Equilibrium of Masonry Domes

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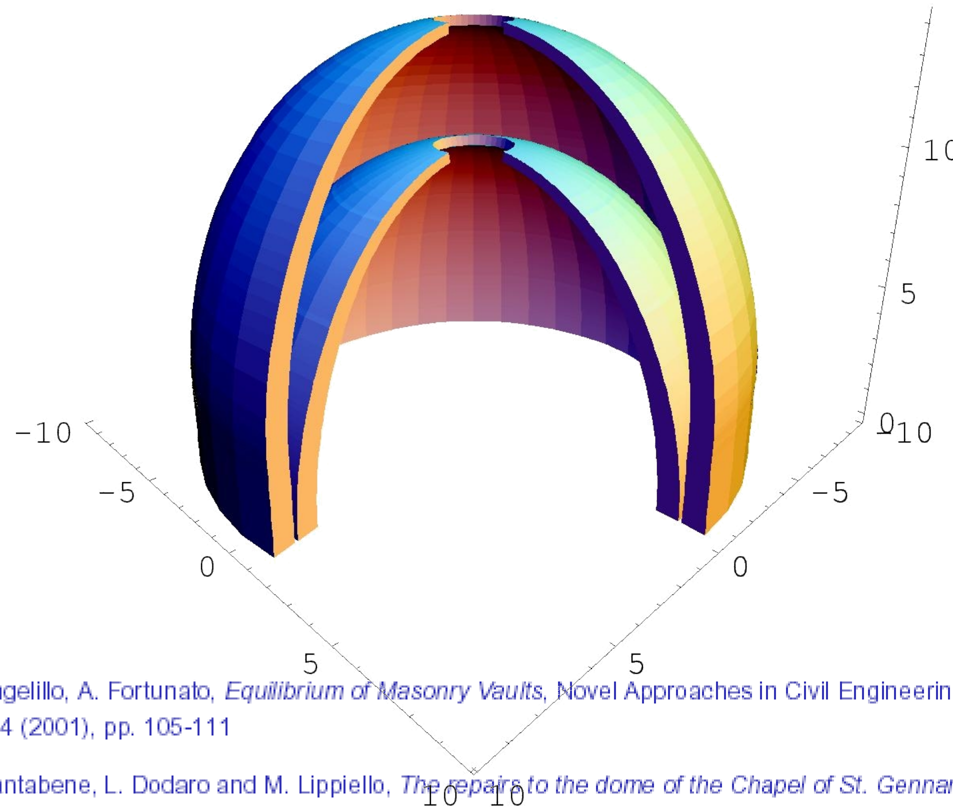

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- G. Cantabene, L. Dodaro and M. Lippiello, *The repairs to the dome of the Chapel of St. Gennaro's Treasure: the Eighteenth-century dispute between F. Sanfelice and G. Lucchese*, Proc. of the 2nd International Congress on Construction History, p. 561- 577. Cambridge (2006).
- M. Lippiello,L. Dodaro and M.R. Gargiulo *Strengthening of Neapolitan Domes Between the XVII and XVIII Century: Historical and Structural Analysis*, Proc. of Structural Analysis of Historical Constructions, p. 1463-1470. New Delhi (2006).

