INSTABILITY AND TRANSITION OF THE ROTATING DISK BOUNDARY LAYER OVER HOMOGENIZED TEXTURED SURFACES

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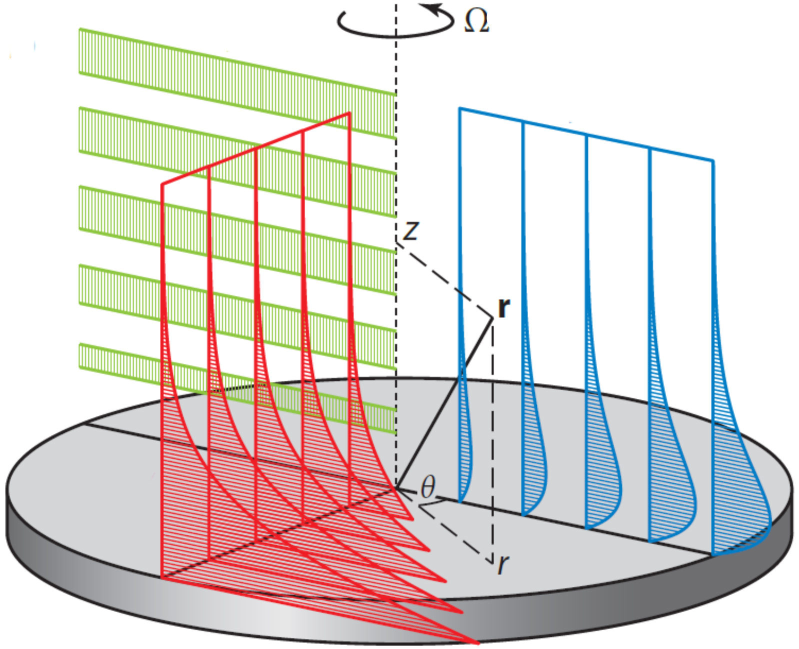

Alfredsson et al., ARFM 2024

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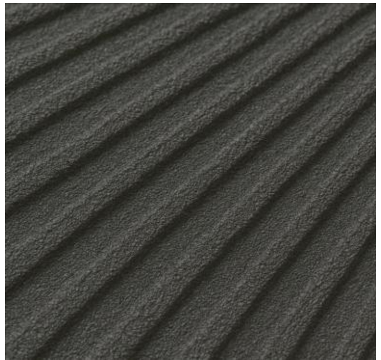
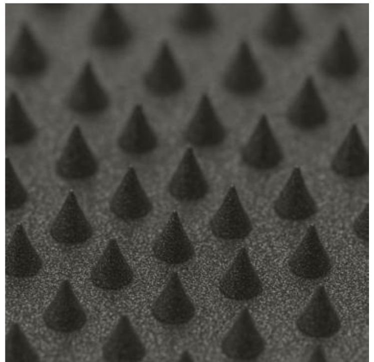
Euromech Colloquium 658, Bari, September 15-17, 2025

Textured disks

Circumferential | **Radial** | **Isotropic**



Homogenization theory

	Inner/micro-scales	Outer/macro-scales
Length	ℓ	$(\nu/\Omega)^{1/2}$
Velocity	$\Omega\ell$	$(\nu\,\Omega)^{1/2}$
Time	Ω^{-1}	Ω^{-1}
Pressure	$\mu\,\Omega$	$\mu\,\Omega$

von Kármán scales

To set ideas:

Imagine a disk rotating at $\Omega = 100$ rad/s in water, the macroscopic length scale is 0.1 mm, the laminar boundary layer is about 0.5 mm thick. The Reynolds number, $Re = \frac{\hat{r}}{\sqrt{\nu/\Omega}}$, attains the value of 500 when the radius is $\hat{r} = 5$ cm.

ℓ: characteristic dimension of the wall texture (periodicity of the pattern)

Expand all terms in powers of $\epsilon = \frac{\ell}{\sqrt{\nu/\Omega}}$ and solve the Stokes equations at different orders of ϵ in a unit cell for each of the textured disks consiered. Observe that, when $\epsilon = 1$, the laminar boundary layer thickness is ten times the height of the roughness.

To first order in ϵ the solutions in the unit cells yield the slip lengths, λ_Θ and λ_R, of the **effective wall conditions**

Effective conditions of the *macroscopic* problem

$$U_R|_{Z=0} = \epsilon\,\lambda_R\;\; S_{RZ}|_{Z=0} + \mathcal{O}(\epsilon^2)$$

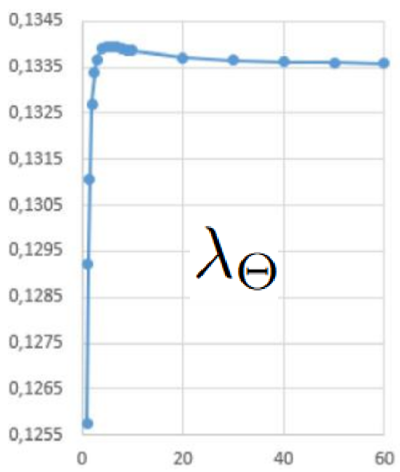
$$U_\Theta|_{Z=0} = R + \epsilon\,\lambda_\Theta\;\; S_{\Theta Z}|_{Z=0} + \mathcal{O}(\epsilon^2)$$

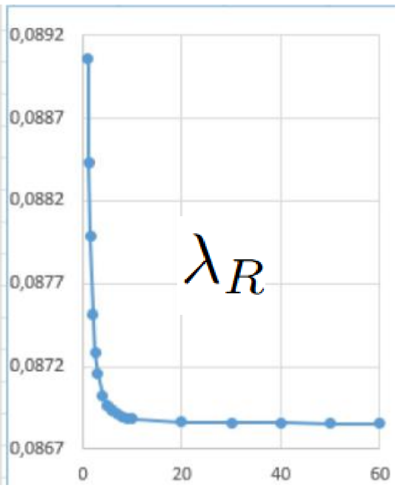
$$U_Z|_{Z=0} = \mathcal{O}(\epsilon^2)$$

$Z = 0$: fictitious surface where the effective wall conditions are applied

$$S_{RZ} = \left.\frac{\partial U_R}{\partial Z} + \frac{\partial U_Z}{\partial R}\right|_{Z=0},\; S_{\Theta Z} = \left.\frac{\partial U_\Theta}{\partial Z} + \frac{1}{R}\,\frac{\partial U_Z}{\partial \Theta}\right|_{Z=0},\; S_{ZZ} = -P + 2\left.\frac{\partial U_Z}{\partial Z}\right|_{Z=0}$$

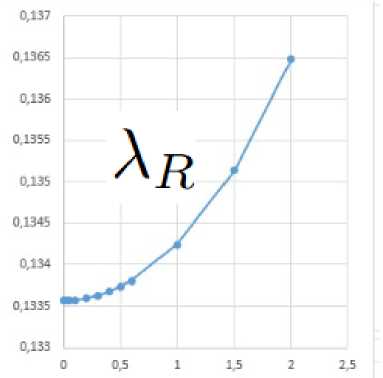
Circumferential ribs

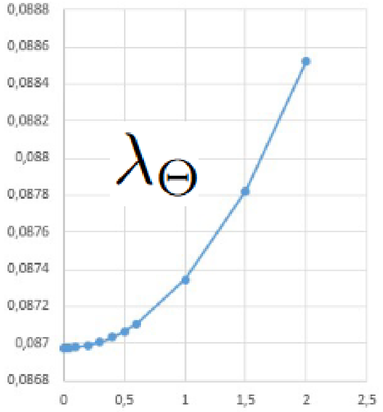




The coefficients vary with the radial position $\frac{\hat{r}}{\ell}$. Assuming $\epsilon = 1$ the radial position at which the slip lengths are independent of the radius is $R = Re \approx 20$.

Radial ribs

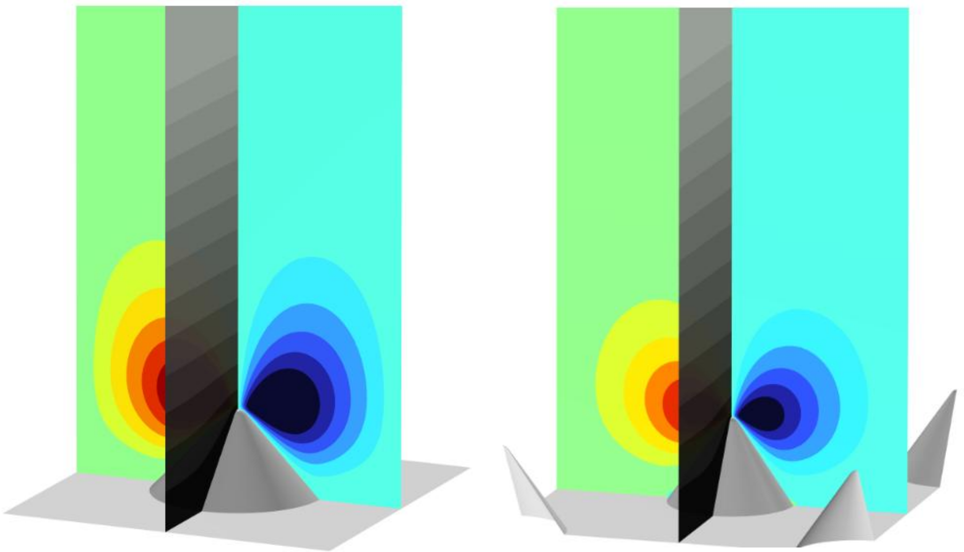




They vary with the angular opening of the ribs, $\Delta\theta$

Isotropic ribs

online/staggered



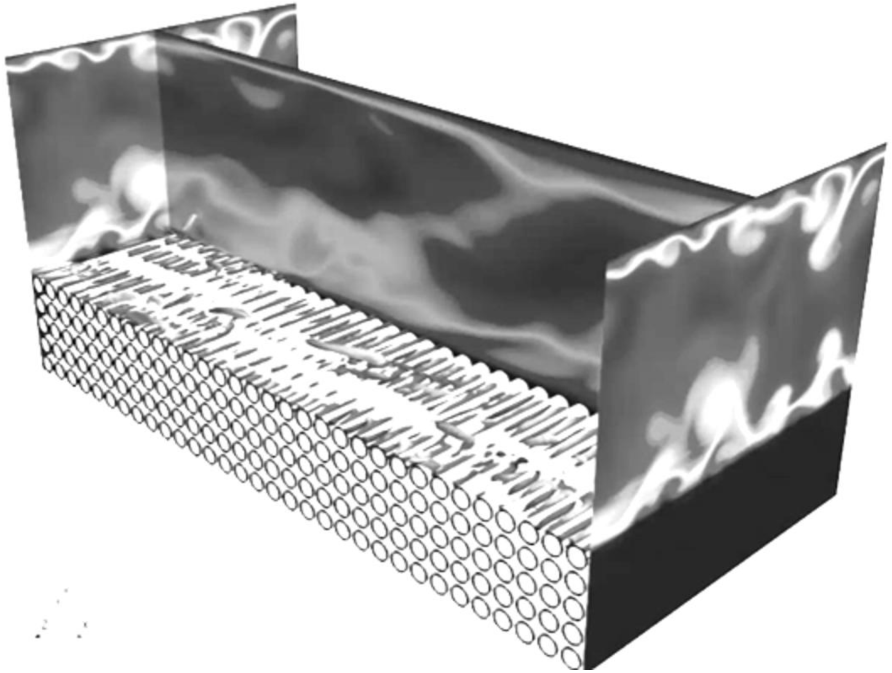
	λ_Θ	λ_R
Conical roughness (inline arrangement)	0.17839	0.17839
Conical roughness (staggered arrangement)	0.15880	0.15880

