



BIOSKINS:

From very small to macroscopic:
Random thoughts on the no-slip condition

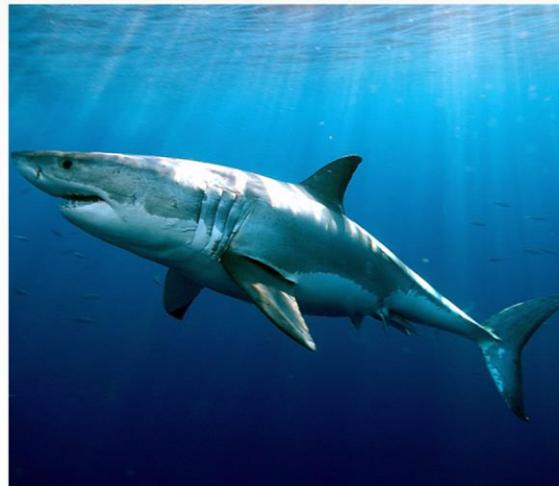
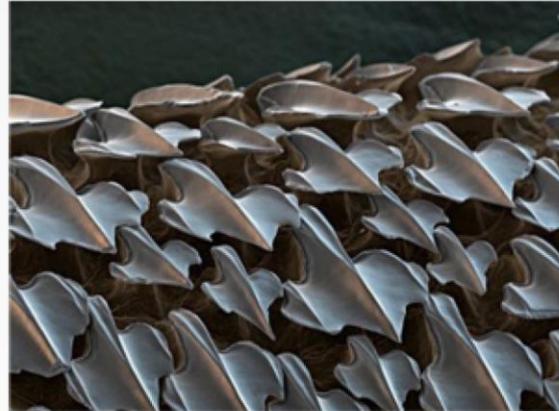
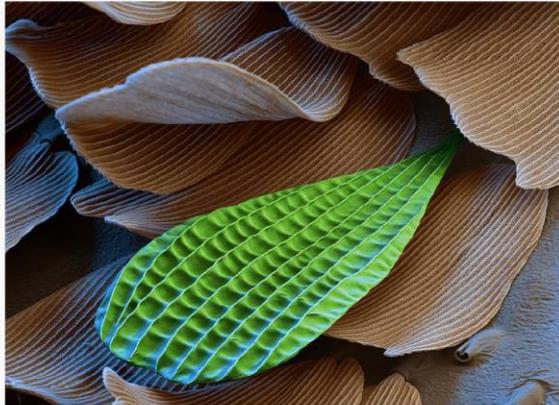
A. Bottaro, UNIGE & IMFT

Toulouse, 14 Septembre 2017

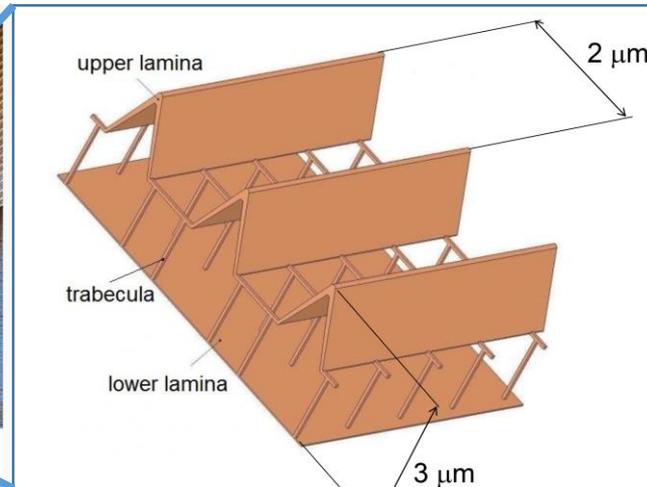
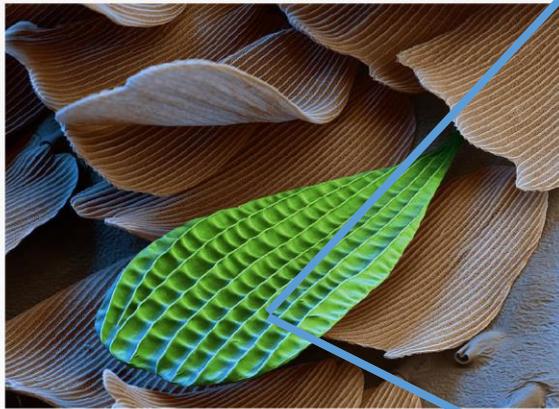




“*Life*” is not smooth, but anisotropic, multiscale, heterogeneous, rough, porous, flexible, etc.



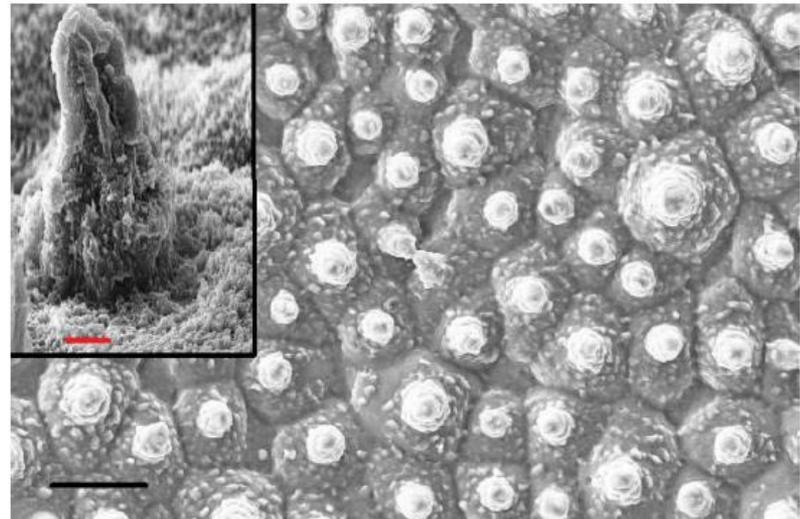
“*Life*” is not smooth, but anisotropic, multiscale, heterogeneous, rough, porous, flexible, etc.