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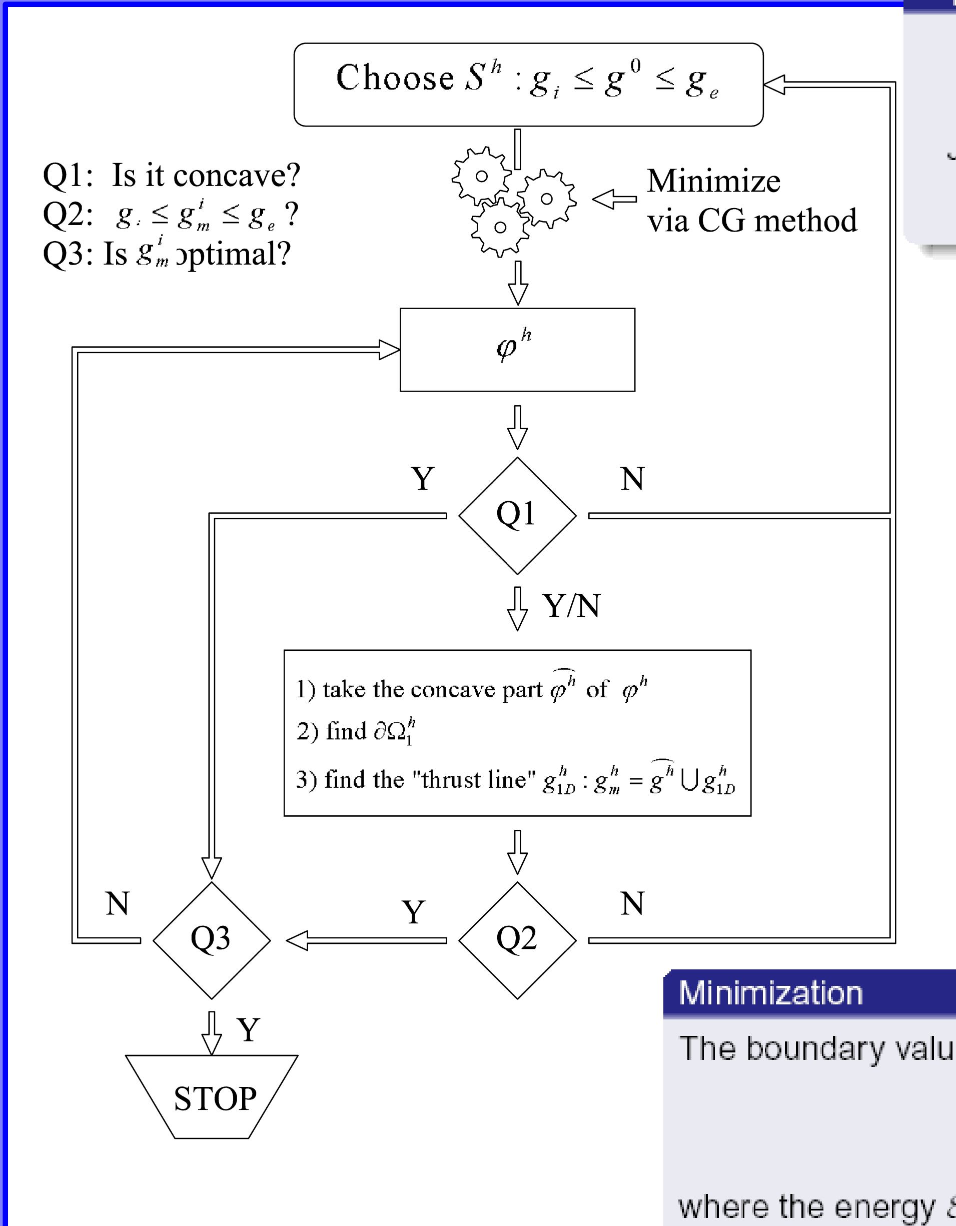


Equilibrium equations and projected Pucher-like form

$$
\begin{aligned}
JT^{\alpha\beta}_{,\beta} + Jb_\alpha &= S^{\alpha\beta}_{,\beta} + b^0_\alpha = 0,\\
JT^{\alpha\beta} f_{,\alpha\beta} - f_{,\alpha} Jb_\alpha + Jb_3 &= S^{\alpha\beta} f_{,\alpha\beta} - f_{,\alpha} b^0_\alpha + b^0_3 = 0.
\end{aligned}
$$

Stress tensor restrictions:

$$
\mathrm{tr}\mathbf{S} \le 0\ ,\qquad \det\mathbf{S} \ge 0\ .
$$

The stress surface (whose graph is φ) is concave:

$$
\begin{aligned}
\varphi_{,11} + \varphi_{,22} &\le 0,\\
\varphi_{,11}\varphi_{,22} - (\varphi_{,12})^2 &\ge 0.
\end{aligned}
$$

The domain Ω can be splitted into:

$$
\Omega_1 \equiv \{\mathbf{x} \in \Omega,\ \mathrm{tr}\mathbf{S} < 0\ \&\ \det\mathbf{S} > 0\}\ ,\ (\mathbf{S}\ \text{biaxial})
$$

$$
\Omega_2 \equiv \{\mathbf{x} \in \Omega,\ \mathrm{tr}\mathbf{S} < 0\ \&\ \det\mathbf{S} = 0\}\ ,\ (\mathbf{S}\ \text{uniaxial})
$$

