Flow through anisotropic poroelastic media

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In collaboration with

G. Zampogna *(DICCA, Genoa)*

Man often tries to achieve technical surfaces which are rigid and smooth …

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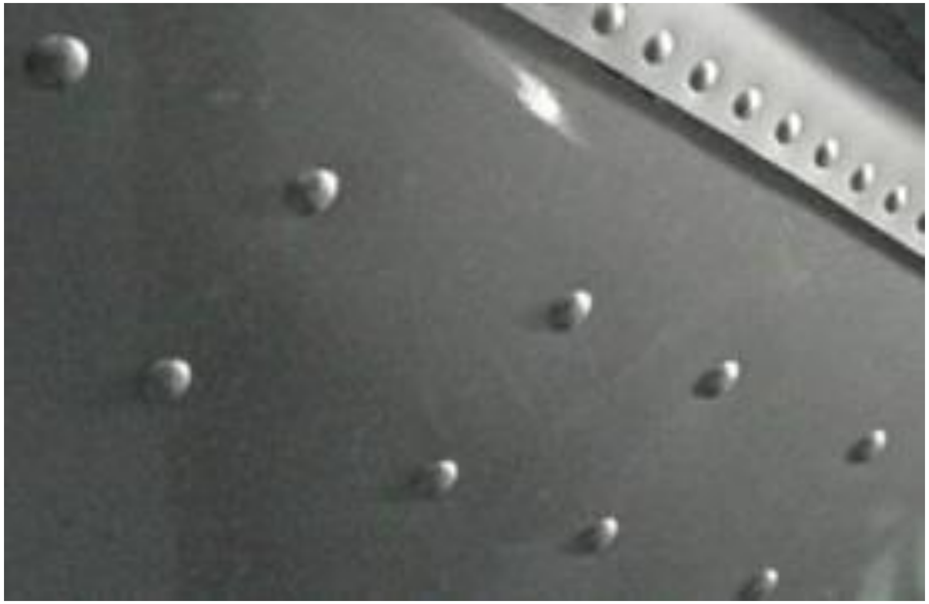
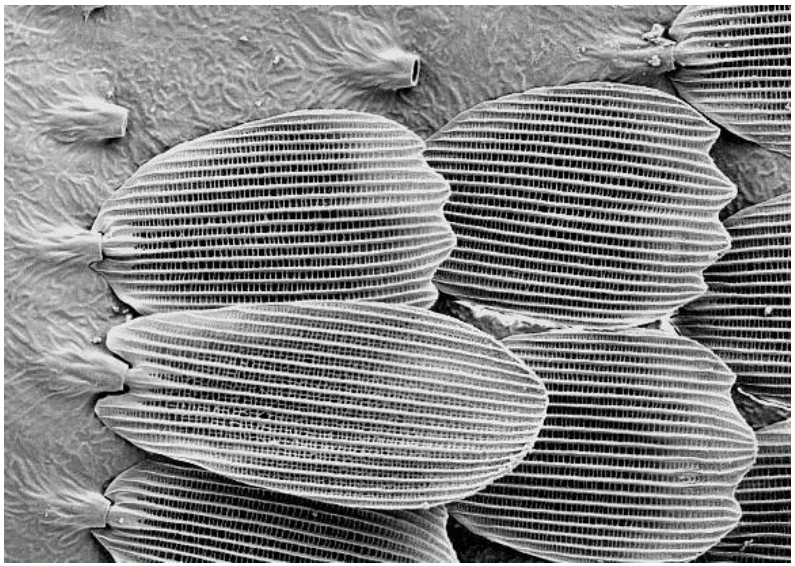
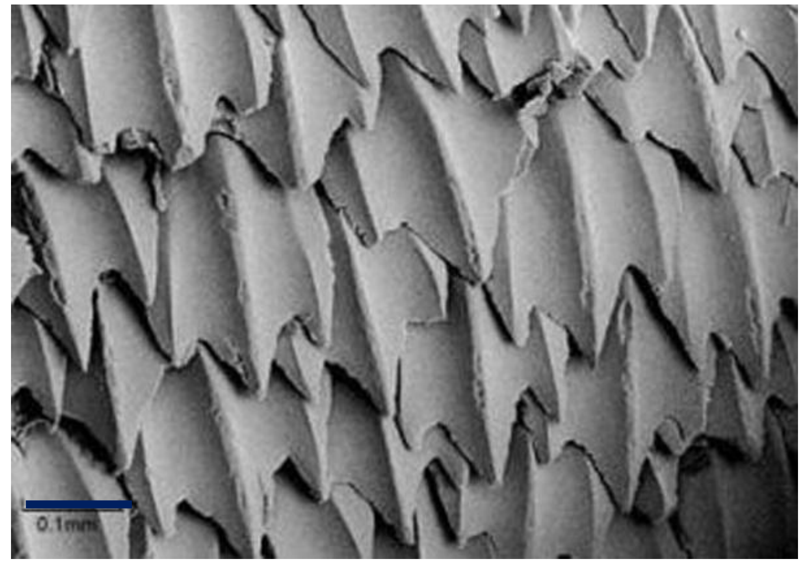