— 2 μm

— 20 μm



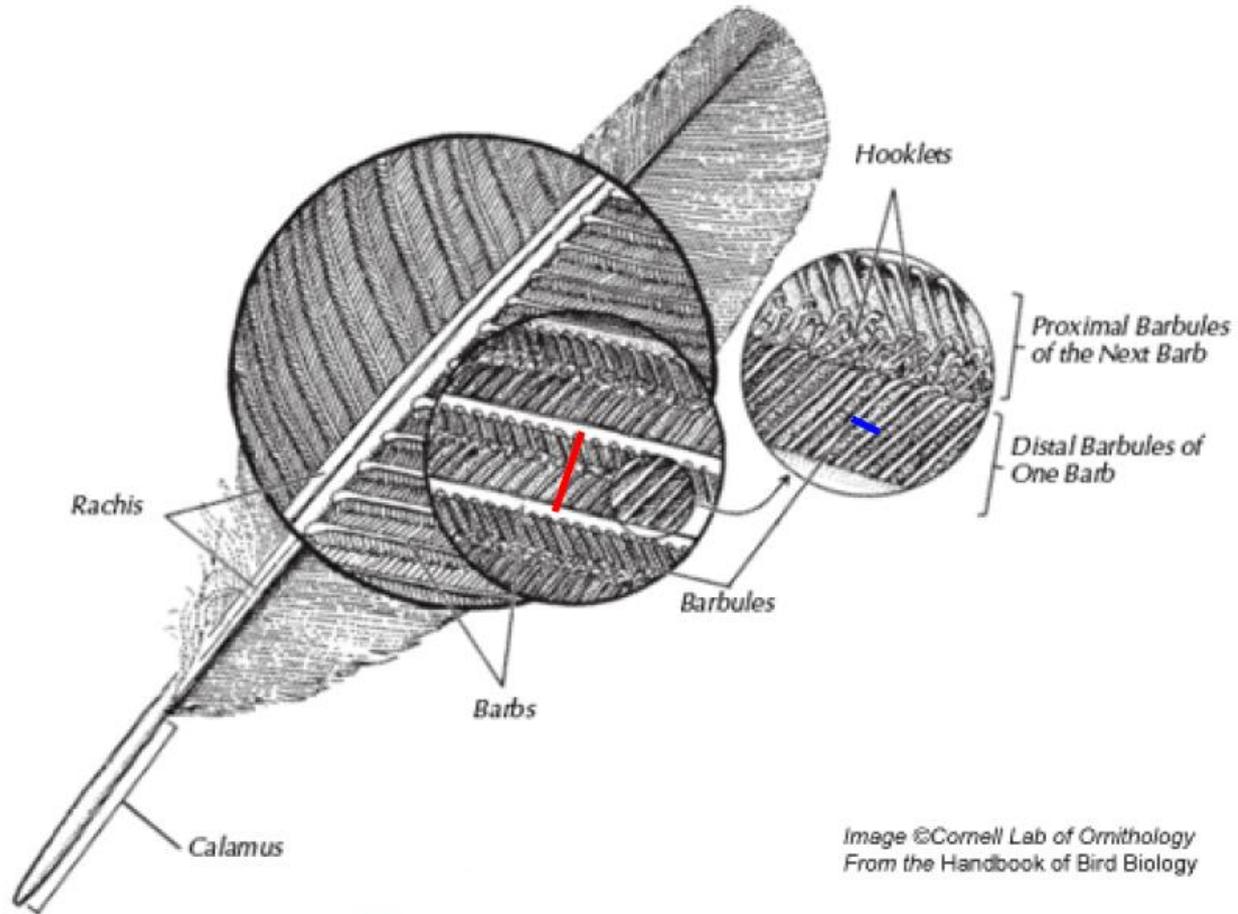


Image ©Cornell Lab of Ornithology
From the Handbook of Bird Biology

200 μm

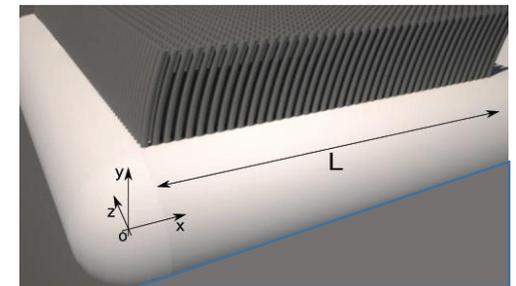
10 μm

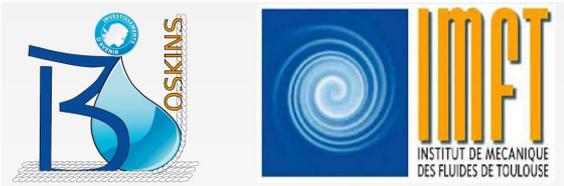


Sometimes a microscopic description of the flow around such multiscale surfaces is impossible ...

GOALS of BIOSKINS:

- to model *apparent slip* over realistic surfaces, interfaces and layers (poro-elastic and poro-plastic) using the theory of homogenisation to average out the fine details around the surface/interface/layer, and
- go beyond the macroscopic “*Navier’s slip condition*” ...

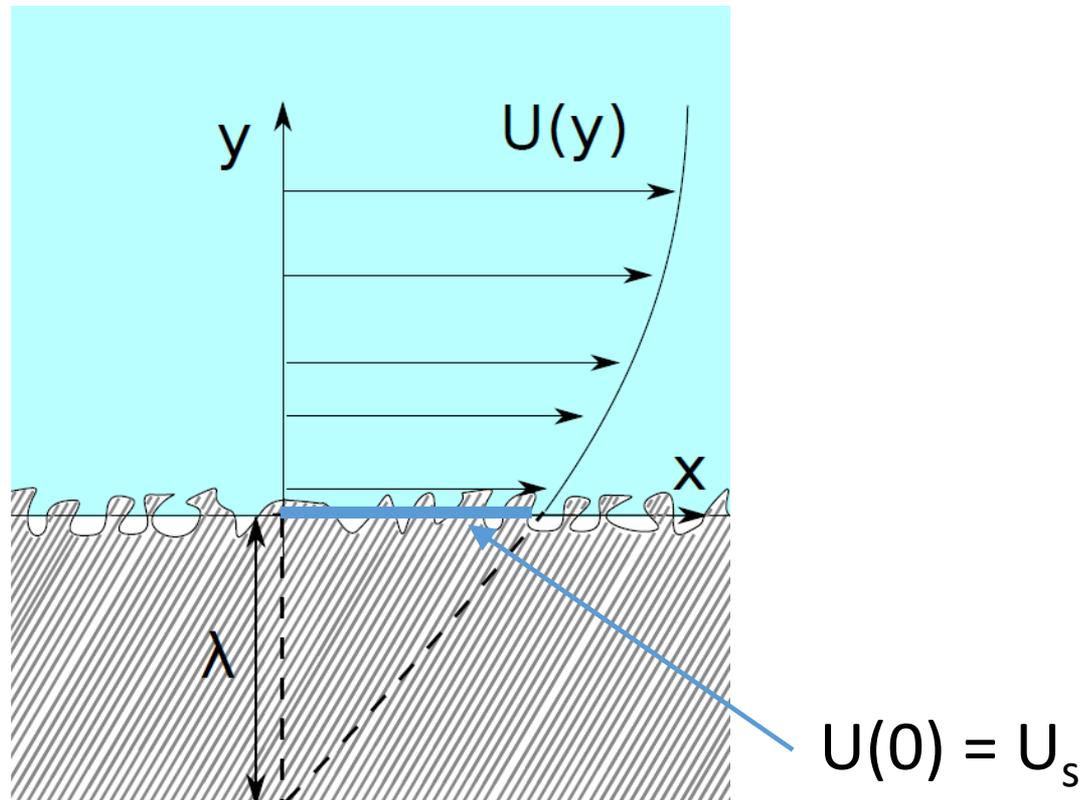




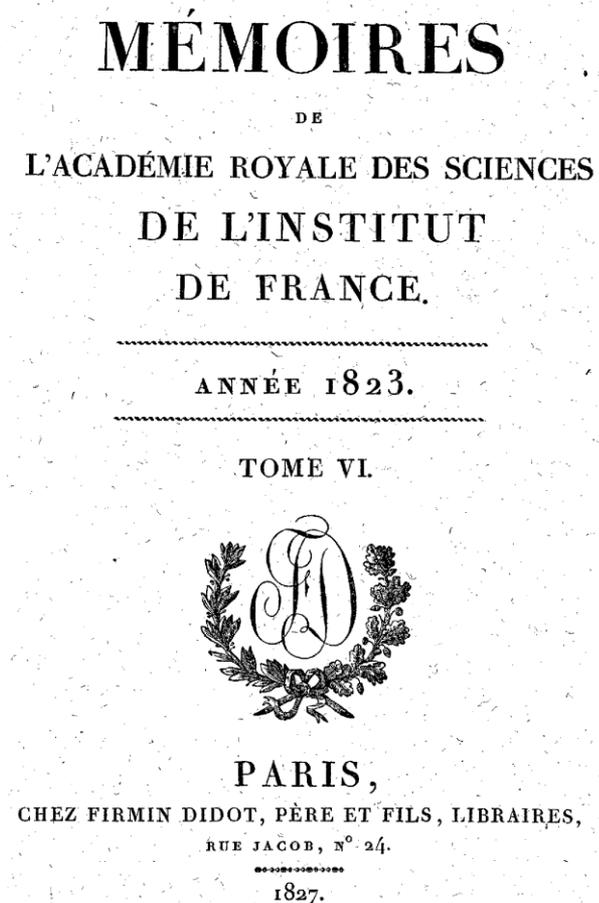
Some definitions:

- *True slip* refers to the actual slippage of fluid molecules over a solid surface
- *Apparent slip* occurs typically when a liquid slides over a less viscous layer (gas layer, surface covered by micro/nano bubbles, density depleted layer adjacent to the surface)
- *Non-Newtonian slip* is attributed to the disentanglement of surface-anchored polymer chains (as opposed to the bulk chains)

A BIT OF HISTORY



Claude Louis Marie Henri Navier (1785-1836)





Claude-Louis Navier

MÉMOIRE

SUR LES LOIS DU MOUVEMENT DES FLUIDES;

PAR M. NAVIER.

Lu à l'Académie royale des Sciences, le 18 mars 1822.



Claude-Louis Navier

MÉMOIRE

SUR LES LOIS DU MOUVEMENT DES FLUIDES;

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Sir George Gabriel Stokes
(1819-1903)



Pour exprimer les conditions de l'équilibre d'une portion de fluide conformément aux notions établies ci-dessus, on considérera une molécule placée au point M dont les coordonnées sont x, y, z ; et une molécule placée au point M' très-voisin du premier, dont les coordonnées sont $x + \alpha, y + \beta, z + \gamma$. On nommera ρ la distance des deux points, en sorte que $\rho = \sqrt{\alpha^2 + \beta^2 + \gamma^2}$. La force répulsive qui s'établit entre ces deux molécules dépend de la situation du point M, puisqu'elle doit balancer la pression, qui peut varier dans les diverses parties du fluide. Elle dépend de la distance ρ , et, comme toutes les actions moléculaires, décroît très-rapidement quand cette distance augmente. On désignera cette force par la fonction $f(\rho)$, à laquelle on attribuera cette propriété, et qui doit être regardée aussi comme dépendante des coordonnées x, y, z .



