A UNIFIED MODELLING OF FULLY AND PARTIALLY SATURATED POROUS MATERIALS BY CONSIDERING AIR DISSOLVED IN WATER

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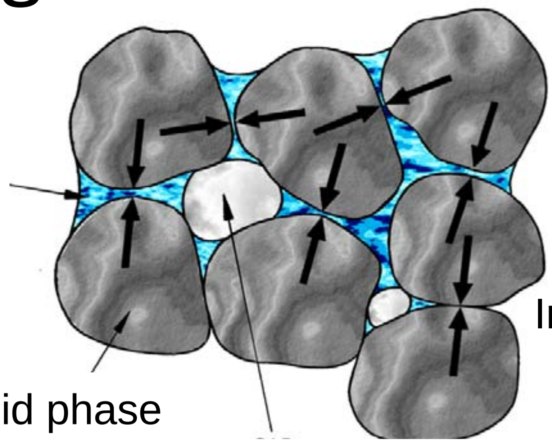
Multiphase modelling of non-isothermal solid porous materials



Microscopic view of a three-phase material

(partially saturated media)

Mathematical model (<u>fully</u> saturated materials):

- linear momentum balance eq.

- mass conservation equation for water

-                 ?

- energy balance equation of mixture

Mathematical model (<u>partially</u> saturated materials):

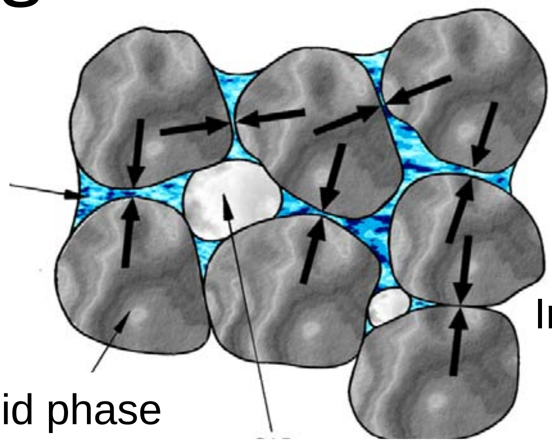
- linear momentum balance eq.

- mass conservation equation for water and vapour

- dry air mass balance equation

- energy balance equation of mixture

Multiphase modelling of non-isothermal solid porous materials



liquid phase: water

solid phase

gas phase: dry air and water vapour

Intergranular forces due to capillary effects

Microscopic view of a three-phase material

(partially saturated media)

Mathematical model (<u>fully</u> saturated materials):

$$div\left(\boldsymbol{\sigma}' - p^w \mathbf{1}\right) + \rho \mathbf{g} = \mathbf{0}$$

Mathematical model (<u>partially</u> saturated materials):

$$div\left(\boldsymbol{\sigma}' - \left[p^g - S_w p^c\right]\mathbf{1}\right) + \rho \mathbf{g} = \mathbf{0}$$

Linear momentum balance equation

Fully saturated:

$$\rho^w div\, \mathbf{v}^s - \beta_{sw} \frac{\partial T}{\partial t} - div\left(\rho^w \frac{\mathbf{k}}{\mu^w}\left[-grad\left(p^c\right) - \rho^w \mathbf{g}\right]\right) = 0,$$

Partially saturated:

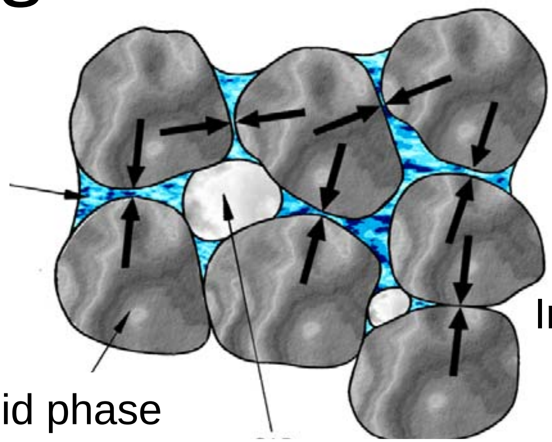
$$n\left[\rho^w - \rho^{gw}\right]\frac{\partial S_w}{\partial t} + \left[\rho^w S_w + \rho^{gw} S_g\right] div\, \mathbf{v}^s + n S_g \frac{\partial \rho^{gw}}{\partial t} - \beta_{sgw}\frac{\partial T}{\partial t}$$

$$-div\left(\rho^g \frac{M_a M_w}{M_g^2} \mathbf{D}_g^{gw} grad\left(\frac{p^{gw}}{p^g}\right)\right) - div\left(\rho^{gw} \frac{\mathbf{k} k^{rg}}{\mu^g}\left[grad\left(p^g\right) - \rho^g \mathbf{g}\right]\right)$$

$$-div\left(\rho^w \frac{\mathbf{k} k^{rw}}{\mu^w}\left[grad\left(p^w\right) - \rho^w \mathbf{g}\right]\right) = 0,$$

Mass balance equation for liquid species

Multiphase modelling of non-isothermal solid porous materials



Microscopic view of a three-phase material

(partially saturated media)

Mathematical model (<u>fully</u> saturated materials):

?

