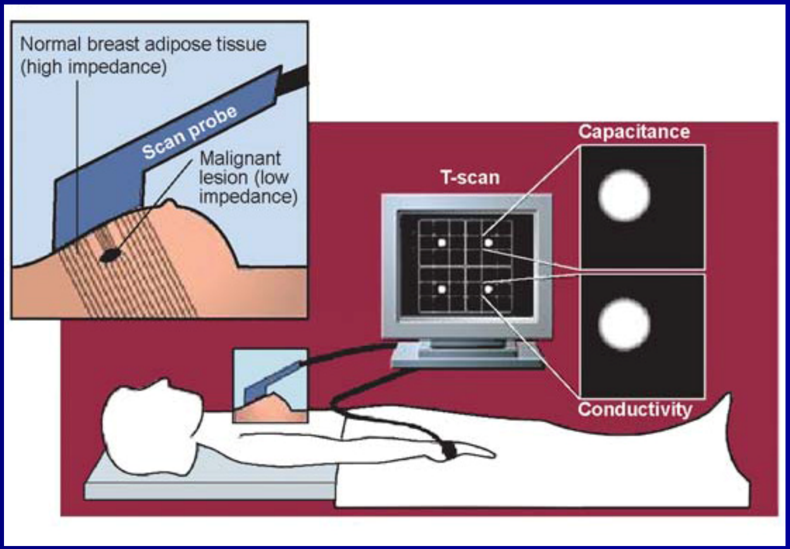
Effective complex conductivity of periodic fibrous composites with interfacial impedance and applications to biological tissues



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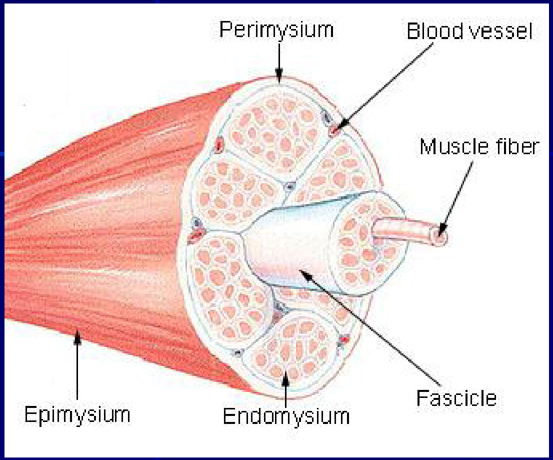
Micromechanics

