Turbulent flow over a porous wall

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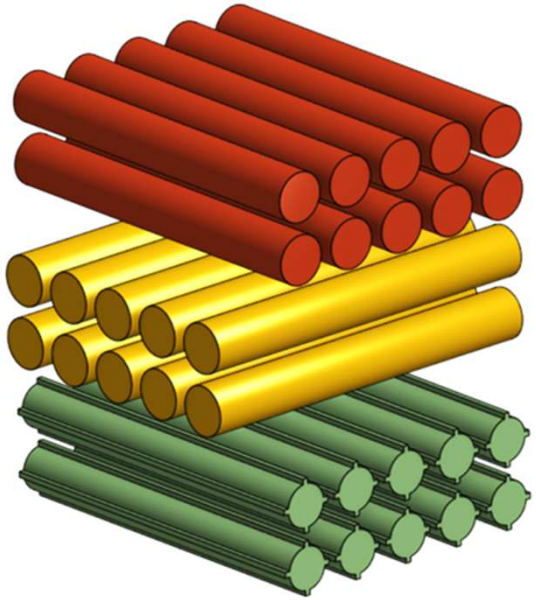
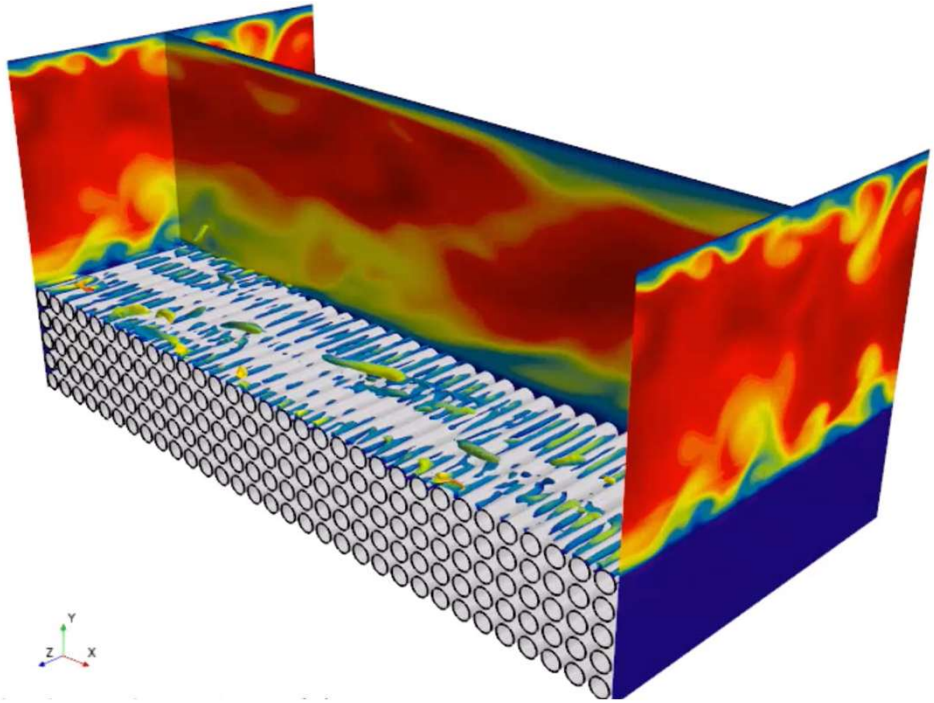
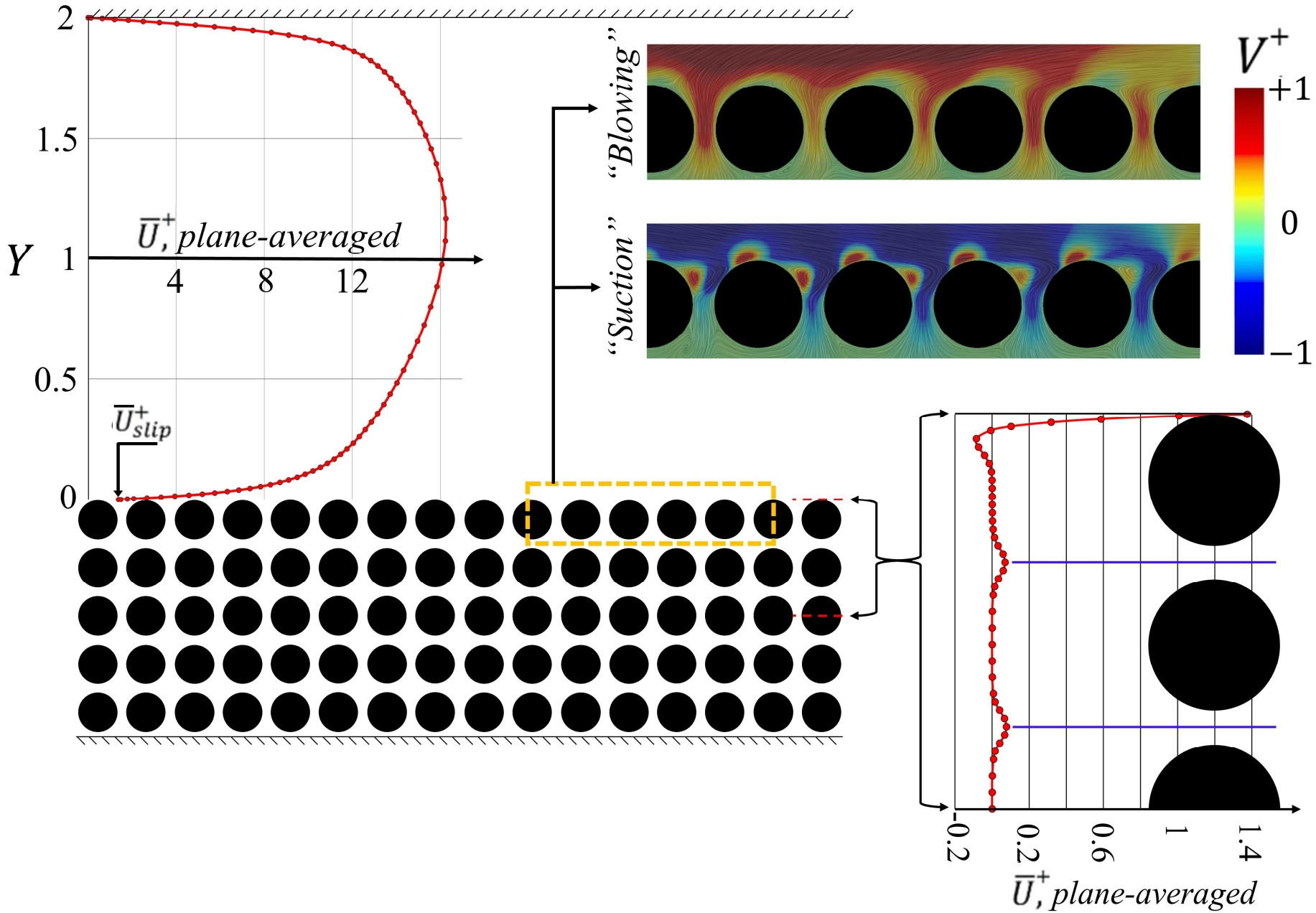


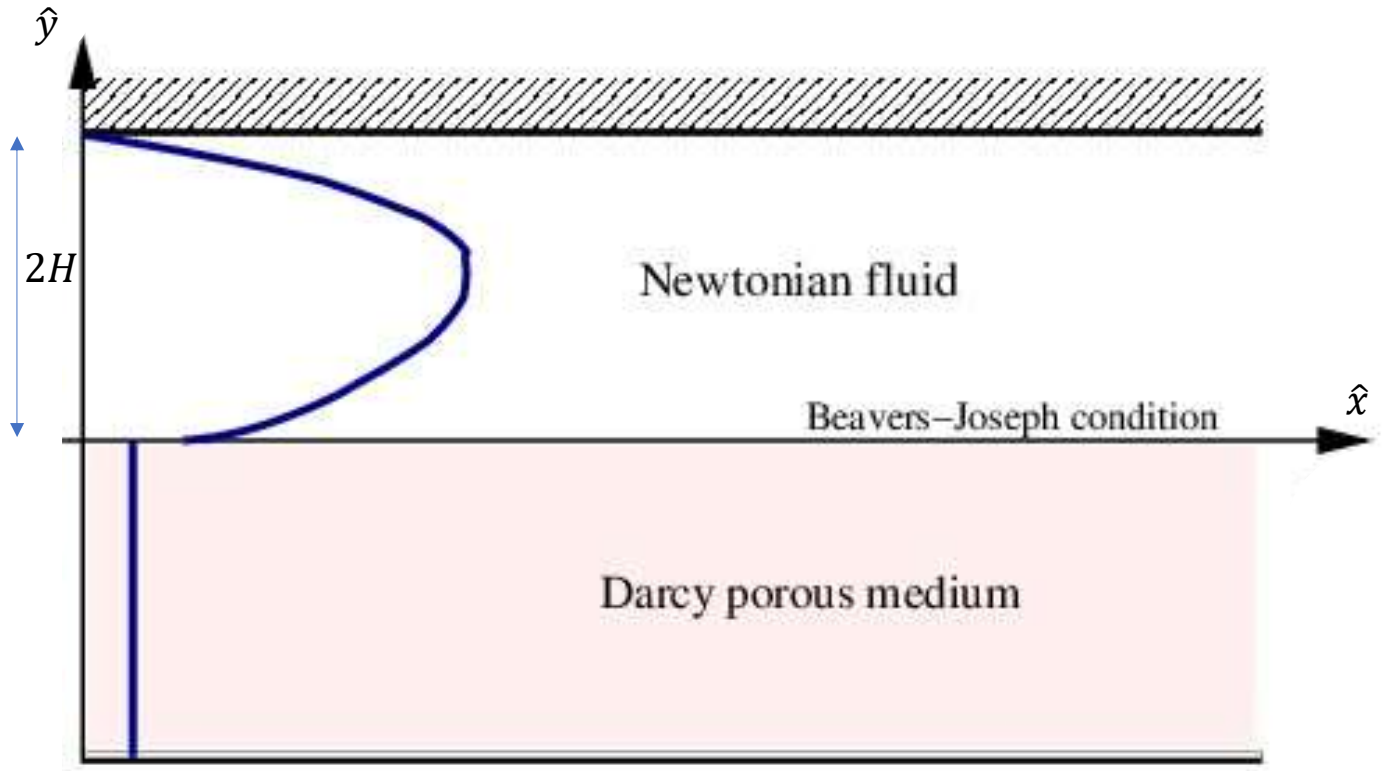



2
1.5
1
0.5
0
Y
4
8
12
$\overline{U}^+$, plane-averaged
$\overline{U}^+_{slip}$
"Blowing"
"Suction"
V^+
+1
0
−1
-0.2
0.2
0.6
1
1.4
$\overline{U}^+$, plane-averaged

Can we model the presence of the permeable substrate without solving for the flow through the porous medium? Can we find *effective* boundary conditions at a virtual wall which

(1) can be derived rigorously and

(2) actually work (and not only in the laminar case)?!



Accounting for transpiration at the fictitious wall is essential

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Generalized slip condition over rough surfaces

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$$u_i = \epsilon \mathcal{L}_{ilk} \left(\frac{\partial u_l}{\partial x_k} + \frac{\partial u_k}{\partial x_l} \right) \Big|_{\mathbb{ES}}$$

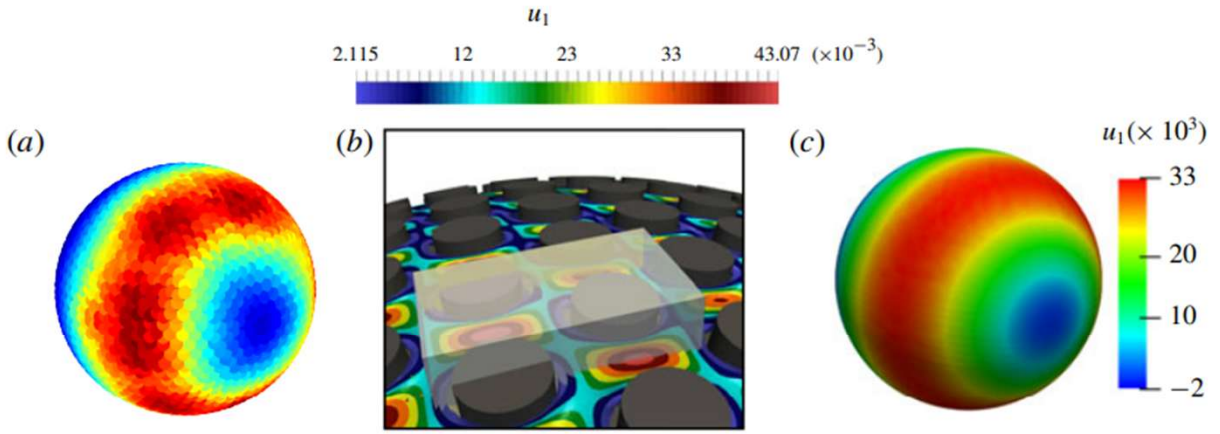

FIGURE 9. (Colour online) Comparison of the streamwise velocity component, u_1, on the sphere, computed with the macroscopic model (2.28) (*c*), and extracted from the reference DNS (*a*). The procedure used to pass from one approach to the other is detailed in the text. (*b*) The microscopic cell, $\mathbb{V}$, (in grey), with the microscopic distribution of the u_1 iso-contours on a surface located slightly above $R = 1$.

