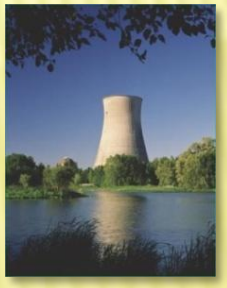
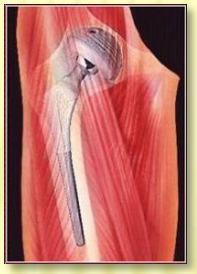
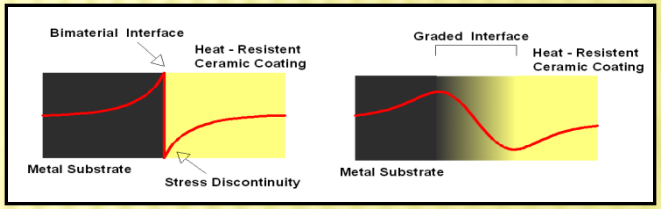
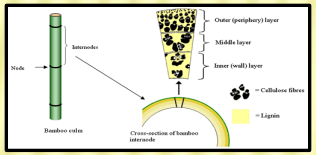
# ELASTIC SOLUTIONS FOR FUNCTIONALLY GRADED MATERIALS

- Background
- Origin of FGMs
- Potential applications
- Bulk and Coating








Roberta Sburlati
DICAT- Facoltà di Ingegneria
Università degli Studi di Genova

Gruppo Materiali Aimeta
29 febbraio-1 marzo 2008
Genova

# RESEARCH TOPIC

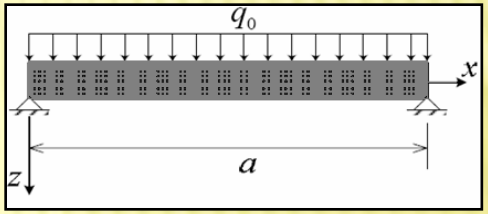
***Analytical or semi-analytical solutions***

***FGMs - Plate (FGMP) - Coating** (FGMC)*

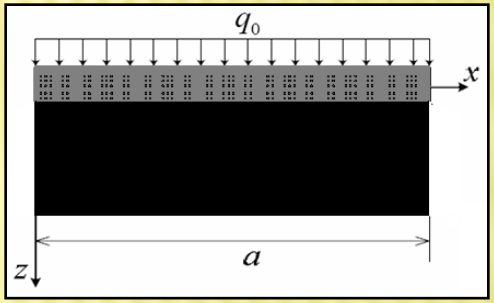
Li, X.Y. et all. (2008) *Int. J. Solids and Structures* 45 (s)
Chung, Y.L. et all. (2008) *J. Mech.Mat.Struct.* (p)
Kashtalyan, M., Menshykova, M. (2007) *Int. J. Solids and Structures* 44
Chi, S.H. et all. (*2006*) *Int. J. Solids and Structures* 43 (p)
Kashtalyan, M. (2004) *Europ. J.Mech. A/Solids* 23 (s)
Reddy, J.N. et all. (1999) *Europ. J.Mech. A/Solids* 23 (p)
Plevako, V.P. (1971) *J. Appl. Math.Mech.* 35(5) (s)

***Graded Contact***

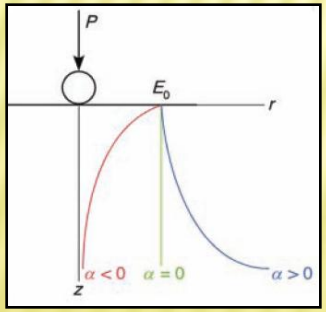
Giannakopoulos, A.E., Pallot, P. (2000) *J. Mech.Phys.Solids* 48
Suresh, S. (2001) *Science*, 292
Erdogan, F. and Guler, M.A. (2004) *Int. J. Solids and Structures* 41