Mathematical model (<u>partially</u> saturated materials):

$$-n\rho^{ga}\frac{\partial S_w}{\partial t}+\rho^{ga}\left[1-S_w\right]div\,\mathbf{v}^s+n\left[1-S_w\right]\frac{\partial\rho^{ga}}{\partial t}-div\left(\rho^g\frac{M_aM_w}{M_g^2}\mathbf{D}_g^{ga}grad\left(\frac{p^{ga}}{p^g}\right)\right)$$
$$-div\left(\rho^{ga}\frac{\mathbf{k}k^{rg}}{\mu^g}\left[grad\left(p^g\right)-\rho^g\mathbf{g}\right]\right)-\beta_s\rho^{ga}\left[1-n\right]\left[1-S_w\right]\frac{\partial T}{\partial t}=\dot{m}_{ga}.$$

Mass balance equation for dry air

$$\left(\rho C_p\right)_{eff}\frac{\partial T}{\partial t}-\left(n\rho^wC_p^w\mathbf{v}^w\right)\cdot grad\,T-div\left(\chi_{eff}\,grad\,T\right)=0$$

$$\left(\rho C_p\right)_{eff}\frac{\partial T}{\partial t}+\left[nS_w\rho^wC_p^w\mathbf{v}^w+nS_g\rho^gC_p^g\mathbf{v}^g\right]\cdot grad\,T-div\left(\chi_{eff}\,grad\,T\right)=-\dot{m}\Delta H_{vap}$$

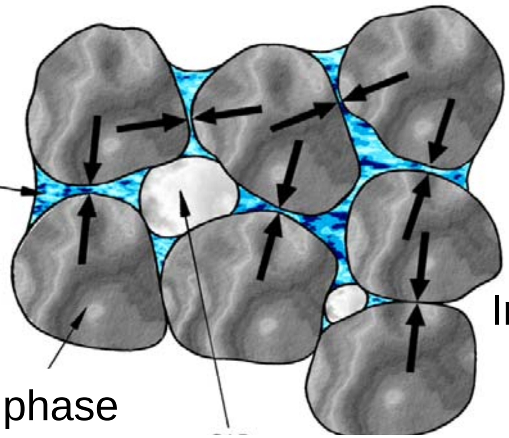
Energy balance equation for the mixture

Multiphase modelling of non-isothermal solid porous materials *(Gawin and Sanavia, TIPM submitted)*

Microscopic view of a three-phase material

(partially saturated media)

liquid phase: water and **dissolved air**

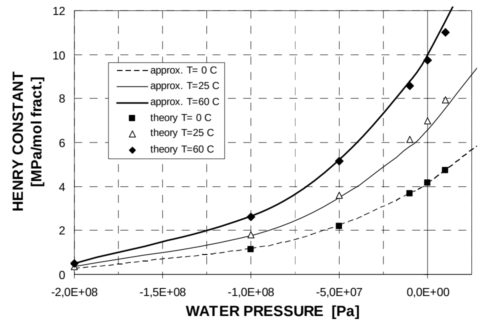


solid phase

Intergranular forces due to capillary effects

gas phase: dry air, water vapour **and released air**

Mathematical model (for <u>fully and partially</u> saturated materials):



- linear momentum balance eq.
- mass conservation equation for water and **vapour**
- dry air **and dissolved air** mass balance equation
- energy balance equation of mixture

$$n\left[\rho^w \frac{p^{ga}}{K_{ca}} - \rho^{ga}\right]\frac{\partial S_w}{\partial t} + \left[\rho^w \frac{p^{ga}}{K_{ca}} S_w + \rho^{ga} S_g\right] div\, \mathbf{v}^s - \beta_{saw}\frac{\partial T}{\partial t} + nS_g \frac{\partial \rho^{ga}}{\partial t}$$

$$+n\rho^w S_w \frac{\partial}{\partial t}\left(\frac{p^{ga}}{K_{ca}}\right) - div\left(\rho^g \frac{M_a M_w}{M_g^2}\mathbf{D}_g^{ga}\, grad\left(\frac{p^{ga}}{p^g}\right)\right) - div\left(\rho^{ga}\frac{\mathbf{k}k^{rg}}{\mu^g}\left[grad\left(p^g\right) - \rho^g \mathbf{g}\right]\right)$$

$$-div\left(\rho^w \mathbf{D}_w^{ga}\, grad\left(\frac{p^{ga}}{K_{ca}}\right)\right) - div\left(\frac{p^{ga}}{K_{ca}}\rho^w \frac{\mathbf{k}k^{rw}}{\mu^w}\left[grad\left(p^w\right) - \rho^w \mathbf{g}\right]\right) = 0.$$