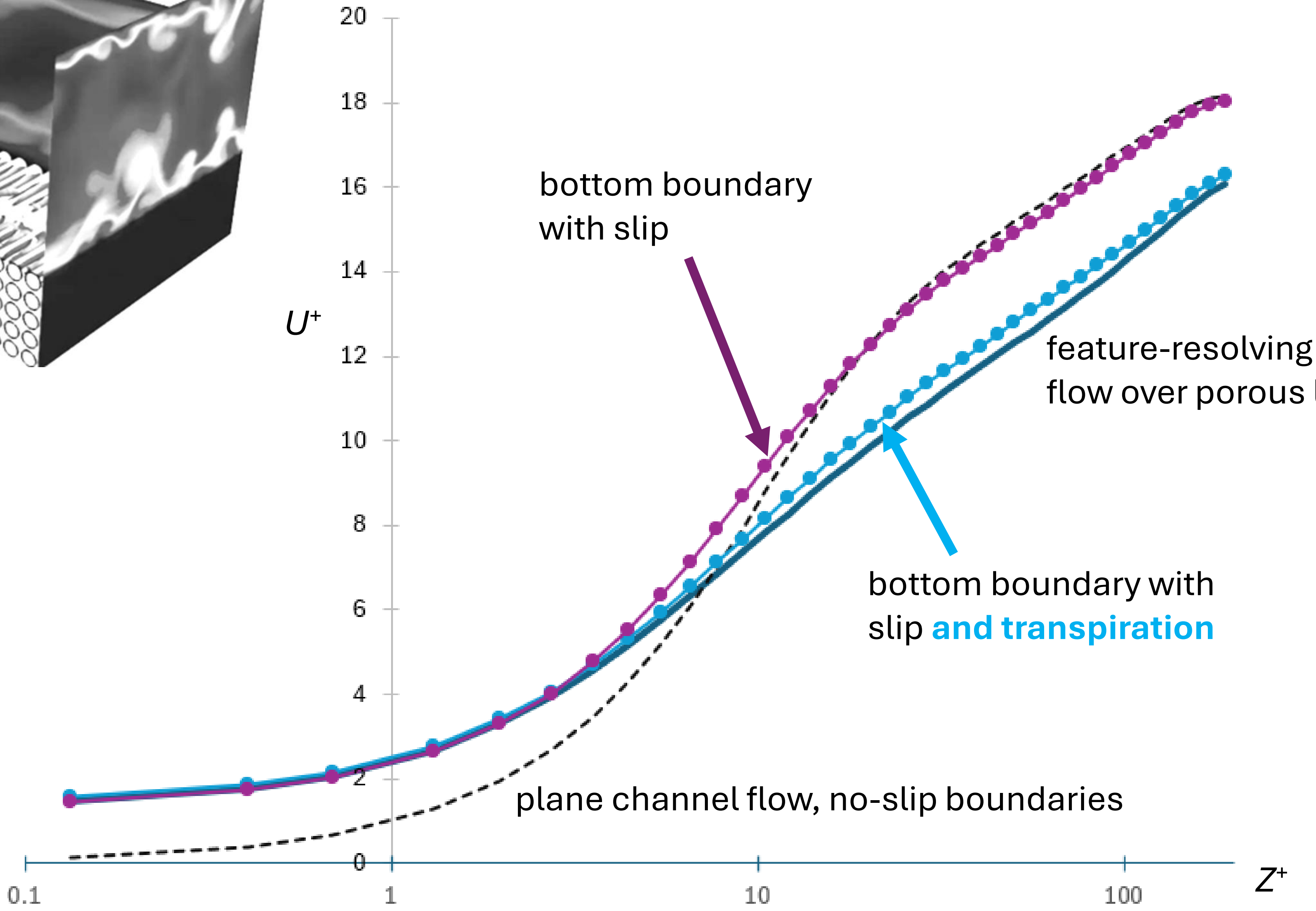
Are the boundary conditions at leading order in ϵ sufficient?

Are the boundary conditions at leading order in ϵ sufficient?

The answer is **yes**, if the flow is laminar,

but **no** after transition to turbulence has occurred





Ahmed & Bottaro
JFM 2025

THE TRANSPIRATION BOUNDARY CONDITION IN THE DISK CASE

$$U_Z|_{Z=0} = -\epsilon^2 \left[\mathcal{K}^{itf}_{\Theta Z} \frac{\partial}{\partial Z} \left(\frac{1}{R} \frac{\partial U_\Theta}{\partial \Theta} + \frac{U_R}{R} \right) + \mathcal{K}^{itf}_{RZ} \frac{\partial^2 U_R}{\partial Z \, \partial R} \right] \Bigg|_{Z=0} + \mathcal{O}(\epsilon^3)$$

For isotropic roughness elements the condition can be simplified to read

$$U_Z|_{Z=0} = \epsilon \, \lambda_Z \frac{\partial U_Z}{\partial Z} \Bigg|_{Z=0} + \mathcal{O}(\epsilon^3),$$

with λ_Z a small, formally $O(\epsilon)$, wall-normal protrusion height.

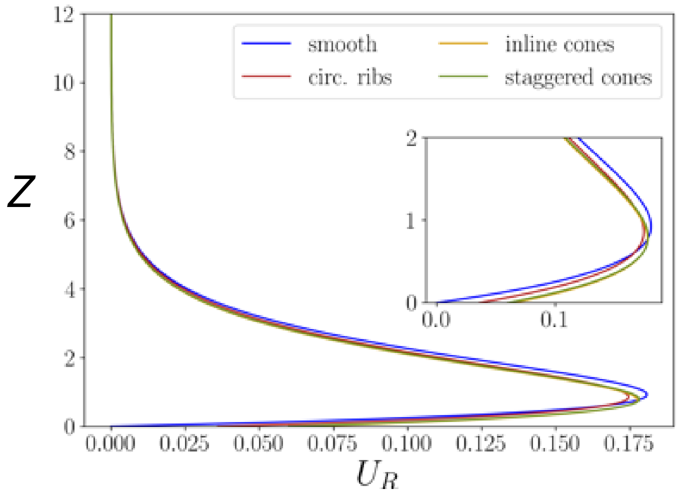
Back to the disk …

Laminar base flow

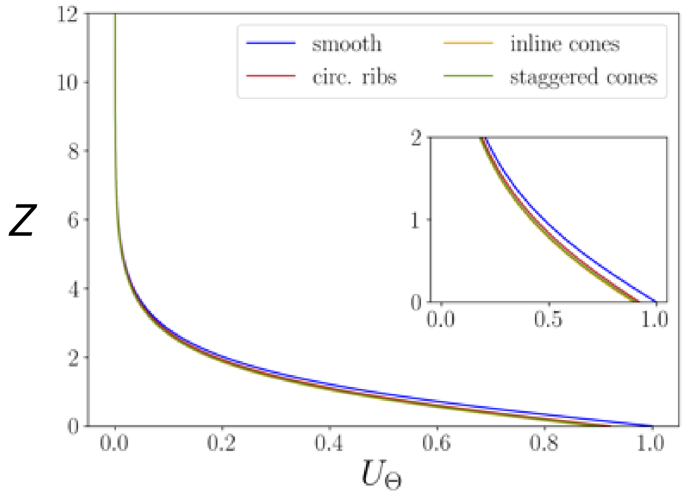
von Kármán solution

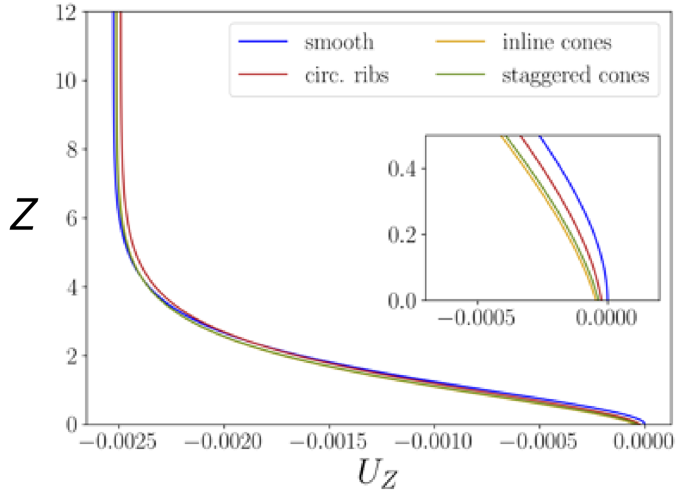
(with and without

slip/transpiration)