Rivets, nuts and bolts are ‘negative’ features …

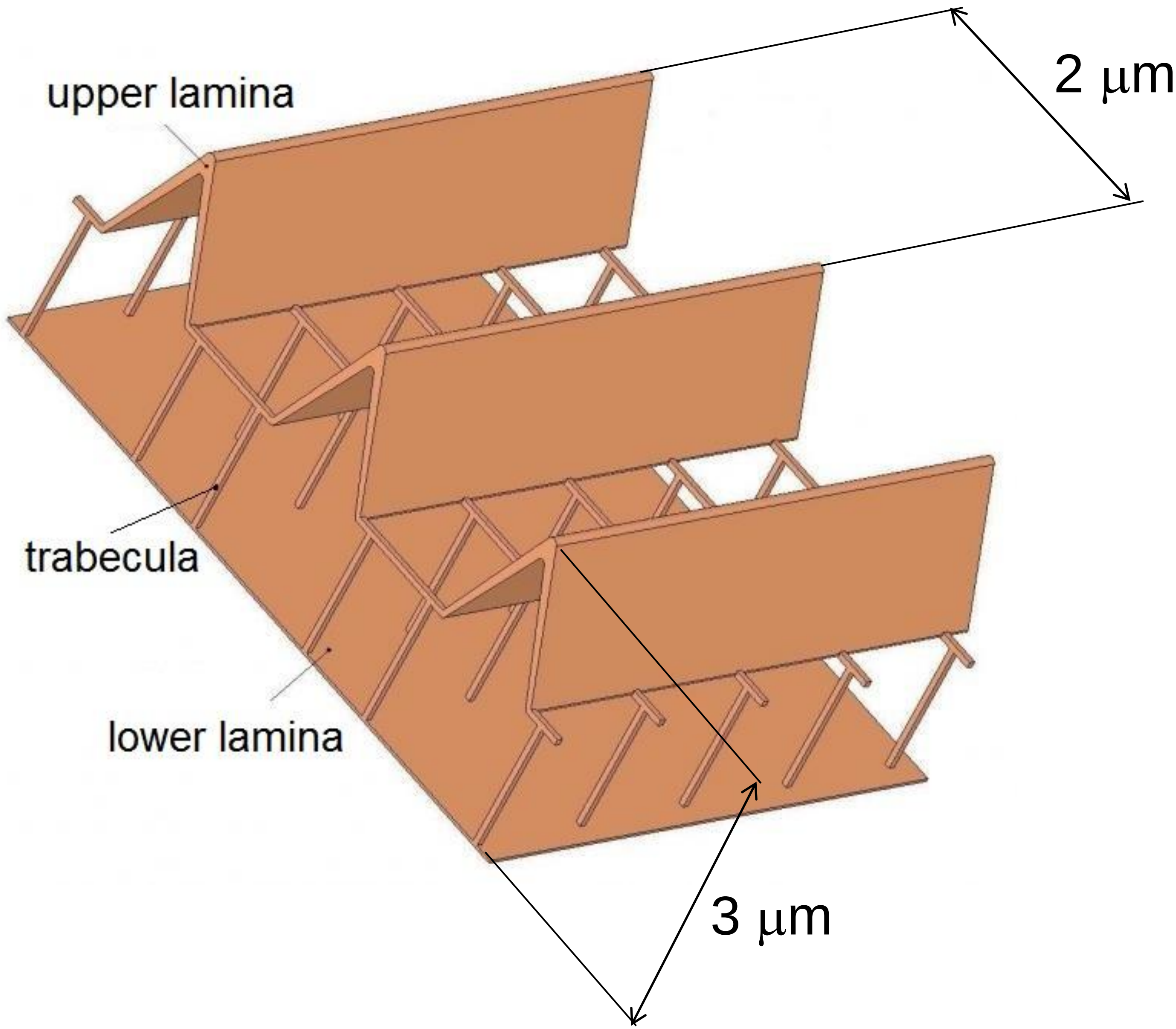
In Nature, *porous*, *anisotropic*, *compliant* , *irregular*, *rough* … at different length scales is the norm!