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- Extend to **higher order** in ϵ

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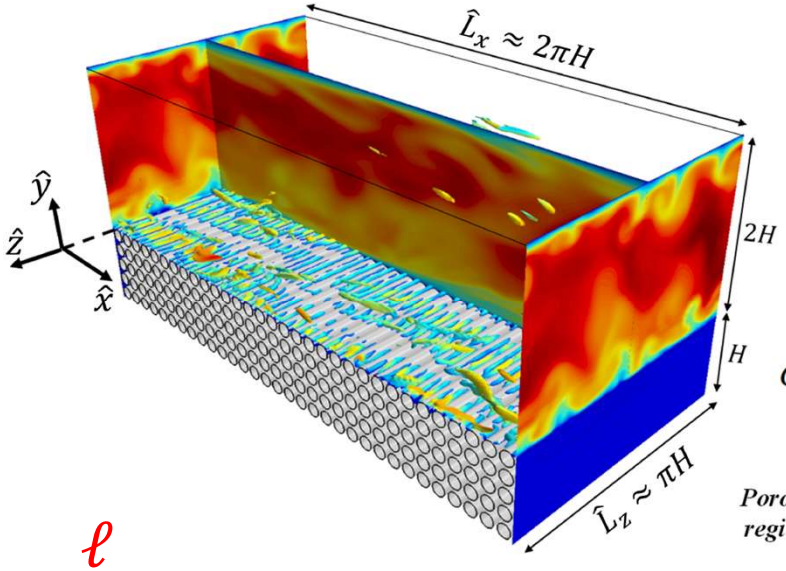
- Extend to **higher order** in ϵ
- Account for **near-wall inertia** and make the simplified procedure (with *effective* boundary conditions at the wall) work for the case of turbulent flow
- **Find a relation that permits to determine ΔU^+ (roughness function) and the slip velocity $\overline{U}^+_{slip}$ directly from macroscopic upscaling coefficients**

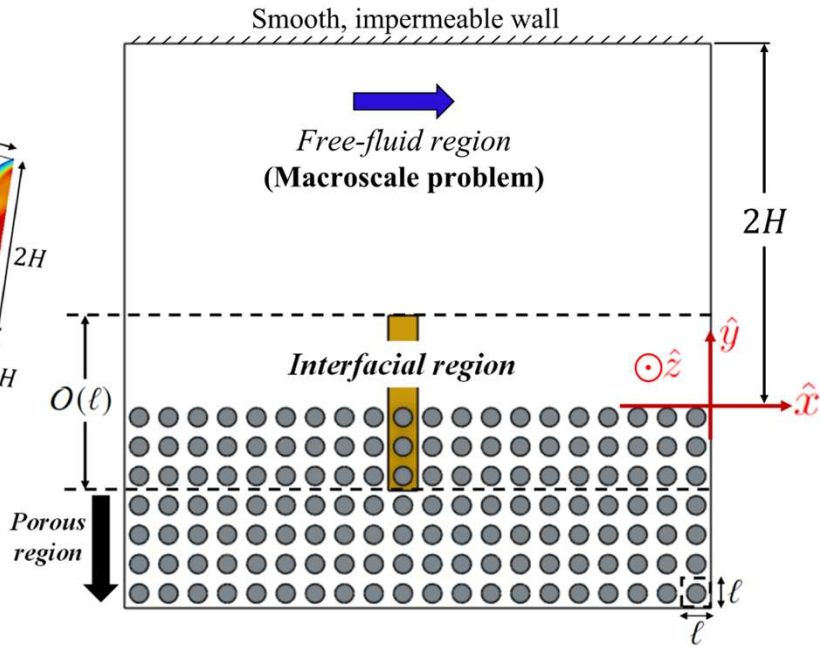
Rapidly-varying properties
(related to surface heterogeneity)

Homogenization framework

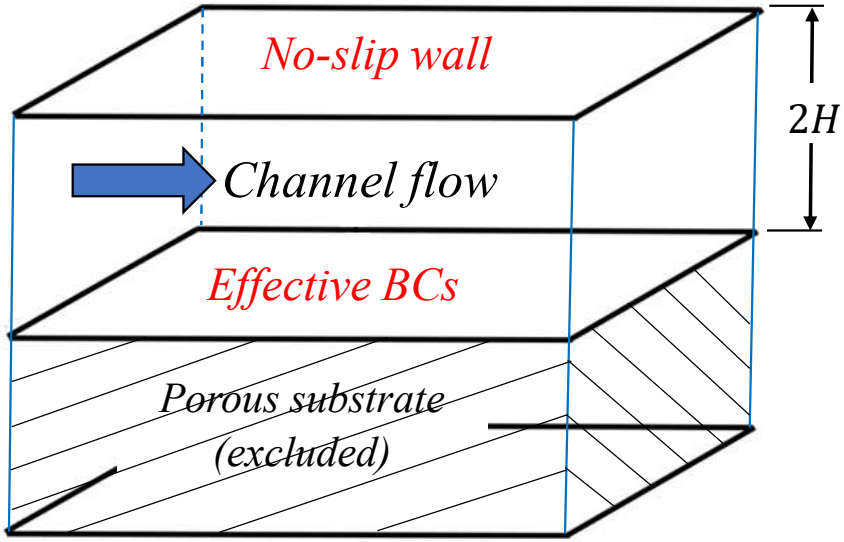
Upscaled, homogeneous properties
(for *effective* boundary conditions)

$$\epsilon = \frac{\ell}{H} \ll 1$$





Navier-slip coefficients,
intrinsic permeability,
interfacial permeability,
etc.



Macroscopic *effective* boundary conditions

$$U\big|_{Y=0^+} = \underbrace{\epsilon\, \boldsymbol{\lambda_x}\, S_{12}\big|_{Y=0^+}}_{\text{Navier-slip condition}} + \underbrace{\epsilon^2\, \boldsymbol{\mathcal{K}_{xy}^{itf}}\, \frac{\partial S_{22}}{\partial X}\Big|_{Y=0^+}}_{\text{Second-order correction}} + \mathcal{O}(\epsilon^3)$$

$$V\big|_{Y=0^+} = \underbrace{-\epsilon^2\, \boldsymbol{\mathcal{K}_{xy}^{itf}}\, \frac{\partial S_{12}}{\partial X}\Big|_{Y=0^+} - \epsilon^2\, \boldsymbol{\mathcal{K}_{zy}^{itf}}\, \frac{\partial S_{32}}{\partial Z}\Big|_{Y=0^+}}_{\text{Second-order, effect of interface permeabilities}} + \underbrace{\epsilon^2\, \boldsymbol{\mathcal{K}_{yy}}\, \frac{\partial S_{22}}{\partial Y}\Big|_{Y=0^+}}_{\text{Second-order, effect of medium permeability}} + \mathcal{O}(\epsilon^3)$$

$$W\big|_{Y=0^+} = \underbrace{\epsilon\, \boldsymbol{\lambda_z}\, S_{32}\big|_{Y=0^+}}_{\text{Navier-slip condition}} + \underbrace{\epsilon^2\, \boldsymbol{\mathcal{K}_{zy}^{itf}}\, \frac{\partial S_{22}}{\partial Z}\Big|_{Y=0^+}}_{\text{Second-order correction}} + \mathcal{O}(\epsilon^3)$$

$$S_{12} = \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X},$$

$$S_{22} = -Re\, P + 2\frac{\partial V}{\partial Y},$$

$$S_{32} = \frac{\partial W}{\partial Y} + \frac{\partial V}{\partial Z}.$$

$\boldsymbol{\lambda_{x,z}}$: Navier-slip coefficients

$\boldsymbol{\mathcal{K}_{xy,zy}^{itf}}$: Interface permeability coefficents

$\boldsymbol{\mathcal{K}_{yy}}$: medium permeability

Near-wall advection modeling

- An **Oseen-like linearization** was proposed to include the effects of near-interface advection in the homogenization scheme (Bottaro A., *J. Fluid Mech*. (2019), vol. 877, P1)
- We define a spatially invariant velocity $\hat{\boldsymbol{u}}_{\boldsymbol{\phi},\boldsymbol{j}} = (\hat{\boldsymbol{u}}_{\boldsymbol{\phi}}, \boldsymbol{0}, \boldsymbol{0})$, representative of the near-wall velocity level.
- Governing equations of the microscale problem: $\dfrac{\partial \hat{u}_i}{\partial \hat{x}_i} = 0, \quad \rho \hat{u}_{\phi,j} \dfrac{\partial \hat{u}_i}{\partial \hat{x}_j} = -\dfrac{\partial \hat{p}}{\partial \hat{x}_i} + \mu \dfrac{\partial^2 \hat{u}_i}{\partial \hat{x}_j^2}$
- The microscopic Reynolds number, defined as $\boldsymbol{Re_\phi = \dfrac{\rho\, \hat{u}_\phi\, \ell}{\mu} = \epsilon\, Re_\tau\, U_\phi^+}$, is assumed to be of order 1.

The closure problems

$$\begin{cases} \partial_i u^\dagger_{i1} = 0, \\ -\partial_i p^\dagger_1 + \partial^2_k u^\dagger_{i1} = Re_\phi\, \partial_1 u^\dagger_{i1}, \\ u^\dagger_{i1} = 0 \qquad \text{at } \mathcal{I}_{\beta\sigma}, \\ -p^\dagger_1 \delta_{i2} + \partial_2 u^\dagger_{i1} + \partial_i u^\dagger_{21} = \delta_{i1} \qquad \text{at } y = y_\infty. \end{cases}$$

$$\lambda_x = \frac{1}{\mathcal{A}} \int_{\mathcal{S}_0} u^\dagger_{11}\, dA \qquad \mathcal{K}^{itf}_{xy} = \frac{1}{\mathcal{A}} \int_{\mathcal{V}_0} u^\dagger_{11}\, dV$$

$$\begin{cases} \partial_i u^\dagger_{i3} = 0, \\ -\partial_i p^\dagger_3 + \partial^2_k u^\dagger_{i3} = Re_\phi\, \partial_1 u^\dagger_{i3}, \\ u^\dagger_{i3} = 0 \qquad \text{at } \mathcal{I}_{\beta\sigma}, \\ -p^\dagger_3 \delta_{i2} + \partial_2 u^\dagger_{i3} + \partial_i u^\dagger_{23} = \delta_{i3} \qquad \text{at } y = y_\infty. \end{cases}$$

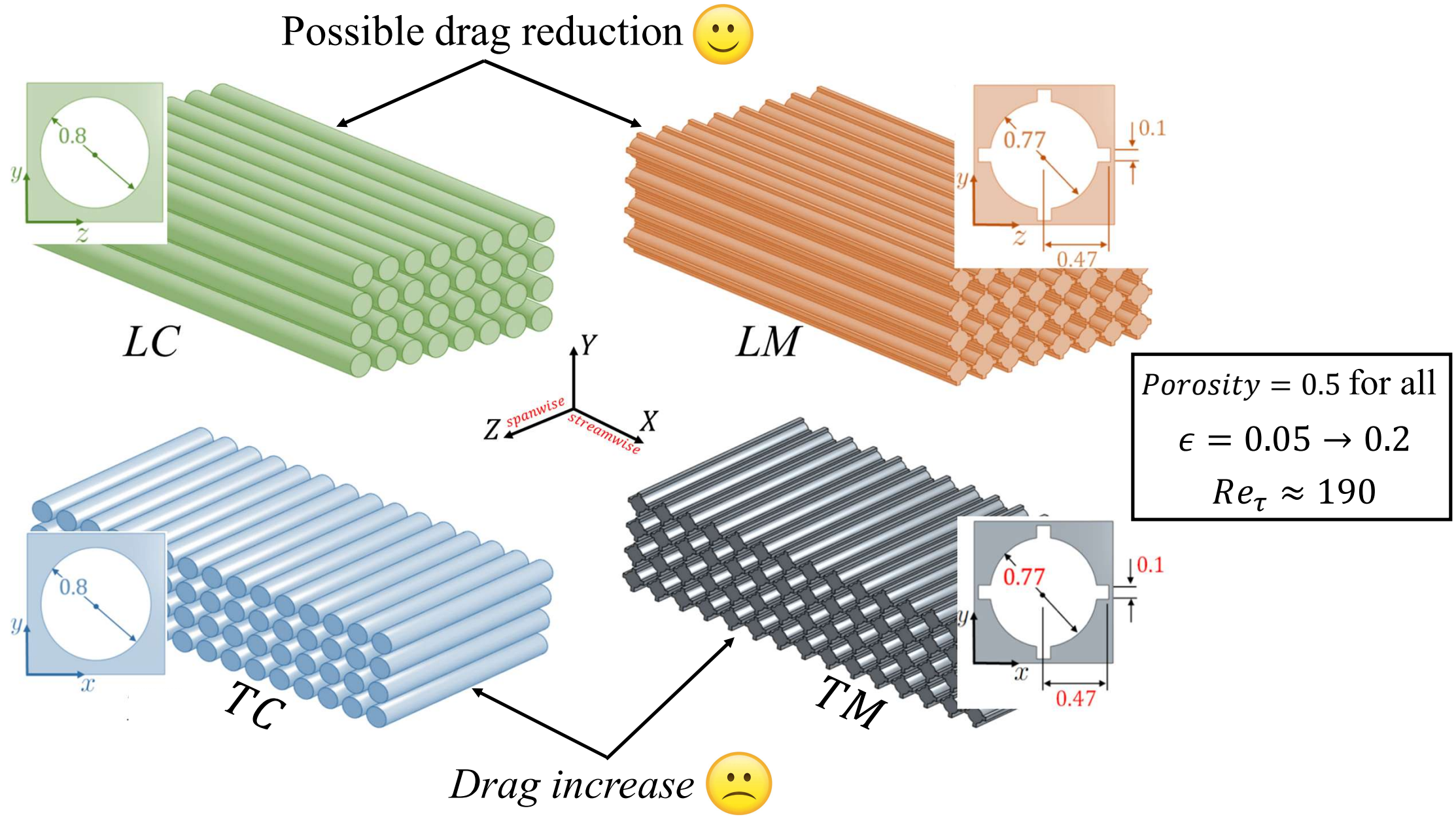
$$\lambda_z = \frac{1}{\mathcal{A}} \int_{\mathcal{S}_0} u^\dagger_{33}\, dA \qquad \mathcal{K}^{itf}_{zy} = \frac{1}{\mathcal{A}} \int_{\mathcal{V}_0} u^\dagger_{33}\, dV$$

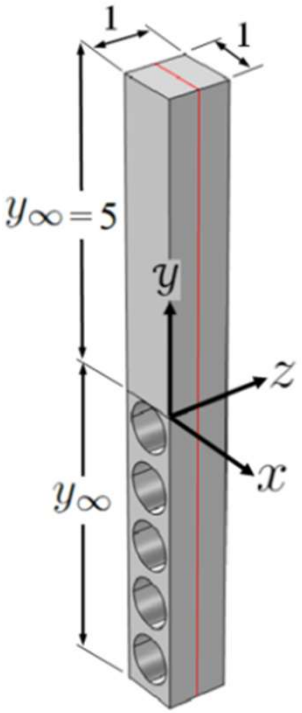
$$U\Big|_{Y=0^+} = \epsilon\, \boldsymbol{\lambda_x} \frac{\partial U}{\partial Y}\Big|_{Y=0^+} - \epsilon^2\, Re\, \boldsymbol{\mathcal{K}_{xy}^{itf}}\, \frac{\partial P}{\partial X}\Big|_{Y=0^-} + \mathcal{O}(\epsilon^3)$$