Main results *(un-published)*

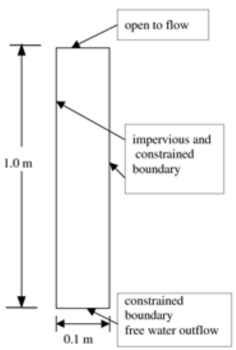
Air dissolved in water formulation permits a unified CTHM modelling of saturated/unsaturated porous materials, <u>without</u>:

• application of a residual saturation, which means the presence of gas in the “fully water saturated” material;

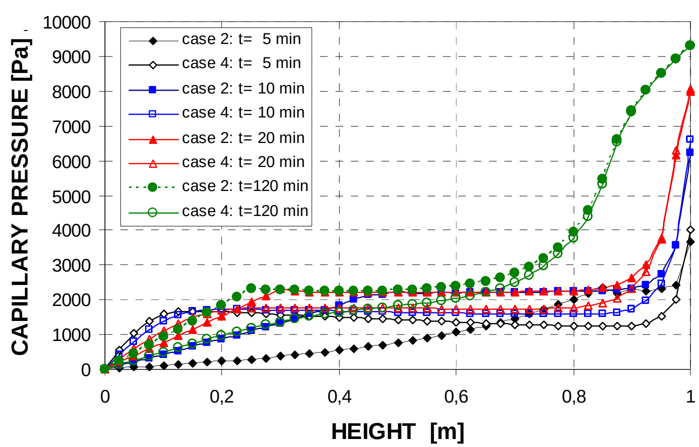
• application of an appropriate switching procedure to eliminate the gas mass conservation equation for fully saturated material (Gawin and Schrefler, EC 1996);

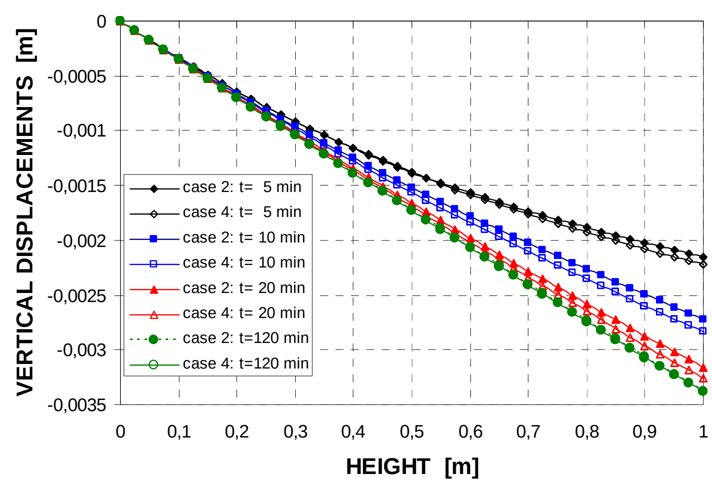
• formulating the related Stefan problem (Nocchetto et al., MC 1991).

This new model, solved by FEM for coupled problems, shows that:



Liakopoulos test

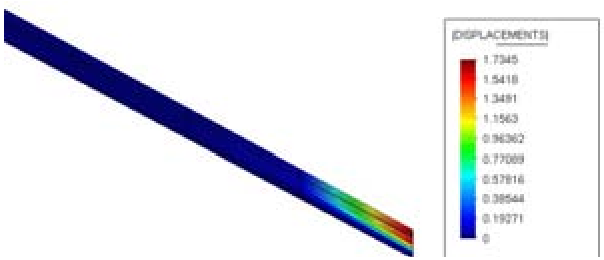


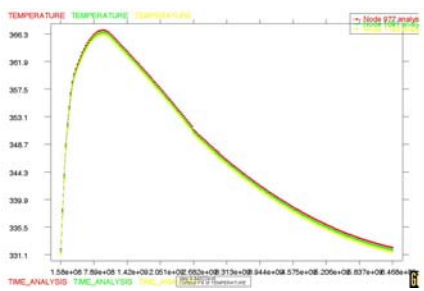


Conclusions and future developments

- A deeper insight in the multi-physics modelling of solid porous materials (taking into account the air dissolved in water and its desorption at lower water pressure – usually neglected in literature), allows for a unified modelling of partially and fully saturated media,
- without application of any 'unphysical' numerical techniques.

Future developments: applications





Onset of Sarno landslides (May 1998)

THM behaviour of radioactive disposal