A minimal model for flow control with a poro-elastic coating

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WHY “POROELASTIC”? 

BECAUSE IN NATURE ROUGH, COMPLIANT, FUZZY, ETC. IS THE RULE, WHEREAS RIGID AND SMOOTH IS NOT!
Passive flow control

Problem motivation

Examples in nature abound

leading edge undulations, i.e. tubercles on whales’ flippers
Passive flow control

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multi-winglets, spiroid winglets, i.e. primary remiges
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*porous* riblets on butterfly and moth scales (on the wings)
Passive flow control
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*porous* riblets on butterfly and moth scales (on the wings)
denticles on shark skin
Passive flow control
Problem motivation

Examples in nature abound

leading edge undulations, i.e. tubercles on whale’s flippers
multi-winglets, i.e. primary remiges
**porous** riblets on butterfly and moth scales (on the wings)
denticles on shark skin
as well as in sports
fuzz on a tennis ball
dimples on a golf ball
...

...
Passive flow control
Problem motivation

- **Focus** of this work: *covert feathers* (layer of self-actuated flaps).

- **Passive** “pop-up” of coverts on wings of some birds during
  - landing and gliding phases of flight, perching manoeuvres;
  - in general - high angle-of-attack/ low-lift regimes.

[Images of a bird in flight and a pelican] the Mykonos pelican
Passive flow control with a poro-elastic coating
A rapid research survey

**AIM:** Determine structure parameters of feathers that yield “optimal” fluid-dynamical performance.

<table>
<thead>
<tr>
<th>Experiments</th>
<th>Poroelasticity theory</th>
<th>NS IBM simulations</th>
<th>Low order model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berlin, Rechenberg,</td>
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<td>Freiberg, Brücker,</td>
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<td>Orléans, Kourta,</td>
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<td>Genova</td>
<td>Genova (at low Re number) : Favier et al., 2009</td>
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<td>Oxford, Taylor,</td>
<td>- Favier (AMU), Revell (Manchester), Pinelli (City U.)</td>
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<td>Palaiseau, de Langre,</td>
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</table>

Gopinath & Mahadevan, 2010

Present work + Ongoing research...
Outline

- **Computational modeling of fluid-structure interaction**
  - Highlights of numerical procedure
  - Key computational results

- **Theoretical modeling for vortex-shedding**
  - **Smooth airfoil**
    - Development of the minimal model
    - Calibration against CFD results
  - **Airfoil with poro-elastic coating (“hairfoil”)**
    - Motivation & development
    - Results, comparison with CFD & physical indications

- **Summary & future extensions**
Computational modeling of fluid-structure interaction

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Key computational results

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Airfoil with poro-elastic coating ("hairfoil")
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Computational model

Fluid solver  (developed by Antoine Dauptain & Julien Favier)

- 2-D computations – **NACA0012 airfoil**.
- \( \text{Re} = 1100 \) for this study – **low Reynolds number regime**.
- **Immersed boundary forces** – for airfoil, buffer zone, coating.
- Hence, **fixed Cartesian grid** (fine on and near airfoil).
- **Numerical scheme**:
  - Convective part - explicit Adams-Bashforth
  - Viscous part - semi-implicit Crank-Nicolson
  - **Pressure Poisson** - conjugate gradient

![Diagram showing solid body (airfoil), mixed fluid-solid part (poro-elastic coating), and buffer zone.](image)
Validation of fluid solver
Case: 10° angle of attack

Comparison of frequency spectra

- **Qualitative analysis:**
  - Periodic solutions *sinusoidal*
  - similar frequency spectra – peak at 2\textsuperscript{nd} superharmonic of fundamental frequency.

- **Quantitative analysis:** Close values of
  - mean lift
  - frequency of oscillations.
Fluid $\rightarrow$ structure forcing & vice-versa

- Modeling all the feathers – too heavy.... Hence,

Homogenized approach

Varying porosity & anisotropy

- Normal component of the force: *Koch & Ladd* (*JFM*, 1997)
- Tangential component: *Stokes' flow* approx (*Favier et al. JFM*, 2009)
For each reference feather, equation for momentum balance solved.

\[ Ml_c^2 \ddot{\theta} + K_r f_1(\theta) + K_i f_2(\theta) + K_d \dot{\theta} = l_c F_{\text{ext}} \]

Different frequency scales (\(\equiv\) time scales):

\[ \omega_r = \sqrt{\frac{K_r}{Ml_c^2}}; \omega_i = \sqrt{\frac{K_i}{Ml_c^2}}; \omega_d = \frac{K_d}{Ml_c^2} \]

In present problem, rigidity effects dominant - i.e.,

\[ \omega_d < \omega_i < \omega_r \]
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RESULTS : Smooth airfoil case

Mean lift vs. angle of attack

Mean drag vs. angle of attack

When using feathers, structure (i.e., rigidity) and fluid time scales synchronized.