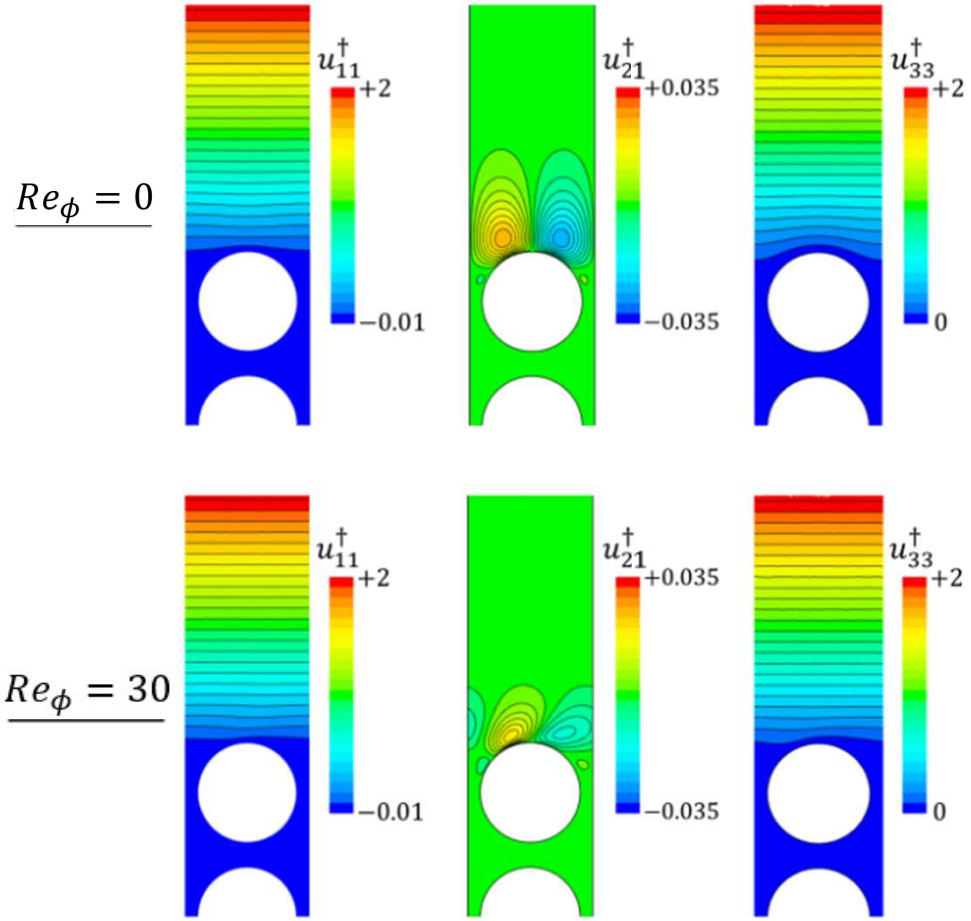
$$V\Big|_{Y=0^+} = -\epsilon \frac{\boldsymbol{\mathcal{K}_{xy}^{itf}}}{\boldsymbol{\lambda_x}} \frac{\partial U}{\partial X}\Big|_{Y=0^+} - \epsilon \frac{\boldsymbol{\mathcal{K}_{zy}^{itf}}}{\boldsymbol{\lambda_z}} \frac{\partial W}{\partial Z}\Big|_{Y=0^+} - \epsilon^2 Re\, \boldsymbol{\mathcal{K}_{yy}}\, \frac{\partial P}{\partial Y}\Big|_{Y=0^-} + \mathcal{O}(\epsilon^3)$$

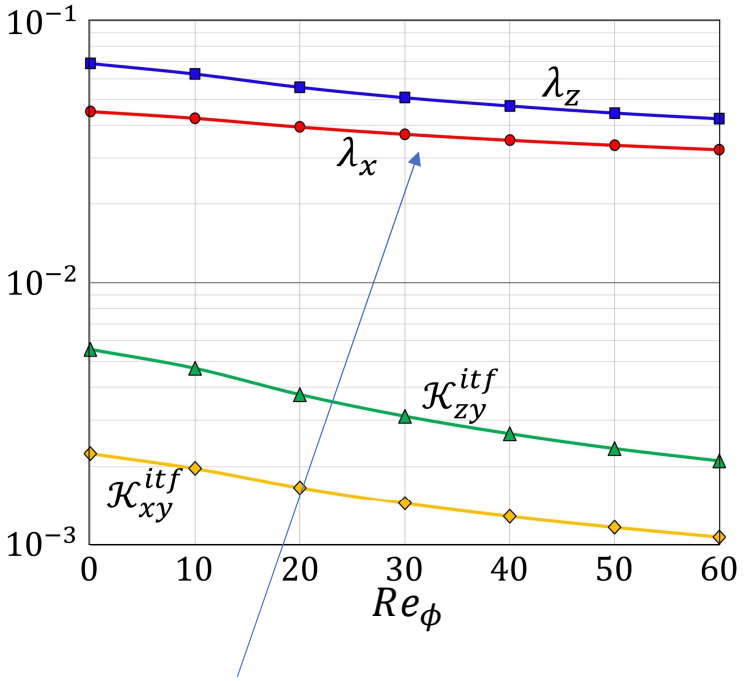
$$W\Big|_{Y=0^+} = \epsilon\, \boldsymbol{\lambda_z} \frac{\partial W}{\partial Y}\Big|_{Y=0^+} - \epsilon^2\, Re\, \boldsymbol{\mathcal{K}_{zy}^{itf}}\, \frac{\partial P}{\partial Z}\Big|_{Y=0^-} + \mathcal{O}(\epsilon^3)$$

Extended Beavers-Joseph-Saffman conditions with NO empirical coefficients

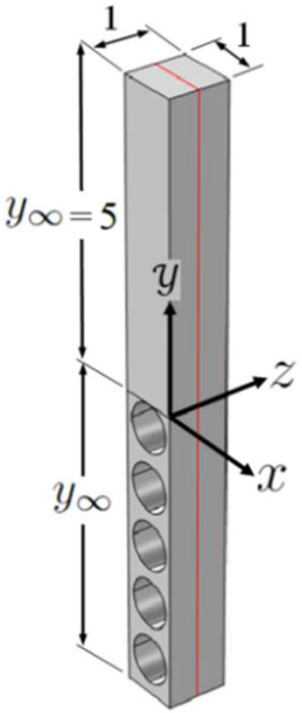

Possible drag reduction
0.8
y
z
LC
LM
0.77
0.1
0.47
Y
Z spanwise
streamwise
X
Porosity = 0.5 for all
$\epsilon = 0.05 \rightarrow 0.2$
$Re_\tau \approx 190$
0.8
y
x
TC
TM
0.77
0.1
0.47
Drag increase

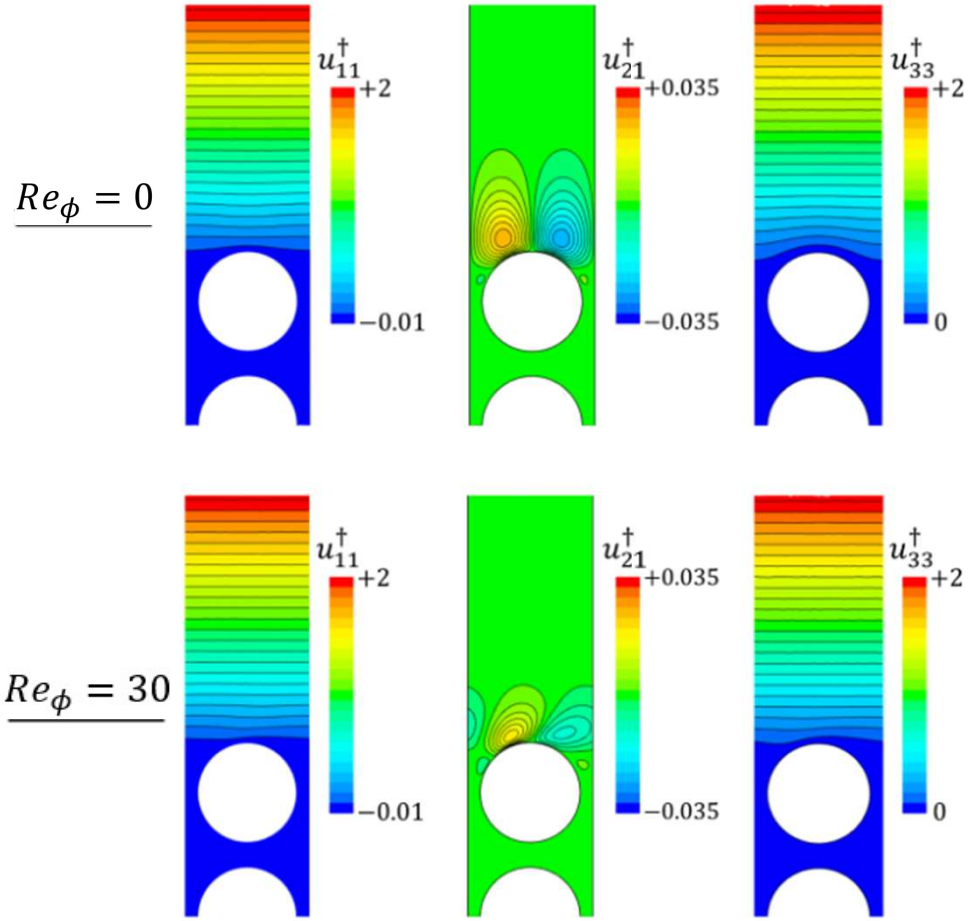


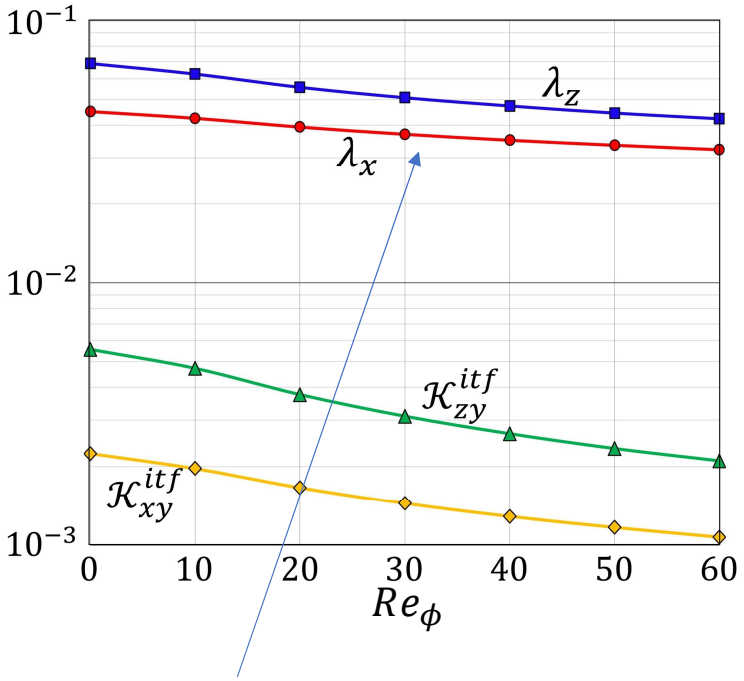




Two different virtual origins for longitudinal and transverse flow, i.e. for mean flow and for turbulence





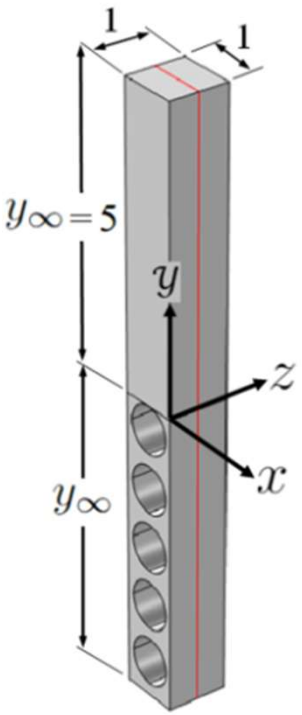


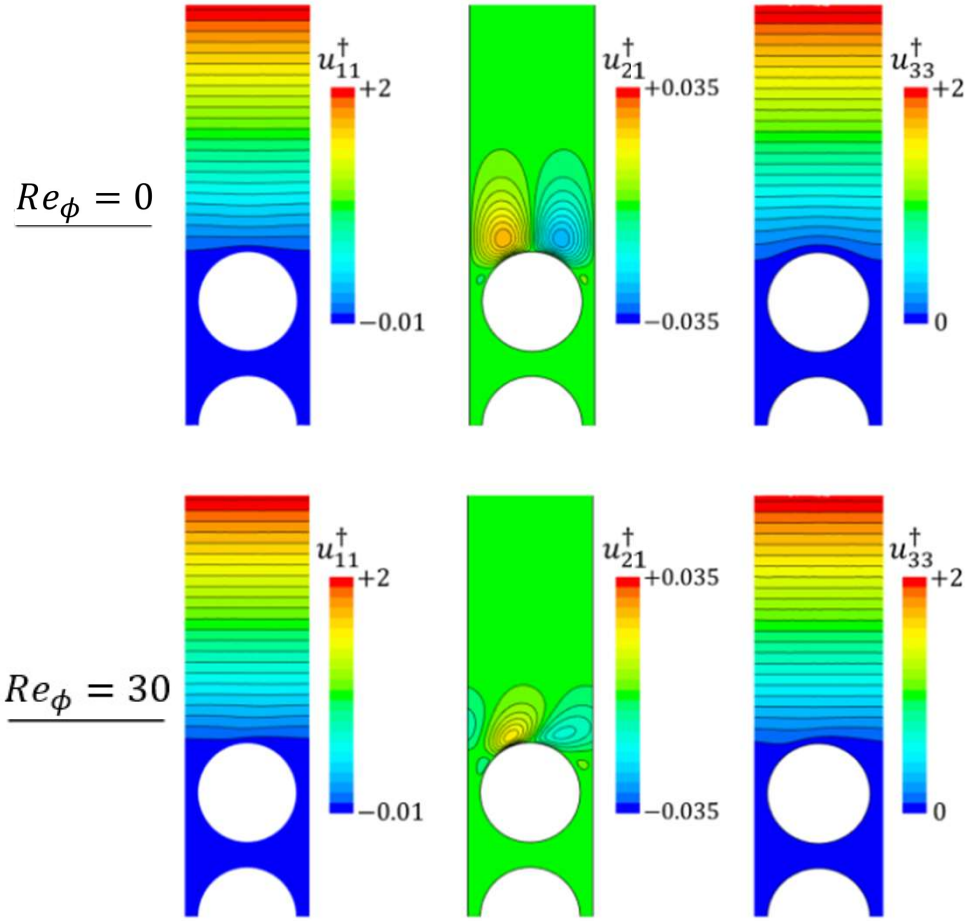
Two different virtual origins for longitudinal and transverse flow, i.e. for mean flow and for turbulence