..... densité du fluide, on aurait ainsi les trois équations

$$P - \frac{dp}{dx} = \rho \left(\frac{du}{dt} + u \frac{du}{dx} + v \frac{du}{dy} + w \frac{du}{dz} \right) - \epsilon \left(\frac{d^2 u}{dx^2} + \frac{d^2 u}{dy^2} + \frac{d^2 u}{dz^2} \right),$$

$$Q - \frac{dp}{dy} = \rho \left(\frac{dv}{dt} + u \frac{dv}{dx} + v \frac{dv}{dy} + w \frac{dv}{dz} \right) - \epsilon \left(\frac{d^2 v}{dx^2} + \frac{d^2 v}{dy^2} + \frac{d^2 v}{dz^2} \right),$$

$$R - \frac{dp}{dz} = \rho \left(\frac{dw}{dt} + u \frac{dw}{dx} + v \frac{dw}{dy} + w \frac{dw}{dz} \right) - \epsilon \left(\frac{d^2 w}{dx^2} + \frac{d^2 w}{dy^2} + \frac{d^2 w}{dz^2} \right).$$

On devrait avoir également $p=0$ dans tous les points de la surface libre du fluide. Il faudrait exprimer que les molécules contiguës aux parois solides ne peuvent se mouvoir que dans le sens de ces parois. Enfin l'on doit joindre aux équations précédentes celle qui exprime que le volume des parties du fluide est invariable, qui est

$$0 = \frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz}.$$



... la constante ε représente en unités de poids la résistance provenant du glissement de deux couches quelconques l'une sur l'autre, pour une étendue égale à l'unité superficielle.

$$\varepsilon = \frac{8\pi}{3\alpha} \int_0^{\infty} d\rho \cdot \rho^4 f(\rho)$$



(the wall)

Si elle était perpendiculaire à l'axe des y ,

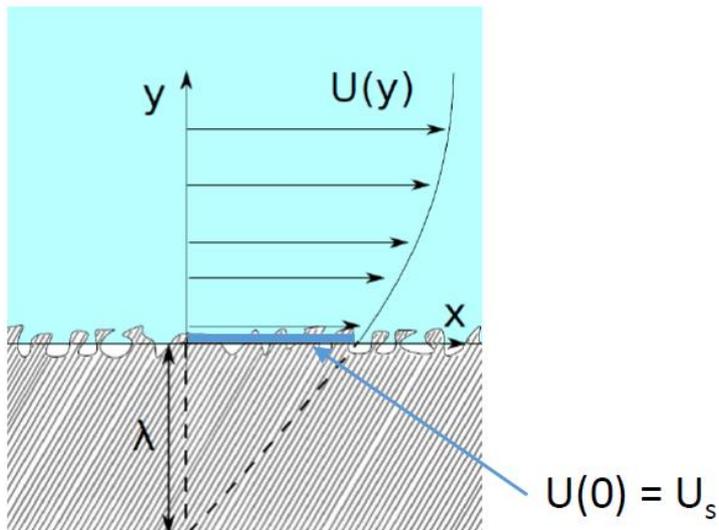
$$Eu + \varepsilon \frac{du}{dy} = 0, \quad Ew + \varepsilon \frac{dw}{dy} = 0;$$

La valeur de la constante E doit varier suivant la nature des corps avec lesquels le fluide est en contact, et (ce qui est physiquement impossible) s'il y avait un espace vide au-dessus de la portion libre de la surface du fluide, ces équations devraient encore être satisfaites pour les points appartenant à cette portion, en y supposant $E = 0$.

In today's terminology, Navier's argument was that

- (i) there is *partial slip*, with
- (ii) the resistance of the wall proportional to the slip velocity U_s :

$$E U_s = -\varepsilon \left. \frac{dU}{dy} \right|_{y=0} \rightarrow U(0) = (-\varepsilon/E) \left. \frac{dU}{dy} \right|_{y=0} = \lambda \left. \frac{dU}{dy} \right|_{y=0}$$



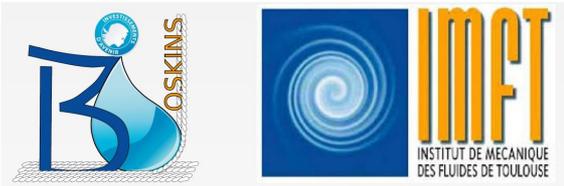
λ = slip length



- Navier's slip condition is a first-order development around a fictitious wall (the position $y = 0$ is arbitrary) applicable when either
 - (i) the surface geometry is microstructured or
 - (ii) the continuum approximation breaks down.
- There is a unique slip length λ for U and W only for isotropic (in x, z) walls. The general (anisotropic) case requires (to first order) that:

$$\begin{bmatrix} U(x, 0, z) \\ W(x, 0, z) \end{bmatrix} = \Lambda \frac{\partial}{\partial y} \begin{bmatrix} U(x, 0, z) \\ W(x, 0, z) \end{bmatrix},$$

with Λ a slip tensor (plus a *non-penetration* condition for V).



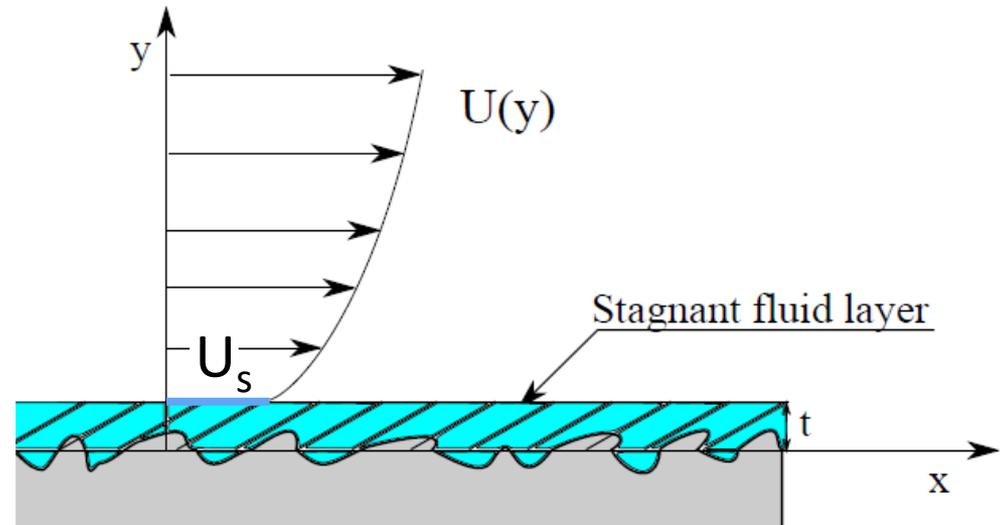
In the 18th century Daniel Bernoulli, Du Buat and Coulomb had already argued for *partial slip* or *no-slip* ...

Later on, the situation became somewhat confusing ...

1816: A thin layer of fluid remains attached to the wall and the bulk of the fluid slips over the outer surface of this stagnant layer

Mémoire sur l'écoulement linéaire de diverses substances liquides par des tubes capillaires de verre

Pierre-Simon Girard



Hg: $t \rightarrow 0$



Girard's idea and experimental results stuck for a while, as beautifully described by Sydney Goldstein in a four-page historical Appendix entitled: “*Note on the conditions at the surface of contact of a fluid with a solid body*” of the book:

MODERN DEVELOPMENTS IN FLUID DYNAMICS

AN ACCOUNT OF THEORY AND EXPERIMENT
RELATING TO
BOUNDARY LAYERS, TURBULENT MOTION
AND WAKES

Composed by the
FLUID MOTION PANEL OF THE
AERONAUTICAL RESEARCH COMMITTEE
AND OTHERS
and edited by
S. GOLDSTEIN

VOLUME I

OXFORD
AT THE CLARENDON PRESS

1938



Girard's stagnant fluid layer is also reminiscent of the near-wall “*Knudsen layer*” (of thickness of the order of the mean free path) which is established in compressible flows when $0.01 \leq \text{Kn} \leq 0.1$ (*slip flow regime*). This can be studied using NS in the bulk, while rarefaction effects are modeled through *partial slip* at the wall (Maxwell slip velocity, 1879):

$$U_s = \frac{2 - \sigma_v}{\sigma_v} \text{Kn} \frac{\partial U_s}{\partial n}$$

σ_v : *tangential momentum accommodation coefficient*
(TMAC)

(more later ...)



Navier's equations have later been re-discovered and/or extended to the compressible case by:

1828: Augustin-Louis **Cauchy** (1789 - 1857)

Exercices de Mathématiques, p. 187

1829: Siméon-Denis **Poisson** (1781 – 1840)

Journal de l'Ecole Polytechnique, XX^e cahier, p. 152, 1831

1843: Adhémar Jean Claude Barré **de Saint Venant** (1797 -1886)

Comptes Rendus de l'Academie des Sciences, v. 17, p.1243

1845: George Gabriel **Stokes** (1819-1903)

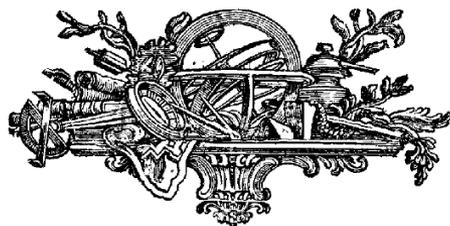
Transactions of the Cambridge Philosophical Society,
Vol VIII, 1849, p. 287



JOURNAL
DE
L'ÉCOLE POLYTECHNIQUE,
PUBLIÉ
PAR LE CONSEIL DE CET ÉTABLISSEMENT.

