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# **Enhanced beam models for delamination toughness tests: mixed-mode fracture tests**



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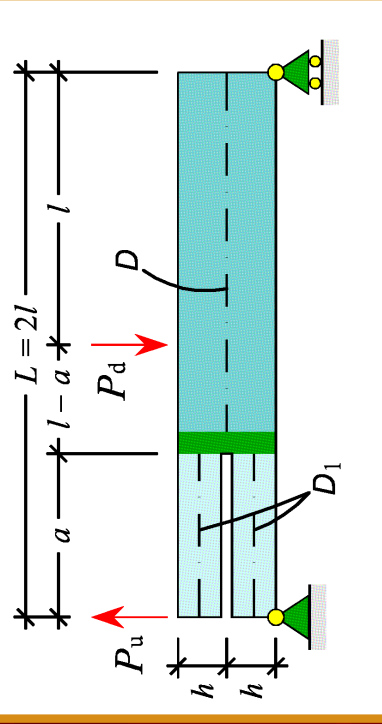
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# Mixed-mode delamination tests

Mixed-mode bending (MMB)



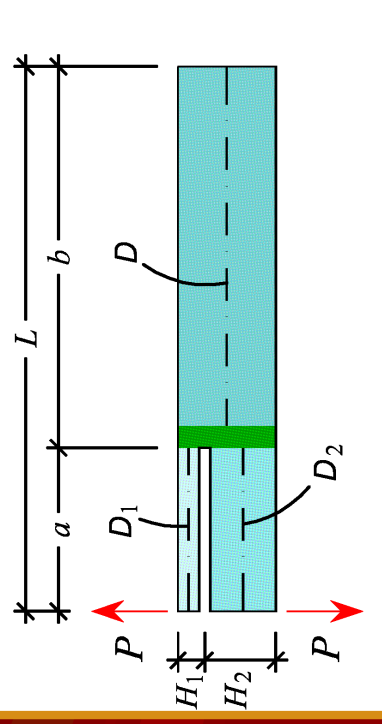
Experimental setup



Simple beam theory model

$$G_I^{SBT} = \frac{8(P_u - P_d/4)^2 a^2}{B^2 D}, \quad G_{II}^{SBT} = \frac{3P_d^2 a^2}{8B^2 D}$$

Asymmetric double cantilever beam (ADCB)

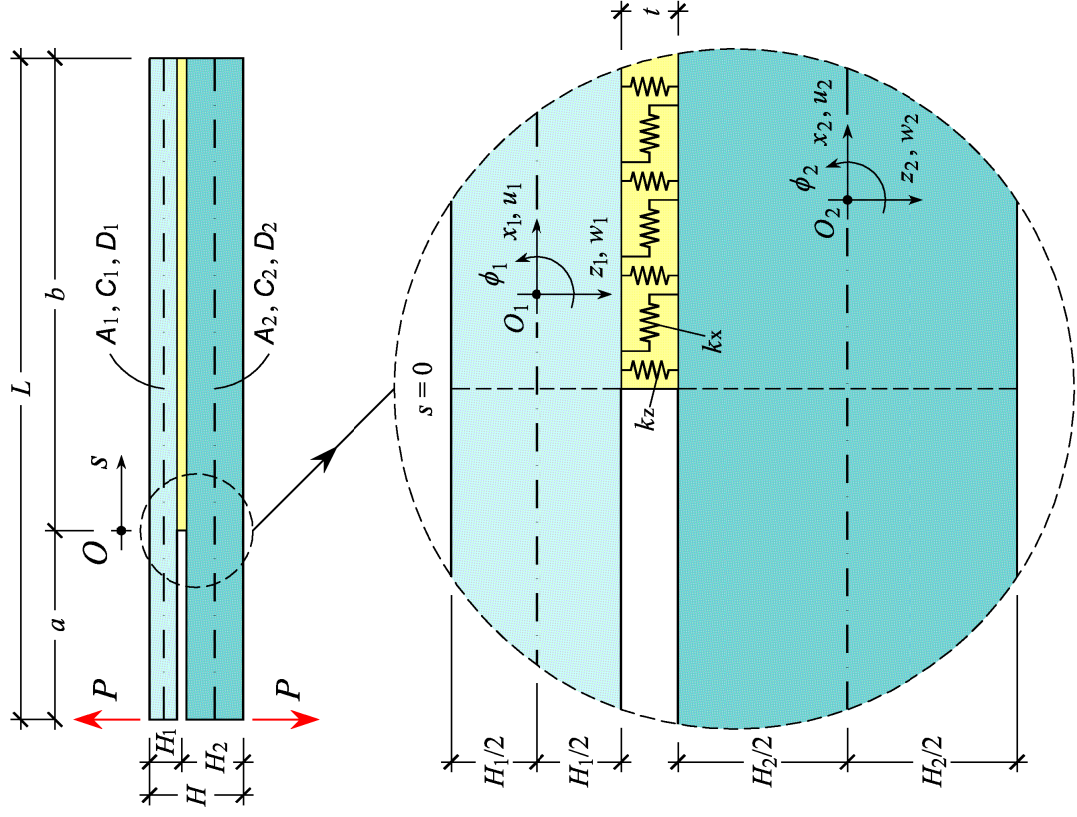


$$G^{SBT} = G_I^{SBT} + G_{II}^{SBT} = \frac{P^2 a^2}{2B^2} \left( \frac{1}{D_1} + \frac{1}{D_2} \right)$$



