ANALISI DIMENSIONALE :

Similitudine geometrica : $\exists \quad \lambda = \frac{l_m}{l_p}$

Similitudine cinematica : $\exists \quad \tau = \frac{t_m}{t_p}$

$\rightarrow$ similitudine dinamica $\quad \pi_1 = f(\pi_2, \pi_3, \pi_4 \dots)$

se $\pi_{2_m} = \pi_{2_p}$; $\pi_{3_m} = \pi_{3_p}$; ... $\Rightarrow \boxed{\pi_{1_m} = \pi_{1_p}}$

Una similitudine **totale** è raramente possibile $\rightarrow$ SIMILITUDINE PARZIALE

$$Ne = \frac{F}{\rho U^2 L^2} \qquad Re = \frac{\rho U L}{\mu} \qquad Ma = \frac{U}{c_s} = \frac{U}{\sqrt{k_s/\rho}} \; ; \; Fr = \frac{U}{\sqrt{gL}}$$

$$We = \frac{U}{\sqrt{\sigma_s/\rho L}}$$

CONDOTTE IN PRESSIONE :

$$Re = \frac{\rho U D_h}{\mu} \qquad D_h = \frac{4A_c}{P} = 4\,\frac{\text{area sezione}}{\text{perimetro bagnato}}$$

Se $Re \gtrsim 2300$: moto turbolento

$$\Delta P_L = f \frac{L}{D} \frac{\rho V^2}{2}$$ perdite di pressione distribuite (attrito) ; f = coef. attrito di Darcy

$h_L = f \frac{L}{D} \frac{V^2}{2g}$: perdite di carico

$$\Delta P_L = K_L \frac{\rho V^2}{2}$$

$h_L = K_L \frac{V^2}{2g}$: perdite di carico concentrate (su gomiti, raccordi, flange, etc.)

K_L : tabulato

$f = \frac{64}{Re}$ (laminare) $\qquad f$ dal diagramma di Moody se moto turbolento

Problemi di verifica : determina ΔP_L dati L, D, V (immediato)

di verifica : " V dati $L, D, \Delta p$ (iterativo)

di progetto : " D dati $L, \Delta p, V$ (iterativo)

Eq. energia : $$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + \underbrace{h_{pump,u}}_{\text{prevalenza pompa}} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \underbrace{h_{turbine,e}}_{\text{salto utile turbina}} + h_L$$

Eq. differenziali, caso incomprimibile :

$\vec{\nabla} \cdot \vec{V} = 0 \quad , \quad \rho\left(\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \vec{\nabla} \vec{V}\right) = -\vec{\nabla} p + \rho \vec{g} + \mu \nabla^2 \vec{V}$

Nel caso cartesiano $\quad \vec{\nabla} \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$$(\vec{V} \cdot \vec{\nabla})\vec{V} = u \frac{\partial \vec{V}}{\partial x} + v \frac{\partial \vec{V}}{\partial y} + w \frac{\partial \vec{V}}{\partial z}$$

In coordinate cilindriche le eq. di Navier-Stokes sono :

Incompressible continuity equation: $\quad \frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\frac{\partial(u_\theta)}{\partial \theta} + \frac{\partial(u_z)}{\partial z} = 0$

r-component of the incompressible Navier–Stokes equation:

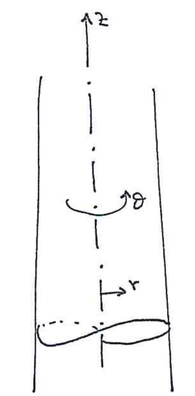
$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z\frac{\partial u_r}{\partial z}\right)$$
$$= -\frac{\partial P}{\partial r} + \rho g_r + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right) - \frac{u_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2}\right]$$

θ-component of the incompressible Navier–Stokes equation:

$$\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z\frac{\partial u_\theta}{\partial z}\right)$$
$$= -\frac{1}{r}\frac{\partial P}{\partial \theta} + \rho g_\theta + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right) - \frac{u_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2}\right]$$

z-component of the incompressible Navier–Stokes equation:

$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right)$$
$$= -\frac{\partial P}{\partial z} + \rho g_z + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right]$$



assialsimmetria: $\frac{\partial}{\partial \theta} = 0$

moto stazionario : $\frac{\partial}{\partial t} = 0$

moto completamente sviluppato: $\frac{\partial \vec{V}}{\partial z} =$

Nota bene : vale l'identità $\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{u}{r^2}\right] = \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r u)\right]$

che semplifica la risoluzione di molti problemi in coordinate cilindriche

In coordinate (r, z, θ) il tensore degli sforzi è $\overline{\overline{\Pi}} = -p\overline{\overline{I}} + \overline{\overline{\tau}}$, con