Minimization

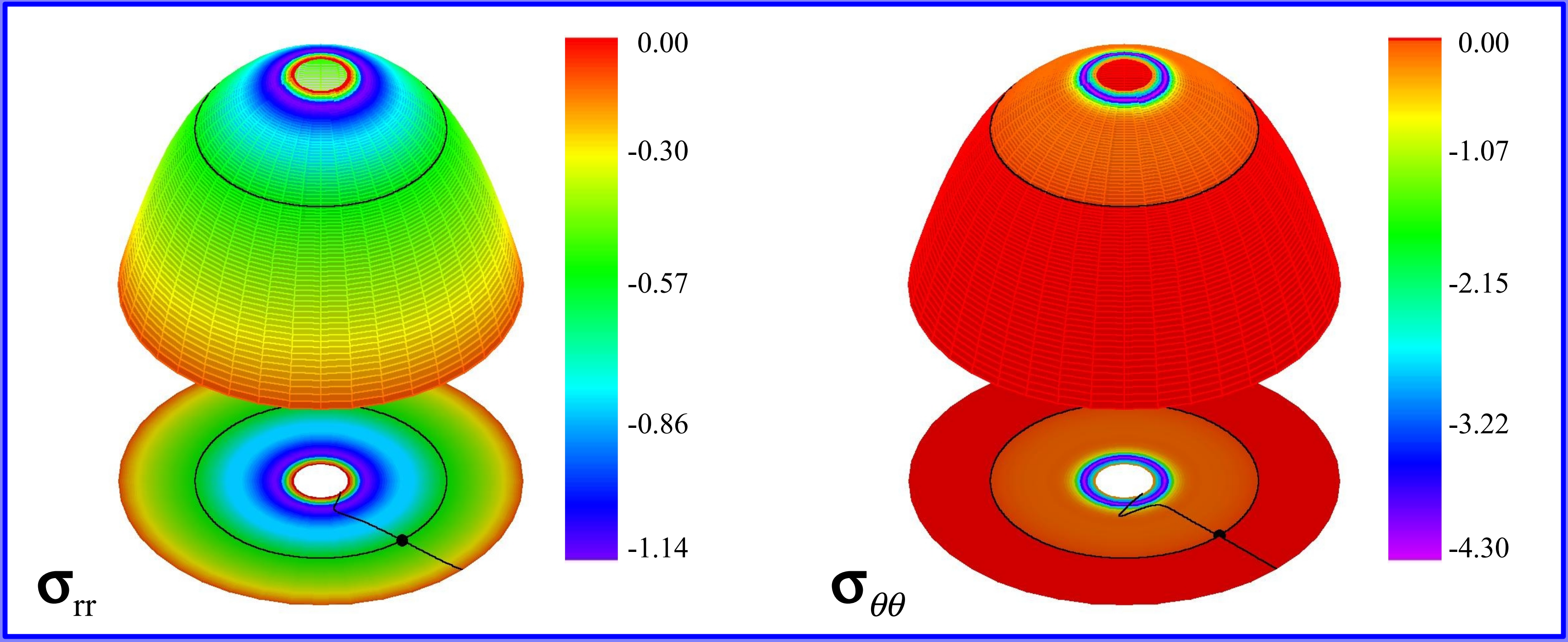
The boundary value problem can be stated in variational form through

$$
\mathcal{E}(g^\circ, \varphi^\circ) = \min_{\varphi\cap\ ,g_i \le g \le g_e} \mathcal{E}(g, \varphi)
$$

where the energy $\mathcal{E}$ is defined as follows

$$
\mathcal{E}(g,\varphi) = \frac{1}{2}\int_\Omega \rho_{\alpha\beta}\varphi_{,\alpha}\varphi_{,\beta}\,da + \int_\Omega p\varphi\,da - \frac{1}{|\Omega|}\int_\Omega p\,da \int_{\partial\Omega} \varphi\,ds
$$