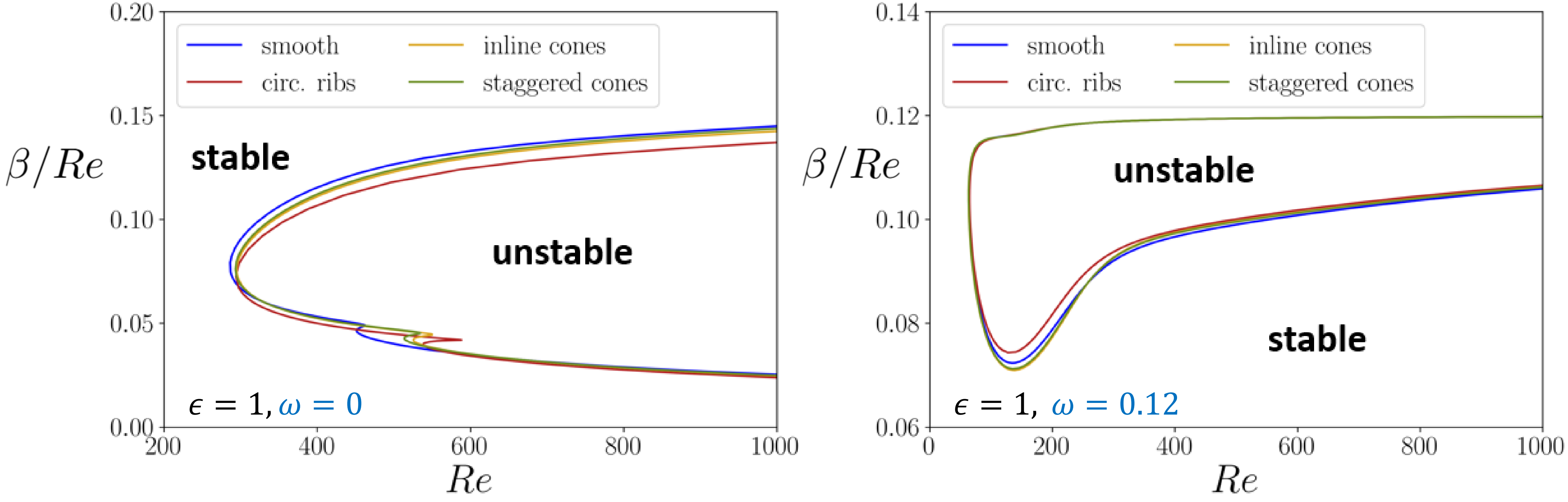
Laminar solution for $\epsilon = 1$ and $R = 350$





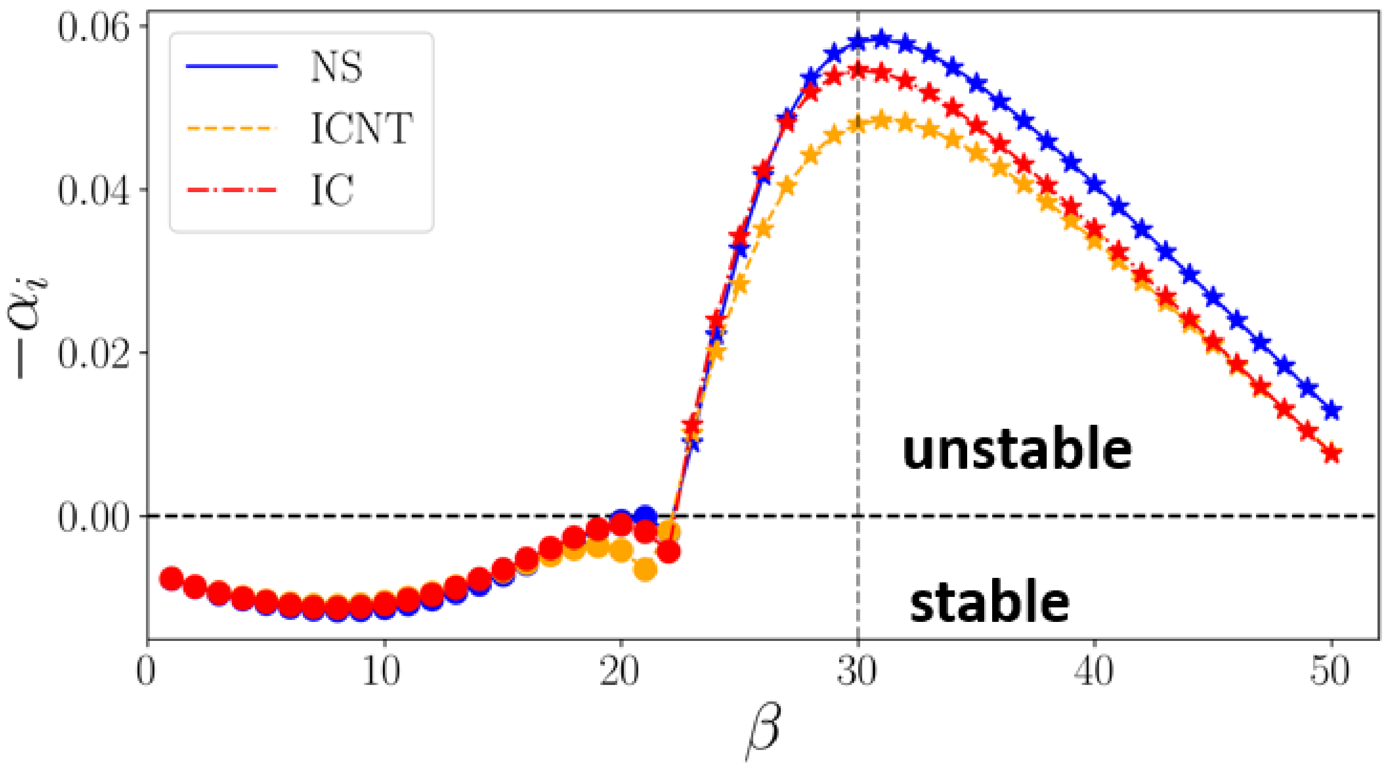
Local linear stability

Take small disturbances of the form: $f(\hat{z})\,\exp[i(\alpha\hat{r} + \beta\theta + \omega\hat{t})]$ and search for complex eigenvalues α



Neutral curves for textured surfaces with and without transpiration are overlapped

… but at $Re = 450$ the growth rates differ!

