Spontaneous Symmetry Breaking of a Hinged Flapping Filament Generates Lift

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Elastic filamentous structures found on swimming and flying organisms are versatile in function, rendering their precise contribution to locomotion difficult to assess. We show in this Letter that a single passive filament hinged on the rear of a bluff body placed in a stream can generate a net lift force without increasing the mean drag force on the body. This is a consequence of spontaneous symmetry breaking in the filament’s flapping dynamics. The phenomenon is related to a resonance between the frequency associated to the von-Kármán vortex street developing behind the bluff body and the natural frequency of the free bending vibrations of the filament.

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Intriguing and unexpected properties of the physical universe can be explained and understood in terms of spontaneously broken symmetry [1], ranging from the spin-spin interactions in ferromagnetism, to superconductivity in the Anderson-Higgs mechanism, to the acquisition of mass of fundamental particles via the Higgs mechanism. In fluid dynamics, examples of symmetry breaking are known since Jacobi’s famous 1834 discovery that a rotating fluid mass could have equilibrium configurations lacking rotational symmetry [2]. A further example is the convective instability that develops in a viscous flow confined between two plates at different temperatures, the well-known Rayleigh–Bénard system [3].

More recently, symmetry-breaking of fluid-structure interaction problems have come to play an important role for our understanding of animal locomotion. For example, it has been found that the periodic vertical movement of a symmetric body, which is free to move in the horizontal direction, can spontaneously generate locomotion due to symmetry breaking [4, 5]. Another example is the symmetry-breaking bifurcation of flagella due to a buckling instability, which has a significant impact on the waveform and swimming trajectory of spermatozoa [6].

Our aim here is to provide evidence and physical support for the emergence of symmetry breaking for a simple, even if non-trivial, fluid-structure interaction system. It consists of an elastic filament free to flap in the wake of a two-dimensional (2D) circular cylinder. The filament is anchored to the cylinder and the unperturbed upstream fluid flows parallel to the filament axis. A sketch of the system is shown in figure 1. This particular 2D configuration can be experimentally realized using soap-film flows [7]. The following important features characterize all numerical experiments we have performed. When invested by the unperturbed stream, the cylinder alone does not show any symmetry breaking; the flow behind it consists of the celebrated von Kármán street of alternating vortices. Note that the up-down symmetry is not broken and, after half a period, the upper eddies are mirror images of the lower ones. The same up-down symmetry is observed for the filament alone when it undergoes a regular flapping motion [7, 8]. When we let the two symmetry-preserving systems interact a surprising feature arises: after a transient we observe a clear symmetry breaking (see figure 2), with the filament oscillating in either the upper or the lower part of the cylinder wake. The filament length plays the role of a bifurcation parameter. As we will see, symmetry breaking is associated to a net generation of lift, which would cause self-propulsion in the transverse direction (with respect to the direction of the unperturbed velocity) if cylinder plus filament were let free to move. Describing and finding possible explanations for the above scenario is the main concern of the present Letter.

We consider a two-dimensional inextensible elastic filament of length \(L_s\), mass per unit length \(\rho_s\) and flexural rigidity \(B\) – as shown in figure 1 – attached to the rear of a rigid circular cylinder of diameter \(D\) and surrounded by a viscous incompressible fluid of density \(\rho_f\), kinematic viscosity \(\nu\) and free stream velocity \(U\). Scaling space and time with \(D\) and \(U\) respectively, four dimensionless parameters arise,

\[
Re = \frac{UD}{\nu}, \quad R_1 = \frac{\rho_s}{\rho_f D}, \quad R_2 = \frac{B}{\rho_f U^2 D^3}, \quad L = \frac{L_s}{D}.
\]
inextensibility, the transverse component of the filament tail position \( y_t \) is the filament curvature and \( \tau \) and \( \hat{n} \) are unit vectors pointing in the tangential and normal direction of the filament, respectively. The fluid and the filament are coupled at their interface by the no-slip condition \( \mathbf{X}_t = \mathbf{U}(\mathbf{X}(s,t),t) \), with \( \mathbf{U}(\mathbf{X}(s,t),t) = \int \mathbf{u}(x,t)\delta(x - \mathbf{X}(s,t)) \, dx \) the Lagrangian filament velocity and \( f(x,t) = \int \mathbf{F}(s,t)\delta(x - \mathbf{X}(s,t)) \, ds \), with \( f(x,t) \) the Eulerian force density and \( \mathbf{F}(s,t) \) the Lagrangian force density.