External cap



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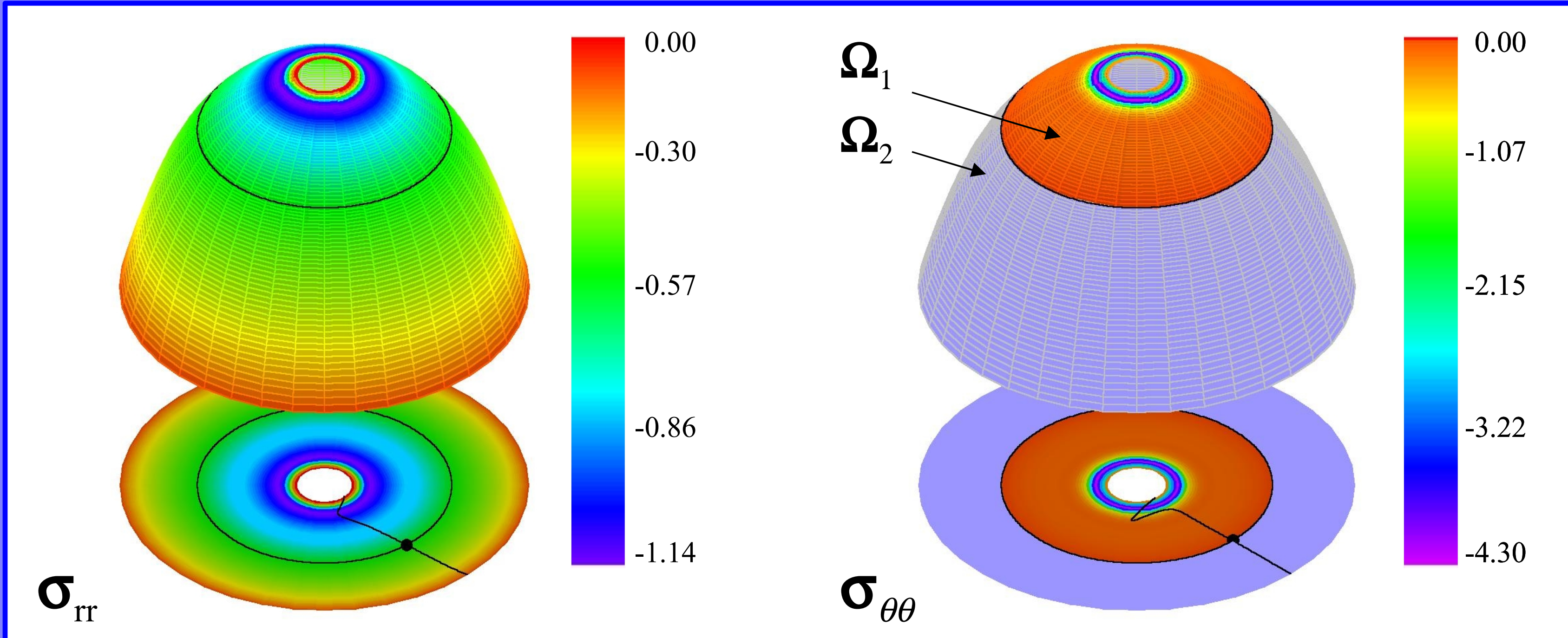
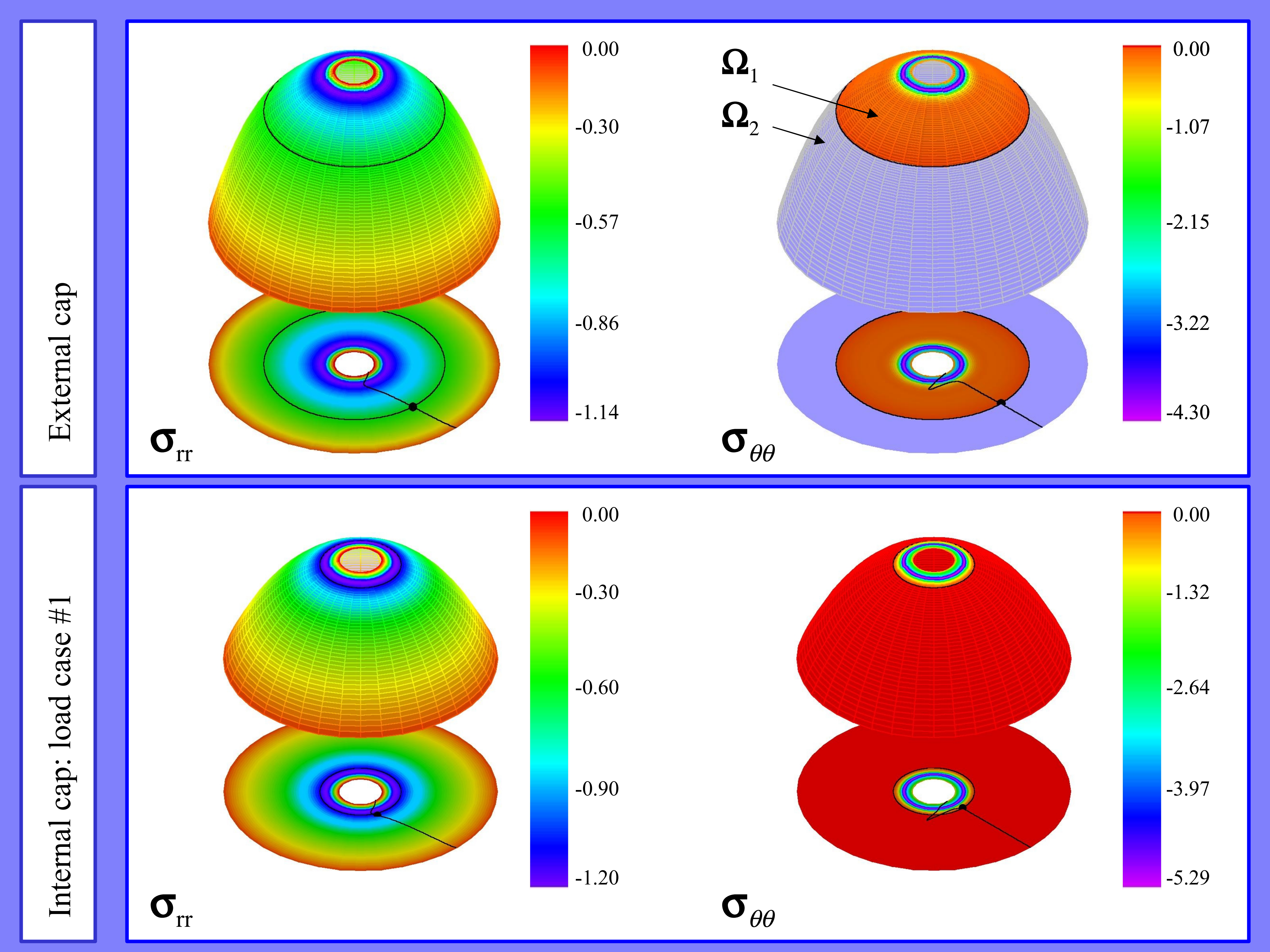