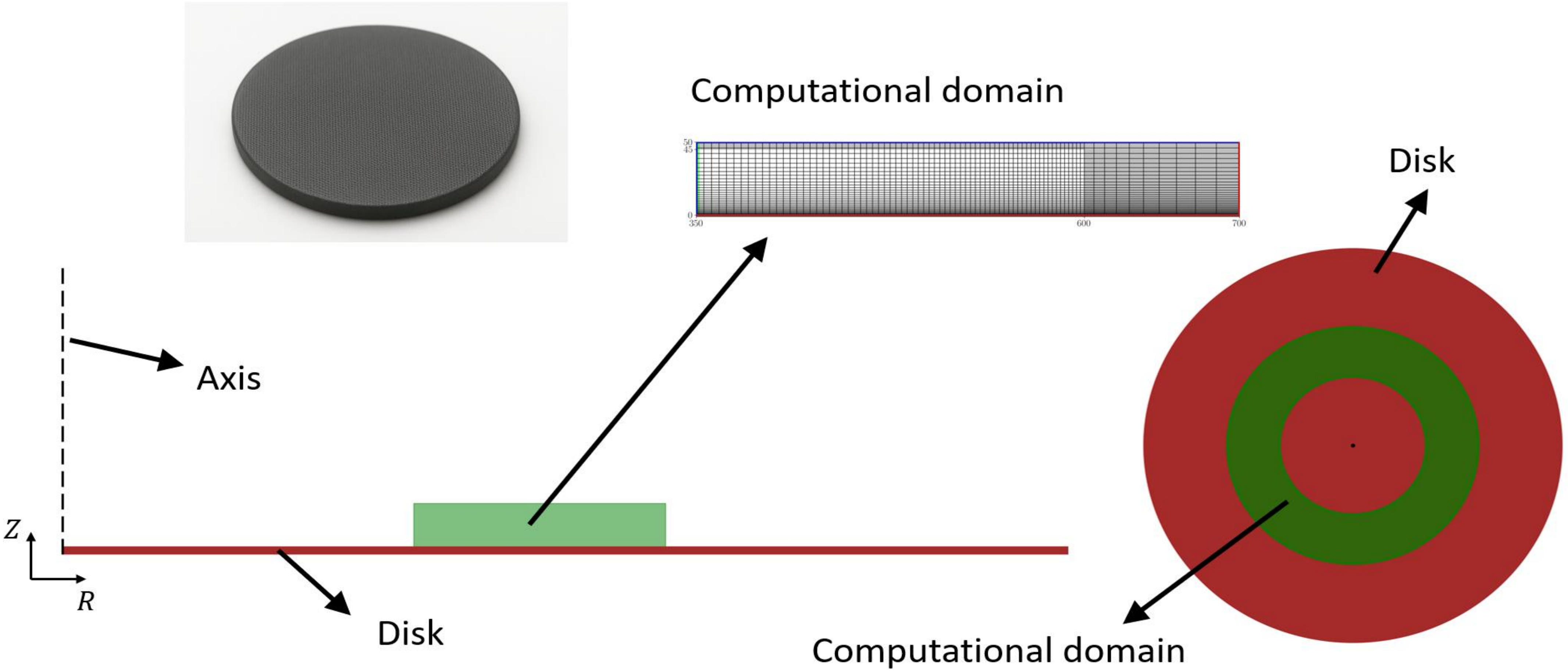


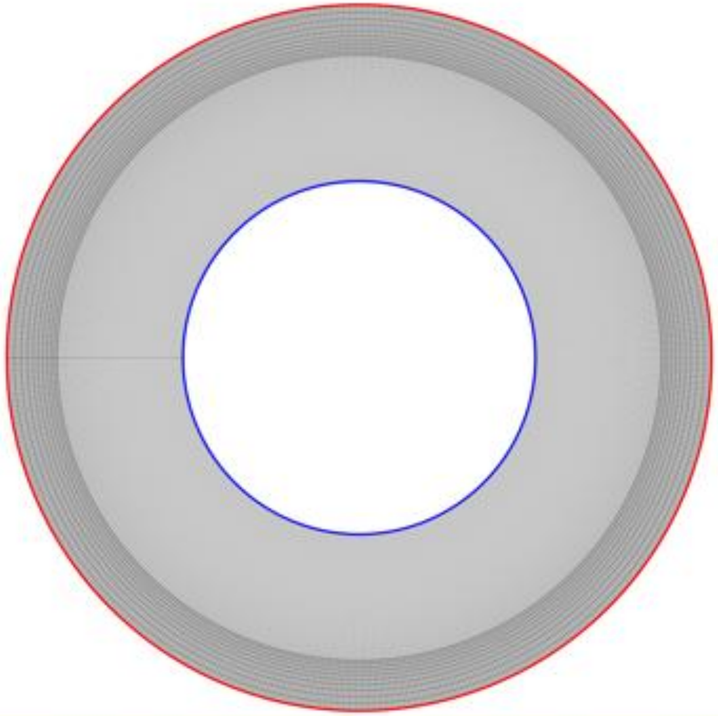
NS = no-slip

ICNT = inline cones, no transpiration

IC = inline cones, w/ transpiration

DNS

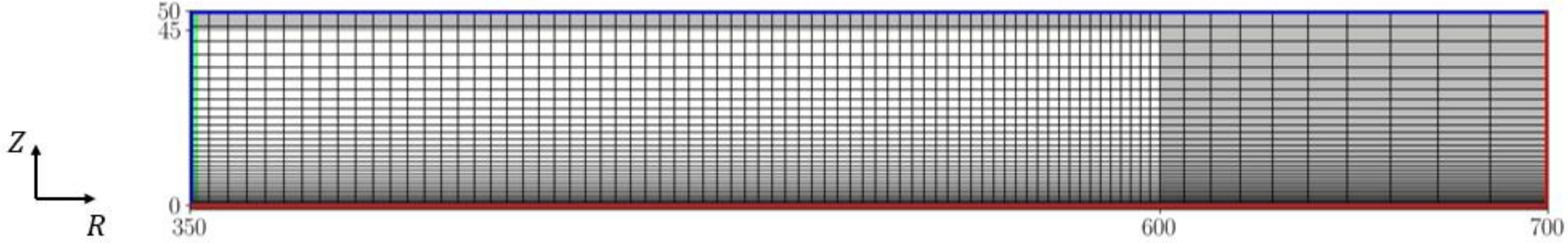

Computational domain
50
45
0
350
600
700
Disk
Axis
Z
R
Disk
Computational domain



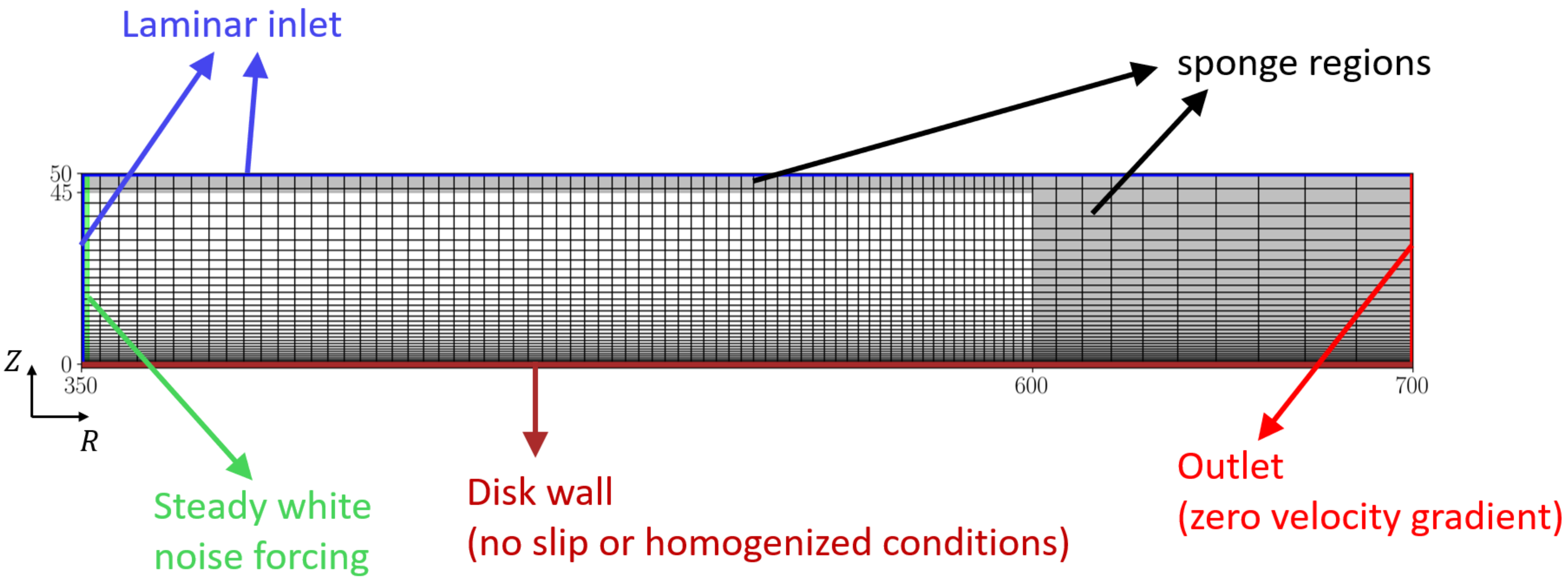
Nek5000 spectral element code
(7^{th} order Legendre polynomials)

$\approx 2 \cdot 10^6$ spectral elements

$\approx 10^9$ grid points

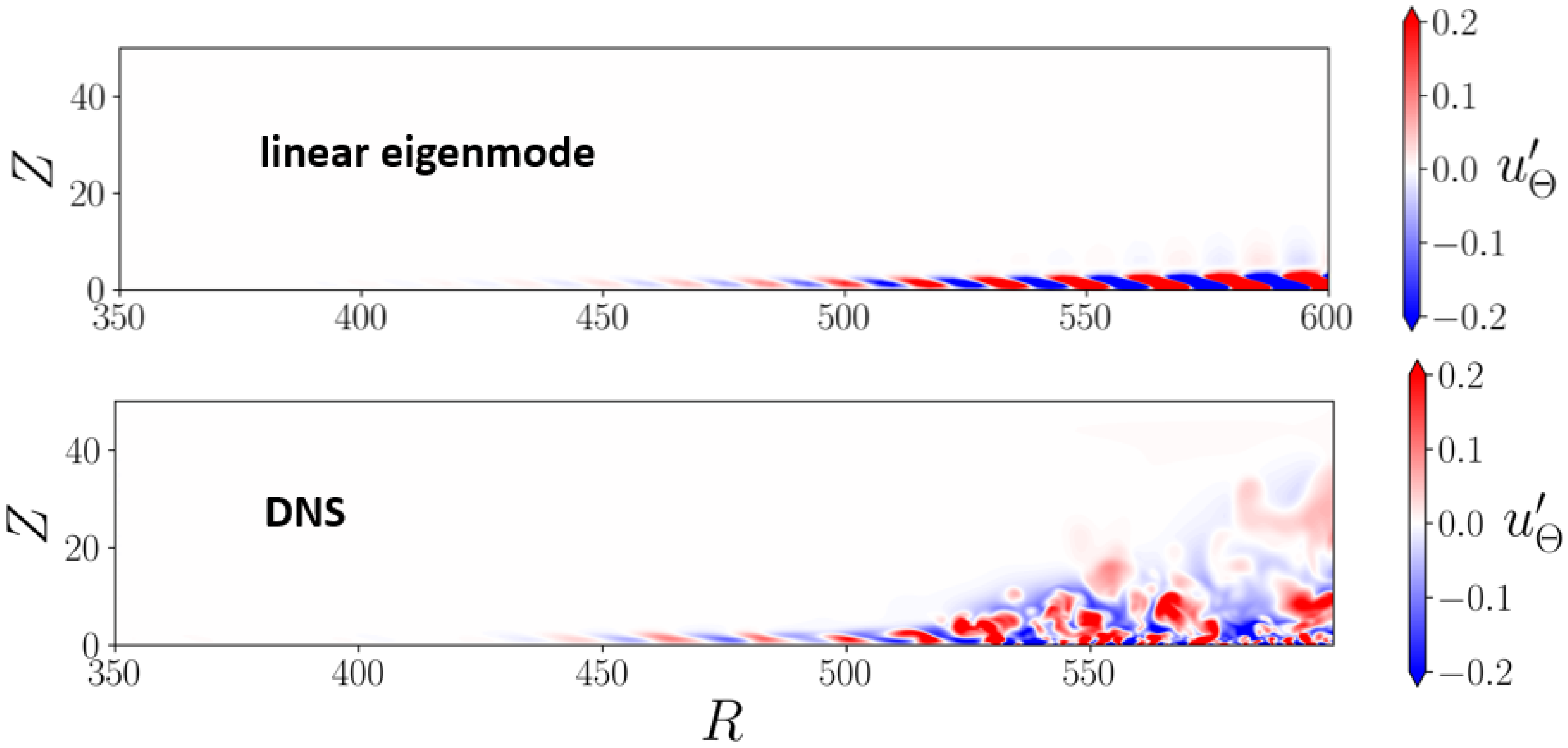


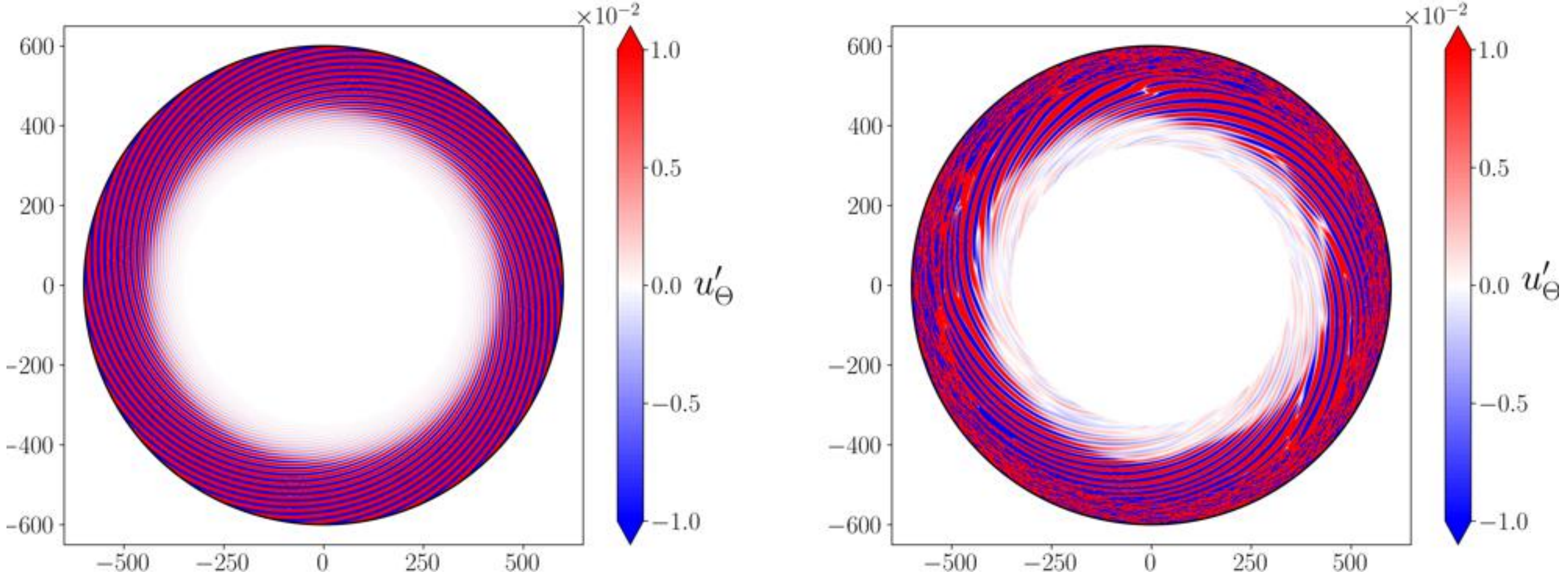
Fischer et al., *2008,* http://nek5000.mcs.anl.gov