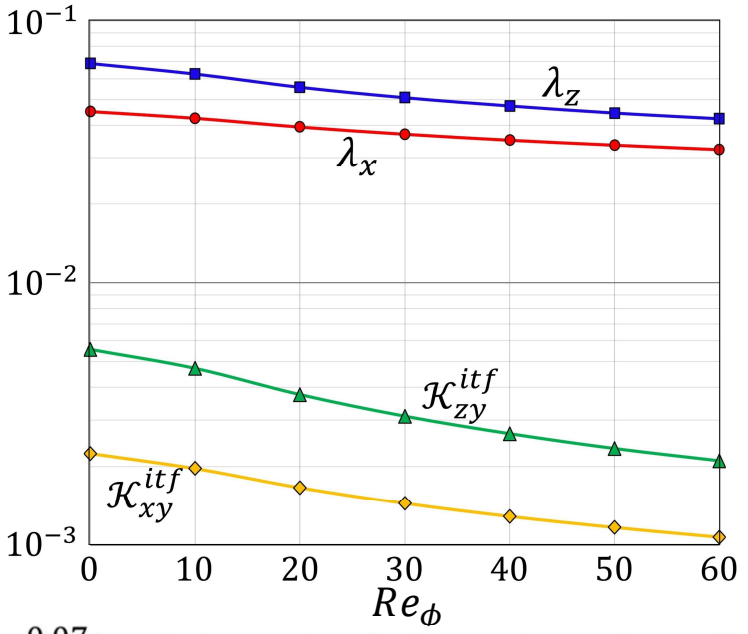
$$Assume\ \hat{u}_\phi = \hat{u}_{slip} \rightarrow Re_\phi = \epsilon Re_\tau \bar{U}^+_{slip}$$

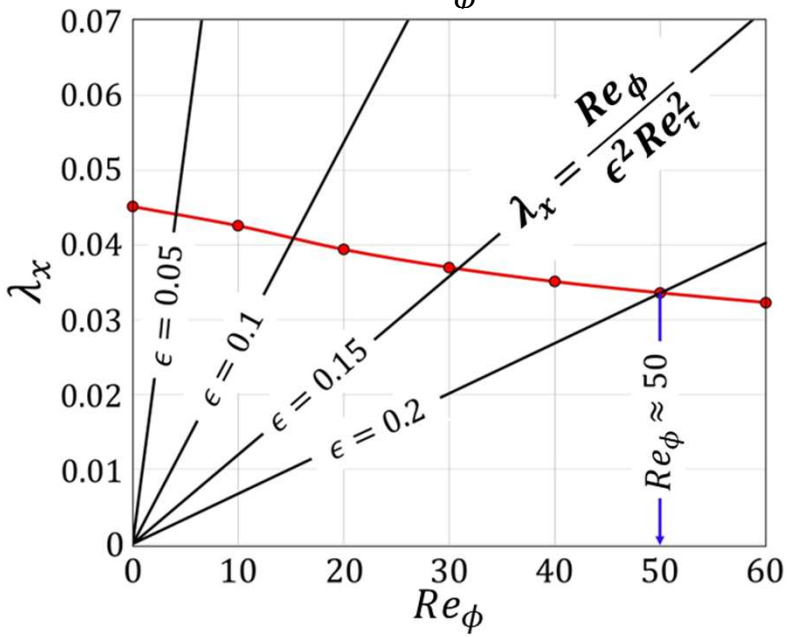
First-order: $$\bar{U}^+_{slip} \approx \lambda_x^+ \left.\frac{\partial \bar{U}^+}{\partial Y^+}\right|_{Y=0} \approx \lambda_x^+ = \frac{\rho u_\tau \times \ell \lambda_x}{\mu} = \epsilon\, Re_\tau\, \lambda_x$$

$$\therefore \boldsymbol{Re_\phi \approx \epsilon^2 Re_\tau^2\, \lambda_x}$$









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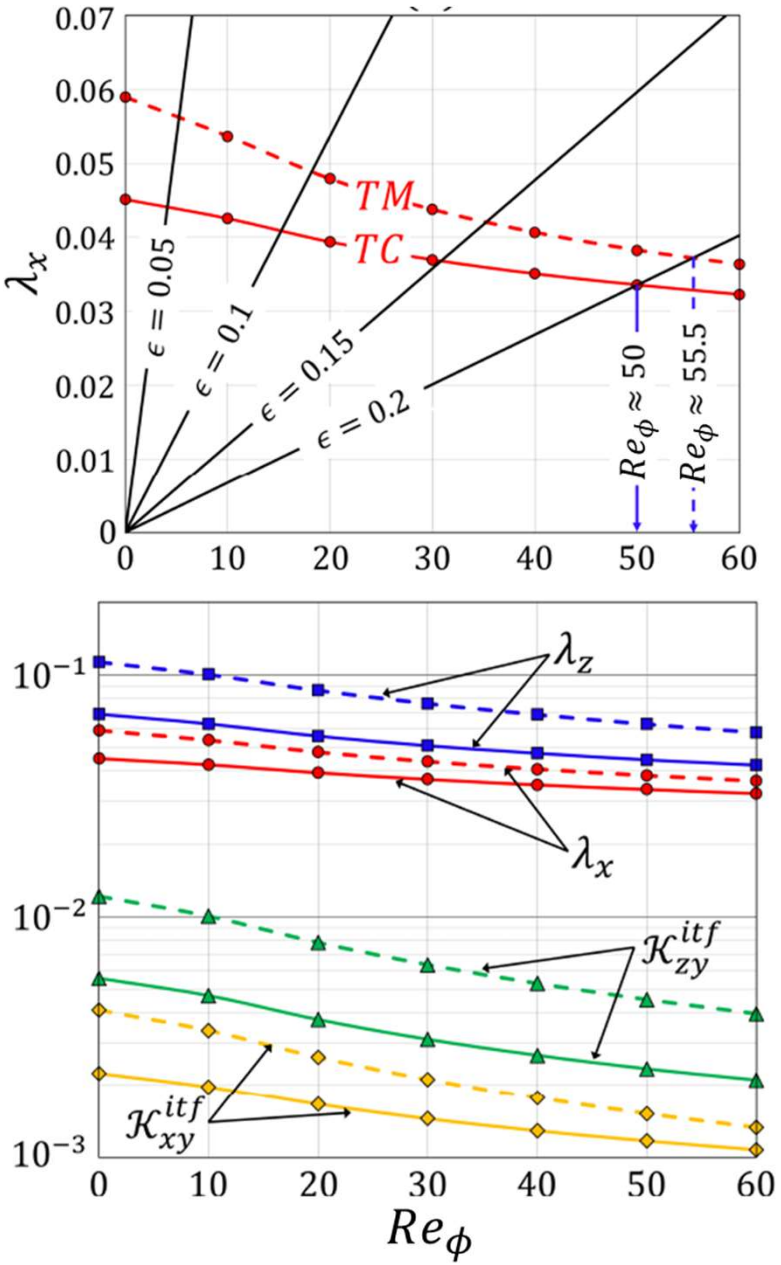
$$\boldsymbol{\therefore Re_\phi \approx \epsilon^2 Re_\tau^2\, \lambda_x}$$

Substrate	Re_ϕ (intersection)	Dimensionless macroscopic coefficients				
		λ_x	λ_z	$\mathcal{K}_{xy}^{itf}$	$\mathcal{K}_{zy}^{itf}$	$\mathcal{K}_{yy}$
TC_5	4.1	0.0440	0.0663	0.0021	0.0052	0.0018
TC_{10}	15.2	0.0409	0.0591	0.0018	0.0042	0.0018
TC_{15}	30.9	0.0368	0.0506	0.0014	0.0031	0.0018
TC_{20}	50.0	0.0336	0.0445	0.0012	0.0023	0.0018
LC	Any	0.0688	0.0451	0.0056	0.0022	0.0018
TM_5	5.3	0.0562	0.1062	0.0037	0.0110	0.00012
TM_{10}	18.2	0.0489	0.0888	0.0028	0.0082	0.00012
TM_{15}	35.3	0.0421	0.0721	0.0019	0.0058	0.00012
TM_{20}	55.5	0.0372	0.0599	0.0014	0.0042	0.00012
LM	Any	0.1130	0.0590	0.0121	0.0041	0.00012



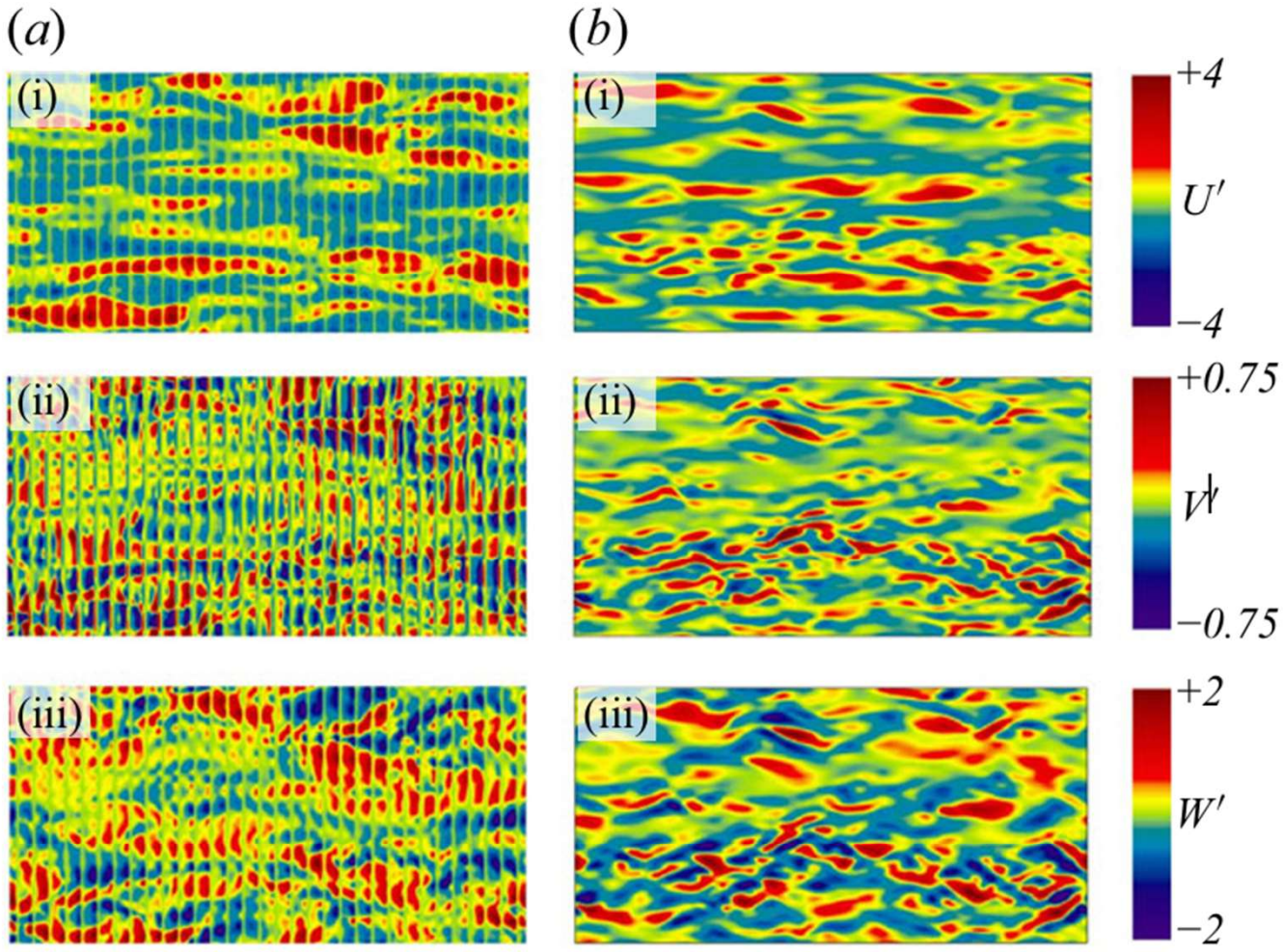


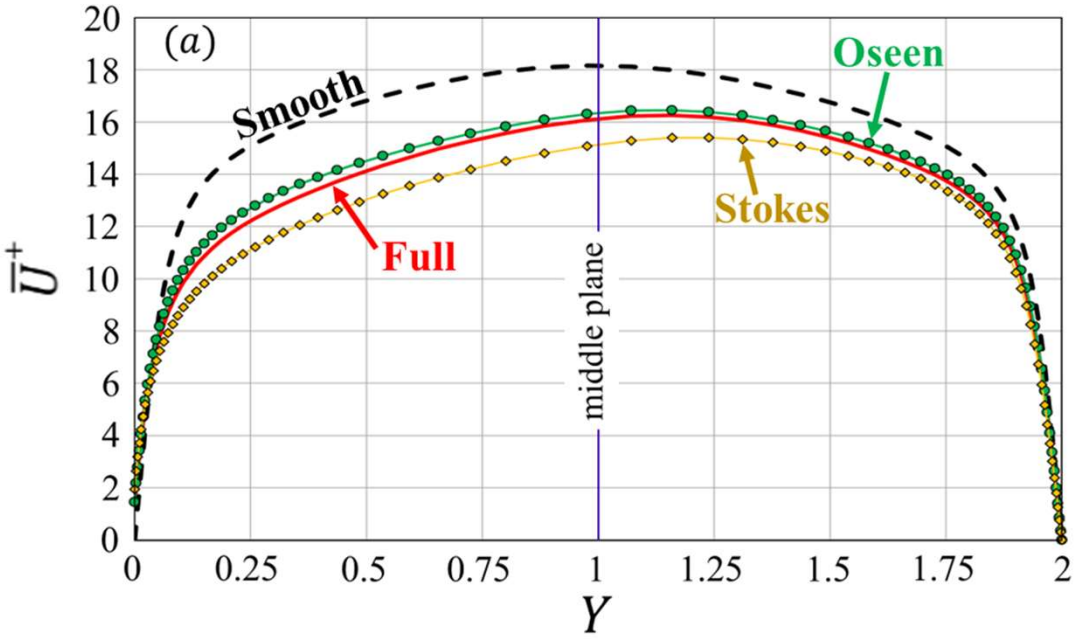
Figure 8. From (i) to (iii), instantaneous distributions of U', V' and W' at the porous–free-fluid interface ($Y = 0$) for case TC_{20}. The fully resolved results (a) are compared with the homogenized ones (b).