40 μm





Butterfly scale



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Shark denticle

RESEARCH ARTICLE

The hydrodynamic function of shark skin and two biomimetic applications

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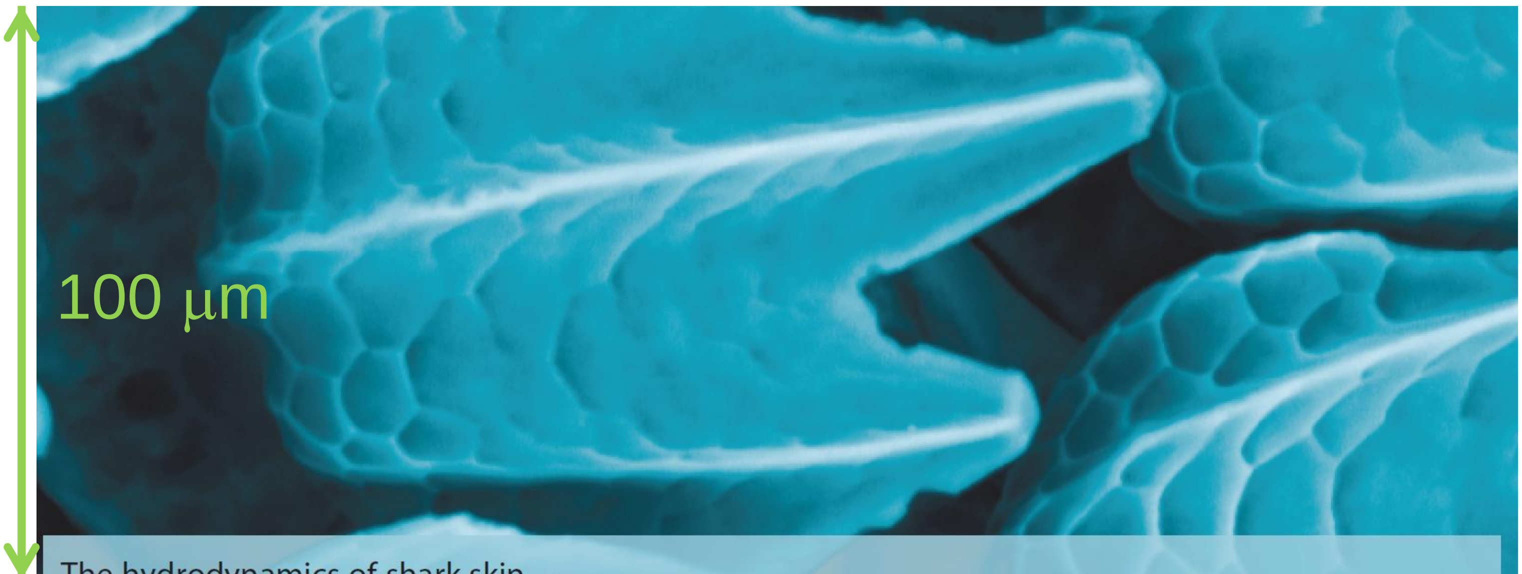


The hydrodynamics of shark skin

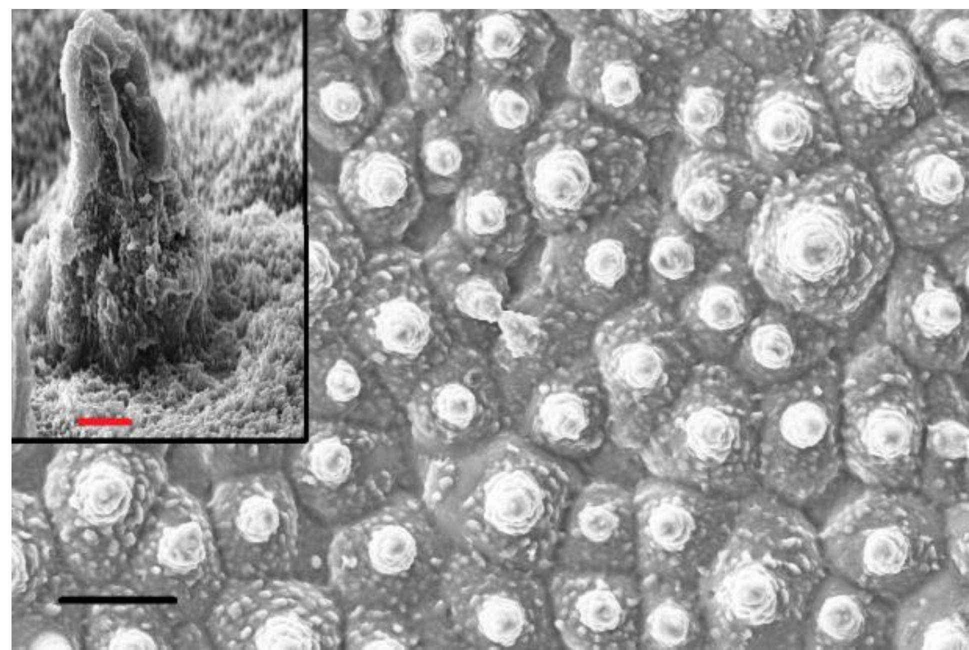
The skin of sharks is covered with scales called denticles. This image, 250 μm across, of the skin of a bonnethead shark shows the details of typical denticles, with three surface ridges leading to three prongs oriented toward the tail. Related in structure to teeth, denticles have long been suspected of reducing hydrodynamic drag on sharks as they swim. Indeed, shark skin has inspired a variety of materials engineered to reduce drag on submerged bodies; swimsuits are perhaps the best-known example. (For more on swimsuit technology, see PHYSICS TODAY, August 2008, pages 32 and 84.)