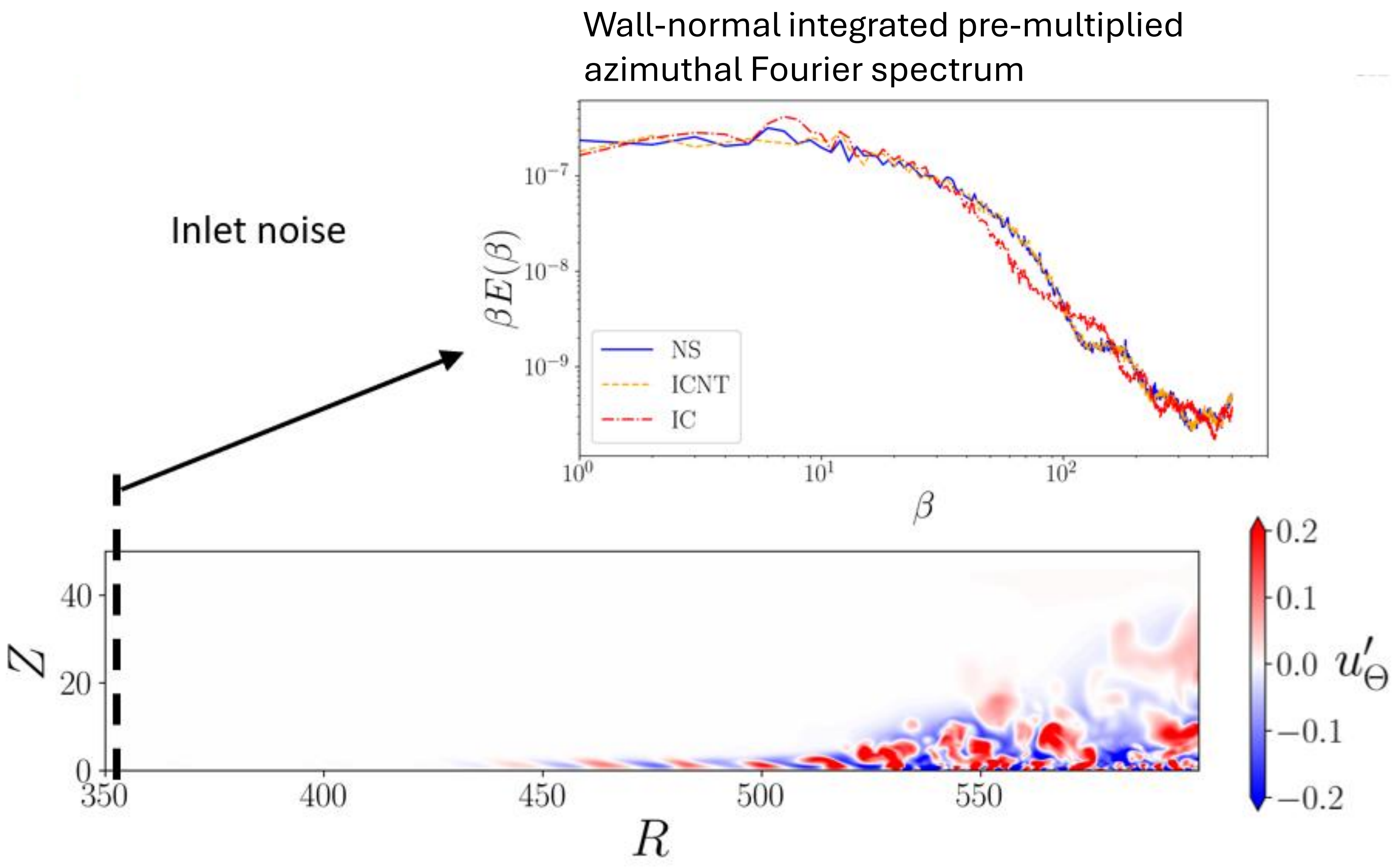
Laminar inlet
sponge regions
50
45
0
Z
R
350
600
700
Steady white
noise forcing
Disk wall
(no slip or homogenized conditions)
Outlet
(zero velocity gradient)

IC case: inline cones, with transpiration

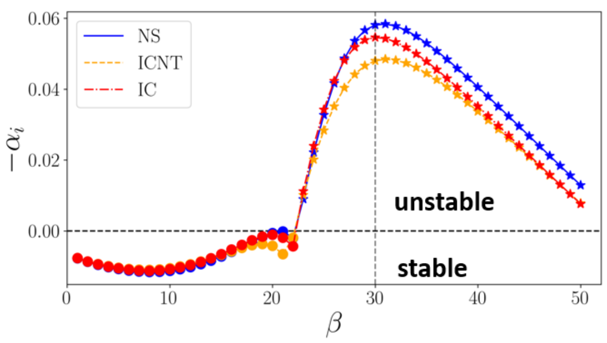




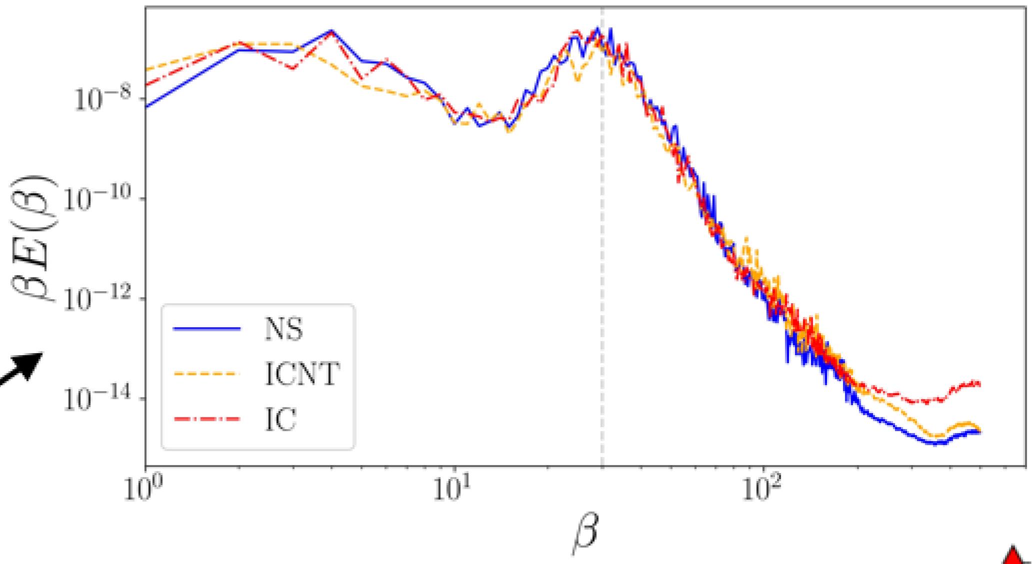
600
400
200
0
−200
−400
−600
−500
−250
0
250
500
×10^−2
1.0
0.5
0.0
−0.5
−1.0
u'_Θ