Skeletal muscle

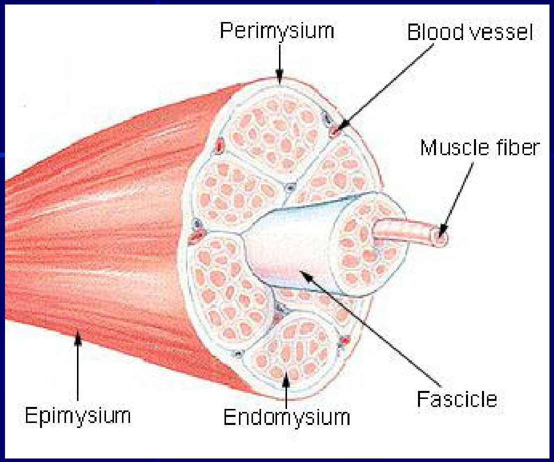


Electrical conduction

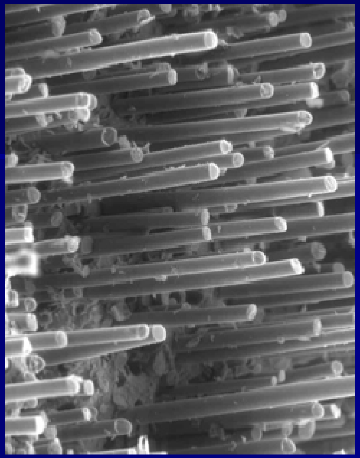
Electric potential w_ε

Micromechanics

Skeletal muscle



Fibrous composite



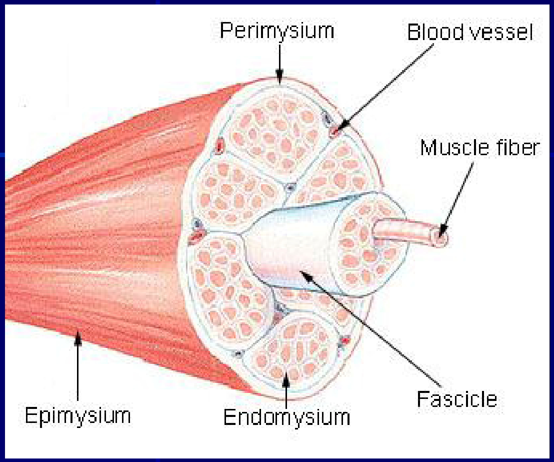
Electrical conduction

Anti-plane problem

Electric potential w_ε ⟺ Longitudinal displacement w_ε

Micromechanics

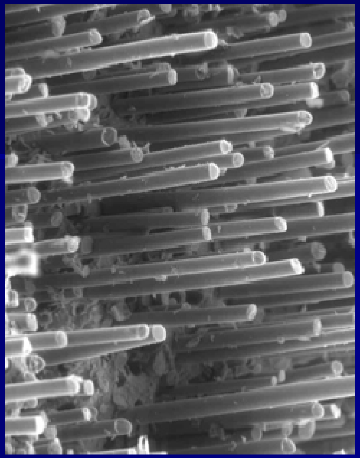
Skeletal muscle



Electrical conduction

Electric potential w_ε

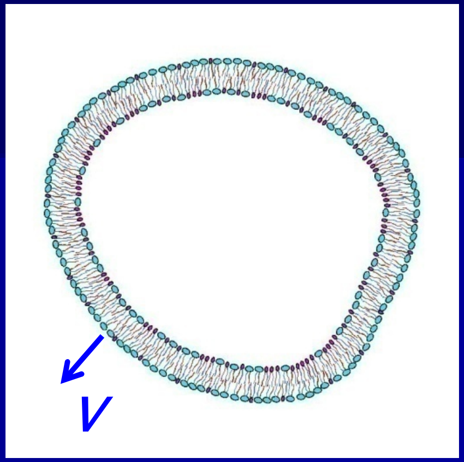
Fibrous composite



Anti-plane problem

Longitudinal displacement w_ε

Cell membrane



Imperfect interface

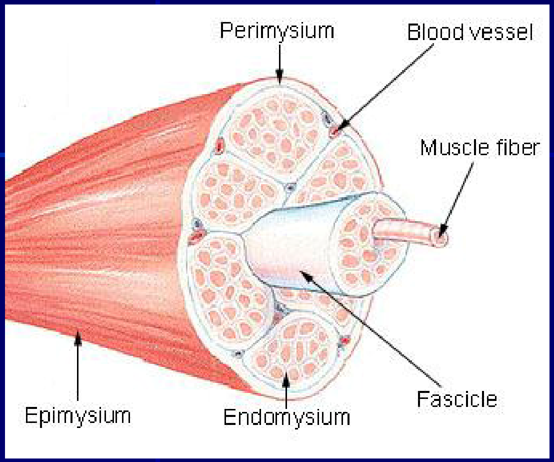
$$\frac{Y}{\varepsilon}[\![w_\varepsilon]\!] = \boldsymbol{\tau}_\nu = G\nabla w_\varepsilon \cdot \nu$$

Lord Rayleigh, Phil Mag, 1892
Nicorovici et al, Proc R Soc Lond A, 1993
Barbero et al, 1995
Rodrìguez-Ramos et al, Mech Mater, 2001
Jiang at al, Mech Mater, 2004
Bonnet, JMPS, 2007

Gu et al, J Phys D: Appl Phys, 1992
Sangani et al, JMPS, 1997
Cheng et al, Proc R Soc Lond A, 1997
Bigoni et al, IJSS 1998
Andrianov et al, IJMS, 2007