VINGTIÈME CAHIER.

TOME XIII.



A PARIS,
DE L'IMPRIMERIE ROYALE.

~~~~~  
Février 1831.

MÉMOIRE

*Sur les Équations générales de l'Équilibre et du Mouvement  
des Corps solides élastiques et des Fluides;*

Lu à l'Académie des sciences, le 12 Octobre 1829.

PAR M. POISSON.



$$\left. \begin{aligned} \rho \left( X - \frac{d^2 x}{dt^2} \right) &= \frac{dU_1}{dz} + \frac{dU_2}{dy} + \frac{dU_3}{dx}, \\ \rho \left( Y - \frac{d^2 y}{dt^2} \right) &= \frac{dV_1}{dz} + \frac{dV_2}{dy} + \frac{dV_3}{dx}, \\ \rho \left( Z - \frac{d^2 z}{dt^2} \right) &= \frac{dW_1}{dz} + \frac{dW_2}{dy} + \frac{dW_3}{dx}; \end{aligned} \right\} \quad (8)$$

$$\frac{d\rho}{dt} + \frac{d(\rho u)}{dx} + \frac{d(\rho v)}{dy} + \frac{d(\rho w)}{dz} = 0. \quad (10)$$

[68.] Les diverses équations que nous venons d'obtenir sont relatives à tous les points de la masse fluide situés à une distance sensible de la surface, ou du moins à une distance plus grande que le rayon d'activité des molécules ; pour compléter le système des équations différentielles du mouvement des fluides, il nous reste à former les équations particulières qui ont lieu à leurs surfaces ; et c'est ce que nous allons faire, en considérant d'abord le cas d'un fluide en contact avec une partie solide. Nous avons déjà remarqué qu'alors il est possible qu'une couche très-mince du fluide devienne adhérente à cette paroi et perde sa fluidité (n.º 41) ; dans ce cas, nous regarderons cette couche comme faisant partie de la paroi qui aura pour surface celle de cette même couche où se termine le fluide qui sera resté mobile.

**Girard's slip**



TRANSACTIONS  
OF THE  
CAMBRIDGE  
PHILOSOPHICAL SOCIETY.

ESTABLISHED NOVEMBER 15, 1819.

VOLUME THE EIGHTH.

CAMBRIDGE:  
PRINTED AT THE UNIVERSITY PRESS:  
AND SOLD BY  
JOHN WILLIAM PARKER, WEST STRAND, LONDON;  
J. DEIGHTON; AND MACMILLAN & CO., CAMBRIDGE.

M.DCCC.XLIX.

XXII. *On the Theories of the Internal Friction of Fluids in Motion, and of the Equilibrium and Motion of Elastic Solids.* By G. G. STOKES, M.A., Fellow of Pembroke College.

---

[Read April 14, 1845.]



**IMFT**  
INSTITUT DE MECANIQUE  
DES FLUIDES DE TOULOUSE



$$\frac{d\rho}{dt} + \frac{d\rho u}{dx} + \frac{d\rho v}{dy} + \frac{d\rho w}{dz} = 0 \dots\dots\dots(11),$$

$$\rho \left( \frac{Du}{Dt} - X \right) + \frac{dp}{dx} - \mu \left( \frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} \right) - \frac{\mu}{3} \frac{d}{dx} \left( \frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} \right) = 0, \text{ \&c. } \dots\dots (12).$$

\* The same equations have also been obtained by Navier in the case of an incompressible fluid (*Mém. de l'Académie*, t. vi. p. 389), but his principles differ from mine still more than do Poisson's.



The next case to consider is that of a fluid in contact with a solid. The condition which first occurred to me to assume for this case was, that the film of fluid immediately in contact with the solid did not move relatively to the surface of the solid.

having calculated, according to the conditions which I have mentioned, the discharge of long straight circular pipes and rectangular canals, and compared the resulting formulæ with some of the experiments of Bossut and Dubuat, I found that the formulæ did not at all agree with experiment. I then tried Poisson's conditions in the case of a circular pipe, but with no better success. In fact, it appears from experiment that the tangential force varies nearly as the square of the velocity with which the fluid flows past the surface of a solid, at least when the velocity is not very small. It appears however from experiments on pendulums that the total



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**Turbulence!!**





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**NO SLIP?**

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$\tau_w \propto U_s^2$   