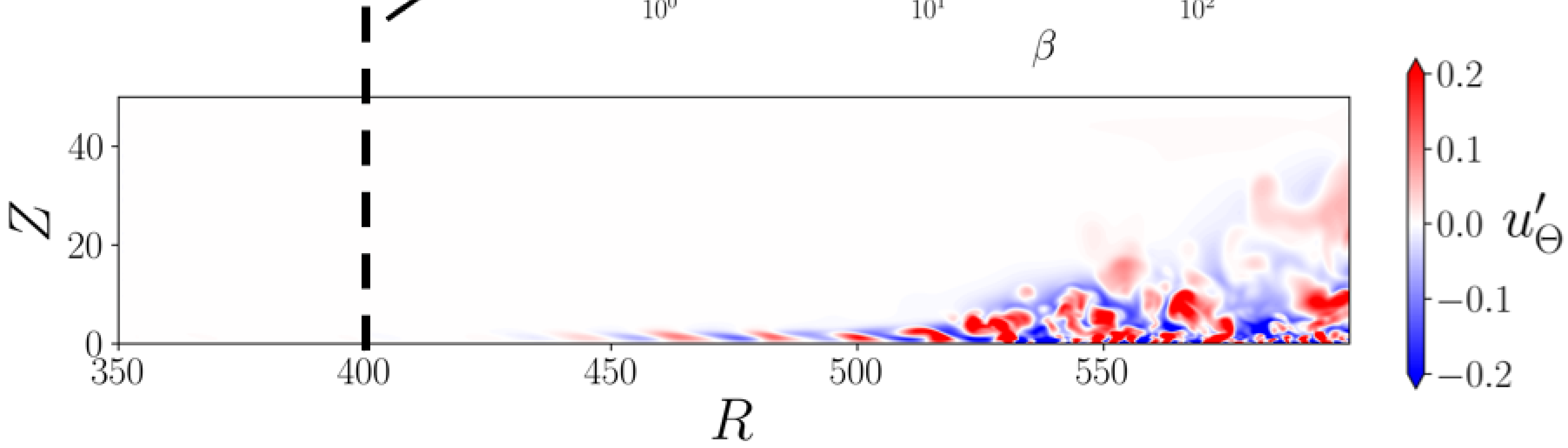


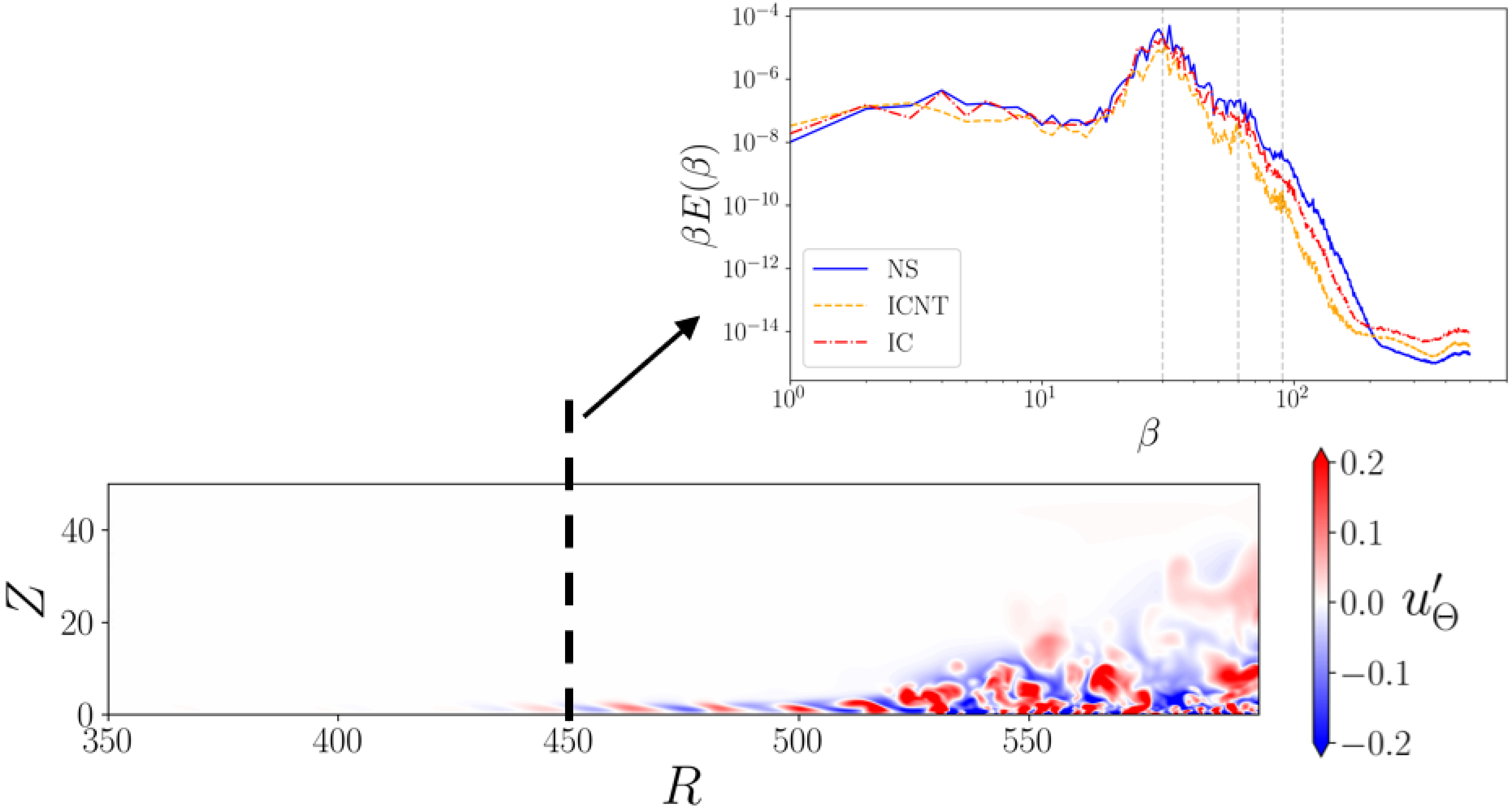
Wall-normal integrated pre-multiplied azimuthal Fourier spectrum
Inlet noise
$\beta E(\beta)$
10^{-7}
10^{-8}
10^{-9}
NS
ICNT
IC
10^{0}
10^{1}
10^{2}
β
Z
40
20
0
350
400
450
500
550
R
0.2
0.1
0.0
−0.1
−0.2
u'_Θ