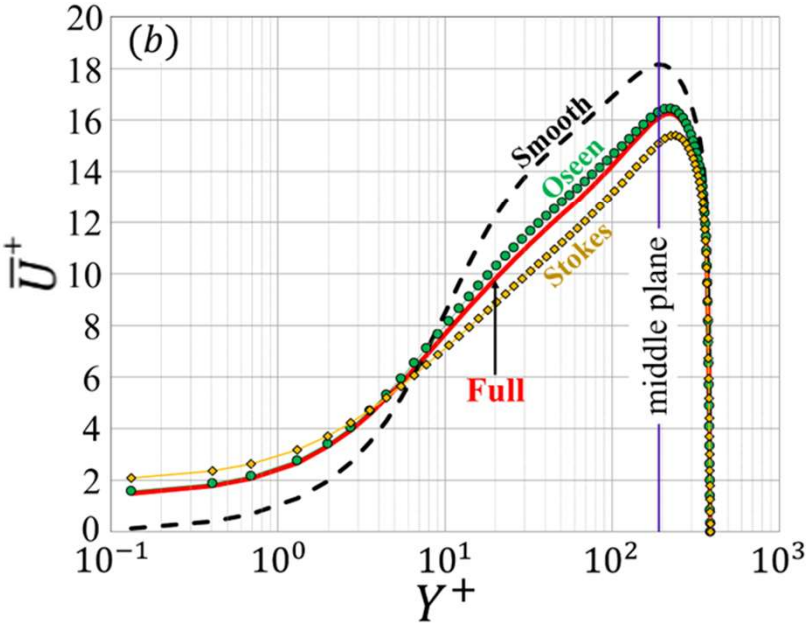
Model validation

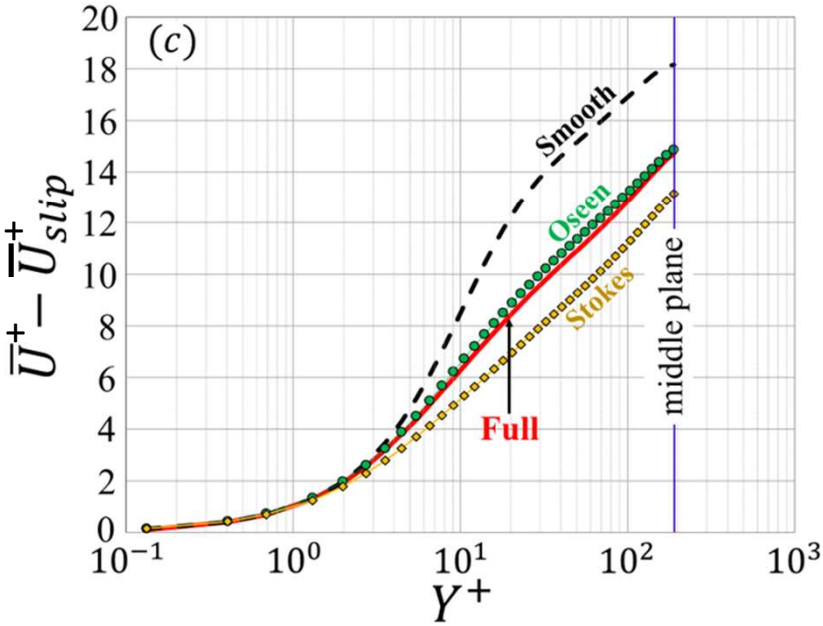
$Transverse\ cyl., \theta = 0.5,$

$\epsilon = 0.2, Re_\tau \approx 190$

* **Stokes' model over-estimates drag increase**
* **Oseen's model perfectly predicts slip velocity**







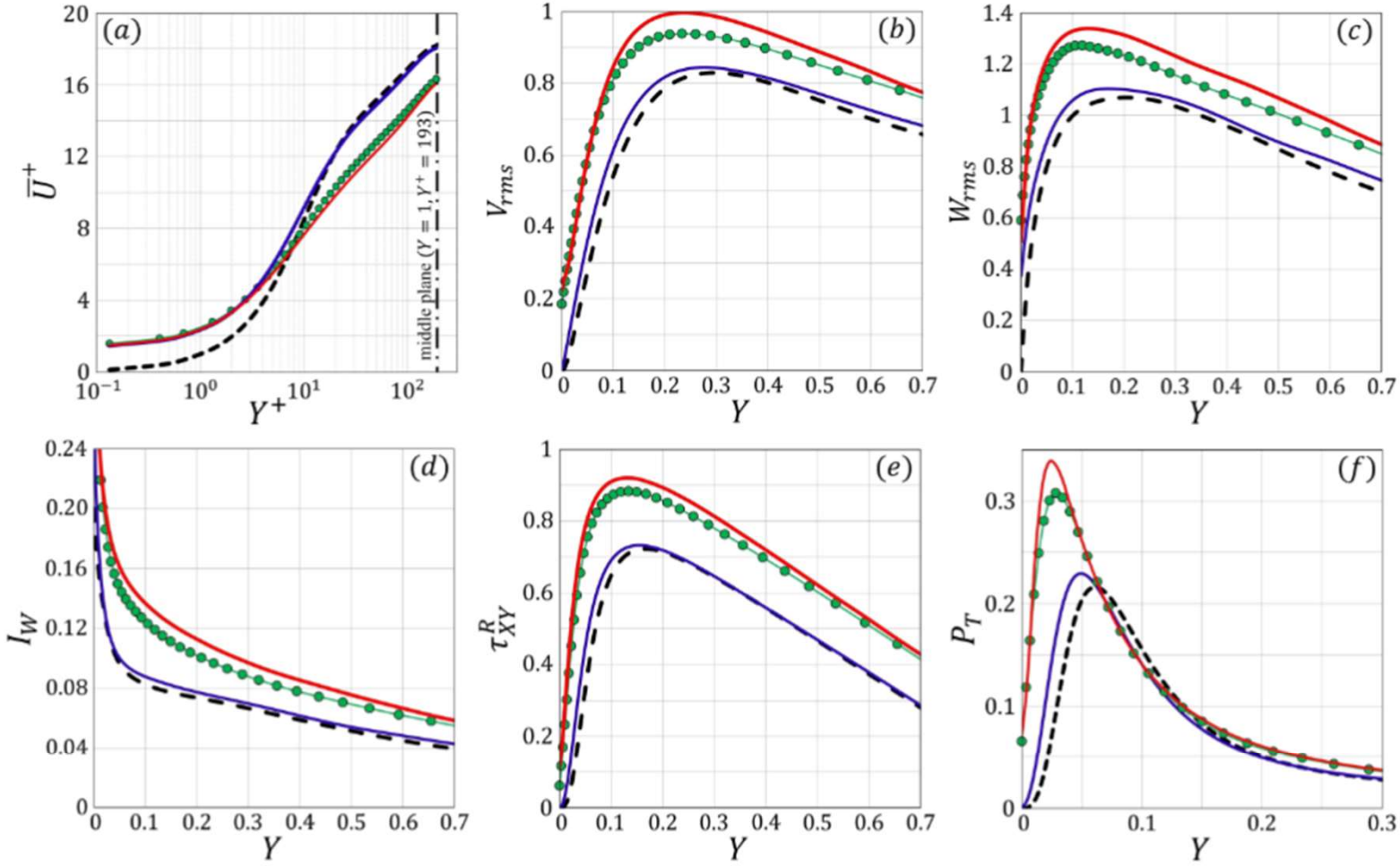
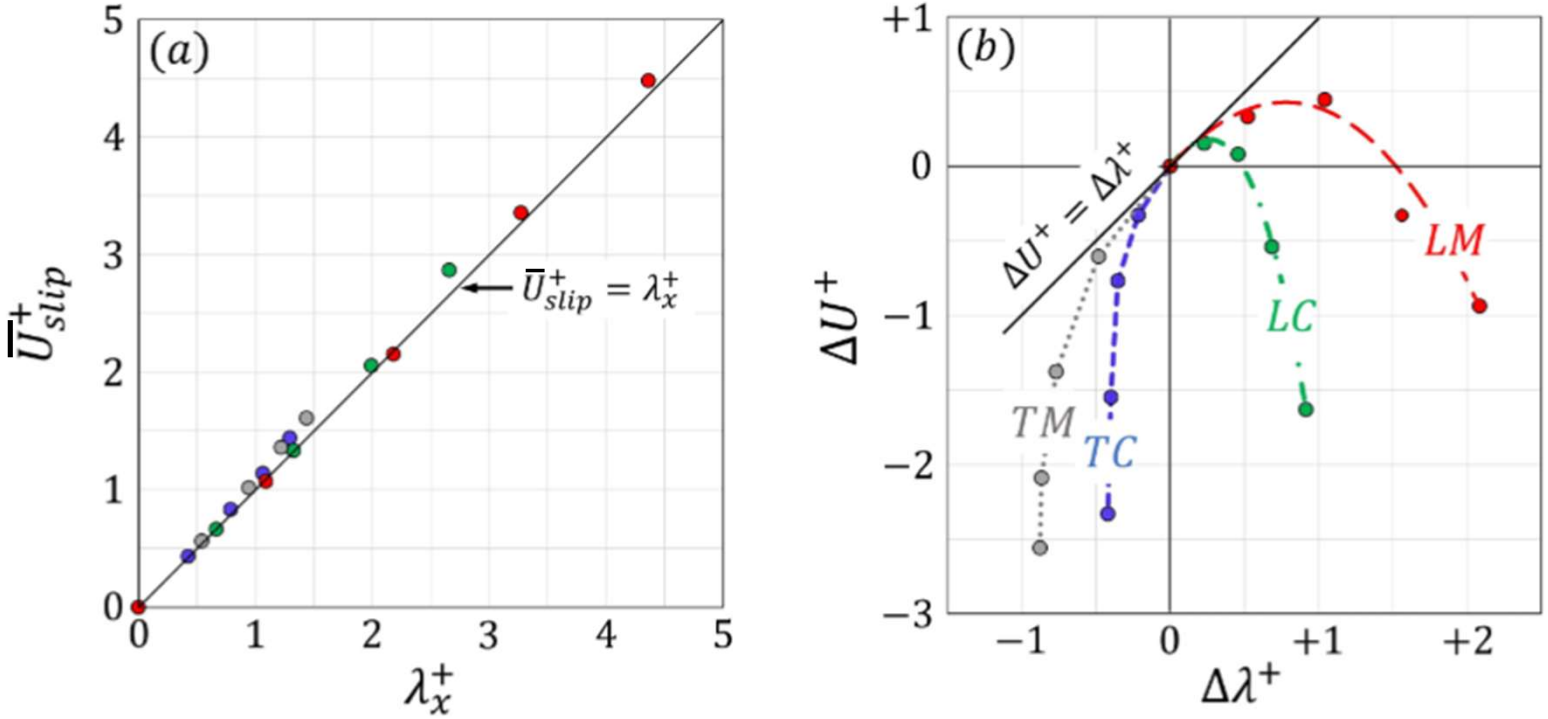
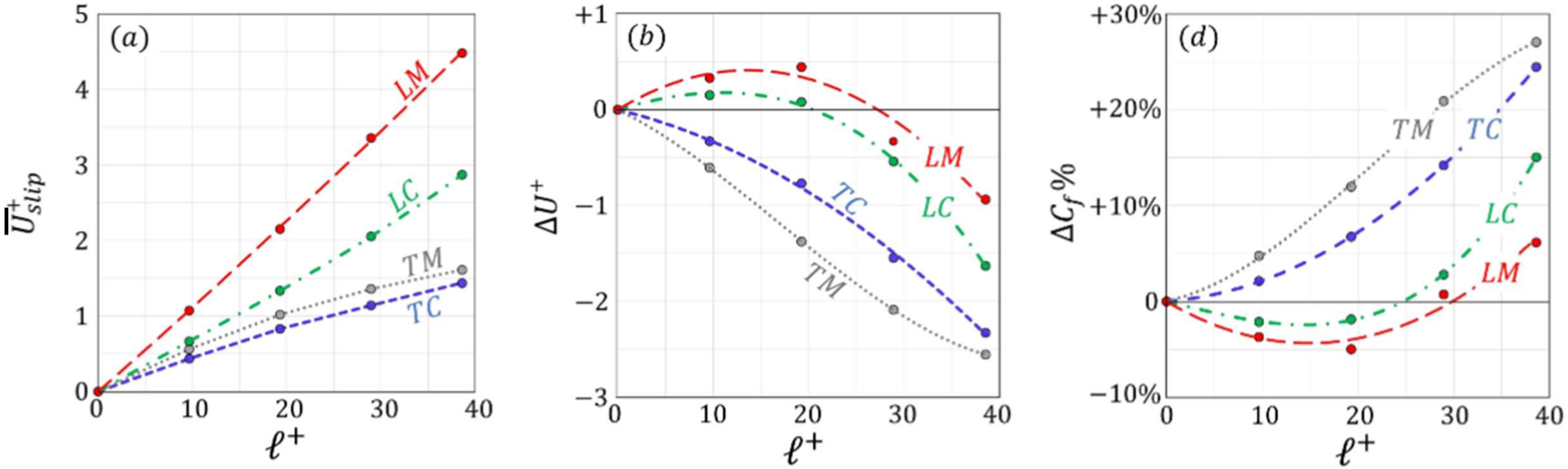
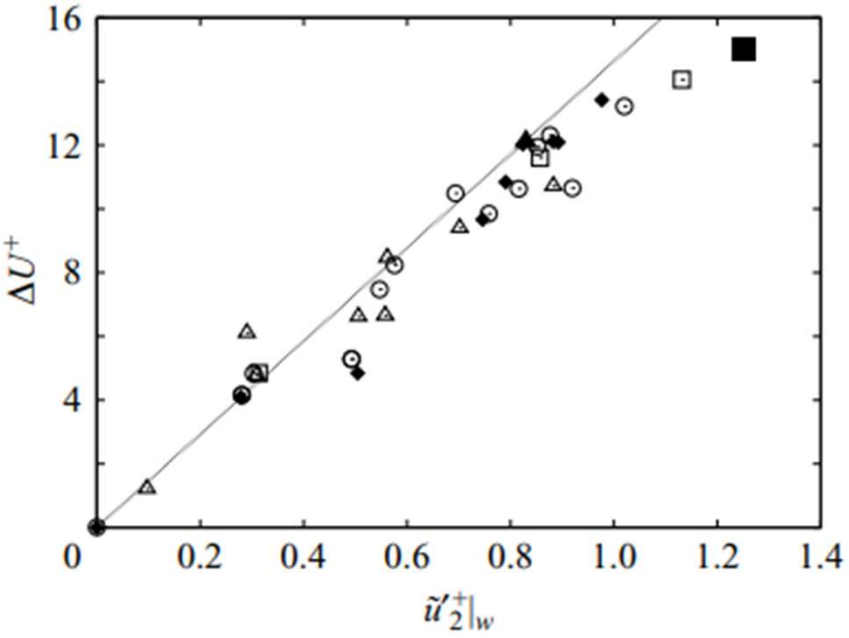


Figure 7: Distribution of the mean velocity (a) and behaviors of quantities of interest related to turbulence statistics (b–f) over the porous substrate TC_{20}: predictions of the homogenized simulation when the effective boundary conditions of the three velocity components are imposed (green lines with filled circles) or when transpiration is neglected (blue lines) are validated against results of the fine-grained simulation (red lines), while the dashed profiles are related to the smooth, impermeable channel case.