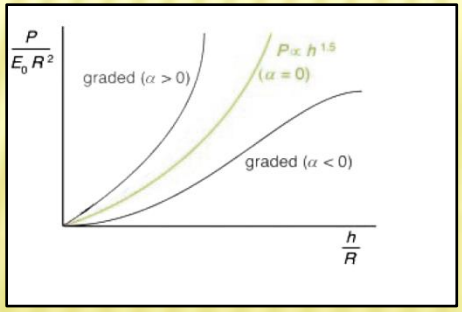



**FGMP**

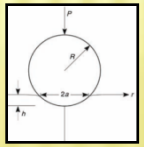



**FGMC**




$E(z)=E_0 e^{\alpha z}$



da Suresh, S. (2001) *Science*, 292

**Questions?**

**material inhomogeneity effects on..**

- **elastic response**
- **impact resistance**
- **temperature resistance**

# MATHEMATICAL SOLUTION

Plevako, V.P. (1971) *Journal of Applied Mathematics and Mechanics* 35(5).
Graded circular plate, axisymmetric transverse load.

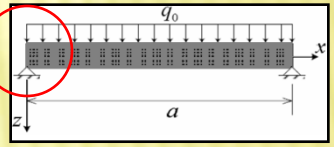
$$u(r,z) = -\frac{1}{2G(z)}\frac{\partial}{\partial r}\left(\nu\Delta - \frac{\partial^2}{\partial z^2}\right)L\ \ r,z$$

$$w(r,z) = -\frac{1}{G(z)}\frac{\partial}{\partial z}\left(\Delta - \frac{\partial^2}{\partial z^2}\right)L\ \ r,z\ \ + \frac{\partial}{\partial z}\left(\frac{1}{2G(z)}\left(\nu\Delta - \frac{\partial^2}{\partial z^2}\right)L\ \ r,z\right)$$

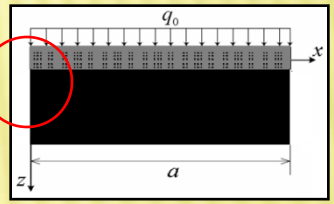
$\nu(z)=\nu$

$$\Delta\left(\frac{1}{G(z)}\Delta L(r,z)\right) - \frac{1}{(1-\nu)}\left(\Delta - \frac{\partial^2}{\partial z^2}\right)L(r,z)\frac{d^2}{dz^2}G(z) = 0$$

$$L(r,z) = \sum_{j=1}^{\infty} q_j\ \ z\ \ J_0\ \ \phi_j r$$




FGMP




FGMC

$G(z)=G_0 e^{\alpha z}$

$$\frac{d^4}{dz^4}q_j(z) - 2\alpha\frac{d^3}{dz^3}q_j(z) + \ \alpha^2 - 2\phi_j^2\ \ \frac{d^2}{dz^2}q_j(z) + 2\alpha\phi_j^2\frac{d}{dz}q_j(z) + \left(\phi_j^2 + \alpha^2\frac{\nu}{1-\nu}\right)\phi_j^2 q_j(z) = 0$$

*Analytical solutions for FGMP and FGMC*

$$\sigma_r(r,z) = \frac{\nu}{r}\frac{\partial}{\partial r}\Delta L\ \ r,z\ \ + \frac{\partial^4 L(r,z)}{\partial r^2\partial z^2}$$

$$\sigma_\vartheta(r,z) = \nu\frac{\partial^2}{\partial r^2}\Delta L\ \ r,z\ \ + \frac{1}{r}\frac{\partial}{\partial r}\frac{\partial^2 L(r,z)}{\partial z^2}$$

$$\sigma_z(r,z) = \left(\Delta - \frac{\partial^2}{\partial z^2}\right)^2 L\ \ r,z$$