For instance - Lift coefficient for 22° - time and frequency domains
# Efficient structure parameters

### Parameters varied during the course of the study

<table>
<thead>
<tr>
<th>Angle of attack, $\alpha$ (degrees)</th>
<th>Rigidity moment, $K_r$</th>
<th>Interaction moment, $K_i$</th>
<th>Dissipation moment, $K_d$</th>
<th>Packing density, $\phi$</th>
<th>Angular sector of movement, $[\theta_{\text{min}}, \theta_{\text{max}}]$ (degrees)</th>
<th>Flow frequency, $\omega_{\text{fluid}}$</th>
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<tbody>
<tr>
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<td>8.9905</td>
<td>0.2034</td>
<td>0.0909</td>
<td>0.0085</td>
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<tr>
<td>70</td>
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<td>0.2034</td>
<td>0.0909</td>
<td>0.0085</td>
<td>$[-60, 60]$</td>
<td>0.4772</td>
</tr>
</tbody>
</table>

### Parameters fixed throughout the course of the study

- Mass of reference beam, $M$ = 12
- Length of reference beam, $l$ = $8.5 \times 10^{-2}$
- Diameter of reference beam, $d_c$ = $2 \times 10^{-3}$
- Equilibrium angle/Initial orientation of reference beams, $\theta_{eq}$ (degrees) = 0
- Extent of the coating = 70% of suction side, starting 0.1 units of length after the leading edge and ending 0.2 units before the trailing edge
- Number of reference beams used, $N$ = 8
Summary of computational results [Phys. Fluids, 2012]

- \( \alpha = 22^\circ \):
  - Mean lift \( \uparrow \): 34.36%, Lift fluctuations' \( \downarrow \): 7.15%, Drag fluctuations' \( \downarrow \): 35.47%, Mean drag \( \uparrow \): 6.6%

- \( \alpha = 45^\circ \):
  - Mean drag \( \downarrow \): 8.92%, Drag fluctuations' \( \downarrow \): 10.46%, Mean lift \( \downarrow \): 1.47%

- \( \alpha = 70^\circ \):
  - Mean lift \( \uparrow \): 7.5%, Drag fluctuations' \( \downarrow \): 9.71%, Mean drag \( \downarrow \): 4.92%
Computational modeling of fluid-structure interaction

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Key computational results

Theoretical modeling for vortex-shedding

Smooth airfoil

→ Development of the minimal model
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Airfoil with poro-elastic coating ("hairfoil")

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Results, comparison with CFD & physical indications
Minimal models: (Airfoil) Vortex-shedding

FINAL AIM:
(a) predict “optimal” structure parameters at a fraction of the cost
(b) explain physical mechanism behind such optimal coatings

Some facts

- For unsteady flows over bodies, for fixed set of parameters, long time history of lift/drag forces periodic + independent of initial conditions
  
i.e, lift/drag can be represented as self-excited oscillator, yielding limit cycle

- Autonomous equations with negative linear damping and positive non-linear damping can produce limit cycles (as in present case)
  
i.e, small disturbances allowed to grow; large disturbances pushed back to equilibrium.
**Minimal models**: periodic forces in the flow past a cylinder

Hartlen & Currie (1970); Currie and Turnbull (1987)

**Rayleigh oscillator**

\[ \frac{d^2 x}{dt^2} + x = \frac{dx}{dt} - \left( \frac{dx}{dt} \right)^3 \]

Skop & Griffin (1973)

**Van der Pol-like oscillator**

\[ \frac{d^2 x}{dt^2} + x = \frac{dx}{dt} - x^2 \frac{dx}{dt} \]

Nayfeh et al (2005); Akthar, Marzouk & Nayfeh (2009)

**Van der Pol + Duffing-type cubic nonlinearity**

\[ \frac{d^2 x}{dt^2} + x = \frac{dx}{dt} - x^2 \frac{dx}{dt} - x^3 \]
Crucial physics: smooth airfoil

- **Super-harmonics** of flow frequencies - peak at **twice the fundamental frequency** – unlike the case of a cylinder.

