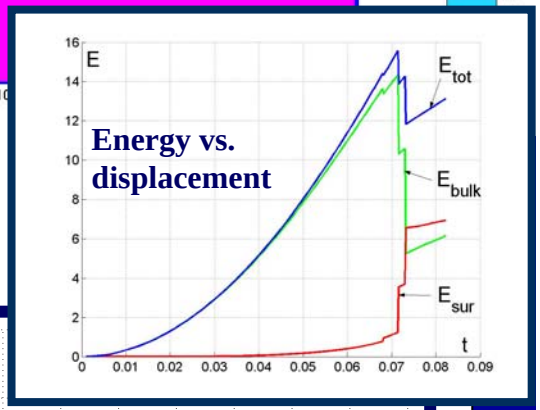
Lancioni & Royer-Carfagni, 2007

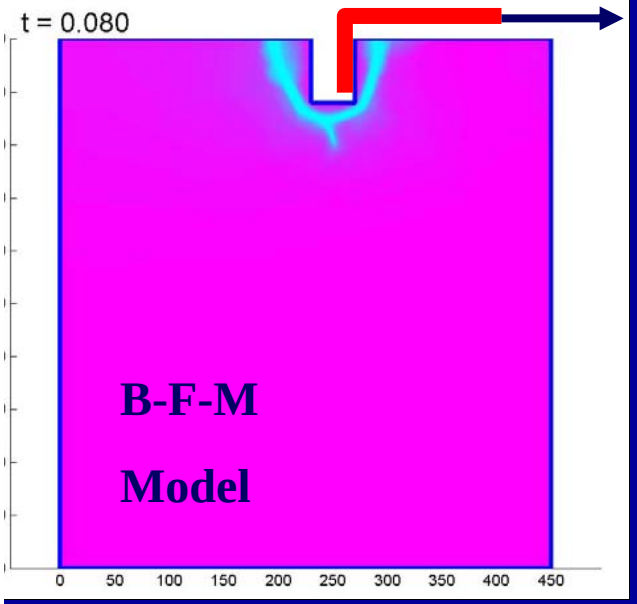
Comparison with observed damage

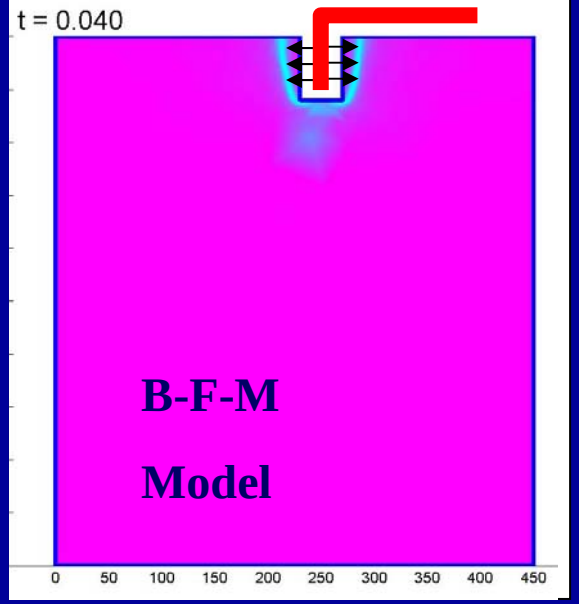


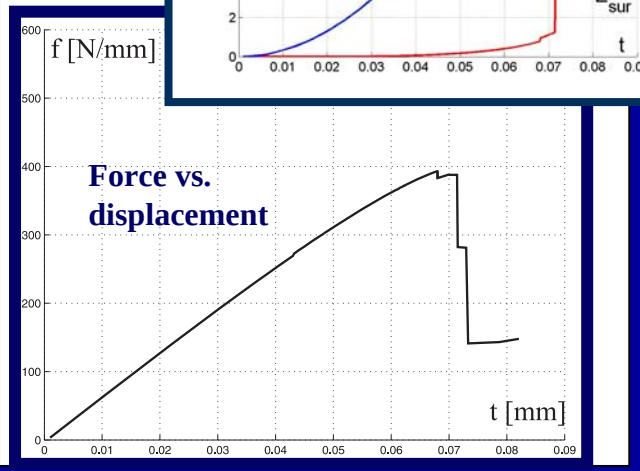












Pull-out

Oxidation expansion

Wedge contact