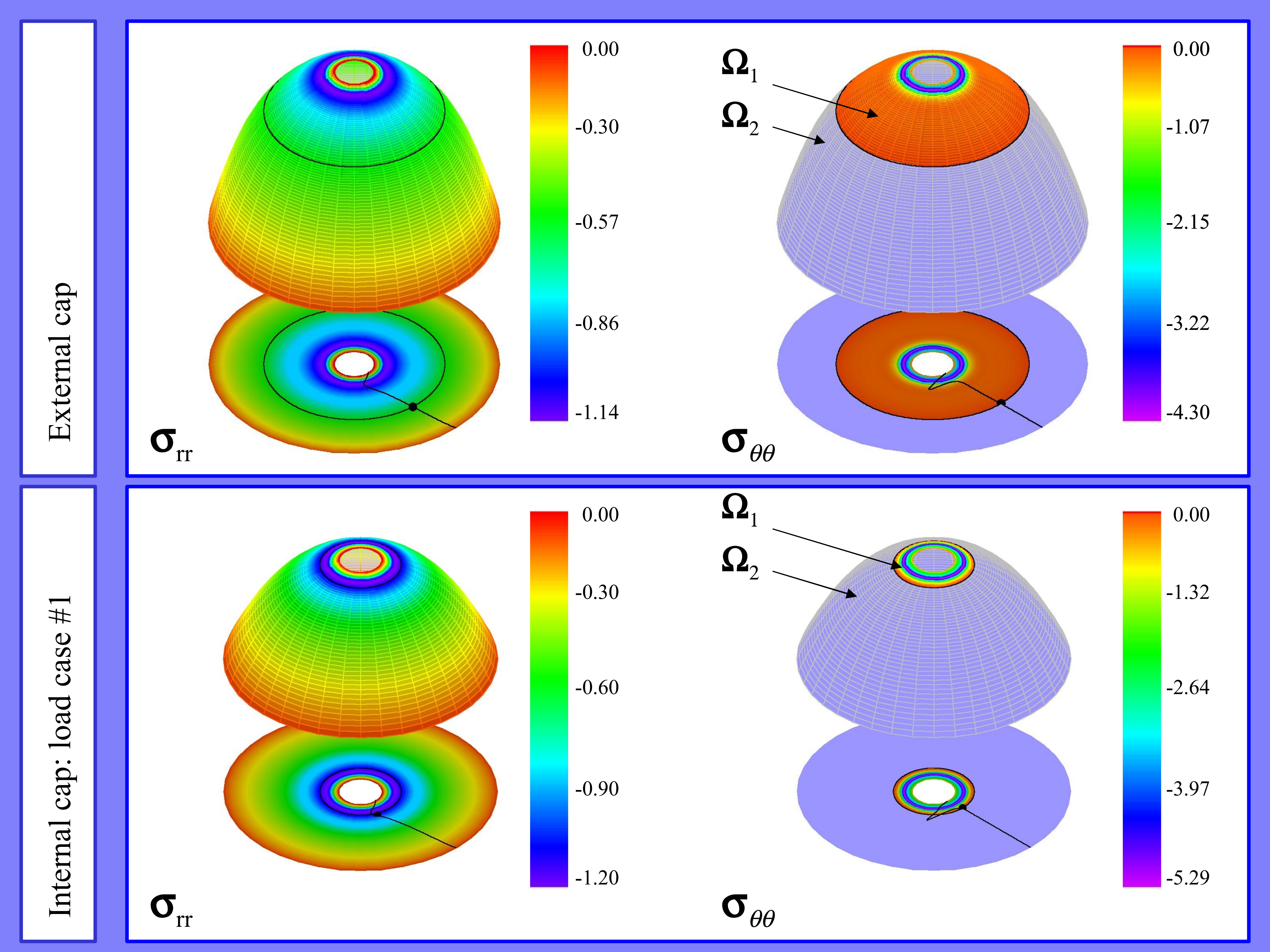
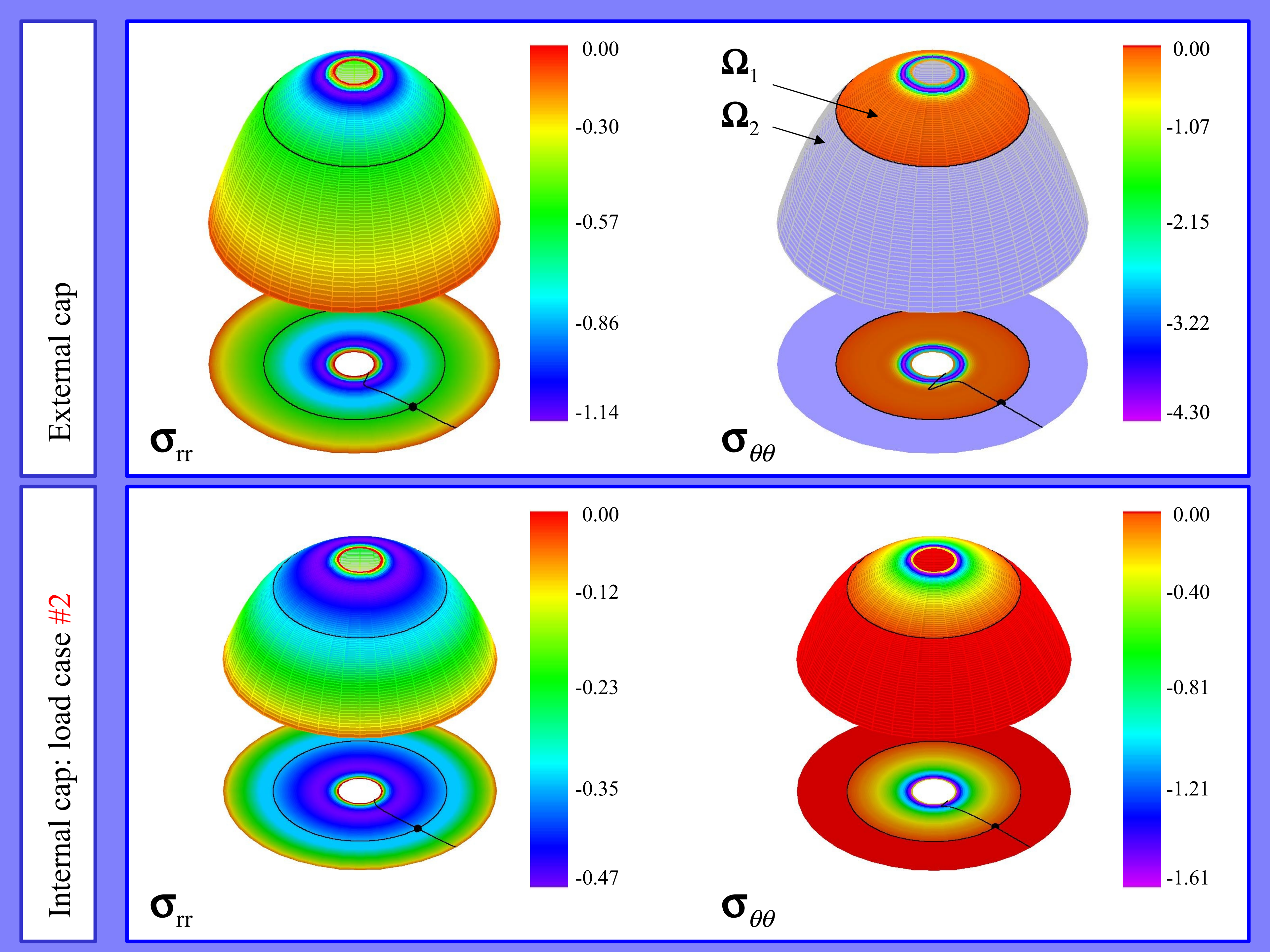