Lift coefficient for 10° - time and frequency domains

- Indicates presence of **quadratic non-linearity in model** equation.
- Can a generic equation with all possible quadratic terms be a model?
Crucial physics: smooth airfoil

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Lift coefficient for 10° - time and frequency domains

- Indicates presence of **quadratic non-linearity in model** equation.
- Can a generic equation with all possible quadratic terms be a model?
- No, at least one higher-order non-linear term is needed to obtain a self-excited oscillator (i.e. independent of initial forcing conditions).
When can a limit cycle exist?

- Most general system with all possible quadratic and cubic nonlinearities, with negative linear damping:

\[
\ddot{x} + x = c \dot{x} + \alpha_1 x^2 + \alpha_2 x \dot{x} + \alpha_3 \dot{x}^2 + \beta_1 x^3 + \beta_2 x^2 \dot{x} + \beta_3 x \dot{x}^2 + \beta_4 \dot{x}^3
\]
When can a limit cycle exist?

- **A necessary condition**: For most general system with all possible quadratic and cubic non-linearities with negative linear damping:
  \[
  \ddot{x} + x = c \dot{x} + \alpha_1 x^2 + \alpha_2 x \dot{x} + \alpha_3 \dot{x}^2 + \beta_1 x^3 + \beta_2 x^2 \dot{x} + \beta_3 x \dot{x}^2 + \beta_4 \dot{x}^3
  \]

- **Poincaré-Lindstedt's method** guarantees the existence of a limit cycle only if
  \[
  \alpha_2 (\alpha_1 + \alpha_3) + \beta_2 + 3\beta_4 < 0
  \]

- Coefficients of cubic terms with odd powers of \(x\) – i.e. \(\beta_1\) & \(\beta_3\) – play no role.

(expand dependent and independent variables in powers of a small book-keeping parameter \(\varepsilon\) to have a solution uniformly valid in time, collect like-order equations, impose conditions on order zero amplitude/frequency of the solution ...)
When can a limit cycle exist?

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- Coefficients of cubic terms with odd powers of \(x\) – i.e. \(\beta_1\) & \(\beta_3\) – play no role.

- Other two cubic terms correspond to **Rayleigh** *(as in present low-order model)* & **van der Pol** oscillators resp.

<table>
<thead>
<tr>
<th>Case</th>
<th>(\alpha_1)</th>
<th>(\alpha_2)</th>
<th>(\alpha_3)</th>
<th>(\beta_2)</th>
<th>(\beta_4)</th>
<th>Existence of limit cycle</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0</td>
<td>0</td>
<td>-1</td>
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<td>-1</td>
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<tr>
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<td>-1</td>
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</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>Limit cycle exists only for initial conditions with (\dot{x}) negative or zero.</td>
</tr>
<tr>
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<td>1</td>
<td>-1</td>
<td>0</td>
<td>No</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Comparison of convergence to limit cycles

Since convergence to the limit cycle, from both small and large initial conditions, is faster for case 6, the model equation is taken as:

\[
\ddot{x} + x = \dot{x} + \dot{x}^2 - \dot{x}^3
\]

In the present case, since mean lift ≠ 0, the equation becomes:

\[
\ddot{C}_L + \omega^2 C_L = \mu \dot{C}_L - \alpha \dot{C}_L^3 + \beta \dot{C}_L^2 + \omega^2 \overline{C}_L
\]

For this equation, method of multiple scales used to find right model parameters, which in turn determine the correct model equation.
How to find a (periodic) solution?

Method of multiple scales – key steps:

- **Solutions** sought in form of power series in \( \delta \), where \( \delta \) measures *how strongly non-linear* the system is.

- If only one time scale considered, typical issue is: for large \( t \), perturbation solution does not match with numerical/exact solution.
  
  **Reason:** Appearance of secular terms in perturbation solution.

- In *present problem*, **minimum three time scales** seen to be sufficient.