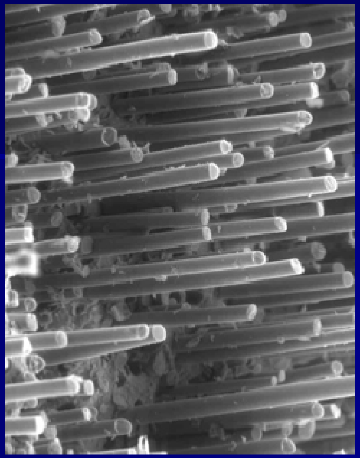
Micromechanics

Skeletal muscle



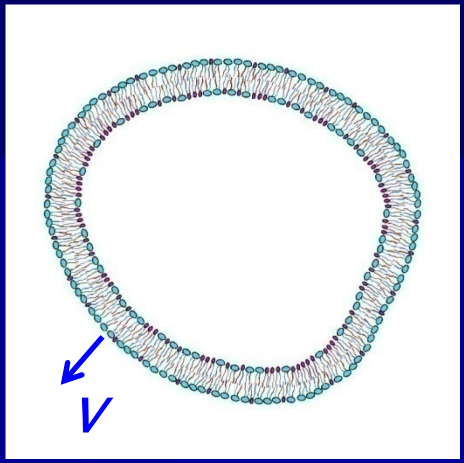
Electrical conduction

Fibrous composite

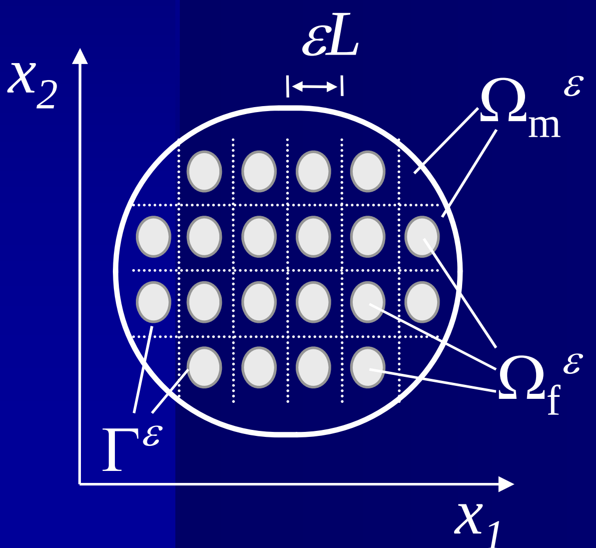


Anti-plane problem

Cell membrane



Imperfect interface



ε-problem

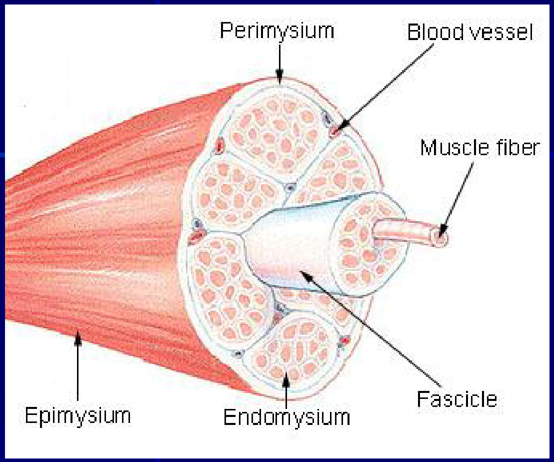
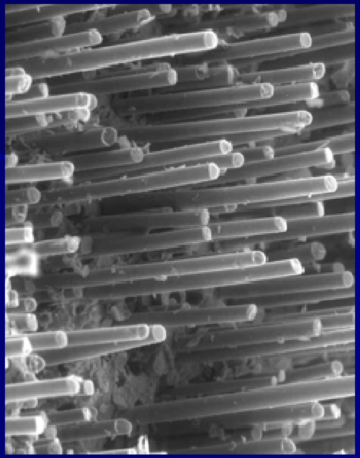
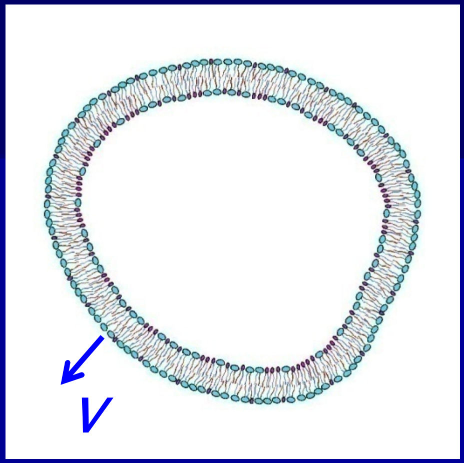
$$-\mathrm{div}(G\nabla w_\varepsilon) = 0 \qquad \text{in } \Omega_f^\varepsilon \cup \Omega_m^\varepsilon$$

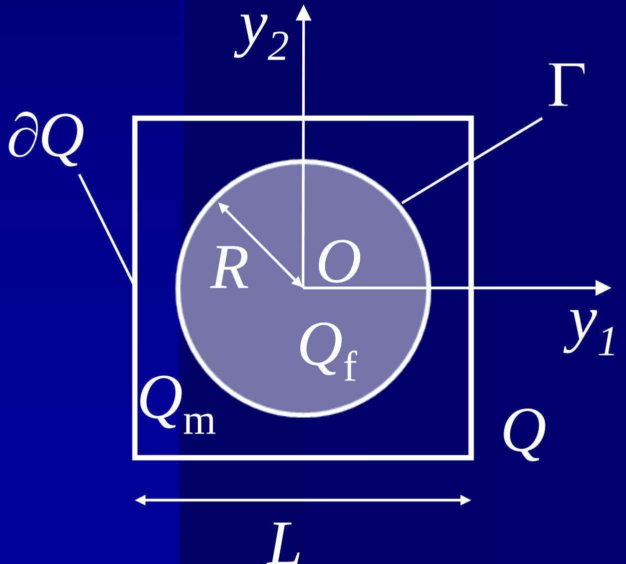
$$[\![G\nabla w_\varepsilon \cdot \nu]\!] = 0 \qquad \text{on } \Gamma^\varepsilon$$