(because of surface irregularities?)



friction varies as the first power of the velocity, and consequently we may suppose that Poisson's conditions, which include as a particular case those which I first tried, hold good for very small velocities.

...

Let  $a$  be the radius of the pipe, and  $U$  the velocity of the fluid close to the surface; then, integrating the above equation, and determining the arbitrary constants by the conditions that  $w$  shall be finite when  $r = 0$ , and  $w = U_s$  when  $r = a$ , we have

$$w = \frac{g\rho \sin \alpha}{4\mu} (a^2 - r^2) + U_s$$



## Later on ...

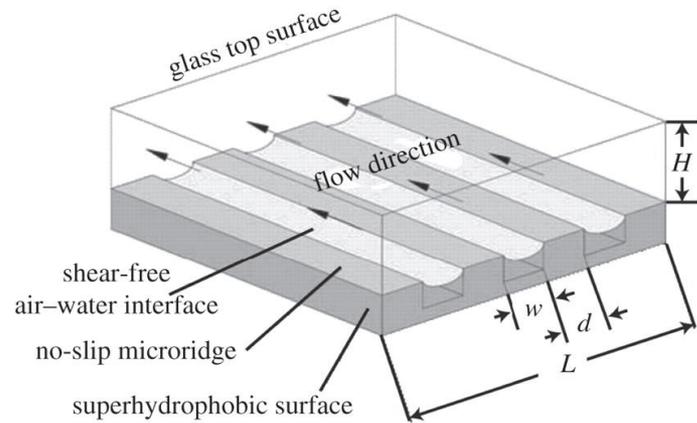
|               |              |                                                               |
|---------------|--------------|---------------------------------------------------------------|
| 1850: Stokes  | $U_s = 0$    | “highly satisfactory”                                         |
| 1879: Lamb    | $U_s \neq 0$ | on account of experiments by<br>Helmholtz & Piotrowski (1860) |
| 1890: Whetham | $U_s = 0$    | through accurate experiments                                  |
| 1895: Lamb    | $U_s = 0$    | “... in all ordinary cases ...”                               |
| 1922: Taylor  | $U_s = 0!$   |                                                               |



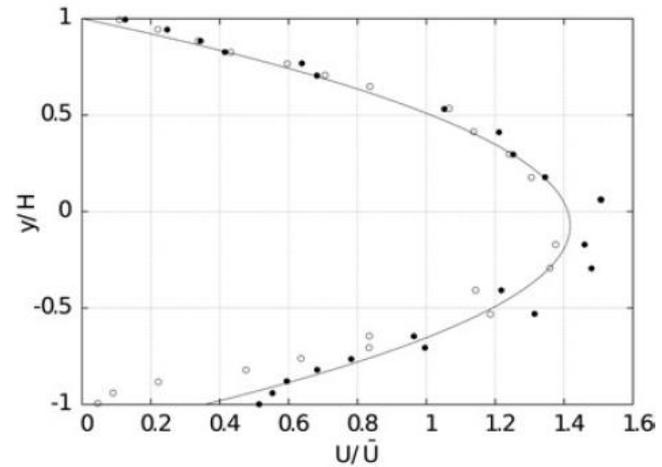
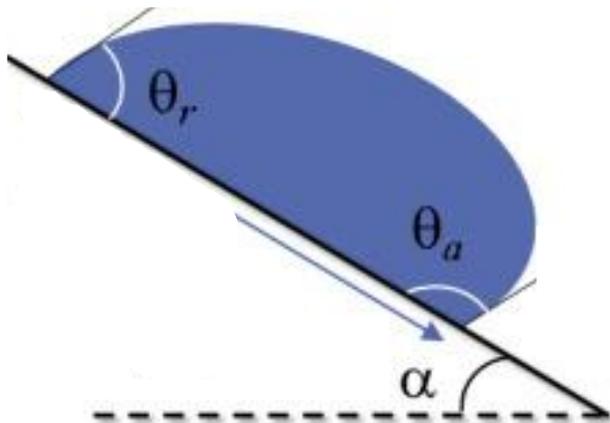
## Later on ...

|               |              |                                                            |
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| 1922: Taylor  | $U_s = 0!$   | <b>CASE CLOSED?</b>                                        |

But what happens near **superhydrophobic** surfaces?



And at the **triple line** of a drop sliding down an incline?



## An excursion into the microscopic world



The **kinetic theory** of gases:

- 1738: Daniel Bernoulli (1700-1782)
- 1857: Rudolf Julius Emanuel Clausius (1822-1888)
- 1867: James Clerk Maxwell (1831-1879)
- 1871: Ludwig Eduard Boltzmann (1844-1906)



## On Stresses in Rarified Gases Arising from Inequalities of Temperature

J. Clerk Maxwell

*Phil. Trans. R. Soc. Lond.* 1879 **170**, 231-256, published 1 January 1879

### APPENDIX.

(Added May, 1879.)

In the paper as sent in to the Royal Society, I made no attempt to express the conditions which must be satisfied by a gas in contact with a solid body, for I thought it very unlikely that any equations I could write down would be a satisfactory representation of the actual conditions, especially as it is almost certain that the stratum of gas nearest to a solid body is in a very different condition from the rest of the gas.

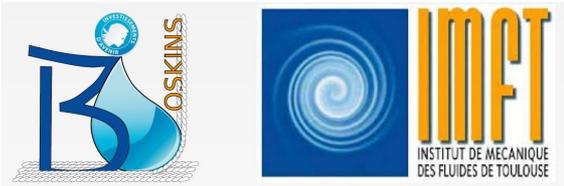
One of the referees, however, pointed out that it was desirable to make the attempt, and indicated several hypothetical forms of surfaces which might be tried. I have therefore added the following calculations, which are carried to the same degree of approximation as those for the interior of the gas.

It will be seen that the equations I have arrived at express both the fact that the gas may slide over the surface with a finite velocity, the previous investigations of which have been already mentioned;\* and the fact that this velocity and the corresponding tangential stress are affected by inequalities of temperature at the surface of the solid, which give rise to a force tending to make the gas slide along the surface from colder to hotter places.



A fraction  $f$  of the molecules impinging on a wall leave it (“*absorbed and then evaporated*” → **diffuse reflexion**), the remaining  $(1 - f)$  are **specularly reflected**. Evaluating mass and momentum of the gas before and after the collision, Maxwell concludes that ...