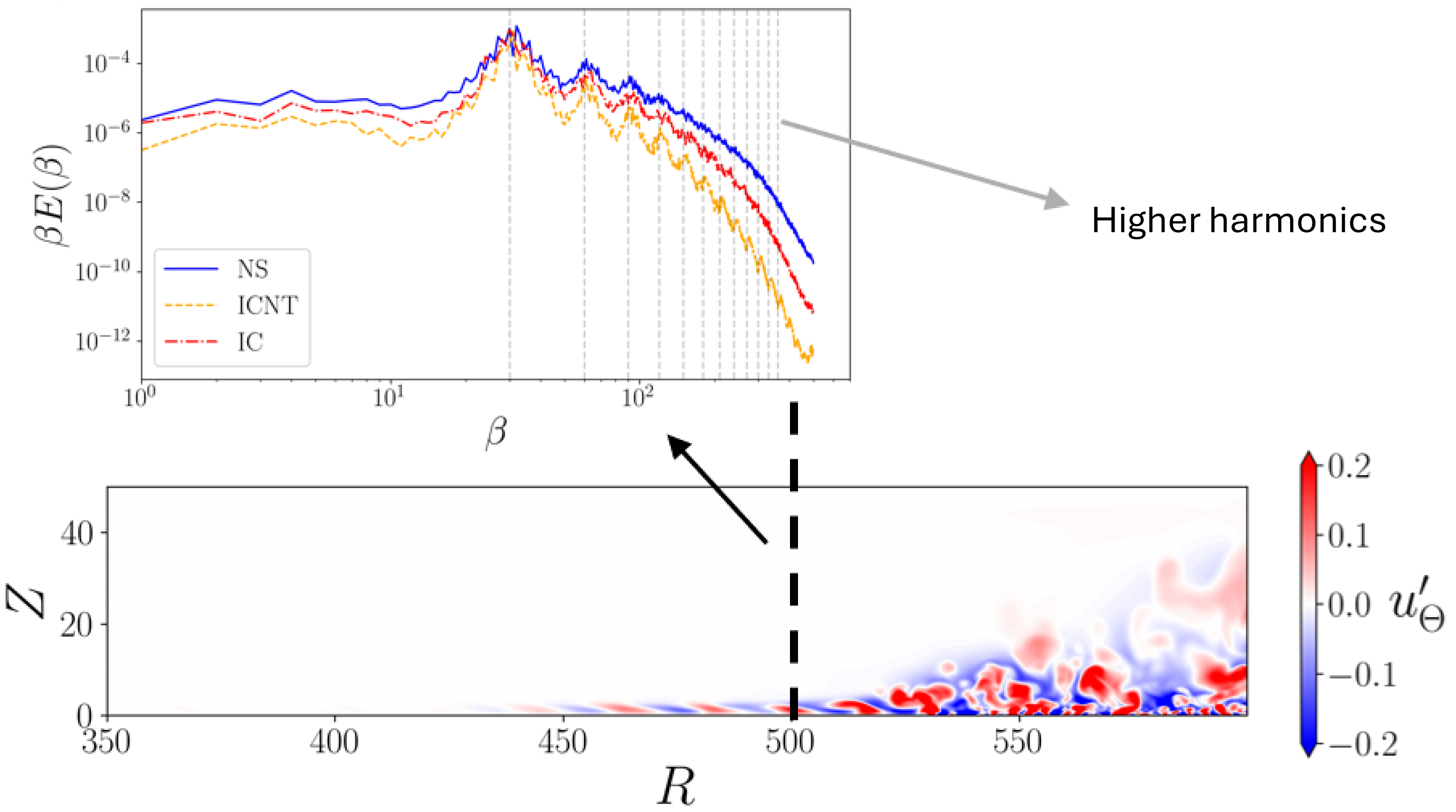
Emergence of coherent structures



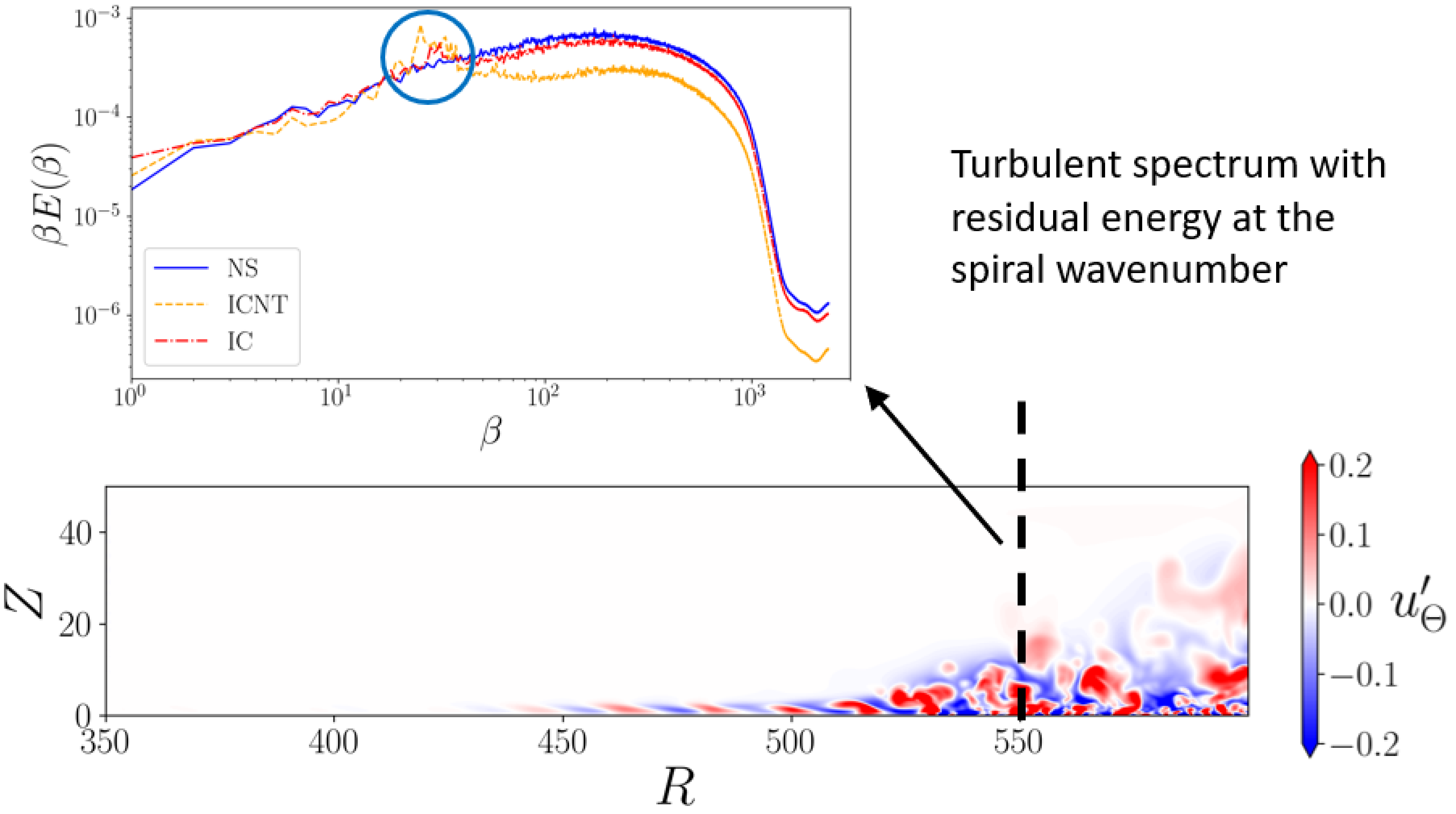




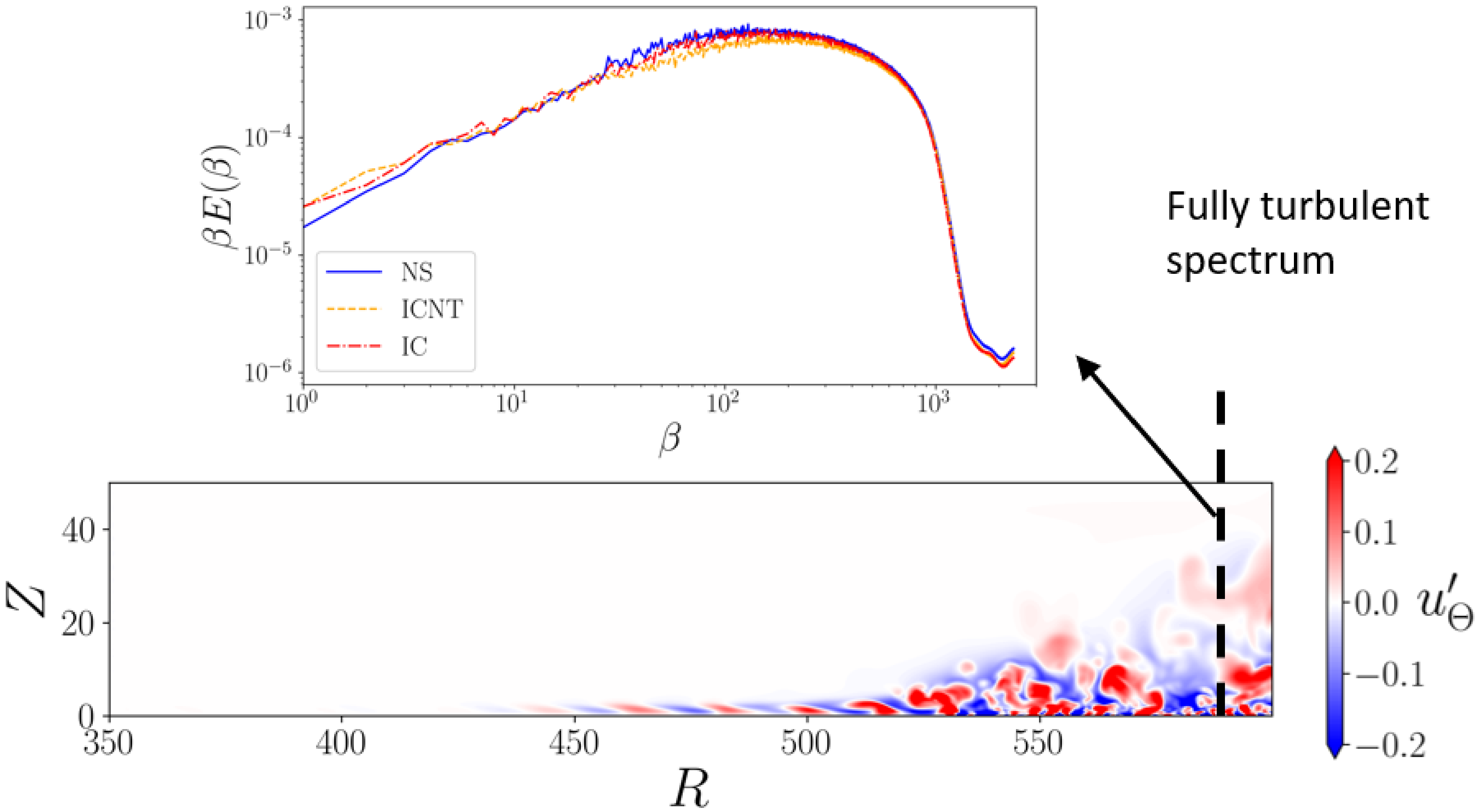
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
$\beta E(\beta)$
NS
ICNT
IC
10^{0}
10^{1}
10^{2}
β
0.2
0.1
0.0
−0.1
−0.2
u'_Θ
40
20
0
Z
350
400
450
500
550
R



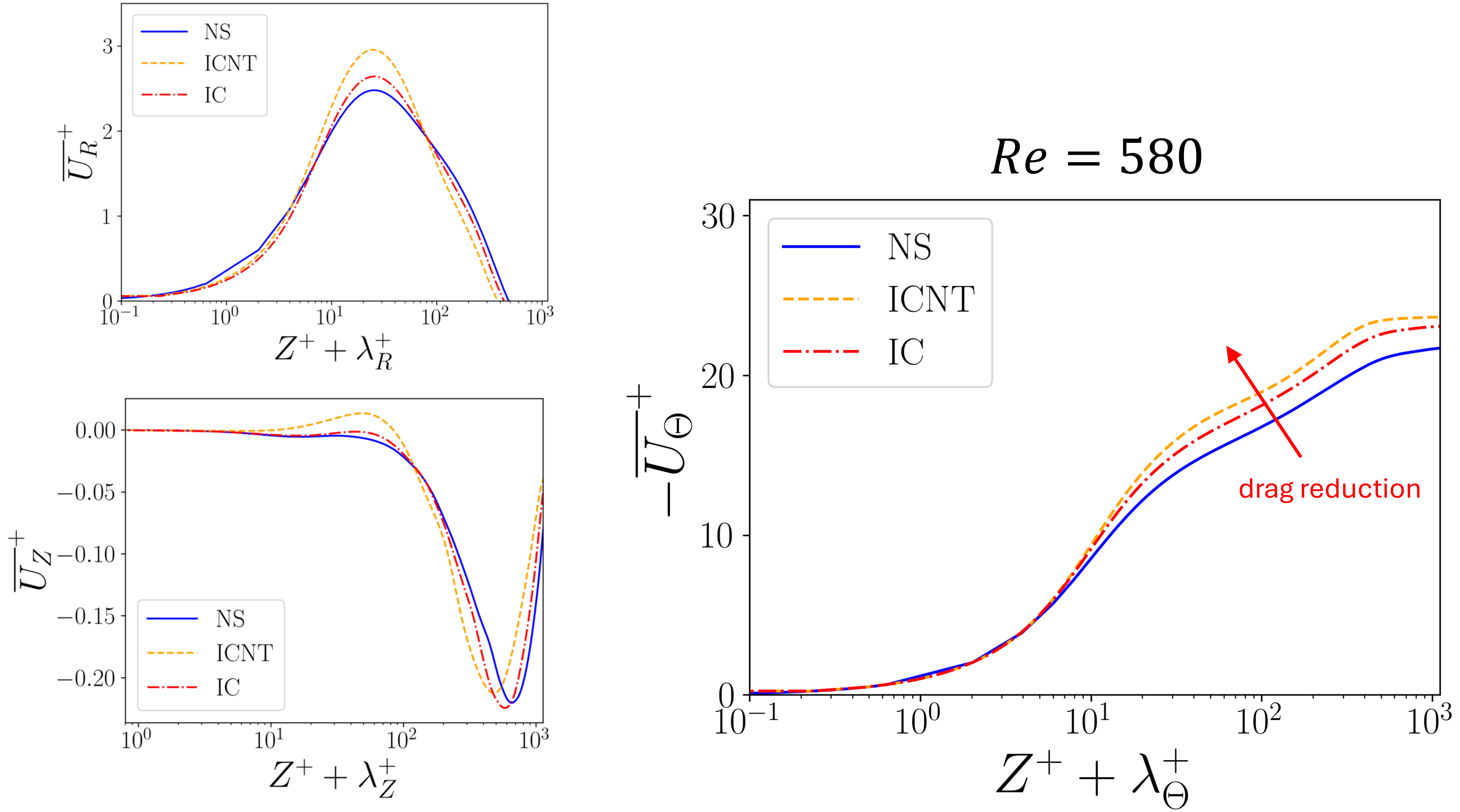
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
$\beta E(\beta)$
NS
ICNT
IC
10^0
10^1
10^2
β
Higher harmonics
40
20
0
Z
350
400
450
500
550
R
0.2
0.1
0.0
−0.1
−0.2
u'_Θ



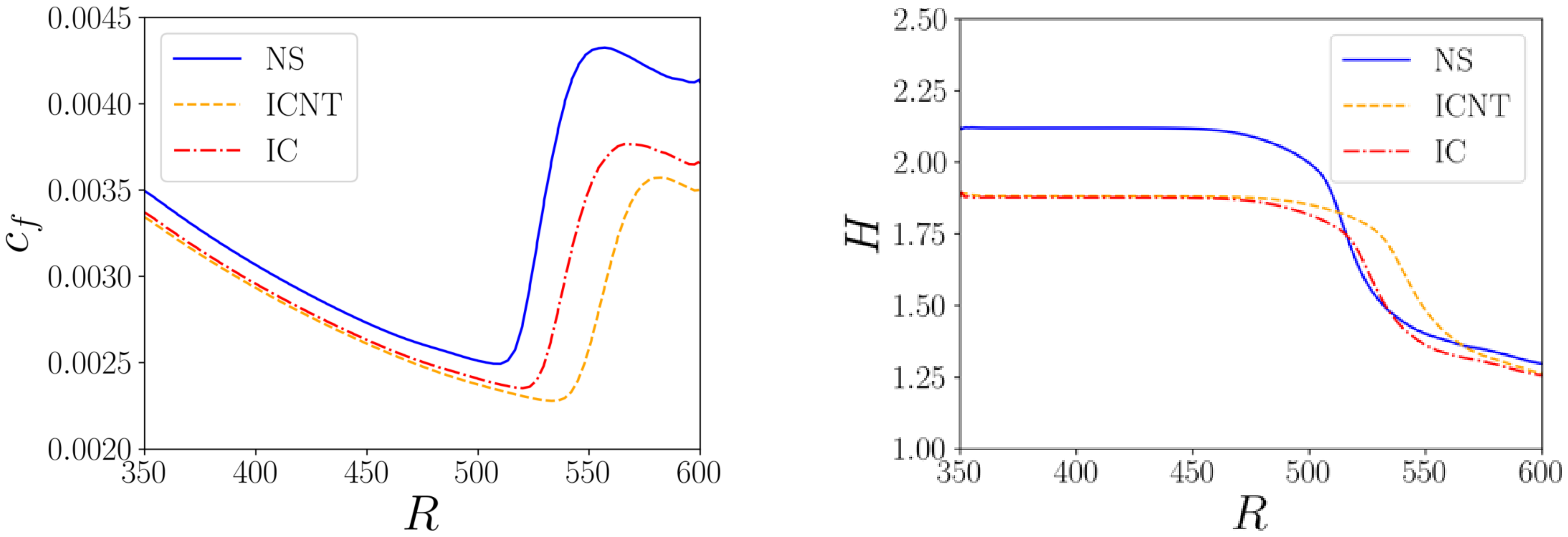
Turbulent spectrum with residual energy at the spiral wavenumber