$$\frac{Y}{\varepsilon}[\![w_\varepsilon]\!] = G\nabla w_\varepsilon \cdot \nu \qquad \text{on } \Gamma^\varepsilon$$

$$w_\varepsilon = w_0(x) - \varepsilon \boldsymbol{\chi}(y) \cdot \nabla w_0(x) + \ \ldots$$

Micromechanics

Skeletal muscle	Fibrous composite	Cell membrane
		
Electrical conduction	Anti-plane problem	Imperfect interface



cell problem

$$-G\Delta_y \chi_h = 0 \quad \text{in } Q_\mathrm{f} \cup Q_\mathrm{m}$$

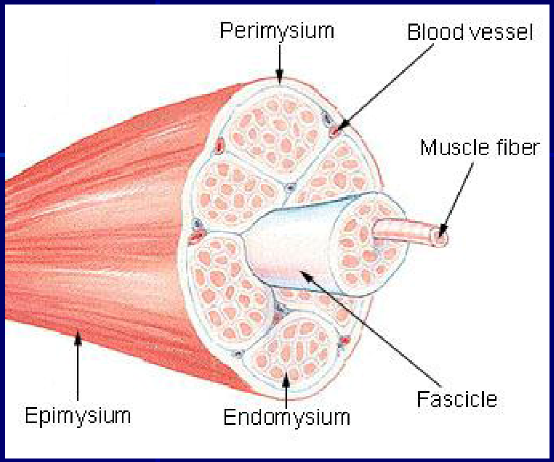
$$[G(\nabla_y \chi_h - \mathbf{e}_h) \cdot \nu] = 0 \quad \text{on } \Gamma$$

$$Y[\![\chi_h]\!] = G(\nabla_y \chi_h - \mathbf{e}_h) \cdot \nu \quad \text{on } \Gamma$$

$$G^\# = \overline{G}\,\mathbf{I} + \frac{1}{|Q|} \int_\Gamma \boldsymbol{\nu} \otimes [\![G\boldsymbol{\chi}]\!]\, \mathrm{d}a$$