# Enhanced BT model of ADCB test

## Mechanical model



## Interfacial stresses

$$\sigma(s) = \frac{P}{B} \sum_{i=1}^6 c_i \exp(\lambda_i s)$$

$$\tau(s) = -2 \frac{B}{B} \frac{\sum_{i=1}^6 c_i \left[ \lambda_i^3 - \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \lambda_i + \left( \frac{1}{D_1} + \frac{1}{D_2} \right) \frac{1}{\lambda_i} \right] \exp(\lambda_i s)}{H_1 - H_2} \frac{H_1 - H_2}{D_1 - D_2}$$

## Modal contributions to Energy release rate

$$G_I^{\text{EBT}} = \frac{1}{2} \frac{(\sigma|_{s=0})^2}{k_z} = \frac{P^2}{2k_z B^2} \left( \sum_{i=1}^6 c_i \right)^2$$

$$G_{II}^{\text{EBT}} = \frac{1}{2} \frac{(\tau|_{s=0})^2}{k_x} = \frac{2P^2}{k_x B^2} \left[ \frac{1}{D_1} + \frac{1}{D_2} + \sum_{i=1}^6 c_i \lambda_i \left( \frac{1}{C_1} + \frac{1}{C_2} - \frac{1}{k_x} \right) \right]^2$$

## Constants

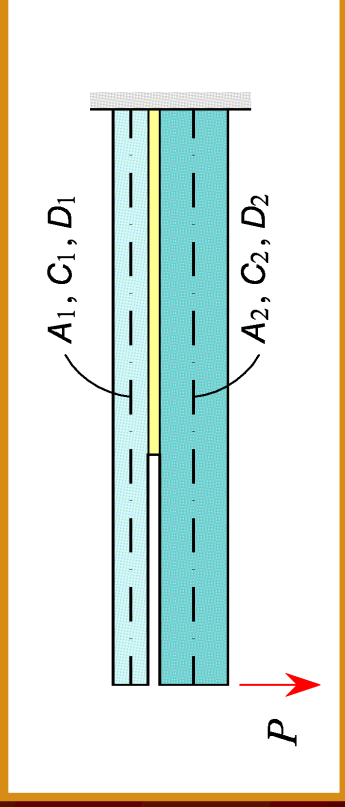
$\lambda_i$  = roots of characteristic equation

$c_i$  = integration constants ( $i = 1, 2, \dots, 6$ )

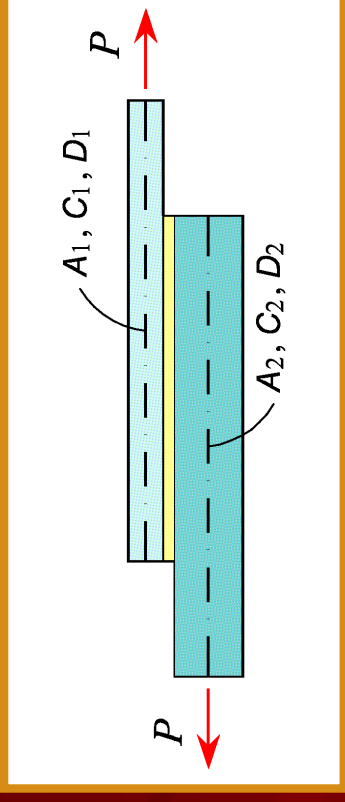


# Developing new EBT models

Asymmetric end loaded split (AELS)

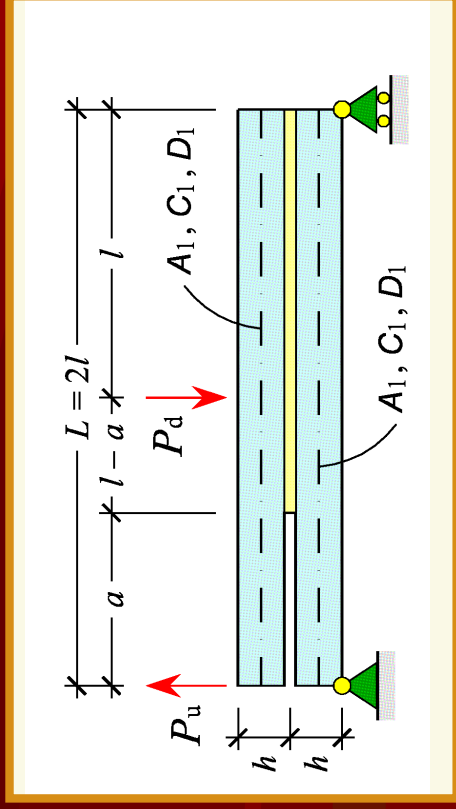


Asymmetric single lap joint (ASLJ)



# Enhanced BT model of MMB test

Mechanical model



Correction factors

$$\left. \begin{aligned} \mu_I &= \frac{G_I^{EBT}}{G_I^{SBT}} = \left[ 1 + \frac{(\lambda_1 + \lambda_2)^2 (1 - Z_1 Z_2 - T_1 T_2) + \frac{\lambda_1^2 - \lambda_2^2}{\lambda_1 \lambda_2 a} (\lambda_1 T_2 - \lambda_2 T_1)}{(\lambda_1^2 + \lambda_2^2) T_1 T_2 - 2 \lambda_1 \lambda_2 (1 - Z_1 Z_2)} \right]^2 \\ \mu_{II} &= \frac{G_{II}^{EBT}}{G_{II}^{SBT}} = \left\{ \coth \lambda_5 (L - a) + \frac{1}{\lambda_5 a} \left[ 1 - 2 \frac{\sinh \lambda_5 (L/2)}{\sinh \lambda_5 (L - a)} \right] \right\}^2 \end{aligned} \right\}$$

where  $Z_j = \operatorname{sech} \lambda_j (L - a)$ ,  $T_j = \tanh \lambda_j (L - a)$ ,  $j = 1, 2$ ,  
and  $\lambda_1, \lambda_2, \lambda_5$  = roots of characteristic equations