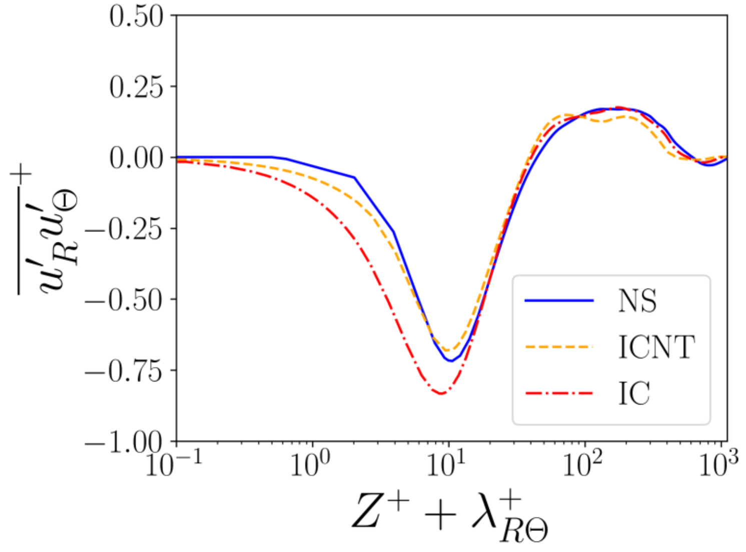


10^{-3}
10^{-4}
10^{-5}
10^{-6}
$\beta E(\beta)$
NS
ICNT
IC
10^{0}
10^{1}
10^{2}
10^{3}
β
Fully turbulent spectrum
40
20
0
Z
350
400
450
500
550
R
0.2
0.1
0.0
−0.1
−0.2
u'_Θ

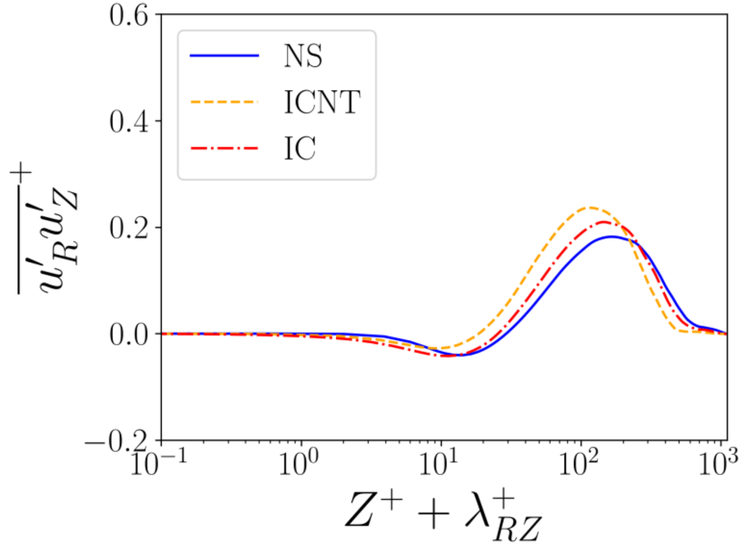


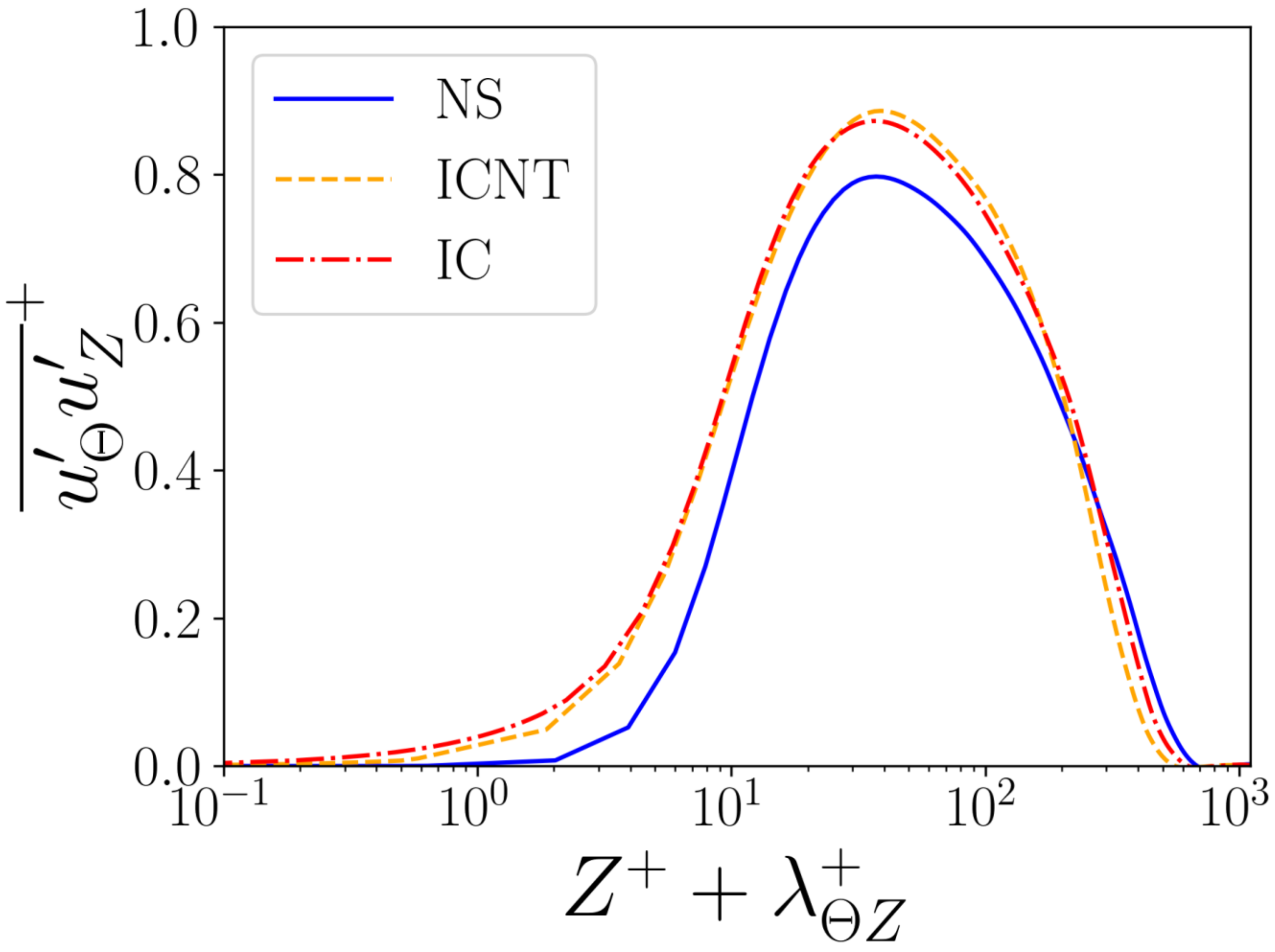
Re = 580
NS
ICNT
IC
drag reduction
$\overline{U_R}^+$
$Z^+ + \lambda_R^+$
$\overline{U_Z}^+$
$Z^+ + \lambda_Z^+$
$-\overline{U_\Theta}^+$
$Z^+ + \lambda_\Theta^+$


0.0045
0.0040
0.0035
0.0030
0.0025
0.0020
c_f
350
400
450
500
550
600
R
NS
ICNT
IC
2.50
2.25
2.00
1.75
1.50
1.25
1.00
H
350
400
450
500
550
600
R
NS
ICNT
IC

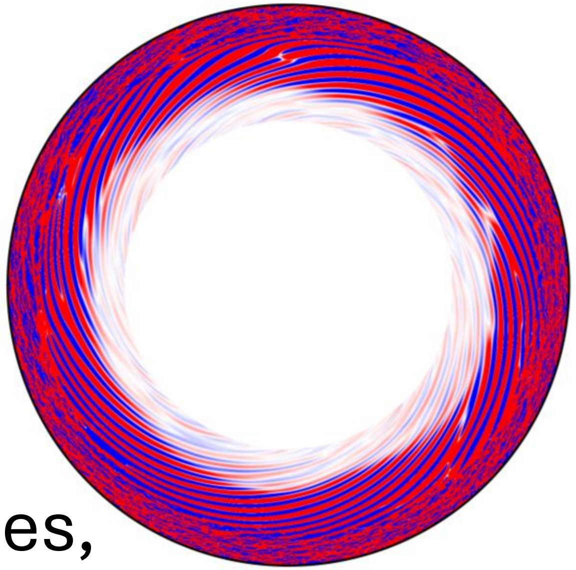


Case	λ^+_R	λ^+_Θ	λ^+_Z	$\lambda^+_{R\Theta}$	λ^+_{RZ}	$\lambda^+_{\Theta Z}$
NS	0.000	0.000	0.000	0.000	0.000	0.000
ICNT	4.377	4.377	0.000	2.184	0.000	0.000
IC	4.483	4.483	3.012	2.242	1.780	1.805





Conclusions



- **Homogenization** is an effective tool to model regularly microstructured rough wall, for quick parametric searches, particularly when coupled to **LSA** which provides immediate and reliable answers as to the performance of the surface
- Slip alone is insufficient to model the behavior of rough walls under transitional/turbulent conditions; a **transpiration** velocity at the fictitious $Z = 0$ boundary is indispensable
- Isotropically arranged cones on the surface, periodicity 0.2 b.l. thicknesses, height 0.1 b.l. thicknesses, appear capable to slightly delay the breakdown to turbulence and reduce **skin friction**