- Transforming model equation into 1\(^{st}\) order complex-varibled equation:

\[
\dot{\zeta} = \iota \omega \zeta - \frac{t}{2} \omega \bar{C}_L + \frac{\delta}{2} \mu (\zeta - \bar{\zeta}) + \frac{\delta}{2} \alpha \omega^2 (\zeta^3 - 3 \zeta^2 \bar{\zeta} + 3 \zeta \bar{\zeta}^2 - \bar{\zeta}^3) + \frac{\delta}{2} \beta t \omega (\zeta^2 - 2 \zeta \bar{\zeta} + \bar{\zeta}^2)
\]

- Introducing three time scales \( T_0 = t, T_1 = \delta t \) and \( T_2 = \delta^2 t \), substituting

\[
\zeta = \sum_{j=0}^{2} \delta^j \zeta_j (T_0, T_1, T_2) + O(\delta^3)
\]

and separating similar coefficients of powers of \( \delta^0 \) (=1), \( \delta^1 \) and \( \delta^2 \) one obtains...
Finding a periodic solution (contd..)

\[ D_0 \zeta_0 - i \omega \zeta_0 = \frac{-l}{2} \omega \bar{C}_L \]  \hspace{2cm} (1)

\[ D_0 \zeta_1 - i \omega \zeta_1 = -D_1 \zeta_0 + \frac{\mu}{2} (\zeta_0 - \bar{\zeta}_0) + \frac{\alpha}{2} \omega^2 (\zeta_0^3 - 3 \zeta_0^2 \bar{\zeta}_0 + 3 \zeta_0 \bar{\zeta}_0^2 - \zeta_0^3) + \frac{\beta}{2} \omega (\zeta_0^2 - 2 \zeta_0 \bar{\zeta}_0 + \bar{\zeta}_0^2) \]  \hspace{2cm} (2)

\[ D_0 \zeta_2 - i \omega \zeta_2 = -D_2 \zeta_0 - D_1 \zeta_1 + \frac{\mu}{2} (\zeta_1 - \bar{\zeta}_1) + \frac{3 \alpha}{2} \omega^2 (\zeta_0^2 \zeta_1 - \zeta_0 \zeta_1^2 \bar{\zeta}_1 + \bar{\zeta}_0^2 \zeta_1 - \bar{\zeta}_0 \bar{\zeta}_1^2 \zeta_1 - 2 \zeta_0 \bar{\zeta}_0 \zeta_1 + 2 \zeta_0 \bar{\zeta}_0 \bar{\zeta}_1) + \beta l \omega (\zeta_0 \zeta_1 - \zeta_0 \bar{\zeta}_1 - \bar{\zeta}_0 \zeta_1 + \bar{\zeta}_0 \bar{\zeta}_1) \]  \hspace{2cm} (3)

- Substituting solution \( \zeta_0 \) from (1) in (2) + eliminating terms proportional to \( \exp(i\omega T_0) \) \[ \implies \text{bounded solution} \]

- Substituting \( \zeta_0 \) and \( \zeta_1 \) in (3), \textbf{solvability conditions} obtained + \textbf{steady-state} assumption on \textbf{amplitude of lift} coefficient \[ \implies \text{parameters of limit cycle} \]

\textbf{SUMMARY:} Given a system, with known model parameters, characteristics of solution (i.e., amplitude, frequency, etc.) can be solved.

\textbf{Conversely}, given a system, with known solution, model parameters can be determined.
Final solution:

\[ C_L(t) = a_0 + a_1 \cos(\omega_s t) + a_2 \cos(2 \omega_s t) + a_3 \sin(3 \omega_s t) \]

where \( a_0, a_1, a_2, a_3 \) and \( \omega_s \) are computational parameters, found in terms of model parameters \( \omega, \mu, \alpha \) and \( \beta \).

Model parameters thus recovered in terms of computational parameters as:

\[ \omega = \frac{a_1^2 a_3 \omega_s}{a_1^2 a_3 - 36 a_3^3 - 6 a_2^2 a_3} \quad ; \quad \delta \mu = \frac{24 a_1 a_3^2 \omega_s}{a_1^2 a_3 - 36 a_3^3 - 6 a_2^2 a_3} \]

\[ \delta \beta = \frac{6 a_2}{a_1^2} \quad ; \quad \delta \alpha = \frac{32 a_1^2 a_3 - 36 a_3^3 - 6 a_2^2 a_3}{a_1^5 \omega_s} \]
RESULTS: Smooth airfoil