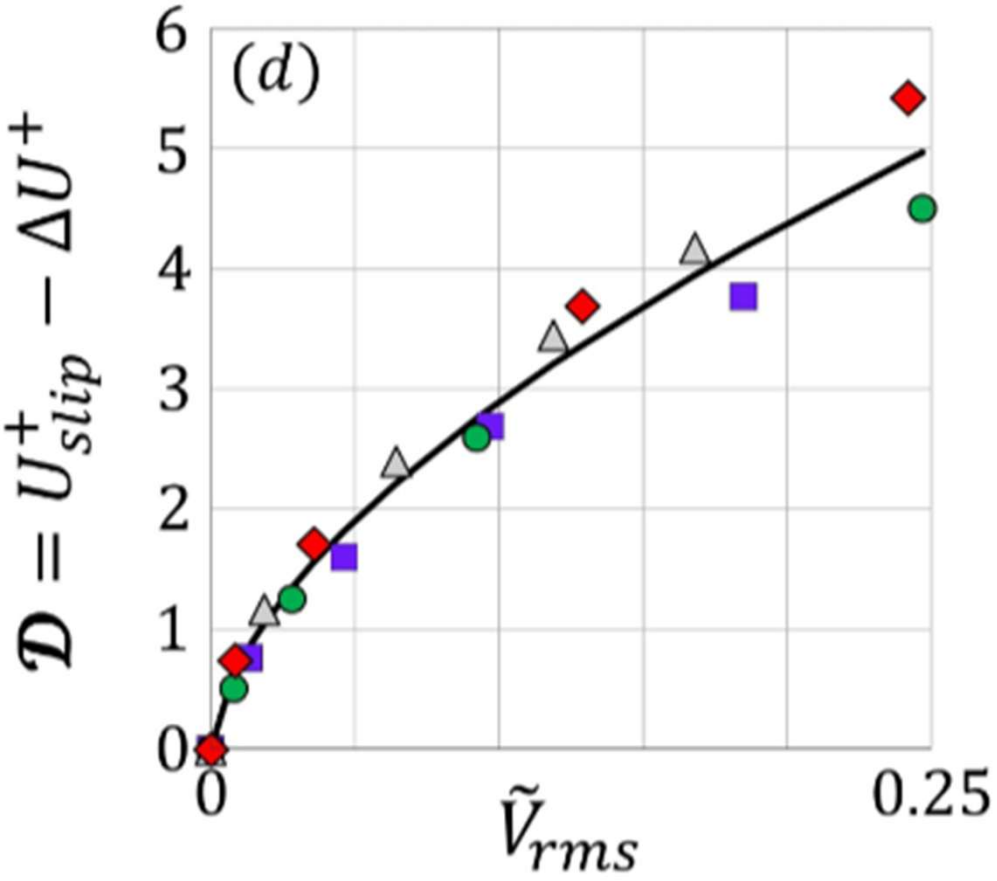

(a)
$\overline{U}^+_{slip}$
$\overline{U}^+_{slip} = \lambda^+_x$
λ^+_x
0
1
2
3
4
5
(b)
ΔU^+
$\Delta U^+ = \Delta\lambda^+$
$\Delta\lambda^+$
TM
TC
LC
LM
+1
0
−1
−2
−3
+2


(a)
$\overline{U}^+_{slip}$
ℓ^+
LM
LC
TM
TC
(b)
ΔU^+
ℓ^+
LM
LC
TC
TM
(d)
ΔC_f %
ℓ^+
TM
TC
LC
LM

Previous work (Orlandi & Leonardi 2006, 2008) has shown that the roughness function is related to the fluctuating vertical velocity at the fictitious wall



$$\tilde{U}^+ = \kappa^{-1} \ln(\tilde{y}^+) + B \left(1 - \frac{\tilde{u}_2'^+|_w}{\kappa}\right)$$



$$\mathcal{D} = 11.5 \times \left[\tilde{V}_{rms}\right]^{0.6}, \qquad NRMS_{error} \approx 11\%,$$

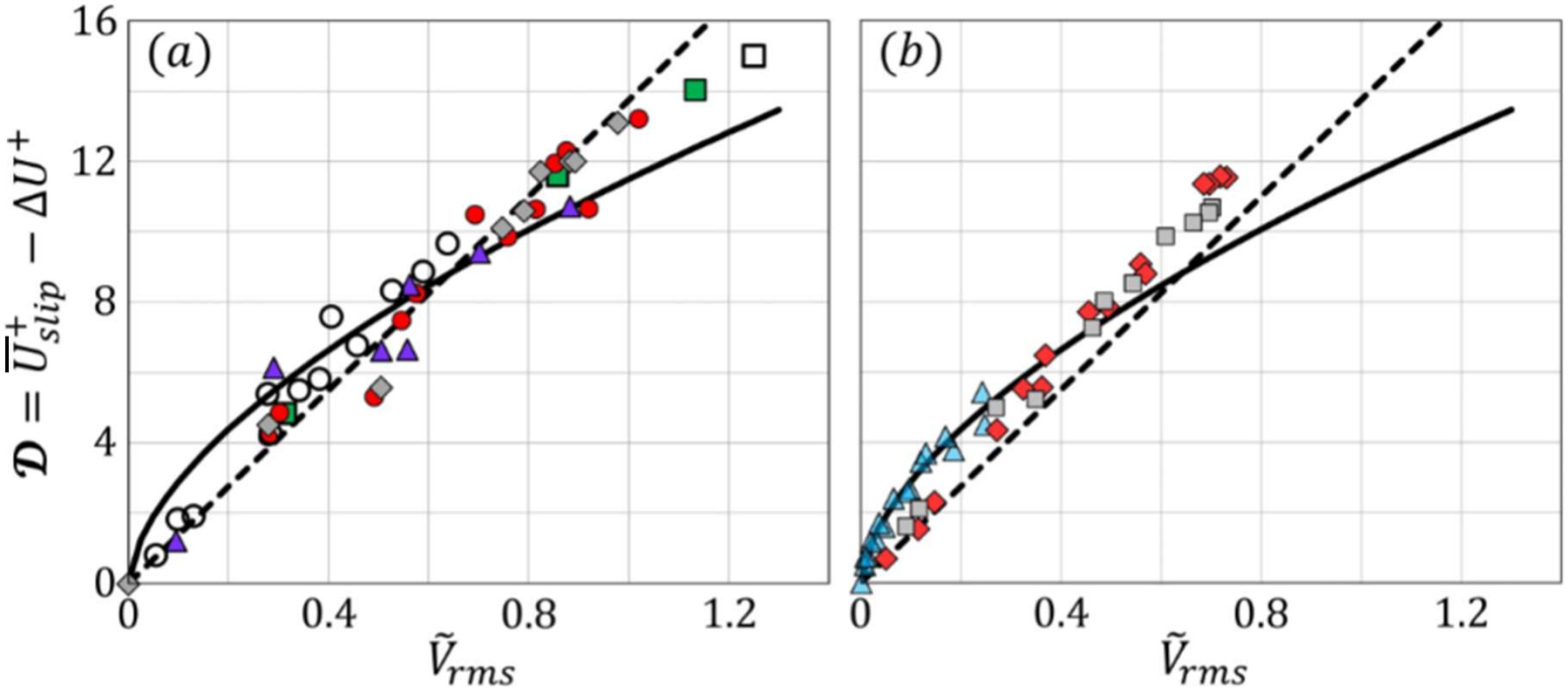


Figure 13: Values of the parameter $\mathcal{D}$ plotted against the r.m.s. of the turbulent fluctuations in the wall-normal velocity at the plane $Y = 0$. In panel (*a*), results from the literature for channels roughened with streamwise-elongated, spanwise-elongated, or three-dimensional elements are shown: blank square, Cheng & Castro (2002); red circles, Leonardi *et al.* (2003); purple triangles, Orlandi & Leonardi (2006); green squares, Burattini *et al.* (2008); gray diamonds, Orlandi & Leonardi (2008); blank circles, Hao & García-Mayoral (2024). In panel (*b*), the results of Hao & García-Mayoral (2024) for symmetric channels bounded by either deep (red diamonds) or shallow (gray squares) porous substrates are plotted, together with the values of the present homogenization-based simulations (light-blue triangles). Solid lines refer to correlation (3.2), while the linear relationship by Orlandi & Leonardi (2008) is plotted with dashed lines.

But …

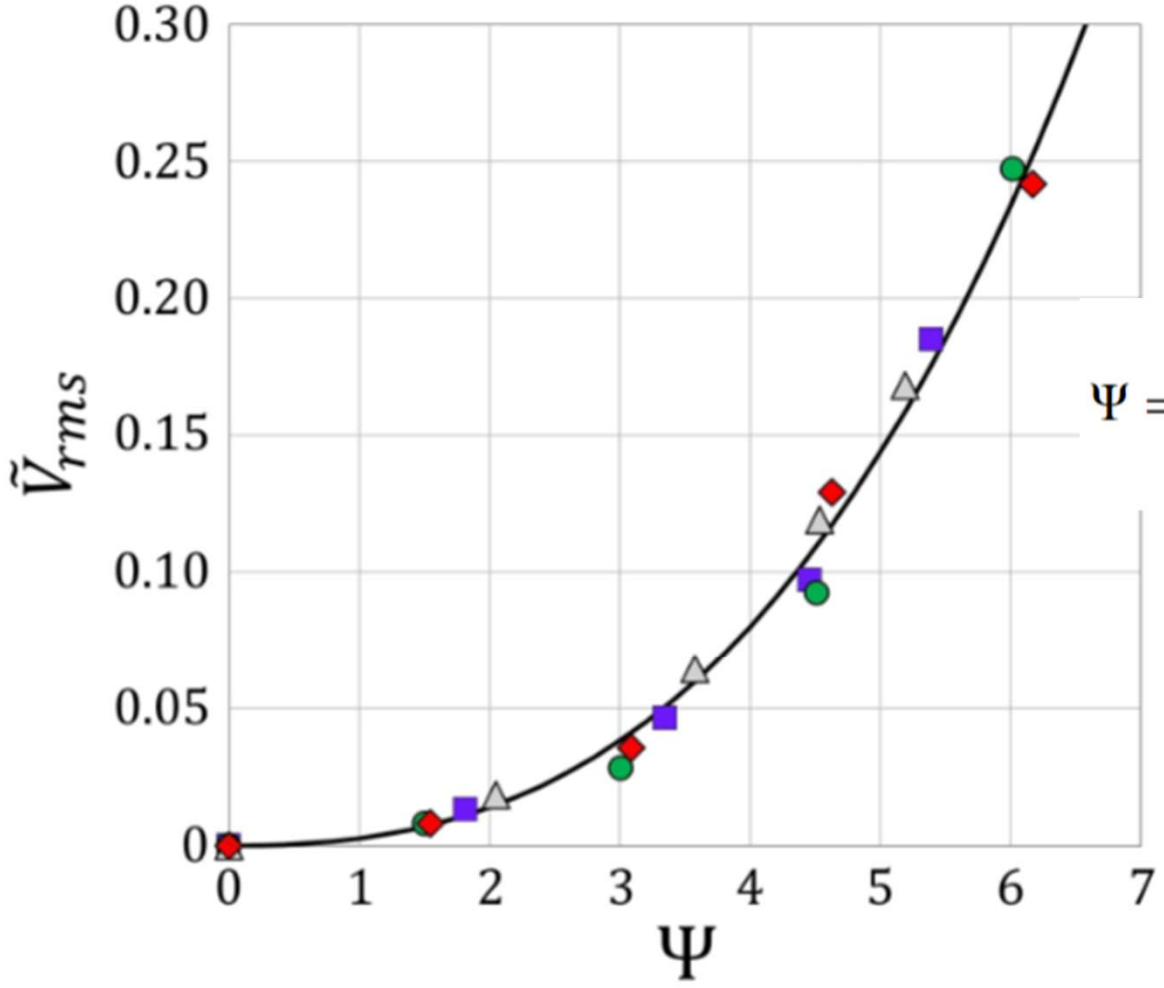
where do we get $\tilde{V}_{rms}$ from???