[ $f$  is the *tangential momentum accommodation coefficient* (**TMAC**), now usually denoted as  $\sigma_v$ ; **TMAC** describes the effective gas-surface interactions]



If there is no inequality of temperature, this equation is reduced to

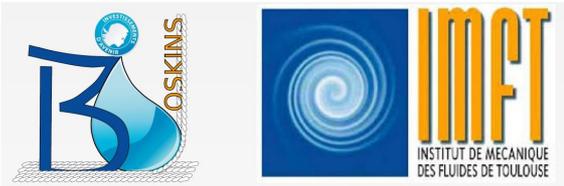
$$v = G \frac{dv}{dx} \dots \dots \dots (69)$$

If, therefore, the gas at a finite distance from the surface is moving parallel to the surface, the gas in contact with the surface will be sliding over it with the finite velocity  $v$ , and the motion of the gas will be very nearly the same as if the stratum of depth  $G$  had been removed from the solid and filled with the gas, there being now no slipping between the new surface of the solid and the gas in contact with it.

The coefficient  $G$  was introduced by HELMHOLTZ and PIOTROWSKI under the name of *Gleitungs-coefficient*, or coefficient of slipping. The dimensions of  $G$  are those of a line, and its ratio to  $l$ , the mean free path of a molecule, is given by the equation

$$G = \frac{2}{3} \left( \frac{2}{f} - 1 \right) l \dots \dots \dots (70)$$

KUNDT and WARBURG found that for air in contact with glass,  $G = 2l$ , whence we find  $f = \frac{1}{2}$ , or the surface acts as if it were half perfectly reflecting and half perfectly absorbent. If it were wholly absorbent,  $G = \frac{2}{3}l$ .



## Later criticisms:

- Approach not valid for the transition and free molecular regime
- Singularity in the absence of diffuse reflections (i.e. when the surface is atomically smooth)
- Neglect of inelastic scattering
- **TMAC** should be determined by the characteristics of both the wall and the incident molecule at the location of impact and not be a simple constant



## A new model for fluid velocity slip on a solid surface

Cite this: *Soft Matter*, 2016, 12, 8388

Jian-Jun Shu,\* Ji Bin Melvin Teo and Weng Kong Chan

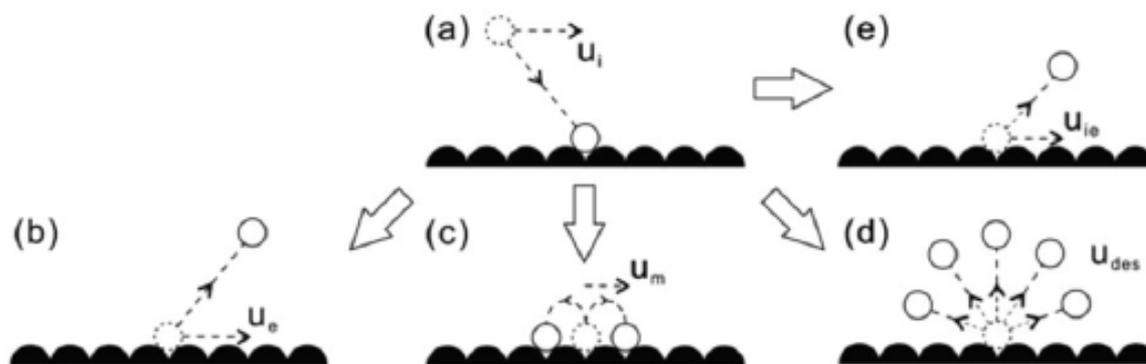


Fig. 1 Molecular interaction at a fluid–solid interface: (a) incident molecule, (b) elastic scattering, (c) surface hopping, (d) desorption and (e) inelastic scattering.



## Soft Matter

PAPER

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### A new model for fluid velocity slip on a solid surface

Cite this: *Soft Matter*, 2016, 12, 8388

Jian-Jun Shu,\* Ji Bin Melvin Teo and Weng Kong Chan

$$u_s = C_1 \dot{\gamma}_s - C_2 + \sqrt{\frac{C_2(C_1 + \lambda)^2}{C_2 + \mu_u \delta} \dot{\gamma}_s^2 - 2C_2(C_1 + \lambda) \dot{\gamma}_s + C_2^2} \quad (34)$$

where it should be emphasised that the coefficients  $C_i$  ( $i = 1, 2$ )  $> 0$  are the representative of the interfacial conditions, adsorption probabilities and properties of the media as follows:

$$C_1 = \frac{1}{1 - p_e^2(1 - p_m)^2} \left[ \frac{1 - p_s}{p_s} \lambda + \frac{p_m u_h}{\dot{\gamma}_0} + p_e^2(1 - p_m)^2 \lambda \right]$$

$$C_2 = \frac{p_e^2(1 - p_m)^2}{1 - p_e^2(1 - p_m)^2} \mu_u \delta. \quad (35)$$

Eqn (34) is the main result for this paper and represents a new general slip velocity model for fluid–solid boundary conditions derived based on the theory of interfacial physics, specifically adsorption and desorption processes. The novelty of this model lies in its applicability to both gas and liquid flows, which has thus far been studied independently in analytical models to the best of our knowledge. Furthermore, the slip velocity expression exhibits non-linearity with respect to the wall shear rate which is in accordance with the prediction of experimental measurements where such phenomena have been observed.

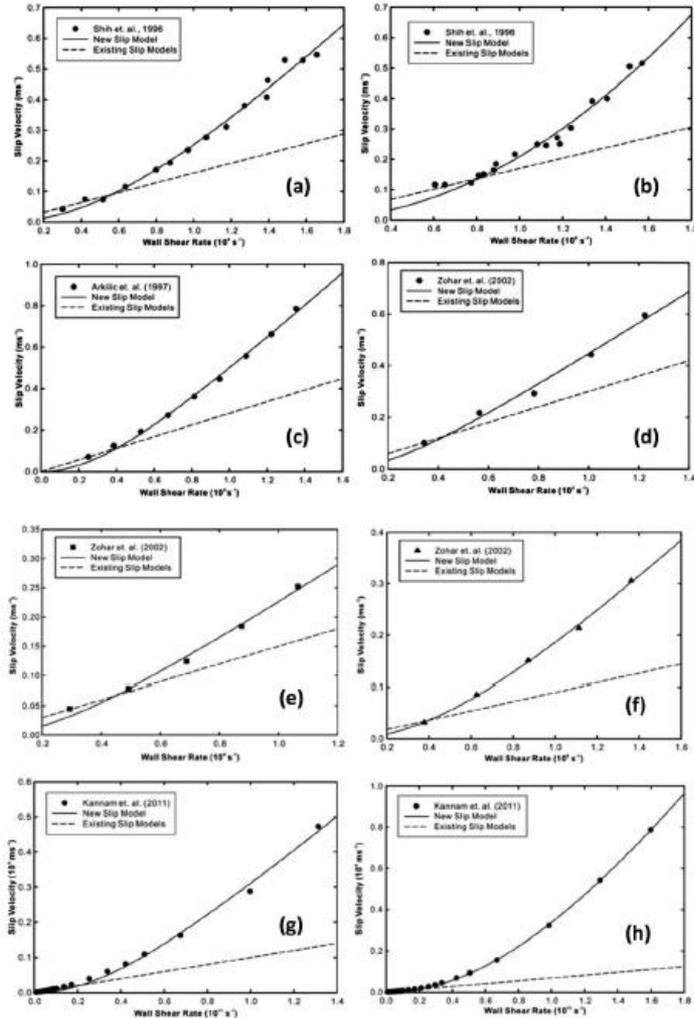


Fig. 2 Comparison of the new (solid lines) and existing (dashed lines) slip models for gas–solid interfaces using experimental results (symbols) from the literature. The fitting coefficients are summarised in Table 1.

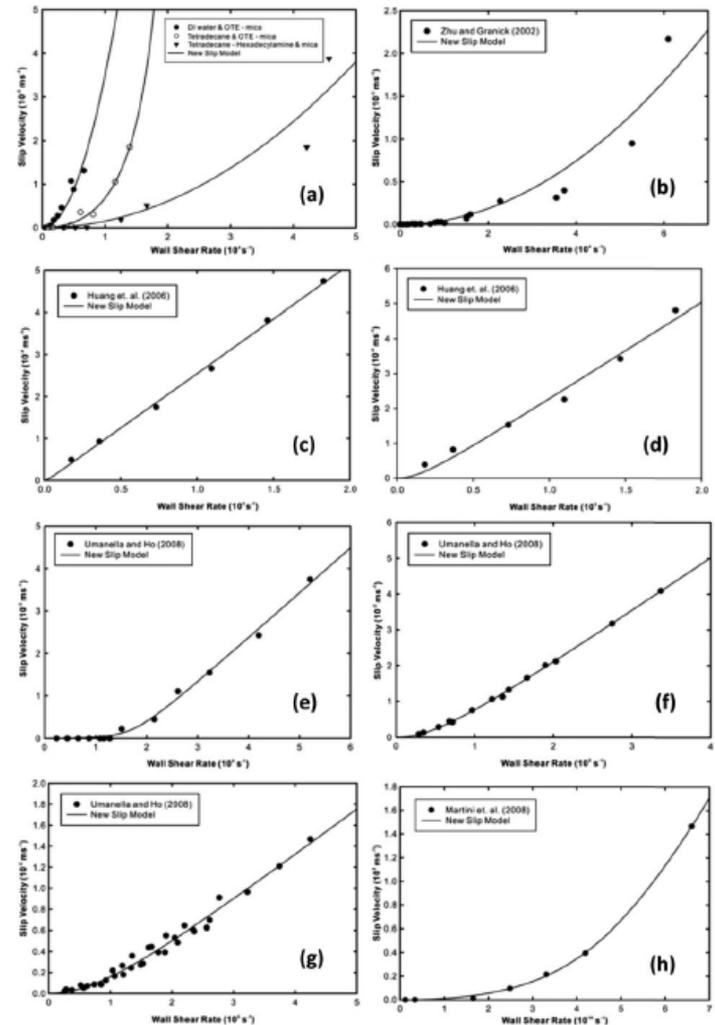
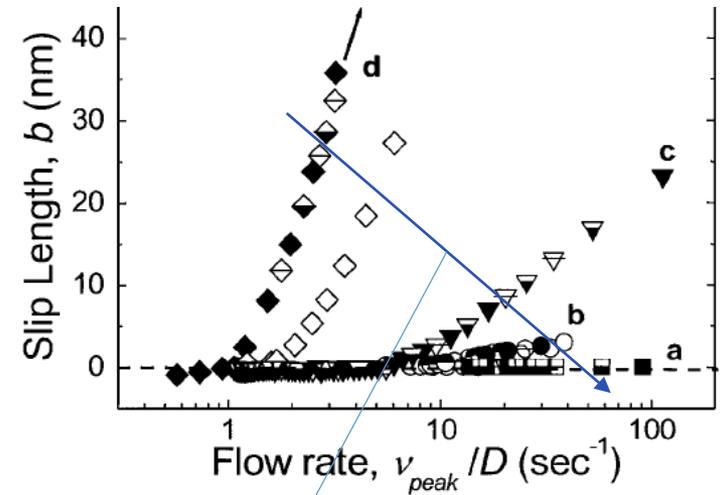