- Final solution:

\[ C_L(t) = a_0 + a_1 \cos(\omega_s t) + a_2 \cos(2 \omega_s t) + a_3 \sin(3 \omega_s t) \]

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\[
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\]
Can do *vice versa* ...
RESULTS: Dependence of amplitude $a_1$ on model parameters

- Size of limit cycle proportional to $\mu / \alpha$.
- Effect of increase in $\mu$ dominates over increase in $\alpha$.
- Oscillations in limit cycle scales as $\sqrt{\mu}$.
- We can easily span a very large parameter space!
Dependence of the frequency $\omega_s$ of the limit cycle on model parameters
Dependence of the frequency $\omega_s$ of the limit cycle on model parameters

we can easily change model parameters and simulate the effect of varying $Re, \alpha$, etc.
Dependence of the frequency $\omega_s$ of the limit cycle on model parameters

we can easily change model parameters and simulate the effect of varying Re, $\alpha$, etc.

... and even uncover unphysical solutions ...
Computational modeling of fluid-structure interaction

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COATED AIRFOIL: *towards a low-order model*

Some questions:

- What are (the) **optimal** structure parameters?
- How are structure **parameters related** to aerodynamic changes?
  - e.g., why do *some* feathers lead to drag reduction *and/or* lift enhancement, etc.?
- Which structure parameters are **most crucial** for realistic physics?
  - e.g., in computations,
    - features modeled with compliance, porosity and anisotropy
    - rigidity effects were predominant.
- Simplest model for coupled fluid-structure system:

\[
\ddot{C}_L + \omega^2 C_L - \omega^2 \bar{C}_L - \mu \dot{C}_L + \alpha \dot{C}_L^3 - \beta \dot{C}_L^2 = \rho_1 \theta \\
\ddot{\theta} + c \dot{\theta} + \omega_1^2 \theta = \rho_2 (C_L - \bar{C}_L)
\]

- The **method of multiple scales** again yields insights!
Figure 1: Fluid-coating interface: (left) - initial undisturbed configuration (i.e., without any forcing from the fluid) - the vertical lines here denote a discrete number of feathers spread uniformly in this layer; (right) - disturbed configuration showing the displacement variable $\theta$. Note here that the colour gradient in this disturbed layer characterizes the non-uniform, time-varying porosity (i.e., darker shades denote clustering of feathers while lighter shades stand for areas with a lower instantaneous concentration of feathers).

\[
\ddot{C}_L + \omega^2 C_L - \omega^2 \bar{C}_L - \mu \dot{C}_L + \alpha \dot{C}_L^3 - \beta \dot{C}_L^2 = \rho_1 \theta
\]

\[
\ddot{\theta} + c \dot{\theta} + \omega_1^2 \theta = \rho_2 (C_L - \bar{C}_L)
\]
Solution of coupled system

- Similar procedure as for smooth airfoil – but now for both equations.
- Three time scales (as before).
- Separating similar coefficients of powers of $\delta^0 (=1)$, $\delta^1$ and $\delta^2$ and solving.
- **Constraints analogous to case of smooth airfoil:**
  - Vanishing of secular terms in closed-form solution of lift.
  - Steady-state assumption on amplitude of lift coefficient $a_1(t)$.

\[
\frac{\mu}{2} a_1(t) - \frac{3}{8} \alpha \omega^2 a_1^3(t) = 0
\]

- **Additional, but similar, constraints** now also on poroelastic coating deformation $a_2(t)$.

\[
ca_2(t) = 0
\]
Case 1: \[ a_1(t) = \frac{2}{\omega} \sqrt{\frac{\mu}{3\alpha}}; \quad a_2(t) = 0 \text{ (i.e, } c \text{ can be arbitrarily large)} \]

\begin{align*}
C_L(t) &= \bar{C}_L + \frac{2\delta\beta\mu}{3\alpha\omega^2} + \sqrt{\frac{4\mu}{3\alpha\omega^2}} \cos(\omega_{s,1}t) + \frac{2\delta\beta\mu}{9\alpha\omega^2} \cos(2\omega_{s,1}t) + \delta \sqrt{\frac{\mu^3}{432\alpha\omega^4}} \sin(3\omega_{s,1}t) \\
\theta(t) &= \frac{-2\delta\rho_2}{\omega(\omega - \omega_1)(\omega + \omega_1)} \sqrt{\frac{\mu}{3\alpha}} \cos(\omega_{s,1}t)
\end{align*}

where \[ \omega_{s,1} = \omega - \frac{(\delta\mu)^2}{16\omega} - \frac{2(\delta\beta)^2\mu}{9\alpha\omega} - \frac{(\delta\rho_1)(\delta\rho_2)}{2\omega(\omega - \omega_1)(\omega + \omega_1)} \]