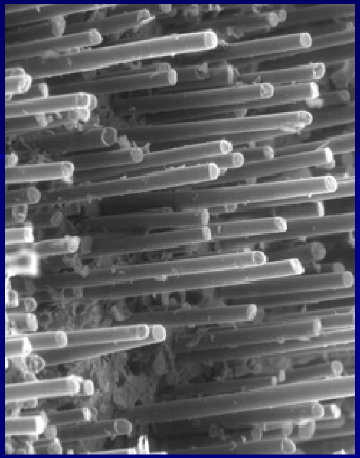
Micromechanics

Skeletal muscle



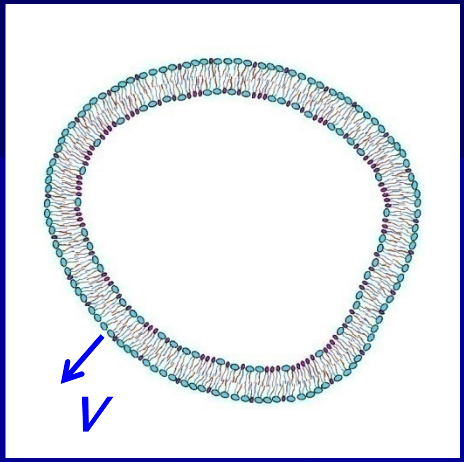
Electrical conduction

Fibrous composite

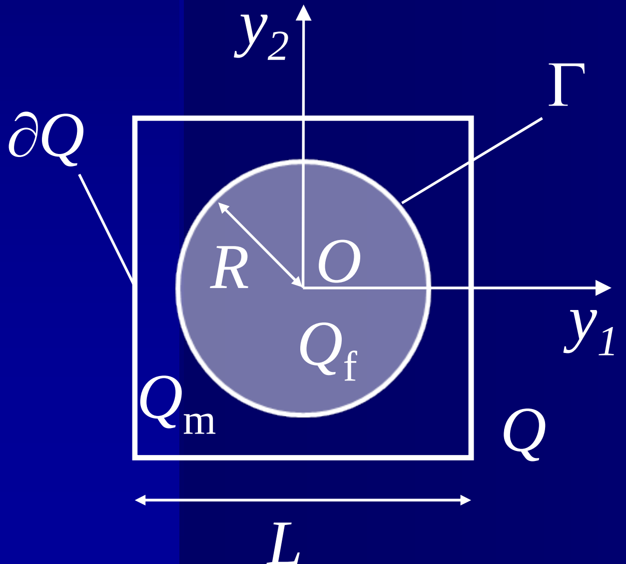


Anti-plane problem

Cell membrane



Imperfect interface



Solution scheme

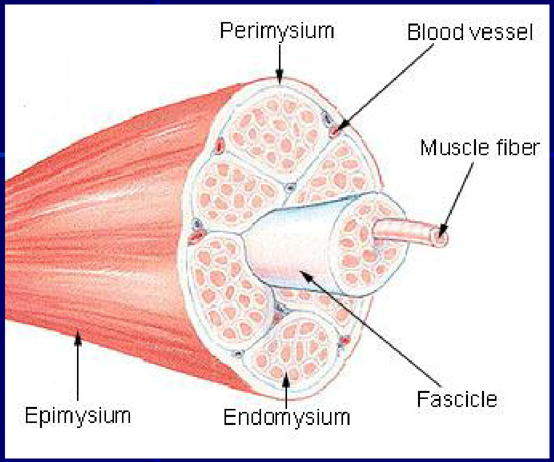
$$\chi_{\mathrm{f}} = \sum_{m=1}^{+\infty}{}^{\mathrm{o}}\, a_m \left(\frac{r}{R}\right)^m \cos m\theta$$

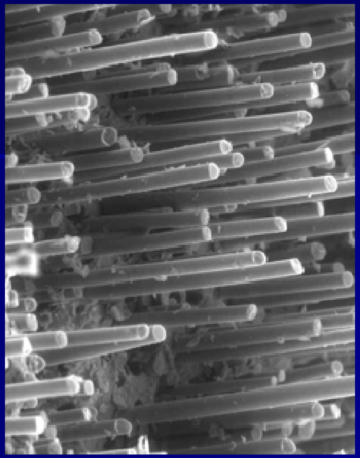
$$\chi_{\mathrm{m}} = \sum_{m=1}^{+\infty}{}^{\mathrm{o}} \left[b_m \left(\frac{r}{R}\right)^m + b_{-m} \left(\frac{r}{R}\right)^{-m} \right] \cos m\theta$$

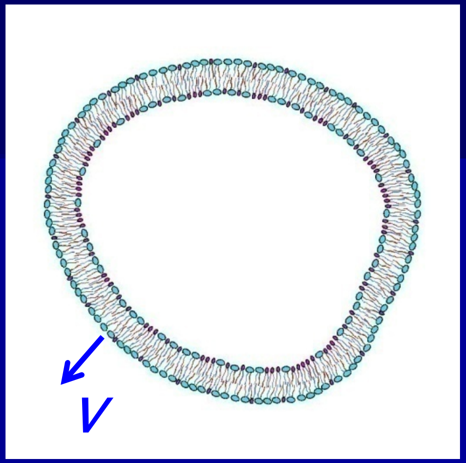
Periodicity: Weierstrass ζ(z) function

Micromechanics

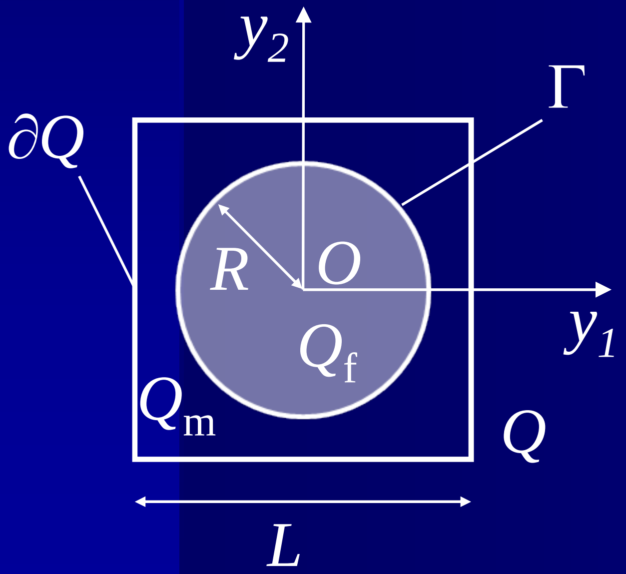
Skeletal muscle — Fibrous composite — Cell membrane