$$U\Big|_{Y=0^+} = \epsilon\, \boldsymbol{\lambda_x} \frac{\partial U}{\partial Y}\Big|_{Y=0^+} - \epsilon^2\, Re\, \boldsymbol{\mathcal{K}_{xy}^{itf}}\, \frac{\partial P}{\partial X}\Big|_{Y=0^-} + \mathcal{O}(\epsilon^3)$$

$$V\Big|_{Y=0^+} = -\epsilon \frac{\boldsymbol{\mathcal{K}_{xy}^{itf}}}{\boldsymbol{\lambda_x}} \frac{\partial U}{\partial X}\Big|_{Y=0^+} - \epsilon \frac{\boldsymbol{\mathcal{K}_{zy}^{itf}}}{\boldsymbol{\lambda_z}} \frac{\partial W}{\partial Z}\Big|_{Y=0^+} - \epsilon^2 Re\, \boldsymbol{\mathcal{K}_{yy}} \frac{\partial P}{\partial Y}\Big|_{Y=0^-} + \mathcal{O}(\epsilon^3)$$

$$W\Big|_{Y=0^+} = \epsilon\, \boldsymbol{\lambda_z} \frac{\partial W}{\partial Y}\Big|_{Y=0^+} - \epsilon^2\, Re\, \boldsymbol{\mathcal{K}_{zy}^{itf}}\, \frac{\partial P}{\partial Z}\Big|_{Y=0^-} + \mathcal{O}(\epsilon^3)$$

Extended Beavers-Joseph-Saffman conditions with NO empirical coefficients

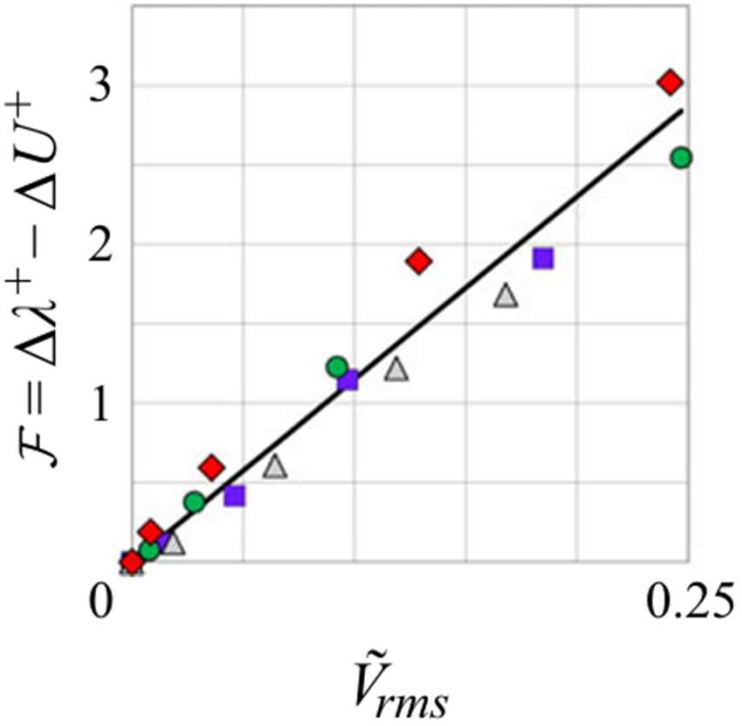


$$\Psi = \left(\frac{\mathcal{K}_{xy}^{itf,+}}{\lambda_x^+} + \frac{\mathcal{K}_{zy}^{itf,+}}{\lambda_z^+} + \sqrt{\mathcal{K}_{yy}^+} \right) \times \left(\frac{\lambda_z^+}{\lambda_x^+} \right)^{0.25}$$

uniquely related to macroscopic upscaled coefficients, and easy to determine in either the approximations of Stokes or Oseen

Final relation for $\mathcal{D}$

$$\mathcal{D} = \overline{U}^{+}_{slip} - \Delta U^{+} = 11.5 \times \left(0.00075\,\Psi^3 + 0.002\,\Psi^2\right)^{0.6}$$



$$\mathcal{F} = 11.5 \times \tilde{V}_{rms}, \qquad NRMS_{error} \approx 18\,\%$$

$$\mathcal{F} = \Delta\lambda^+ - \Delta U^+ = 11.5 \times \left(0.00075\,\Psi^3 + 0.002\,\Psi^2\right)$$

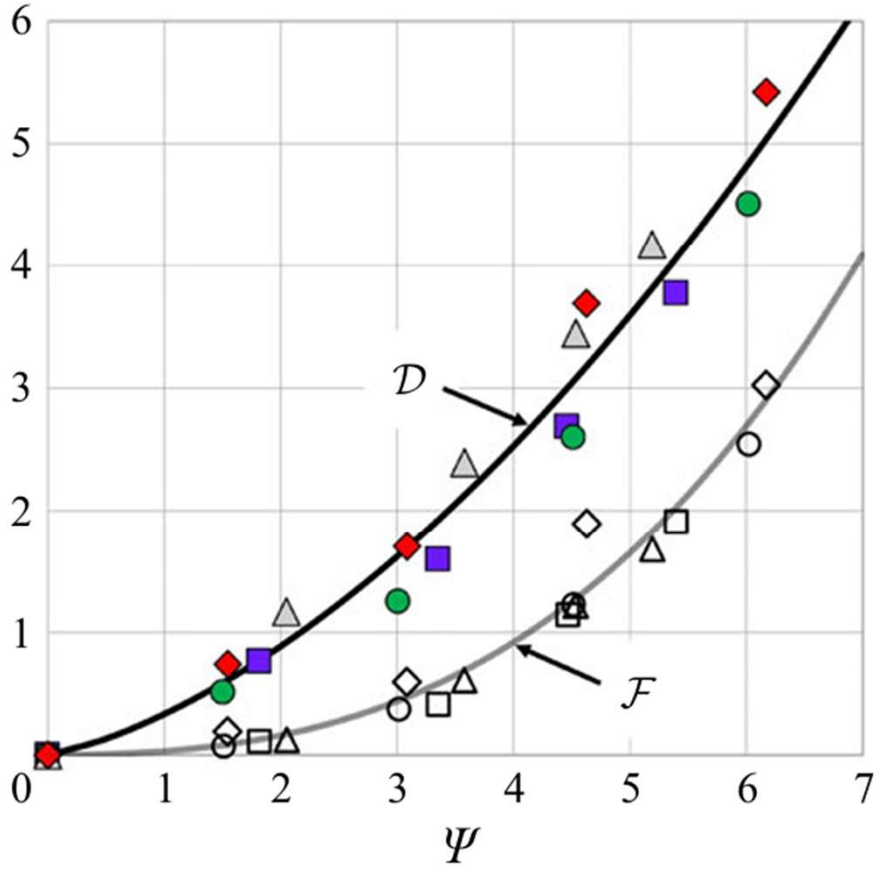


Figure 16. The roughness-function-related quantities $\mathcal{D}$ and $\mathcal{F}$, plotted against the parameter Ψ for the different porous patterns considered (same symbols as in figure 13, filled for $\mathcal{D}$ and empty for $\mathcal{F}$). Correlations (3.12) and (3.13) are plotted with solid lines.

Now we are in the position to make a priori predictions …

Rough wall

$(\mathcal{K}_{yy} = 0)$

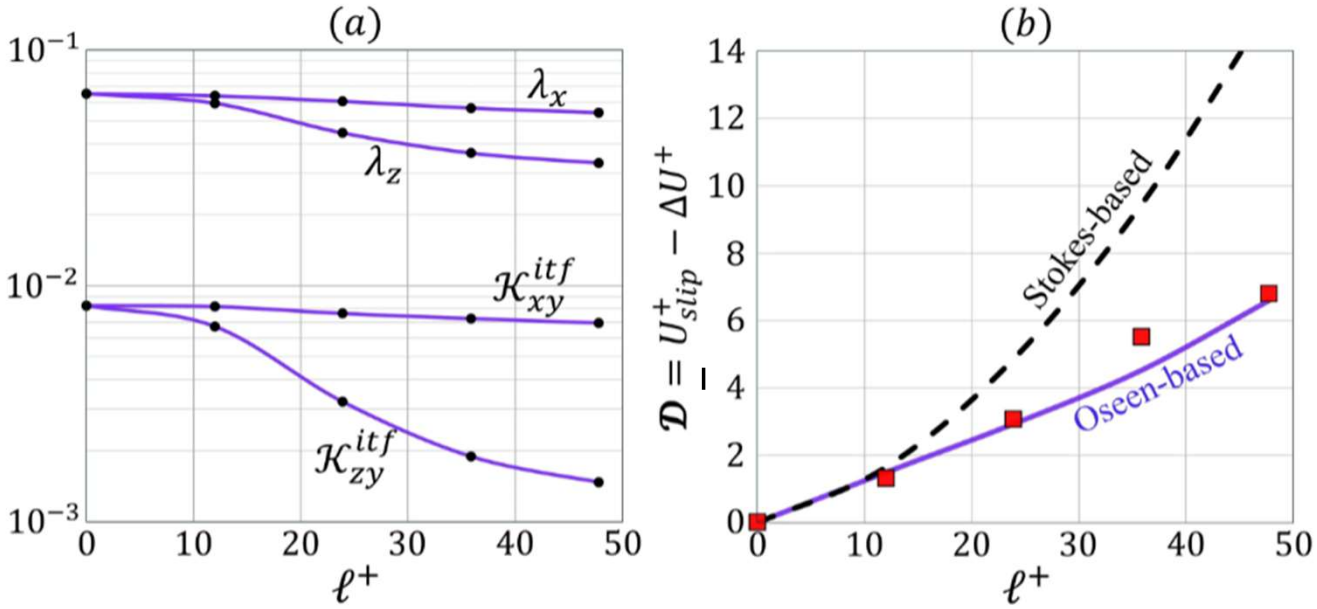


Figure 15: Turbulent flow ($Re_\tau \approx 180$) in a symmetric channel whose top/bottom boundaries are roughened with cubes (in-line arrangement) of size-to-pitch ratio $e/\ell = 0.5$, with the spacing in wall units, $\ell^+ = \epsilon Re_{\tau(M)}$, varied up to 50. Values of the macroscopic coefficients are plotted against ℓ^+ in panel (a). In panel (b), the behavior of the parameter $\mathcal{D}$ based on (3.11) is shown (blue curve), and is validated against the results by Hao & García-Mayoral (2024) obtained from full simulations (squares). The black dashed curve refers to the predictions of (3.11) when Ψ is evaluated with the Stokes-based upscaled coefficients, neglecting near-wall inertia; they are $\lambda_x = \lambda_z \approx 0.0653$ and $\mathcal{K}_{xy}^{itf} \approx \mathcal{K}_{zy}^{itf} = 0.0083$.

Riblets

$(\mathcal{K}_{yy} = 0)$

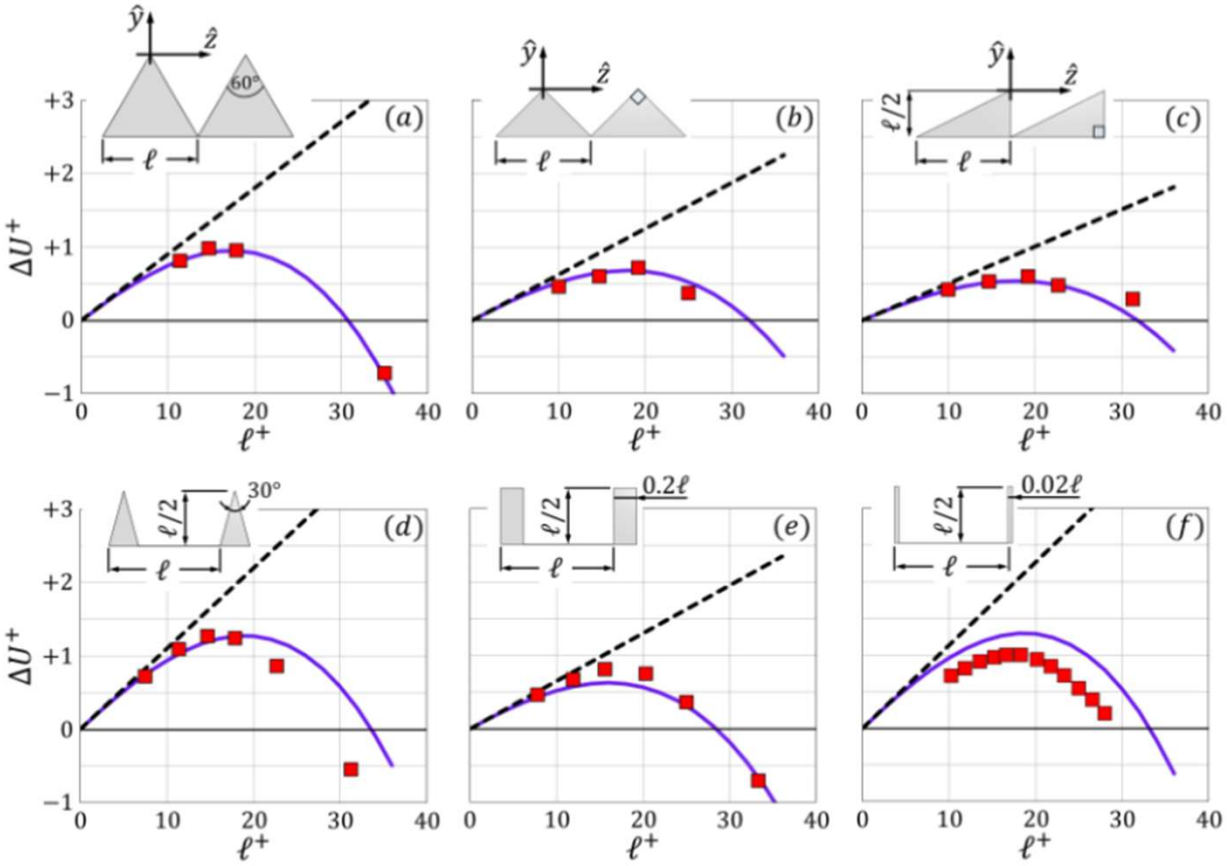
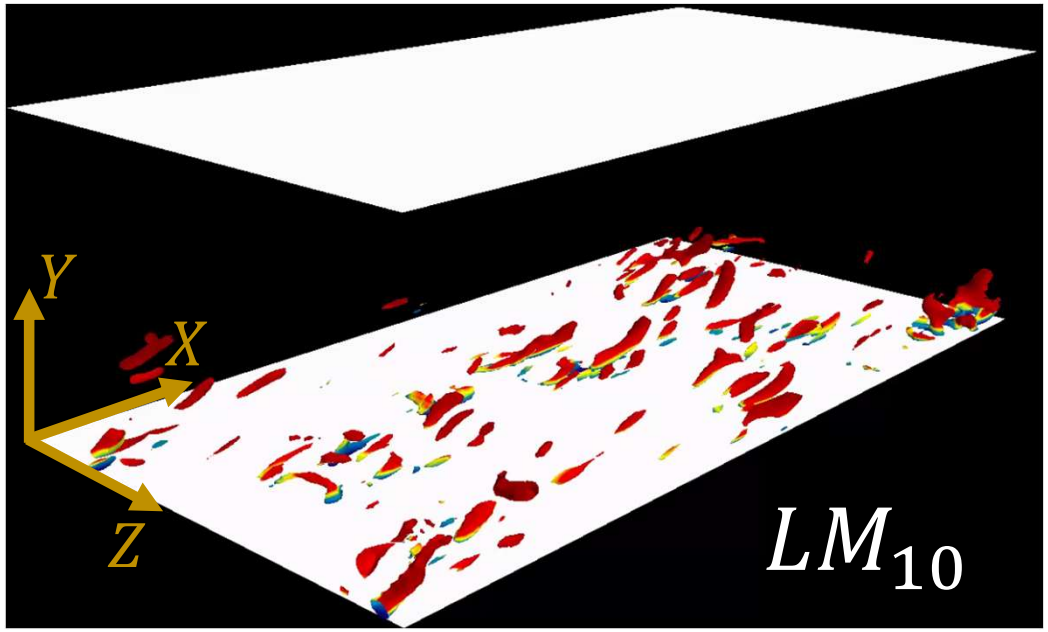