NOTE:
- Form of \( C_L(t) \) exactly similar to case of smooth airfoil (with super-harmonics).
- No super-harmonics of \( \omega_{s,1} \) in dynamics of \( \theta(t) \).
- Resonant condition: If \( \omega_{s,1} \approx 0 \) (i.e, \( \omega \sim \omega_1 \)), \( \sqrt{\frac{4\mu}{3\alpha\omega^2}} \) dominates, mean lift ↑
- Non-resonant condition: Changes in structure parameters do not directly change lift →

THE STRUCTURE IS SLAVED BY THE FLUID
RESULTS: Weak fluid→structure coupling

Case 2: \( a_1(t) = 0 \; ; \; c = 0 \) (i.e, \( a_2(t) \) can be arbitrary \( \Rightarrow C_0 \))

\[
C_L(t) = \widetilde{C}_L + \frac{\delta \rho_1 C_0}{(\omega - \omega_1)(\omega + \omega_1)} \cos(\omega_{s,2} t)
\]

\[
\theta(t) = C_0 \cos(\omega_{s,2} t)
\]

where \( \omega_{s,2} = \omega_1 - \frac{(\delta \rho_1)(\delta \rho_2)}{2\omega_1(\omega - \omega_1)(\omega + \omega_1)} \) (i.e, \( \omega_{s,2} \) a perturbation of \( \omega_1 \)).

NOTE:

- Dynamics of coupled system **dictated by structure frequency**.
- No superharmonics of \( \omega_{s,2} \) in \( C_L(t) \) and \( \theta(t) \).
- **Resonant condition**: If \( \omega_{s,2} \approx 0 \) (i.e, \( \omega \sim \omega_1 \)), mean lift \( \uparrow \) by \( O(\delta) \) when:
  - structure-fluid coupling parameter \( \rho_1 \) increased (decrease porosity).
  - increase compliance so that steady state oscillations of feather \( C_0 \) is large.
- **Non-resonant condition**: Lift fluctuations \( \downarrow \) if

\[
\frac{\delta \rho_1 C_0}{(\omega - \omega_1)(\omega + \omega_1)} < \sqrt{\frac{4\mu}{3\alpha \omega^3}}
\]

NEVER REALISED IN PRACTISE WITH IBM SIMULATIONS
RESULTS: Two-way coupling

Case 3: \( a_1(t) = \frac{2}{\omega} \sqrt{\frac{\mu}{3\alpha}} \); \( c = 0 \) (i.e., \( a_2(t) \) can be arbitrarily large)

\[
C_L(t) = \widetilde{C}_L + \frac{2\delta\beta\mu}{3\alpha\omega^2} + \sqrt{\frac{4\mu}{3\alpha\omega^2}} \cos(\omega_{s,1}t) + \frac{2\delta\beta\mu}{g\alpha\omega^2} \cos(2\omega_{s,1}t) + \delta \sqrt{\frac{\mu^3}{4\cdot32\alpha\omega^4}} \sin(3\omega_{s,1}t)
\]

\[
+ \frac{\delta\rho_1 C_0}{(\omega - \omega_1)(\omega + \omega_1)} \cos(\omega_{s,2}t)
\]

\[
\theta(t) = C_0 \cos(\omega_{s,2}t) - \frac{2\delta\rho_2}{\omega(\omega - \omega_1)(\omega + \omega_1)} \sqrt{\frac{\mu}{3\alpha}} \cos(\omega_{s,1}t)
\]

NOTE:

- Solution – combination of solutions of cases 1 and 2.
- No super-harmonics of \( \omega_{s,1} \) in dynamics of \( \theta(t) \).
- No superharmonics of \( \omega_{s,2} \) in \( C_L(t) \) and \( \theta(t) \).
- Resonant condition: If \( \omega_{s,1} \) and \( \omega_{s,2} \approx 0 \), mean lift \( \uparrow \) by \( O(\delta) \) as in Case 2.
- Non-resonant condition: Increase in lift fluctuations avoided as in Case 2.
Model parameters from CFD results