The flow past a circular cylinder (without the filament) becomes unstable at \( Re \approx 47 \) via a Hopf bifurcation, resulting in the emergence of the von Kármán vortex street. The sustained vortex shedding is periodic up to approximately \( Re \approx 180 \), before the limit cycle becomes unstable to three-dimensional disturbances. Next, let us consider the flexible filament alone, clamped at one end and free at the other in a uniform flow field. If it is sufficiently long or the imposed uniform flow sufficiently strong \( (L_x U/\nu \gtrsim 10^3) \), its motion is steady for low mass ratios \( R_1 \), periodic for intermediate \( R_1 \) and finally chaotic for large \( R_1 \) [10]. The bending rigidity \( R_2 \) has a stabilizing effect on the structure, but for filaments with high bending stiffness \( R_2 \), an additional destabilizing effect becomes significant, due to the upstream influence – through modification of pressure – of the vortices shed from the trailing edge [11].

We have numerically solved the governing equations for the flow past the cylinder in the presence of the filament [25].

Figure 2(a) shows the filament position at different times in the presence of a long filament (label 3), a short filament with small rigidity (label 2) and a short filament with high rigidity (label 1). For \( L = 3 \) sinus waves propagate along the filament and amplify as they approach the free end. In the cases of \( L = 1.5 \) the filament flaps – depending on the initial perturbation – either above or below the \( y = 0 \) axis. Although the flow in the immediate vicinity of the cylinder is unsteady and non-trivial, both the symmetric and asymmetric motions shown in figure 2(a) are periodic (see figure 2b) and synchronized with the von Kármán vortex street. The observed asymmetry develops spontaneously from the interaction between the fluid and the structure and not from any imposed geometrical conditions. As a result of the symmetry breaking there is now a significant net lift force and a reduced mean drag on the composite body compared to the cylinder without the filament (see table 1).

The equation describing the lateral motion \( (Y(s,t)) \) of the unforced inextensible filament is \( R_1 Y_{tt} + R_2 Y_{ssss} = 0 \). The corresponding eigenfrequency is \( f_s = [R_2/(R_1 L^4)]^{1/2} \) and constitutes the characteristic time scale associated with the free vibrations of the elastic filament. On the other hand, the characteristic, dimensionless frequency of the flow behind a cylinder at \( Re = 100 \) is \( f_c = f D/U = 0.164 \), where \( f \) is the vortex shedding frequency. In order to have efficient coupling between the filament’s elastic degree of freedom and vortex structures
The total energy of the filament reads

\[ E(t) = \frac{1}{2} \int_0^L R_1 |X_t|^2 + R_2 |X_{ss}|^2 \, ds \]  

where we have disregarded the term due to tensile force, since its contribution is orders of magnitude smaller than the bending term. Since the filament flapping is synchronized with the vortex shedding, \[ D/U \] is the natural time scale of the filament. However, it is appropriate to consider the non-dimensional filament energy in terms of density \( \rho_s \) and length \( L_s \) of the filament, instead of \( \rho_f \) and \( D \) which are related to the cylinder. This result in the following scaling of the total energy \( \tilde{E} = E/(R_1 L^3) \). In figure 3(a) the mean value of \( \tilde{E} \) shows a distinct peak in the response of the flexible and rigid filaments for \( L \approx 1.25 \) and \( L \approx 2.25 \), respectively. These values are in good agreement with the predicted values given by the resonance condition (2), i.e. \( L_r = 1.2 \) and \( L_r = 2.6 \).