Fig. 3 Validation of the new slip model (solid lines) for liquid–solid interfaces using experimental results (symbols) from the literature. The fitting coefficients are summarised in Table 2.



## True slip also in liquids?

There are two schools of microscopic explanation. The traditional explanation is that since most surfaces are rough, the viscous dissipation as fluid flows past surface irregularities brings it to rest, regardless of how weakly or strongly molecules are attracted to the surface [2–4]. This has been challenged by accumulating evidence that, if molecularly smooth surfaces are wet only partially, hydrodynamic models work better when one uses instead “partial slip” boundary conditions [5–14]. Then the main issue is whether fluid molecules attract the surface or the fluid more strongly [5–12,15]. In this study we tested the limits of both ideas. To the best of our knowledge, this is the first experimental study in which roughness was varied systematically at the nanometer level.



Increasing roughness  
(rms roughness of case a: 6 nm)

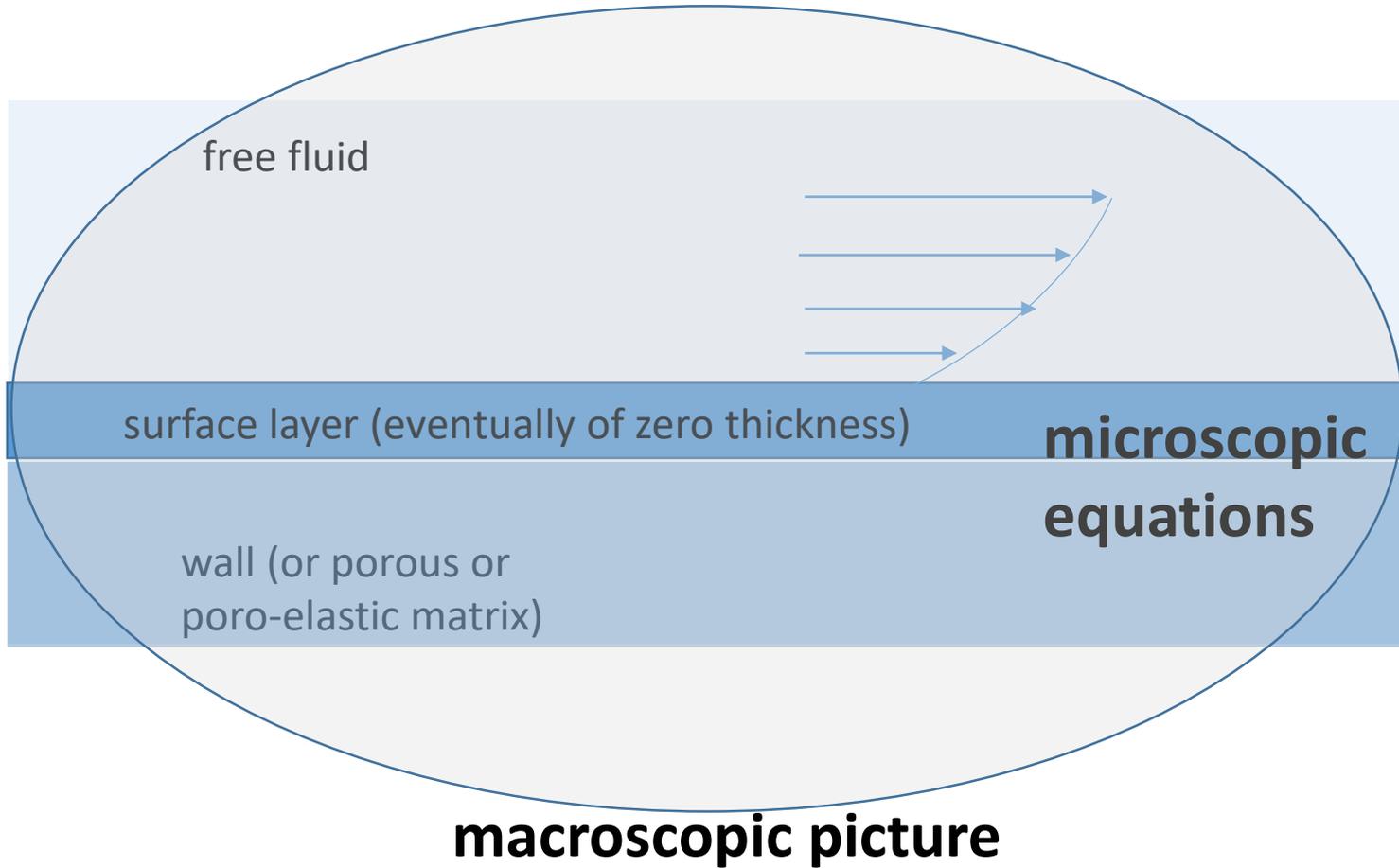
### Limits of the Hydrodynamic No-Slip Boundary Condition

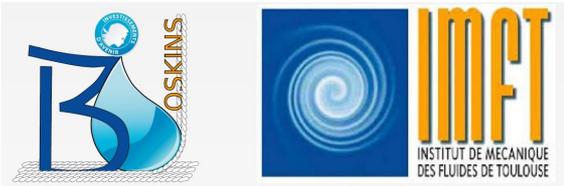


A possible mechanism for rate dependence was proposed by de Gennes [19]. He conceives that shear induces the nucleation of vapor bubbles; once the nucleation barrier is exceeded they grow to cover the surface, and flow of liquid is over this thin vapor film rather than the surface itself. There is evidence to support this picture [20].



**Back to the macroscopic picture ...**





# FLUID FLOW ABOVE A POROUS LAYER

*J. Fluid Mech.* (1967), vol. 30, part 1, pp. 197–207  
*Printed in Great Britain*

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## **Boundary conditions at a naturally permeable wall**

**By GORDON S. BEAVERS AND DANIEL D. JOSEPH**

Department of Aeronautics and Engineering Mechanics,  
University of Minnesota, Minneapolis, Minnesota

(Received 21 February 1967)

Later recovered theretically by P.G. Saffman (1971)  
and demonstrated valid at leading order (when the  
pore size tends to zero) by W. Jäger & A. Mikelić (1990)

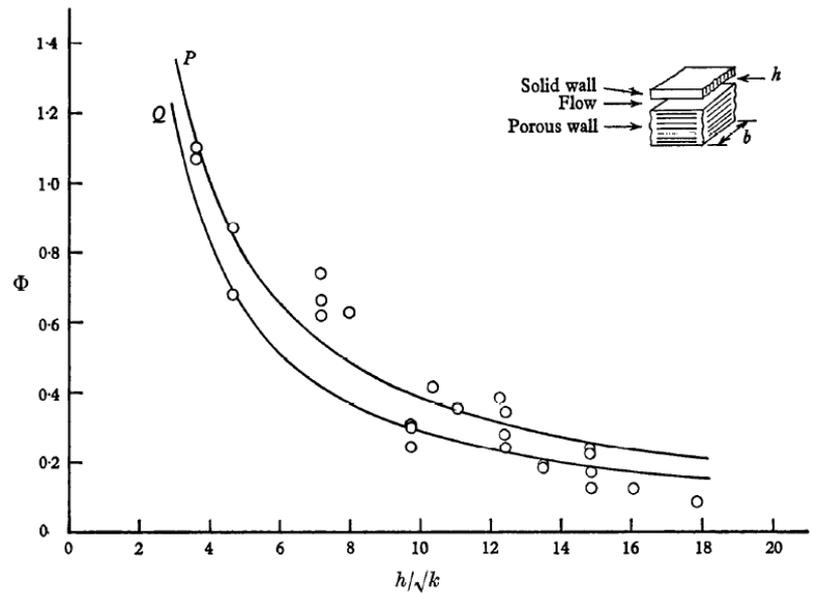
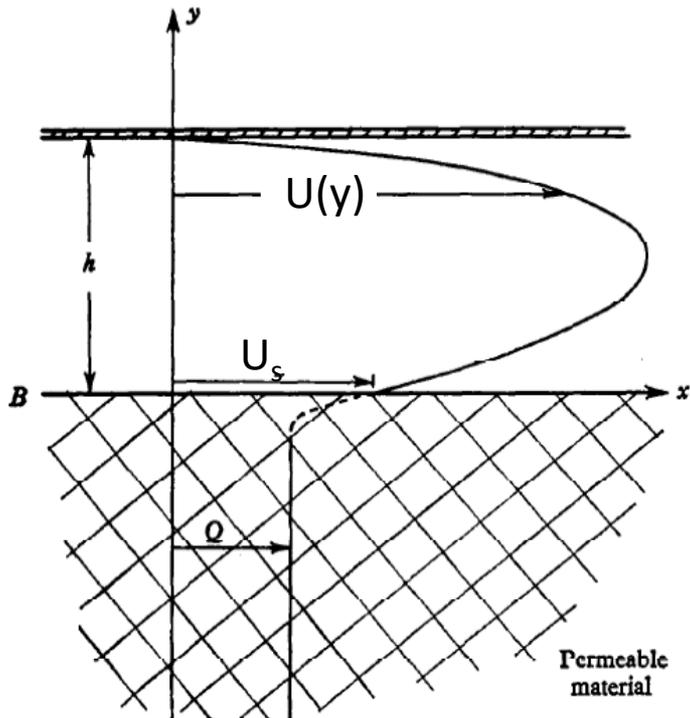


FIGURE 4.  $\Phi$  as a function of  $h/\sqrt{k}$  for foamed metal porous specimens using demineralized water.  
 $k = 1.1 \times 10^{-8} \text{ in.}^2$ . Curve P,  $\alpha = 0.8$ ; curve Q,  $\alpha = 1.2$ .

$$\left. \frac{dU}{dy} \right|_{y=0} = \frac{\alpha}{\sqrt{k}} (U_s - Q)$$

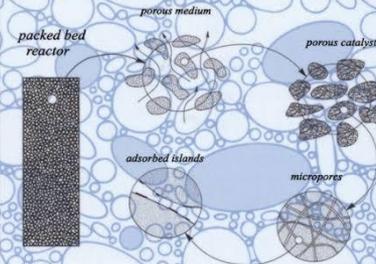
$\Phi$ : fractional increase in mass flow rate

$\frac{\sqrt{k}}{\alpha}$  is a "slip length"  $\lambda$



# THEORY AND APPLICATIONS OF TRANSPORT IN POROUS MEDIA

## The Method of Volume Averaging



Stephen Whitaker

# Most notable further developments by volume averaging:



*Int. J. Heat Mass Transfer.* Vol. 38, No. 14, pp. 2635-2646, 1995  
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 0017-9310/95 \$9.50+0.00

0017-9310(94)00346-7

## Momentum transfer at the boundary between a porous medium and a homogeneous fluid—I. Theoretical development

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Departamento de Ingeniería Química, Instituto Tecnológico de Celaya, 38010 Celaya, Gto., México

and

STEPHEN WHITAKER†

Department of Chemical Engineering, University of California at Davis, Davis, CA 95616, U.S.A.

stress jump  
condition:

B.C. 1

$$\varepsilon_{\beta\omega}^{-1} \frac{\partial \langle v_{\beta} \rangle_{\omega}}{\partial y} - \frac{\partial \langle v_{\beta} \rangle_{\eta}}{\partial y} = \frac{\beta}{\sqrt{(K_{\beta\omega})}} \langle v_{\beta} \rangle_{\omega} \quad y = 0$$

(86)

B.C. 2

$$\langle v_{\beta} \rangle_{\omega} = \langle v_{\beta} \rangle_{\eta} \quad y = 0. \quad (87)$$