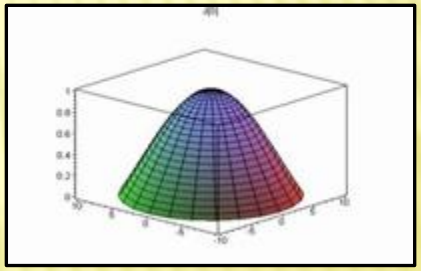
$$\tau_{rz}(r,z) = -\frac{\partial^2}{\partial r\partial z}\left(\Delta - \frac{\partial^2}{\partial z^2}\right)L\ \ r,z$$

$$\tau_{r\vartheta}(r,z) = \tau_{\vartheta z}(r,z) = 0$$

$$q_j\ \ z\ \ = e^{\frac{\alpha z}{2}}\left[A_j\cosh\ \beta_j z\ \cos\ \alpha_j z\ + B_j\cosh\ \beta_j z\ \sin\ \alpha_j z\ + C_j\sinh\ \beta_j z\ \cos\ \alpha_j z\ + D_j\sinh\ \beta_j z\ \sin\ \alpha_j z\right]$$

# FUTURE RESEARCH

*Boundary conditions*
*- for the top and bottom surfaces FGMP*
*- for the top surface and the interface for FGMC*

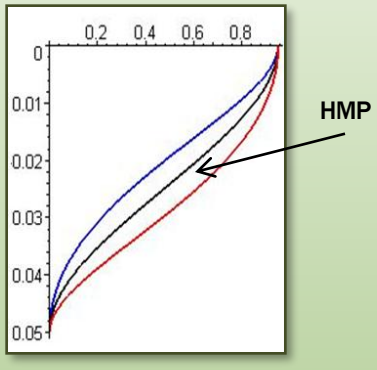


$$u_j(r,z) = -\frac{1}{2G_0 e^{\alpha z}}\left[ -\nu - 1 \frac{d^2}{dz^2} q_j(z) + \nu \phi_j^2 q_j(z) \right] \phi_j J_1 \phi_j r$$

$$w_j(r,z) = -\frac{1}{2G_0 e^{\alpha z}}\left[ \nu - 1 \frac{d^3}{dz^3} q_j(z) - \nu - 1 \alpha \frac{d^2}{dz^2} q_j(z) - \nu - 2 \phi_j^2 \frac{d}{dz} q_j(z) + \alpha \nu \phi_j^2 q_j(z) \right] J_0 \phi_j r$$

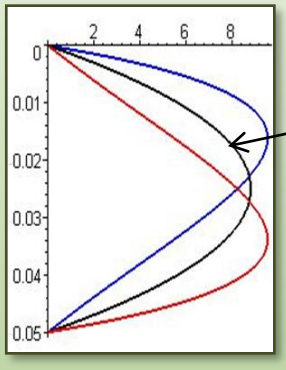
## *Results – FGMs – Plate*

*$G(z)=G_0 e^{\alpha z}$ , loading $J_o$ (j=1) , h=1/20 b, $\nu$=0.3, r=1/2 b, $G_h/G_0$= 10, 1, 0.1*



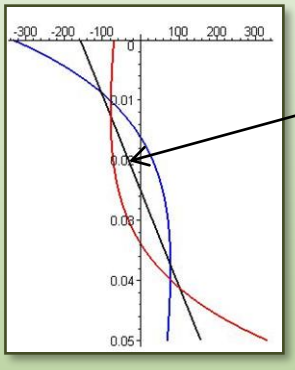

HMP

$\sigma_{zz}$




HMP

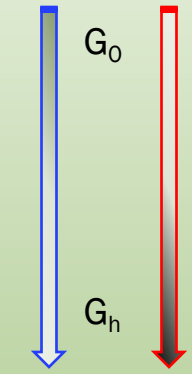
$\sigma_{rz}$




HMP

$\sigma_{rr}$

**Thickness plate**




$G_0 > G_h$ $G_0 < G_h$

Kashtalyan, M. EJM/A(2004) , Li, X.Y. et all. IJSS(2008), Chung, et all. JoMMS (in print)

**Thank you for your attention....**