The actual critical values for symmetry-breaking, \( L_c \) obtained from numerical simulations – are in qualitative agreement with the values inferred from the resonance condition \( L_r \). In figure 3(b), we show the emergence of a sustained asymmetric flapping state at a well-defined threshold, which take the values \( L_c = 1.5 \) and \( L_c = 2.1 \), for the flexible and stiff filaments, respectively. The criterion for asymmetric behavior is a non-zero mean value of the angle \( \Theta \) formed by the straight line connecting the filament anchor point to its tail position with the \( x \)-axis. Note that the angle \( \Theta \) in figure 3(b) does not approach zero for small \( L \), the fingerprint of a possible singularity for \( L \rightarrow 0 \).

The number of filament’s bending modes excited increases with its flexibility, resulting in a dynamically more complex fluid-structure interaction. For flexible structure this results in a third regime for intermediate values of the length ratio (1.6 < \( L < 2.5 \)) where the filament behavior is quasi-periodic or even chaotic. In this regime, the attracting asymmetric state becomes a saddle-point; each shed von Kármán vortex slightly deflects the filament from its asymmetric position towards the opposite side. As a consequence, flexible filaments with 1.6 < \( L < 2.5 \) travel back and forth between the upper and lower regions on a slow time scale as they undergo synchronized undulations with the vortex street on a fast time scale, resulting in a quasi-periodic motion.

The cylinder wake exerts both viscous and pressure drag on the filament; the flapping filament in turn influences the surrounding fluid with restoring tensile and bending forces as well as vortices shed from its trailing edge. In the region of reversed flow behind the cylinder (for \( Re = 100 \) it extends to \( x = 1.9 D \) for the time-averaged flow) the filament is interchangeably compressed and stretched; outside this region, the filament is exposed to stretching. This induces a nonuniform tension on the filament in order to oppose the stretching and compressive viscous stress. The filament also has a restoring force due to its bending rigidity; if it is too flexible (\( R_2 < 10^{-4} \)), it is unable to resist the pressure drag [12] exerted by the back flow and collapses towards the cylinder. Note that flexibility is not necessary for symmetry breaking; therefore – and in contrast to observations made for Stokes flows [6] or non-Newtonian fluids [13] – the trapping of the filament in either the lower or upper half-plane is not due to the buckling instability of a flexible filament.

In order to highlight how the filament interacts with
the unsteady vortical structures in the recirculation region, we compute the maximum value of the finite-time Lyapunov exponent (FTLE). Contours of FTLE correspond to precise vortex boundaries and reveal Lagrangian coherent structures \([14, 15]\) in a similar fashion as visualization techniques based on injection of “tracers”, such as dye or smoke.

Figure 4 shows maximum values of FTLE of instantaneous flow fields. The modification of the cylinder vortex street in the presence of the short and long filaments is observed to be distinctly different. The near field of the vortex street is nearly left unaltered in the presence of the long filament. As the filaments leading part is pushed downward due to the emergence of the upper von Kármán vortex, its trailing edge is simultaneously pulled upward due the action of the lower (and larger) von Kármán vortex. The long filament is thus able to fully adjust its shape and synchronize its flapping with the surrounding fluid vortices; the only significant flow modification is observed at the trailing edge of the filament, where the filament vortices are shed and quickly absorbed by the stronger cylinder vortices. On the other hand, the short filaments shown in figures 4(b) and 4(c) alter the near cylinder wake considerably. The snapshots in figures 4(b) and 4(c) correspond to shortly after the beginning of the upward movement of the filaments. At this instant there is a significant compressive fluid force; in order to resist compression the short filaments induce vorticity with negative (clockwise) circulation. For the flexible filament (figure 4b), the emergence of a co-rotating vortex pair separated by the flexible filament is clearly visible. This filament-induced-vortex propagates along the lower side of the filament before it rapidly merges with the upper negative vortex. As a result, the negative cylinder vortex passes through the filament thus exerting a stretching force on it. The more rigid filament (figure 4c) releases a vortex from its trailing edge with the same rotation as the upper cylinder vortex \([26]\). The filament-induced vorticity in the wake breaks the symmetry by modifying the pressure distribution in the near cylinder wake.