$\overline{\overline{\tau}}$ tensore sforzi viscosi :

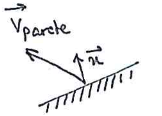
$\overline{\overline{I}}$ tensore identità

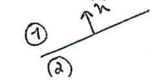
$$\tau_{ij} = \begin{pmatrix} \tau_{rr} & \tau_{r\theta} & \tau_{rz} \\ \tau_{\theta r} & \tau_{\theta\theta} & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \tau_{zz} \end{pmatrix} = \begin{pmatrix} 2\mu\frac{\partial u_r}{\partial r} & \mu\left[r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right] & \mu\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) \\ \mu\left[r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right] & 2\mu\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}\right) & \mu\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) \\ \mu\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) & \mu\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) & 2\mu\frac{\partial u_z}{\partial z} \end{pmatrix}$$

Se $\mu \to 0 \Rightarrow$ equazioni di Eulero

Condizioni al contorno

	Eulero	Navier – Stokes
PARETE IN MOTO CON VELOCITÀ V_{parete}	$\vec{V} \cdot \vec{n} = \vec{V}_{parete} \cdot \vec{n}$	$\vec{V} = \vec{V}_{parete}$
INTERFACCIA TRA DUE FLUIDI ① e ②	$\vec{V}_1 \cdot \vec{n} = \vec{V}_2 \cdot \vec{n}$ $(\overline{\overline{\Pi}}_1 \cdot \vec{n}) \cdot \vec{n} = (\overline{\overline{\Pi}}_2 \cdot \vec{n}) \cdot \vec{n}$	$\vec{V}_1 = \vec{V}_2$ $\overline{\overline{\Pi}}_1 \cdot \vec{n} = \overline{\overline{\Pi}}_2 \cdot \vec{n}$





SOLUZIONI APPROSSIMATE :

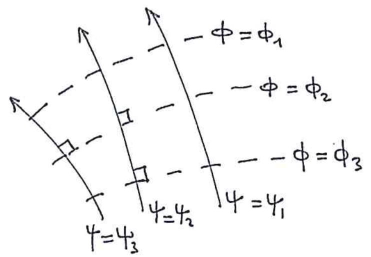
Moti di scorrimento ("creeping flow") ("moti di Stokes") : $Re \ll 1$, $\vec{\nabla} p \cong \mu \nabla^2 \vec{V}$

Resistenza su una sfera in moto con $Re \leq 1$: $\vec{D} = -3\pi\mu D \vec{V}$

Moti potenziali (o irrotazionali): $\vec{\zeta} = \vec{\nabla} \times \vec{V} = \vec{0} \Rightarrow \exists\, \phi$ t.c. $\vec{V} = \vec{\nabla}\phi$

(ϕ = potenziale di velocità)

$\rho \frac{\partial \phi}{\partial t} + p + \frac{1}{2}\rho V^2 + \rho g z = \text{cost.}$ in tutto il campo di moto incomprimibile e irrotazionale. Se anche campo stazionario il primo termine si annulla



Linee di corrente (ψ = cost.) e linee equipotenziali (ϕ = cost.) sono ortogonali tra loro.

N.B. ψ è definita solo per moto piano $[(x,y)$ oppure $(r,\theta)]$ o moto assialsimmetrico $\left[(r,z)\ ,\ \frac{\partial}{\partial\theta} = 0\right]$

Tabella per moto bidimensionale o assialsimmetrico :

Velocity components for steady, incompressible, irrotational, two-dimensional regions of flow in terms of velocity potential function and stream function in various coordinate systems