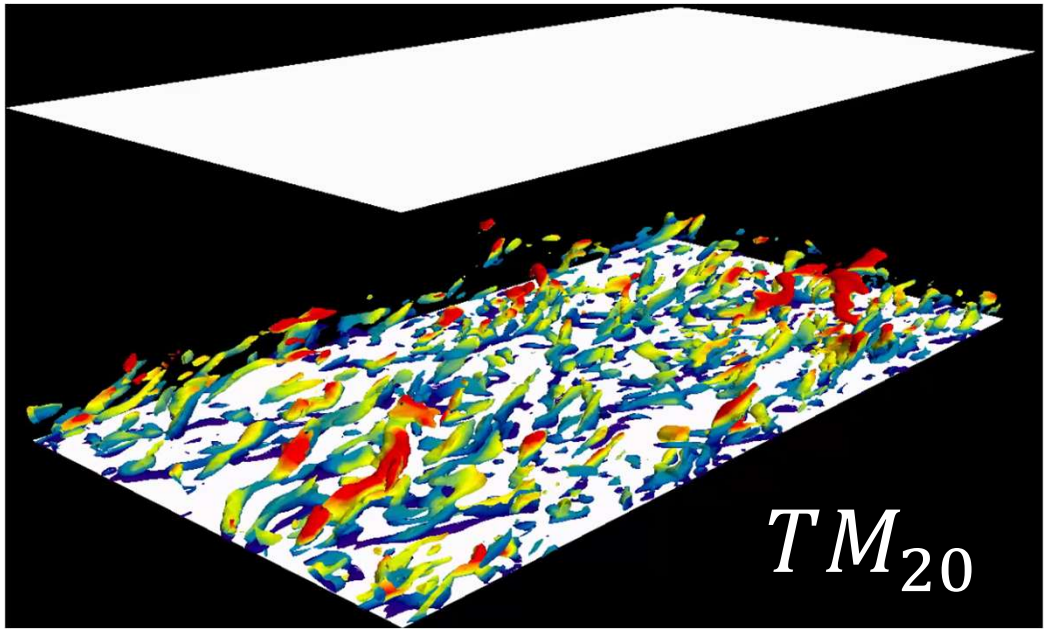
Figure 16: Behavior of ΔU^+ with the increase in ℓ^+, for the turbulent flow over surfaces with different shapes of riblets. The proposed correlation (blue solid lines) is validated against relevant results from the literature (red symbols), while the black dashed lines represent the simple linear dependence $\Delta U^+ = \lambda_x^+ - \lambda_z^+$. The literature results plotted are by (*a–e*) Wong *et al.* (2024) and (*f*) Bechert *et al.* (1997); the latter were reported originally in terms of $\frac{\Delta C_f}{C_{f,smooth}}$ and the corresponding values of ΔU^+ are obtained here employing the relation $\Delta U^+ = -\frac{\Delta C_f}{C_{f,smooth}} \times \left[(2C_{f,smooth})^{-0.5} + 1.25\right]$.

Salient ideas

- Homogenization theory can include advection within a linearized (Oseen-like) framework. The theory yields **macroscopic upscaling coefficients**
- Effective wall conditions appear to mimic very well the effect of the surface/substrate
- The drag-reducing/increasing effect of a porous substrate or of wall roughness ΔU^+ can be reproduced simply by identifying the macroscopic upscaling coefficients, and then computing the **characteristic function** Ψ

Isosurfaces with λ_2 $criterion = 500$

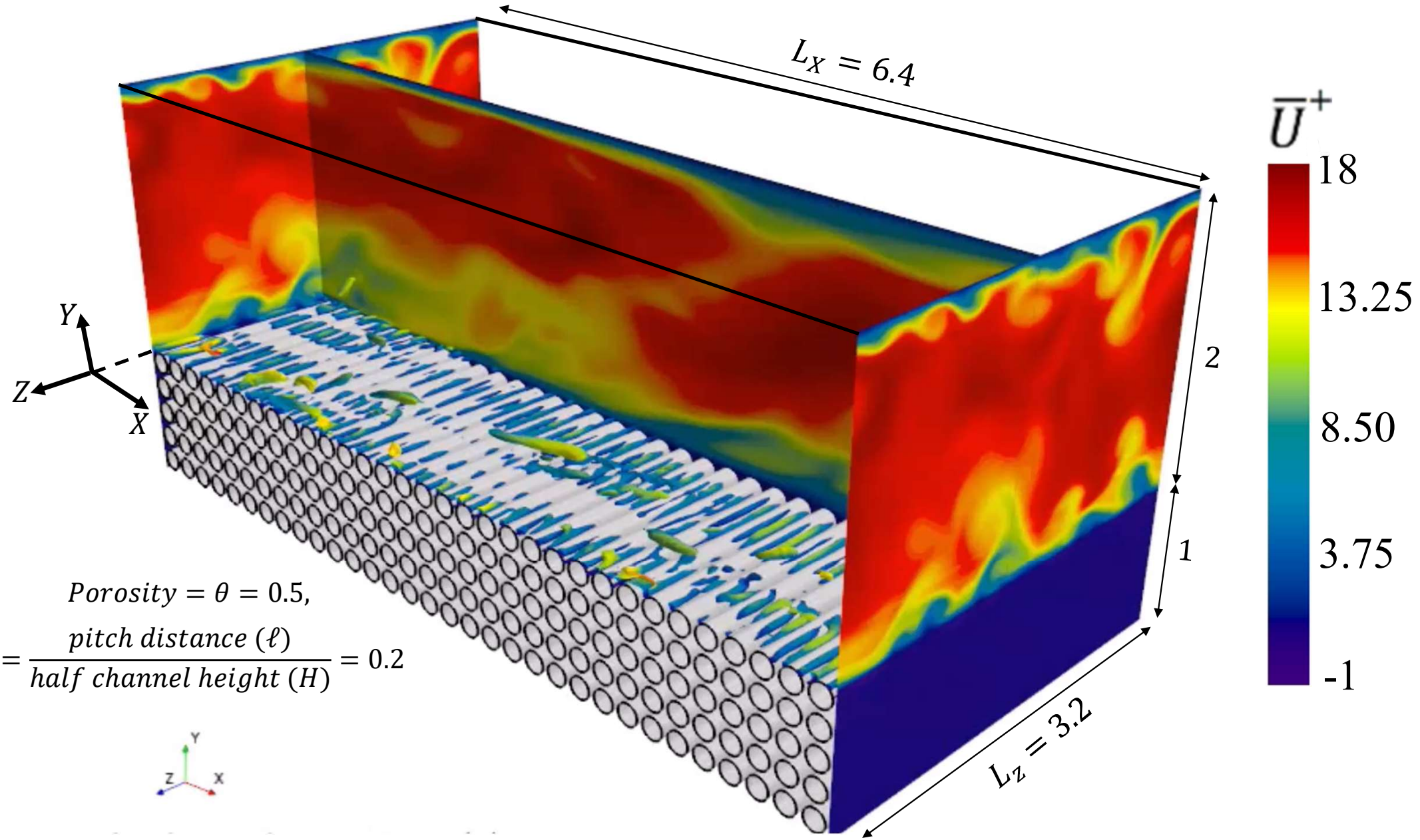




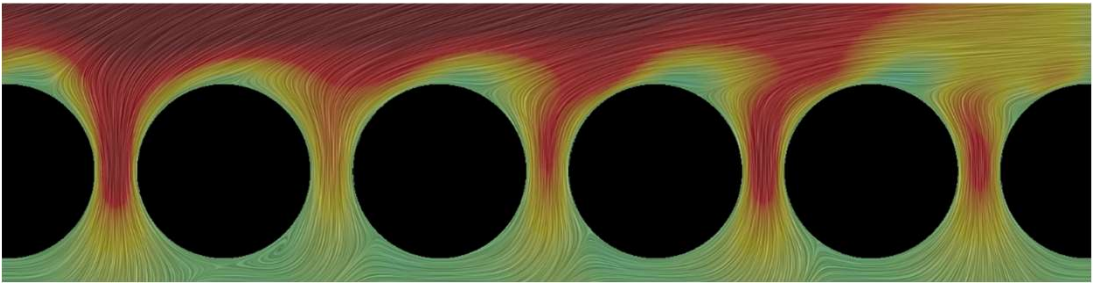
U

0 18

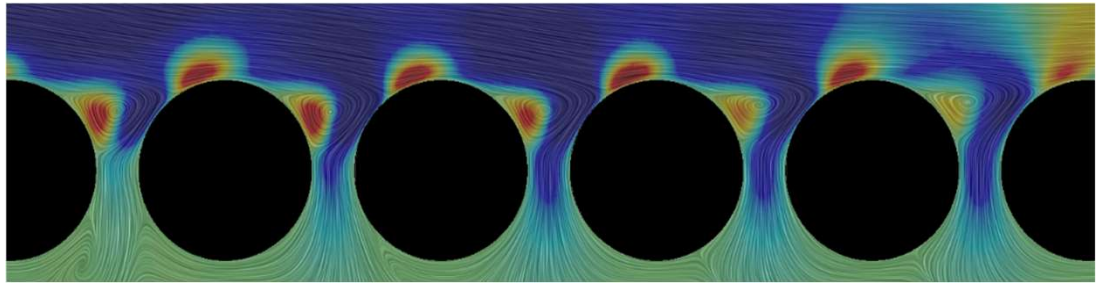
physical time period: $\Delta T^* = 1$

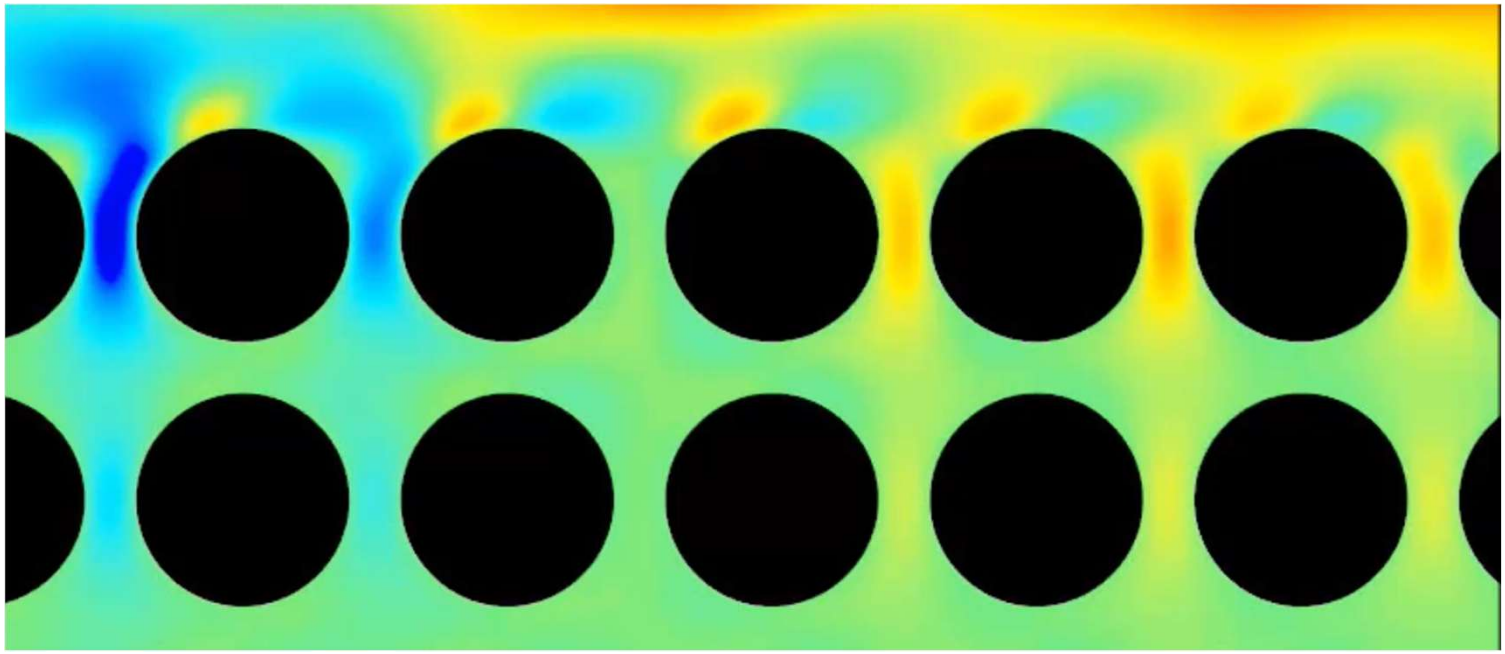

$L_X = 6.4$
$\overline{U}^+$
18
13.25
8.50
3.75
-1
2
1
$L_Z = 3.2$
Y
Z
X
$Porosity = \theta = 0.5,$
$\epsilon = \frac{pitch\ distance\ (\ell)}{half\ channel\ height\ (H)} = 0.2$

Blowing events



Suction events





physical time period: $\Delta T^* \approx 5$