Re-writing the most general form of analytical solution (i.e., Case 3) as:

\[ C_L(t) = l_0 + l_1 \cos(\omega_{s,1} t) + l_2 \cos(2\omega_{s,1} t) + l_3 \sin(3\omega_{s,1} t) + l'_1 \cos(\omega_{s,2} t) \]

\[ \theta(t) = \theta_1 \cos(\omega_{s,2} t) + \theta'_1 \cos(\omega_{s,1} t) \]

one gets the following coupled quadratic equations for the frequencies \( \omega \) and \( \omega_1 \):

\[
(l_1^2 l_3^2 - 6 l_1^2 l_3 l_2) \omega^2 - l_1^2 l_3 \omega_{s,1} \omega - l_1^2 l_3 \omega_{s,1} \omega + l_1^2 l_3 \omega_{s,2} \omega_1 = 0
\]

\[
(2 \theta_1 l_1 - l_1' \theta_1') \omega_1^2 - 2 \omega_{s,2} \theta_1 l_1 \omega_1 + l_1' \theta_1' \omega^2 = 0
\]

and the following six equations:

\[
\delta \mu = \frac{24 l_3 \omega}{l_1} \quad ; \quad \delta \beta = \frac{6 l_2}{l_1^2} \quad ; \quad \delta \alpha = \frac{32 l_3}{l_1^3 \omega} \quad ; \quad C_0 = \theta_1 \quad ;
\]

\[
\delta \rho_1 = \frac{(\omega - \omega_1)(\omega + \omega_1)l_1'}{C_0} \quad ;
\]

\[
\delta \rho_2 = -\omega (\omega - \omega_1)(\omega + \omega_1) \theta_1' \sqrt{\frac{3 \alpha}{\mu}}
\]
Comparison: *minimal model* and CFD

- **CASE**: Airfoil with a poro-elastic coating in the front half of its suction side:

- Lift coefficient – time and frequency domains:

- Correspondence with *Case 1*, i.e. case with only $\omega_{s,1}$ and super-harmonics.
Computational modeling of fluid-structure interaction

Highlights of numerical procedure
Key computational results

Theoretical modeling for vortex-shedding

Smooth airfoil

Theory & development
Results and comparison with CFD results

- Airfoil with poro-elastic coating ("hairfoil")
  Motivation & development
  Results, comparison with CFD & physical indications

- Summary & future extensions
SUMMARY

- Computational modeling of fluid-structure interaction
  - Computational investigation of low Reynolds number flows.
  - Employment of immersed boundary method for complex, moving boundaries.
  - Synchronization of structure frequency with fluid frequency can:
    - affect flow topology near airfoil, by spontaneous adjustment;
    - modify vortex-shedding;
    - change pressure distribution for the better.

Without coating

With coating
SUMMARY

- Theoretical modeling for vortex-shedding
  - Non-linear minimal models developed for vortex-shedding behind:
    - smooth airfoil;
    - airfoil with poro-elastic coating.
  - These models are capable of:
    - reproducing dynamics obtained by heavy computations;
    - giving insights into prediction of optimal structure parameters.
FUTURE EXTENSIONS & PERSPECTIVES

● **Non-linear model** for structure part.

● **Bending feathers**: Bending also neglected since feathers were short enough - usually the case with birds' coverts.

● Effectiveness of coating under *turbulent conditions*, particularly vis-a-vis control of transition to turbulence.

● For higher Reynolds number regimes meaningful to add a third spatial component ...

● Modeling of hairy actuators on *internal flow without vortex-shedding* Eg:- Couette flow.

● How do actuators affect velocity profile in boundary layer ?

● Effectiveness of coating on *more complex configurations* –
  ➢ asymmetric airfoils (with positive camber)
  ➢ dynamic airfoils (with slow pitching and/or heaving, dynamically changing camber).
**Immersed boundary force**

- Feedback forcing term in N-S \( \leftrightarrow \) **Spring-mass system** equilibrium.
  \[
  F = \alpha \int (U^{des} - U) dt + \beta (U^{des} - U)
  \]

- Spring constant \( \alpha \) **not large** – else, spring breaks.

- Damping parameter \( \beta \) **not large** – else, force less reactive.

- Magnitudes of these constants in buffer zone must ensure **no dominant frequency enters inflow**, when domain is streamwise periodic.