Derivazione dell' eq. di Blasius : $f''' + \frac{1}{2} f f'' = 0$ + B.C.'s

$$\begin{cases} u_x + v_y = 0 \\ u u_x + v u_y = \nu u_{yy} \\ P_y = 0 \end{cases} \mapsto \begin{matrix} u = \psi_y \\ v = -\psi_x \end{matrix} \mapsto \boxed{\psi_y \psi_{xy} - \psi_x \psi_{yy} = \nu \psi_{yyy}} \quad (1)$$

$$\text{B.C.} \begin{cases} u = v = 0 & @ \ y = 0 \\ u = U_\infty & @ \ y \to \infty \end{cases} \qquad \boxed{\begin{matrix} \psi_x = \psi_y = 0 & @ \ y = 0 \\ \psi_y = U_\infty & @ \ y \to \infty \end{matrix}} \quad (2)$$

Da $u = \psi_y$ si osserva che $\psi = O(U_\infty \delta)$ con $\delta \propto \sqrt{\frac{\nu x}{U_\infty}}$

quindi $\psi = O\left(\sqrt{\nu U_\infty x}\right)$. Introduco una funzione di corrente adimensionale $f = \frac{\psi}{\sqrt{\nu U_\infty x}}$ e la variabile "simile" $\eta = \frac{y}{\sqrt{\frac{\nu x}{U_\infty}}}$.

$$\psi_y = \sqrt{\nu U_\infty x} \, \frac{df}{d\eta} \frac{\partial \eta}{\partial y} = U_\infty f' \qquad \frac{\partial \eta}{\partial y} = \frac{1}{\sqrt{\frac{\nu x}{U_\infty}}} \ ; \ \frac{\partial \eta}{\partial x} = -\frac{1}{2} \frac{\eta}{x}$$

$$\psi_x = \frac{\partial}{\partial x}\left(\sqrt{\nu U_\infty x} \, f\right) = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}} f + \sqrt{\nu U_\infty x} \frac{df}{d\eta}\frac{\partial \eta}{\partial x} =$$

$$= \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}} f - \frac{1}{2}\eta \sqrt{\frac{\nu U_\infty}{x}} f'$$

$$\psi_{xy} = \left(\cancel{\frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}} f'} - \cancel{\frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}} f'} - \frac{1}{2}\eta \sqrt{\frac{\nu U_\infty}{x}} f'' \right) \frac{1}{\sqrt{\frac{\nu x}{U_\infty}}} =$$

$$= -\frac{1}{2}\eta \frac{U_\infty}{x} f''$$

$$\psi_{yy} = U_\infty f'' \frac{\partial \eta}{\partial y} = \frac{U_\infty^{3/2}}{\sqrt{\nu x}} f''$$

$$\psi_{yyy} = \frac{U_\infty^{3/2}}{\sqrt{\nu x}} f''' \frac{\partial \eta}{\partial y} = \frac{U_\infty^2}{\nu x} f'''$$

Inserendo in (1) :

$$U_\infty f'\left(\cancel{-\frac{1}{2}\eta\frac{U_\infty}{x}f''}\right)-\left(\frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}f-\cancel{\frac{1}{2}\eta\sqrt{\frac{\nu U_\infty}{x}}f'}\right)\frac{U_\infty^{3/2}}{\sqrt{\nu x}}f''=\frac{U_\infty^2}{x}f'''$$

$$-\frac{1}{2}\frac{ff''}{x}=\frac{f'''}{x} \qquad \text{e per } x\neq 0 \qquad \boxed{f'''+\frac{1}{2}ff''=0} \quad (3)$$

Inserendo in (2) : $y=0 \rightarrow \eta=0$ (parete)

$y\rightarrow\infty \rightarrow \eta\rightarrow\infty$ (bordo dello strato limite)

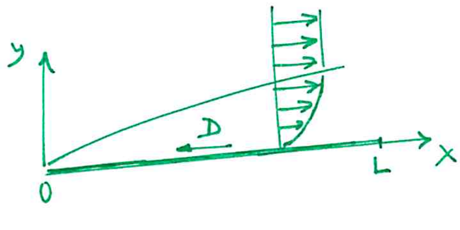
$$\boxed{\begin{array}{lll} f=f'=0 & @ & \eta=0 \\ f'=1 & @ & \eta\rightarrow\infty \end{array}} \quad (4)$$

L' eq. differenziale ordinaria (3) si risolve con un qualunque metodo numerico (ex. Runge-Kutta, metodo di tiro, etc.).

$$\tau_w=\mu\left.\frac{\partial u}{\partial y}\right|_{y=0}=\mu\left.\psi_{yy}\right|_{y=0}=\mu\frac{U_\infty^{3/2}}{\sqrt{\nu x}}f''(0) \qquad \text{con } f''(0)=0.332$$

$$=0.332\frac{\rho U_\infty^2}{\sqrt{Re_x}} \qquad Re_x=\frac{U_\infty x}{\nu} \quad \text{numero di Reynolds locale}$$

La resistenza D esercitata dalla lastra sul fluido (assumendo una lastra lunga L e di apertura w) è:



$$D=w\int_0^L\tau_w\,dx=0.664\frac{\rho U_\infty^2}{\sqrt{Re_L}}(Lw) \qquad Re_L=\frac{U_\infty L}{\nu} \quad \text{numero di Reynolds basato su } L$$

$$c_{f,x}=\frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}=\frac{0.664}{\sqrt{Re_x}}=\frac{\theta}{x} \quad ; \qquad C_F=\frac{D}{\frac{1}{2}\rho U_\infty^2(Lw)}=\frac{1.328}{\sqrt{Re_L}}$$

(θ = spessore di quantità di moto)