Electrical conduction — Anti-plane problem — Imperfect interface



Solution scheme

$$\chi_f = \sum_{m=1}^{+\infty}{}^{o} a_m \left(\frac{r}{R}\right)^m \cos m\theta$$

$$\chi_m = \sum_{m=1}^{+\infty}{}^{o} \left[b_m \left(\frac{r}{R}\right)^m + b_{-m} \left(\frac{r}{R}\right)^{-m} \right] \cos m\theta$$

$$\chi_m = -c_1 \Re\left(\frac{\eta_1}{\omega_1} z\right) + \sum_{s=1}^{+\infty}{}^{o} c_s \Re\left(\frac{\zeta^{(s-1)}(z)}{(s-1)!}\right)$$

Results

Infinite system

$$(\mathbf{\Gamma} + \mathbf{M})\mathbf{c} = \begin{pmatrix} L \\ 0 \\ 0 \\ \dots \end{pmatrix}$$

Material and volume fraction

$$\mathbf{\Gamma} = \begin{bmatrix} (\pi/f)(\gamma_1 + f) & 0 & 0 & 0 & \dots \\ 0 & (\pi/f)^3\gamma_3 & 0 & 0 & \dots \\ 0 & 0 & (\pi/f)^5\gamma_5 & 0 & \dots \\ 0 & 0 & 0 & (\pi/f)^7\gamma_7 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

Geometry

$$\mathbf{M} = \begin{bmatrix} \pi & \mu_{13} & 0 & \mu_{17} & \dots \\ \mu_{31} & 0 & \mu_{35} & 0 & \dots \\ 0 & \mu_{53} & 0 & \mu_{57} & \dots \\ \mu_{71} & 0 & \mu_{75} & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

Interface

$$\gamma_m = \frac{m + RY(G_{\mathrm{f}}^{-1} + G_{\mathrm{m}}^{-1})}{m + RY(G_{\mathrm{f}}^{-1} - G_{\mathrm{m}}^{-1})}$$

Results

Infinite system

$$(\mathbf{\Gamma}+\mathbf{M})\mathbf{c} = \begin{pmatrix} L \\ 0 \\ 0 \\ \dots \end{pmatrix}$$

Material and vol. frac.

$$\mathbf{\Gamma} = \begin{bmatrix} (\pi/f)(\gamma_1+f) & 0 & 0 & 0 & \dots \\ 0 & (\pi/f)^3\gamma_3 & 0 & 0 & \dots \\ 0 & 0 & (\pi/f)^5\gamma_5 & 0 & \dots \\ 0 & 0 & 0 & (\pi/f)^7\gamma_7 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

Geometry

$$\mathbf{M} = \begin{bmatrix} \pi & \mu_{13} & 0 & \mu_{17} & \dots \\ \mu_{31} & 0 & \mu_{35} & 0 & \dots \\ 0 & \mu_{53} & 0 & \mu_{57} & \dots \\ \mu_{71} & 0 & \mu_{75} & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

Interface

$$\gamma_m = \frac{m + RY(G_\mathrm{f}^{-1} + G_\mathrm{m}^{-1})}{m + RY(G_\mathrm{f}^{-1} - G_\mathrm{m}^{-1})}$$

Effective conductivity (N=1)

$$\frac{G^\#}{G_\mathrm{m}} = 1 - \frac{2f}{\gamma_1}\left[1 + \frac{f}{\gamma_1}\right]^{-1}$$

Results

Infinite system

$$(\mathbf{\Gamma}+\mathbf{M})\mathbf{c} = \begin{pmatrix} L \\ 0 \\ 0 \\ \dots \end{pmatrix}$$

Material and vol. frac.

$$\mathbf{\Gamma} = \begin{bmatrix} (\pi/f)(\gamma_1+f) & 0 & 0 & 0 & \dots \\ 0 & (\pi/f)^3\gamma_3 & 0 & 0 & \dots \\ 0 & 0 & (\pi/f)^5\gamma_5 & 0 & \dots \\ 0 & 0 & 0 & (\pi/f)^7\gamma_7 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

Geometry

$$\mathbf{M} = \begin{bmatrix} \pi & \mu_{13} & 0 & \mu_{17} & \dots \\ \mu_{31} & 0 & \mu_{35} & 0 & \dots \\ 0 & \mu_{53} & 0 & \mu_{57} & \dots \\ \mu_{71} & 0 & \mu_{75} & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

Interface

$$\gamma_m = \frac{m + RY(G_{\mathrm{f}}^{-1} + G_{\mathrm{m}}^{-1})}{m + RY(G_{\mathrm{f}}^{-1} - G_{\mathrm{m}}^{-1})}$$