Here we have used  $\langle v_{\beta} \rangle_{\omega}$  and  $\langle v_{\beta} \rangle_{\eta}$  to represent the  $x$ -components of the two volume average velocity vectors and the dimensionless coefficient  $\beta$  is given by

$$\beta = \sqrt{(K_{\beta\omega})} [\delta^{-1} \lambda \cdot \mathbf{A} \cdot \lambda (\varepsilon_{\beta\omega} - 1)^2 (\varepsilon_{\beta\omega}^{-3} + 1) - \delta \lambda \cdot \mathbf{D} \cdot \mathbf{K}_{\beta\omega}^{-1} \cdot \lambda]. \quad (88)$$

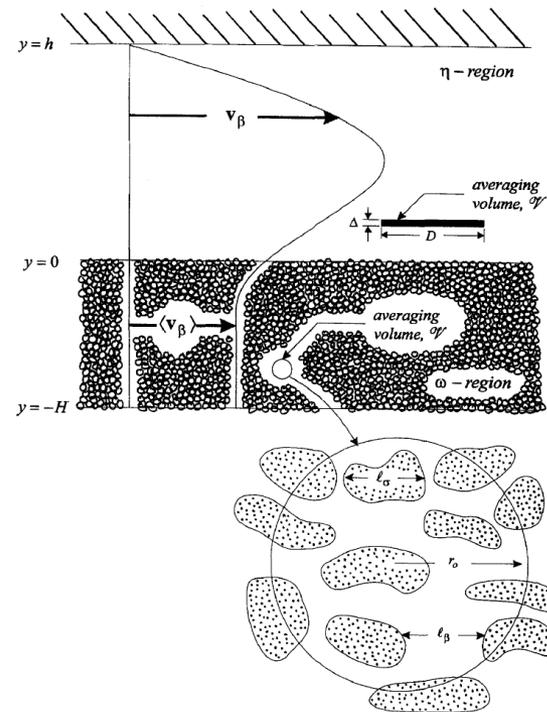
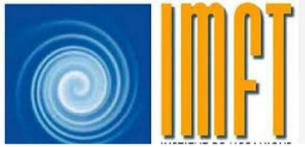


Fig. 1. Flow of a homogeneous fluid parallel to a porous medium.



## Most notable further developments by multiscale:

*J. Fluid Mech.* (2017), vol. 812, pp. 866–889. © Cambridge University Press 2017 866  
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doi:10.1017/jfm.2016.838

### A framework for computing effective boundary conditions at the interface between free fluid and a porous medium

Uģis Lācis<sup>1</sup> and Shervin Bagheri<sup>1,†</sup>

<sup>1</sup>Linné Flow Centre, Department of Mechanics KTH, SE-100 44 Stockholm, Sweden

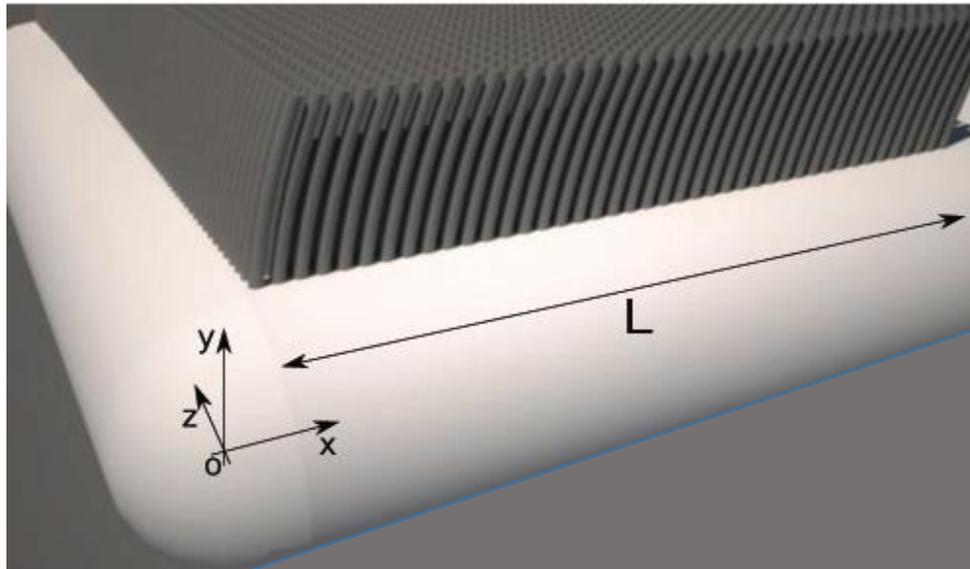
(Received 10 April 2016; revised 6 December 2016; accepted 7 December 2016; first published online 6 January 2017)

$$\hat{u}_s = -\frac{K_s}{\mu} \partial_x \hat{p}^- + L_s (\partial_y \hat{u} + \partial_x \hat{v})$$

$$\hat{v}_p = -\frac{K_{cyl}}{\mu} \partial_y \hat{p}^-$$

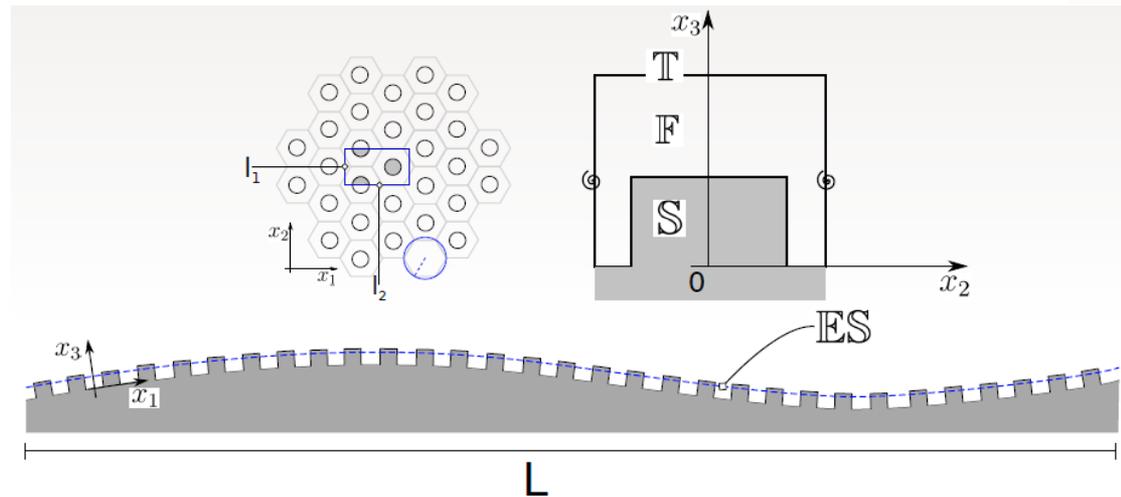
(free fluid: Stokes flow ...)

## FLUID FLOW OVER A POROELASTIC LAYER

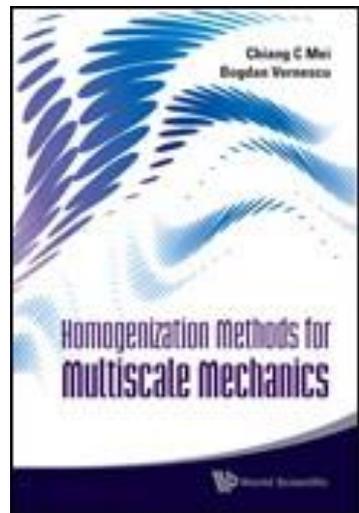


PROBLEM BEING PURSUED BY  
N. LUMINARI and M. PAUTHENET

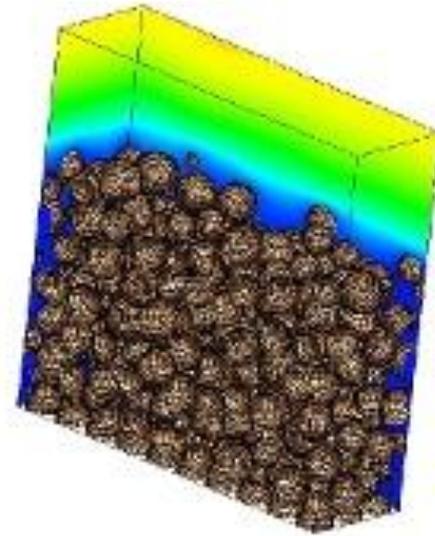
# FLUID FLOW OVER A SURFACE WITH (REGULAR) ROUGHNESS



PROBLEM BEING PURSUED  
BY G. ZAMPOGNA



## FLUID FLOW OVER A BED OF MOBILE GRAINS



Cambridge Monographs on Mechanics

### The Dynamics of Fluidized Particles

ROY JACKSON

**Continuous** nature at the macroscale,  
**discrete** at the microscale

Continuum approach with “surface layer” whose rheological behavior is given by IBM/DEM simulations

- lower layer: Darcy
- surface layer: ANS (volume averaging *a la* Jackson)
- upper layer: NS

PROBLEM BEING PURSUED BY B. FRY



Institut de Mécanique des Fluides

Amphithéâtre Nougaro (Entrée A) - 2 Allée du Pr Camille Soula, Toulouse



# « BIOSKINS »

Mise en scène : A. Bottaro, IMFT & Università di Genova

14 septembre 2017 à 9 h 30

Amphithéâtre Nougaro



### Acteurs principaux:

Martin Pauthenet, GEMP :

«BIOSKINS : Au-delà de Darcy pour les écoulements inertiels en milieux poreux déformables»

Nicola Luminari, EMT2 :

«BIOSKINS : Métamodèles pour systèmes de fibres élastiques»

Giuseppe Zampogna, HEGIE :

«BIOSKINS : L'analogie macroscopique»

Benjamin Fry, HEGIE :

«BIOSKINS : Grains et poro-plasticité»

*Avec l'aimable participation de :*

*C. Airiau, T. Bonometti, F. Charru, Y. Davit, L. Lacaze, J. Magnaudet et M. Quintard*

## Du très petit au macroscopique : Réflexions autour de l'adhérence

Depuis le travail théorique de G.I. Taylor sur l'instabilité de Taylor-Couette (1923), la condition de non-glissement (ou d'adhérence) a été un principe clé pour décrire les écoulements visqueux près de parois solides. La plupart des frontières dans les configurations naturelles sont cependant irrégulières, à l'échelle microscopique ou macroscopique, poreuses et/ou compliantes, et les conditions aux limites à utiliser ne sont pas nécessairement évidentes. Pour de nombreux cas, une approximation acceptable consiste en une condition de glissement de Navier (proposée un siècle avant le travail de Taylor), mais dans plusieurs circonstances cela ne suffit pas. Une solution possible pour dériver des conditions fiables aux frontières et aux interfaces repose sur la théorie de l'homogénéisation ...



Projet mené dans le  
cadre d'une chaire  
d'attractivité