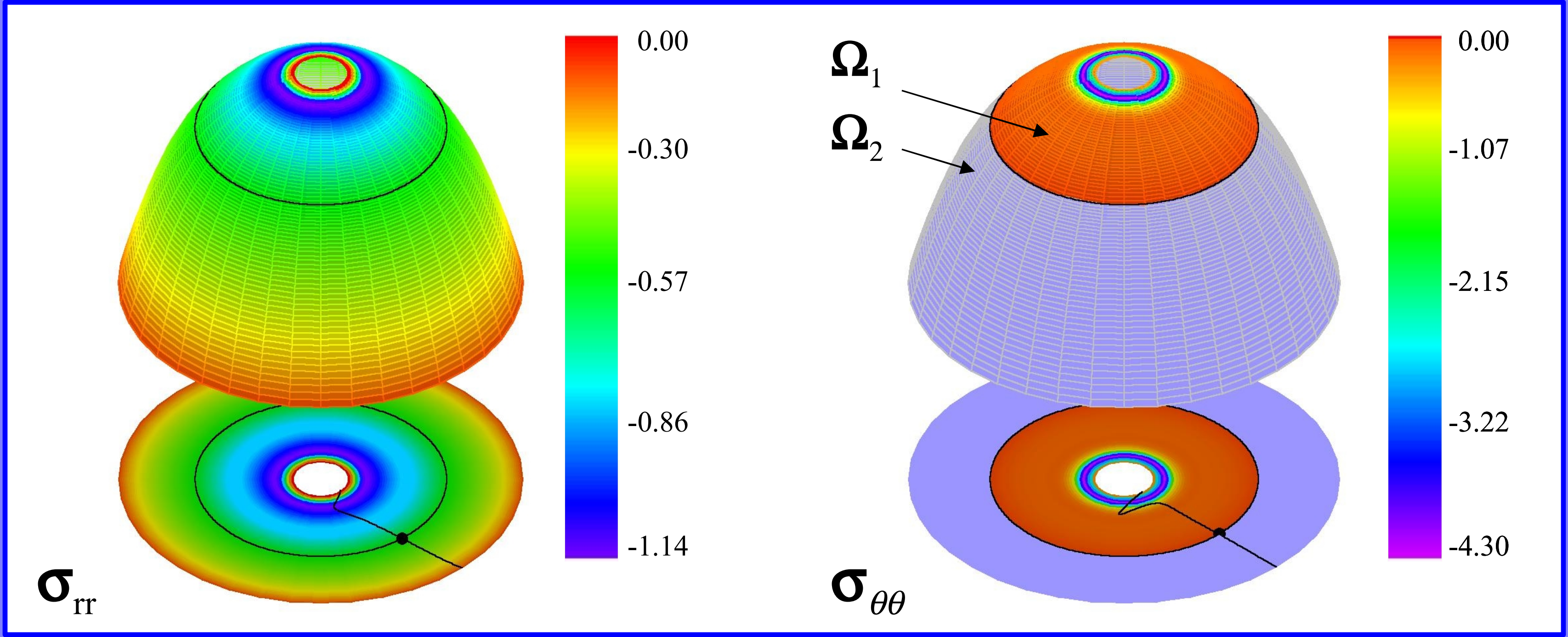

External cap
Internal cap: load case #1
σ_{rr}
0.00
-0.30
-0.57
-0.86
-1.14
Ω_1
Ω_2
$\sigma_{\theta\theta}$
0.00
-1.07
-2.15
-3.22
-4.30
σ_{rr}
0.00
-0.30
-0.60
-0.90
-1.20
$\sigma_{\theta\theta}$
0.00
-1.32
-2.64
-3.97
-5.29


External cap
0.00
-0.30
-0.57
-0.86
-1.14
σ_{rr}
Ω_1
Ω_2
0.00
-1.07
-2.15
-3.22
-4.30
$\sigma_{\theta\theta}$
Internal cap: load case #1
0.00
-0.30
-0.60
-0.90
-1.20
σ_{rr}
Ω_1
Ω_2
0.00
-1.32
-2.64
-3.97
-5.29
$\sigma_{\theta\theta}$