Effective conductivity (N=2)

$$\frac{G^{\#}}{G_{\mathrm{m}}} = 1 - \frac{2f}{\gamma_1}\left[1 + \frac{f}{\gamma_1} - \frac{p_{13}f^4}{\gamma_1\gamma_3}\right]^{-1}$$

Results

Infinite system

$$(\mathbf{\Gamma} + \mathbf{M})\mathbf{c} = \begin{pmatrix} L \\ 0 \\ 0 \\ \dots \end{pmatrix}$$

Material and vol. frac.

$$\mathbf{\Gamma} = \begin{bmatrix} (\pi/f)(\gamma_1 + f) & 0 & 0 & 0 & \dots \\ 0 & (\pi/f)^3\gamma_3 & 0 & 0 & \dots \\ 0 & 0 & (\pi/f)^5\gamma_5 & 0 & \dots \\ 0 & 0 & 0 & (\pi/f)^7\gamma_7 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

Geometry

$$\mathbf{M} = \begin{bmatrix} \pi & \mu_{13} & 0 & \mu_{17} & \dots \\ \mu_{31} & 0 & \mu_{35} & 0 & \dots \\ 0 & \mu_{53} & 0 & \mu_{57} & \dots \\ \mu_{71} & 0 & \mu_{75} & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

Interface

$$\gamma_m = \frac{m + RY(G_\mathrm{f}^{-1} + G_\mathrm{m}^{-1})}{m + RY(G_\mathrm{f}^{-1} - G_\mathrm{m}^{-1})}$$

Effective conductivity (N=3)

$$\frac{G^\#}{G_\mathrm{m}} = 1 - \frac{2f}{\gamma_1}\left[1 + \frac{f}{\gamma_1} - \frac{\frac{p_{13}f^4}{\gamma_1\gamma_3}}{1 - \frac{p_{35}f^8}{\gamma_3\gamma_5}}\right]^{-1}$$

Results

Infinite system

$$(\mathbf{\Gamma}+\mathbf{M})\mathbf{c} = \begin{pmatrix} L \\ 0 \\ 0 \\ \cdots \end{pmatrix}$$

Material and vol. frac.

$$\mathbf{\Gamma} = \begin{bmatrix} (\pi/f)(\gamma_1+f) & 0 & 0 & 0 & \cdots \\ 0 & (\pi/f)^3\gamma_3 & 0 & 0 & \cdots \\ 0 & 0 & (\pi/f)^5\gamma_5 & 0 & \cdots \\ 0 & 0 & 0 & (\pi/f)^7\gamma_7 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

Geometry

$$\mathbf{M} = \begin{bmatrix} \pi & \mu_{13} & 0 & \mu_{17} & \cdots \\ \mu_{31} & 0 & \mu_{35} & 0 & \cdots \\ 0 & \mu_{53} & 0 & \mu_{57} & \cdots \\ \mu_{71} & 0 & \mu_{75} & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

Interface

$$\gamma_m = \frac{m + RY(G_{\mathrm{f}}^{-1} + G_{\mathrm{m}}^{-1})}{m + RY(G_{\mathrm{f}}^{-1} - G_{\mathrm{m}}^{-1})}$$

Effective conductivity (N=4)

$$\frac{G^{\#}}{G_{\mathrm{m}}} = 1 - \frac{2f}{\gamma_1}\left[1 + \frac{f}{\gamma_1} - \frac{\frac{p_{13}f^4}{\gamma_1\gamma_3} + \frac{p_{17}f^8}{\gamma_1\gamma_7} - \frac{p_{1357}f^{16}}{\gamma_1\gamma_3\gamma_5\gamma_7}}{1 - \frac{p_{35}f^8}{\gamma_3\gamma_5} - \frac{p_{57}f^{12}}{\gamma_5\gamma_7}}\right]^{-1}$$

Results

Infinite system

$$(\mathbf{\Gamma}+\mathbf{M})\mathbf{c} = \begin{pmatrix} L \\ 0 \\ 0 \\ \dots \end{pmatrix}$$

Material and vol. frac.

$$\mathbf{\Gamma} = \begin{bmatrix} (\pi/f)(\gamma_1+f) & 0 & 0 & 0 & \dots \\ 0 & (\pi/f)^3\gamma_3 & 0 & 0 & \dots \\ 0 & 0 & (\pi/f)^5\gamma_5 & 0 & \dots \\ 0 & 0 & 0 & (\pi/f)^7\gamma_7 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

Geometry

$$\mathbf{M} = \begin{bmatrix} \pi & \mu_{13} & 0 & \mu_{17} & \dots \\ \mu_{31} & 0 & \mu_{35} & 0 & \dots \\ 0 & \mu_{53} & 0 & \mu_{57} & \dots \\ \mu_{71} & 0 & \mu_{75} & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