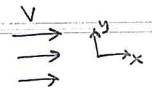
Description and Coordinate System	Velocity Component 1	Velocity Component 2
Planar; Cartesian coordinates	$u = \frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}$	$v = \frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x}$
Planar; cylindrical coordinates	$u_r = \frac{\partial\phi}{\partial r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$	$u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{\partial\psi}{\partial r}$
Axisymmetric; cylindrical coordinates	$u_r = \frac{\partial\phi}{\partial r} = -\frac{1}{r}\frac{\partial\psi}{\partial z}$	$u_z = \frac{\partial\phi}{\partial z} = \frac{1}{r}\frac{\partial\psi}{\partial r}$

cilindrica

$$\underbrace{\vec{\nabla}\times\vec{V} = \vec{e}_r\left(\frac{1}{r}\frac{\partial u_z}{\partial\theta} - \frac{\partial u_\theta}{\partial z}\right) + \vec{e}_\theta\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right) + \vec{e}_z\frac{1}{r}\left[\frac{\partial(r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial\theta}\right]}_{\text{ROTORE}}$$

Moti elementari piani :

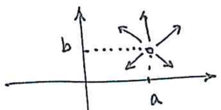
moto uniforme :



$\phi = Vx$

$\psi = Vy$

sorgente di portata $\dot{V}/L$, centrata in $P(a,b)$:

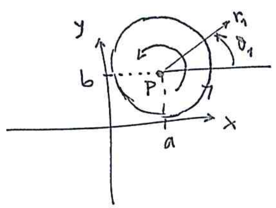


$$\phi = \frac{\dot{V}/L}{2\pi}\ln\frac{r_1}{r_0} = \frac{\dot{V}/L}{2\pi}\ln\left[\sqrt{(x-a)^2+(y-b)^2}\cdot\frac{1}{r_0}\right]$$

$$\psi = \frac{\dot{V}/L}{2\pi}\theta_1 = \frac{\dot{V}/L}{2\pi}\arctan\frac{y-b}{x-a}$$

r_0 = lunghezza di riferimento

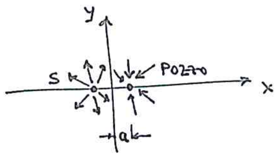
vortice di circolazione Γ , centrato in $P(a,b)$



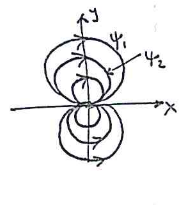
$$\phi = \frac{\Gamma}{2\pi}\theta_1 = \frac{\Gamma}{2\pi}\arctan\frac{y-b}{x-a}$$

$$\psi = -\frac{\Gamma}{2\pi}\ln\frac{r_1}{r_0} = -\frac{\Gamma}{2\pi}\ln\left[\frac{\sqrt{(x-a)^2+(y-b)^2}}{r_0}\right]$$

doppietta : SORGENTE + POZZO di portata $\dot{V}/L$ molto grande e molto vicini tra loro $(a \to 0)$ in modo che $\frac{a\,\dot{V}/L}{\pi} = K$ finito,



K = intensità della doppietta

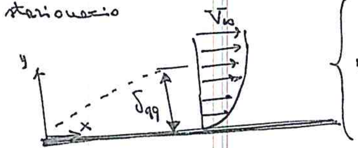


$\psi = -K\frac{\sin\theta}{r}$ $\phi = K\frac{\cos\theta}{r}$

moto uniforme + doppietta : moto attorno a cilindro

Strato limite Incomprimibile, stazionario

lastra piana sottile V_∞ costante



$$\begin{cases} u_x + v_y = 0 \\ u u_x + v u_y = V_\infty\frac{dV_\infty}{dx} + \nu u_{yy} \\ p_y = 0 \end{cases}$$

$u = \psi_y$ $v = -\psi_x$ usando ψ l'equazione di continuità è automaticamente soddisfatta

$\eta = y\sqrt{\frac{V_\infty}{\nu x}}$ variabile di similitudine

funzione di corrente adimensionale : $f(\eta) = \frac{\psi}{\sqrt{\nu V_\infty x}}$, $u = V_\infty f'$

eq. di Blasius : $f''' + \frac{1}{2}ff'' = 0$, cc : $f(0) = f'(0) = 0$

$f'(\infty) = 1$

SPESSORE DI SPOSTAMENTO : $\delta^* = \int_0^\infty\left(1 - \frac{u}{V_\infty}\right)dy$

SPESSORE DI QUANTITÀ DI MOTO : $\theta = \int_0^\infty \frac{u}{V_\infty}\left(1 - \frac{u}{V_\infty}\right)dy$

Aerodinamica esterna : $C_L = \frac{L}{\frac{1}{2}\rho V^2 A}$ $C_D = \frac{D}{\frac{1}{2}\rho V^2 A}$

L = lift (portanza)

D = drag (resistenza)

D è composta da resistenza di attrito, resistenza di pressione ...