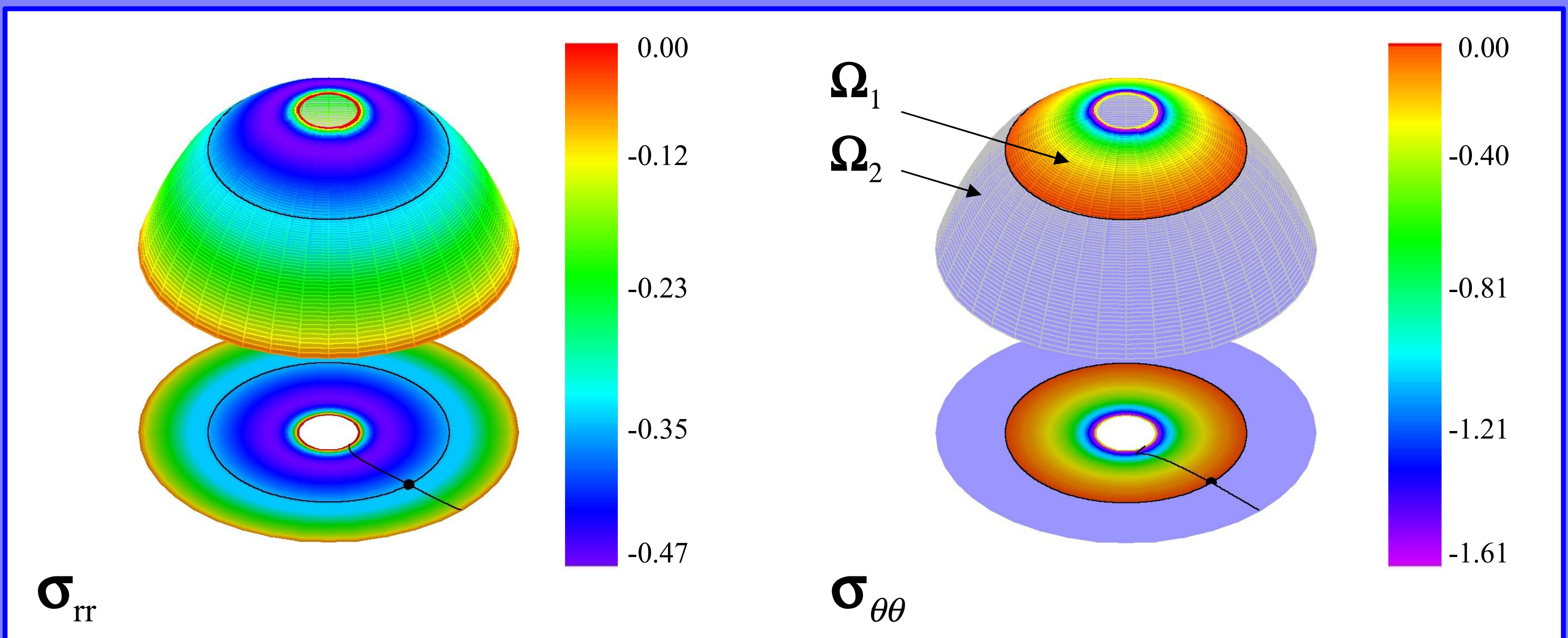


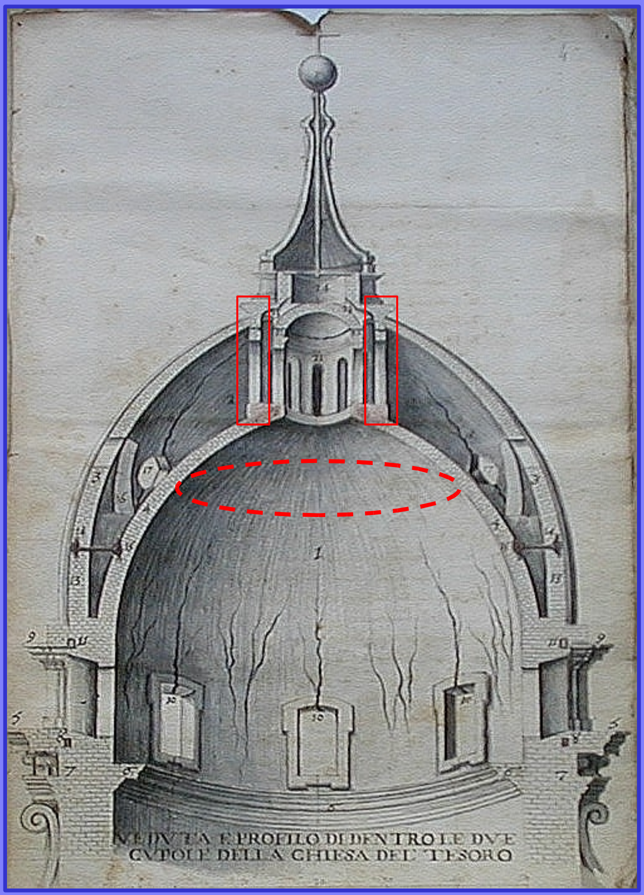
External cap
Internal cap: load case #2
σrr
σθθ
Ω1
Ω2
0.00
-0.30
-0.57
-0.86
-1.14
0.00
-1.07
-2.15
-3.22
-4.30
σrr
σθθ
0.00
-0.12
-0.23
-0.35
-0.47
0.00
-0.40
-0.81
-1.21
-1.61

External cap



Internal cap: load case #2





So...

- The results of our analysis indicate that the shape of the double dome is essentially correct for equilibrium of the structure.
- The perplexities about the dome stability expressed by Lucchese seem not justified. The code suggests that the dome is safe... one can think the other way around... the dome is the check for the code... the code works well!
- The demolition of the first enclosure of the drum, as proposed by Sanfelice, seems appropriate, being the weight of the lantern and of the double drum the crucial factor in widening the Ω_2 region, that is the region of potential radial fractures.

Future works

- Step 1: Insert the effect of iron rings.
- Step 2: Test the method on other domes.
- Step 3: Develop the numerical code in order to make it effective with any kind of vaults.