Interface

$$\gamma_m = \frac{m + RY(G_{\mathrm{f}}^{-1} + G_{\mathrm{m}}^{-1})}{m + RY(G_{\mathrm{f}}^{-1} - G_{\mathrm{m}}^{-1})}$$

Effective conductivity (general)

$$\frac{G^{\#}}{G_{\mathrm{m}}} = \frac{\mathcal{N}}{\mathcal{D}}$$

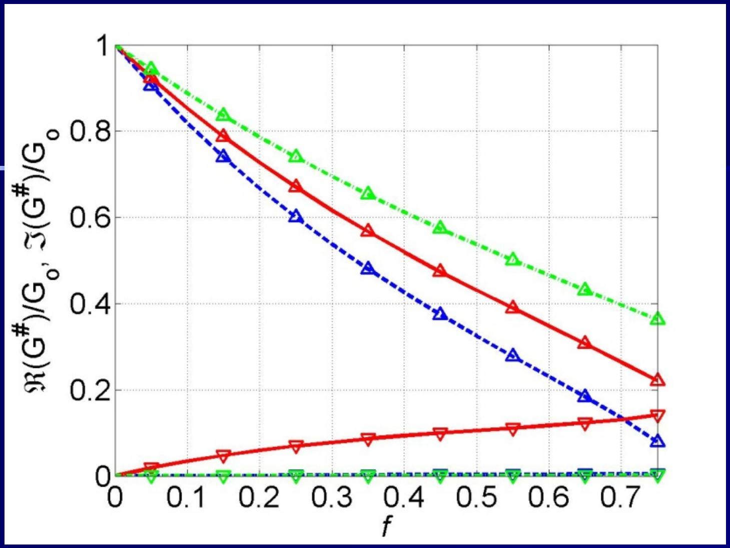
$$\mathcal{D} = \sum_{n=0}^{N}{}^{\mathrm{e}} \sum_{\mathcal{I}\in\{1\dots N\}} \left(\frac{f}{\pi}\right)^{2|\mathcal{I}|} \det(\mathbf{\Gamma})_{\mathcal{I}} \;\det \mathbf{M}_{\overline{\mathcal{I}}}$$

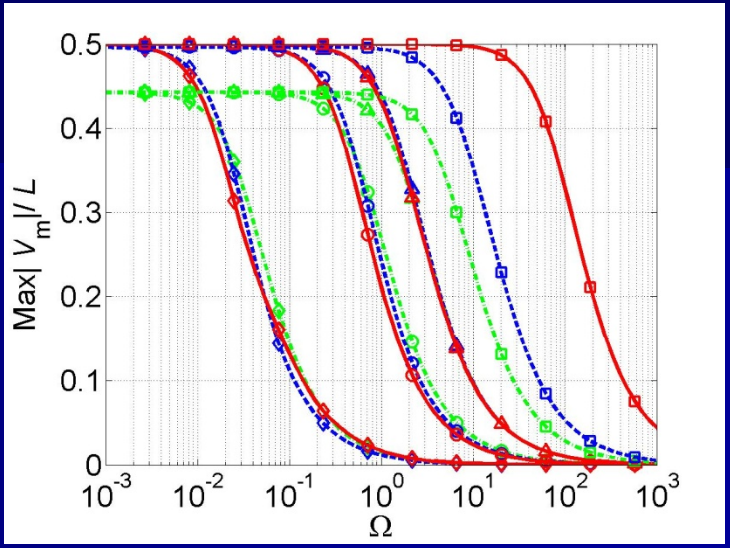
Results

Convergence

N	Ω_c	$\frac{\Re(G^\#)}{G_m}$	$\frac{\Im(G^\#)}{G_m}$	Ω_c	$\frac{\Re(G^\#)}{G_m}$	$\frac{\Im(G^\#)}{G_m}$
		$f=0.25$			$f=0.75$	
1	0.7706	0.6696	0.0696	0.4940	0.2611	0.1182
2	0.7704	0.6693	0.0697	0.4892	0.2277	0.1348
3	0.7704	0.6693	0.0697	0.4871	0.2230	0.1388
4	0.7704	0.6693	0.0697	0.4864	0.2210	0.1403
5	0.7704	0.6693	0.0697	0.4860	0.2203	0.1409
10	0.7704	0.6693	0.0697	0.4857	0.2200	0.1413
15	0.7704	0.6693	0.0697	0.4857	0.2200	0.1413

Results





Perspectives

- Different arrangements
- Three-dimensional geometries
- Functionally-graded materials
- Structural interfaces