The vorticity induced by the short filaments does not only result in a non-zero mean lift force because of broken symmetry, but also in a reduced mean drag force. A similar drag-reducing mechanism has been observed \([16–18]\) when a number of short filaments \((L \approx 1/4)\) is distributed over the rear side of a cylinder. In our case, the induced vortex from a single filament re-organizes momentum inside the recirculation zone, which counteracts the drag force on the cylinder. Significantly, cf. table I, asymmetric flapping always result in a smaller total drag (i.e. including the drag force on the filament) than for a cylinder alone. In this context, it is interesting to mention recent observations \([19, 20]\) that the von Kármán vortex shedding can be significantly altered in the range \(47 < Re < 100\) by placing a second small object approximately at a 45 degree angle from the \(y = 0\) axis and around \(1 – 2\) diameters downstream of the cylinder. In fact, it was found that the vortex shedding can be stabilized altogether up to \(Re = 70\). Our results show that the filament tethered at the rear end of the cylinder, oscillates in the proximity of the most sensitive regions identified in \([20]\), with an ensuing favorable effect on the wake dynamics.

We have shown that the presence of passive short filaments in unsteady wakes can generate lift without in-

\[
\begin{array}{cccccc}
 L & R_2 & C_d & C_l & f_c & y_m \\
 0.0 & 1.36 \pm 0.01 & 0.00 \pm 0.34 & 0.164 & 0.00 \pm 0.34 & 0.164 \pm 0.43 \\
 1.5 & 1.32 \pm 0.08 & 0.18 \pm 0.28 & 0.159 & 0.159 \pm 0.43 & 0.16 \pm 0.43 \\
 3.0 & 1.28 \pm 0.06 & 0.00 \pm 0.23 & 0.157 & 0.00 \pm 0.64 & 0.16 \pm 0.43 \\
 1.5 & 1.23 \pm 0.05 & 0.21 \pm 0.24 & 0.145 & 0.37 \pm 0.13 & 0.16 \pm 0.43 \\
 3.0 & 1.24 \pm 0.08 & 0.00 \pm 0.32 & 0.139 & 0.00 \pm 0.54 & 0.37 \pm 0.13 \\
\end{array}
\]

TABLE I: Total mean drag coefficient \((C_d)\), total mean lift coefficient \((C_l)\), Strouhal number \((f_c)\) and mean-value of filament tail position \(y_m\) at \(Re = 100\) for different filament lengths and stiffness. The first row corresponds to the cylinder without a filament. Values after the ± sign denote the oscillation amplitudes around the means.
creasing drag. Such filaments may thus in many circumstances favor the hydro- or aero-dynamic behavior of bluff body wakes. Numerous observations [21] suggest that animals control the flow around their bodies in order to reduce drag by making use of their pelage, of scales, feathers and other appendages, with a wide range of textures, rigidities, lengths, and active under different Reynolds numbers. In many cases the functional role of appendages is not fully understood, as their complex motion varies greatly depending on the task being carried out. The pteropod *Clione antarctica* is one example: equipped with three bands of cilia as well as a pair of wings, it possesses two distinct modes of swimming: ciliary mode and flapping mode. While flapping, significant – possibly passive – movement of the cilia is observed but the functional role of this movement is not yet clear [22]. Based on the results presented here it can be speculated that even passive, apparently inert cilia have a positive influence on the locomotion of flying and swimming animals.

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[25] We discretize the fluid equations with a staggered-grid, finite-volume formulation using second order semi-implicit time integration. The no-slip boundary condition is enforced at Lagrangian points by appropriate regularized surface forces [9, 23]. A grid size of $h = 1/60$ is sufficient to reproduce previous published work [10, 23] for the flow past a cylinder –in terms of lift/drag coefficients for various $Re$ – and for the 2D “flag” problem – in terms of straight/flapping motion for various $R_1$ and $R_2$.