Many of the experimental studies of such materials—and of shark skin itself—have examined the drag on rigid bodies, a scenario that may be relevant for some applications but not for sharks or swimmers. New work by Johannes Oeffner and George Lauder of Harvard University has now looked at the effects of undulation. The pair mechanically flapped sheets of shark skin in a flowing water tank to determine the speed at which each sheet held its position. Comparing the swimming speed for natural shark skin with that for skin with the denticles sanded off, the team found that denticles actually decreased the swimming speed for rigid sheets but produced a 12% increase for flexible sheets that mimicked typical shark undulations. The team attributes the increase not just to decreased drag but also to increased thrust arising from the altered flow environment observed near the undulating surface. Surprisingly, the researchers saw no clear speed increase in similar experiments with "shark-inspired" swimsuit fabric. (J. Oeffner, G. V. Lauder, J. Exp. Biol.215 , 785, 2012; image submitted by George Lauder.)

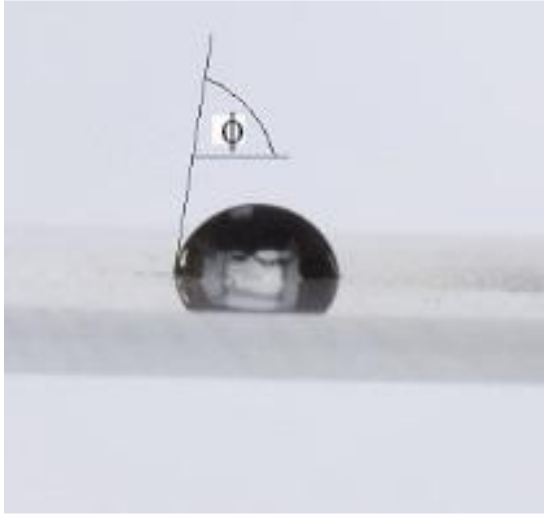
Superhydrophobicity: the Lotus leave

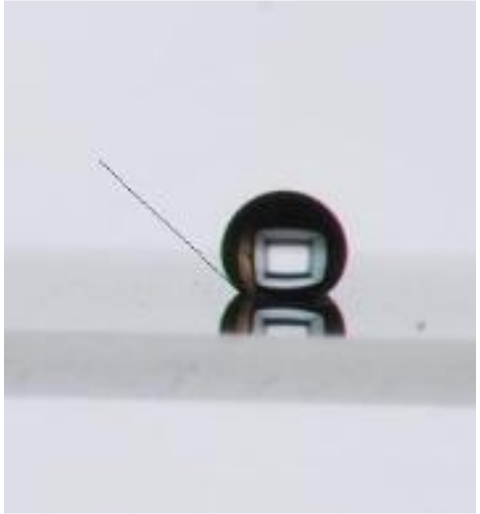
2 μm

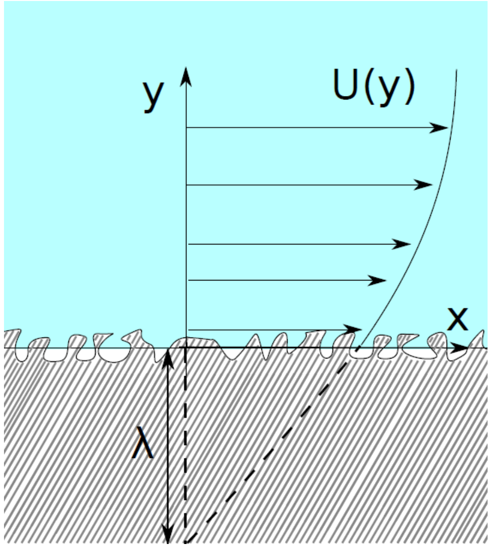
20 μm



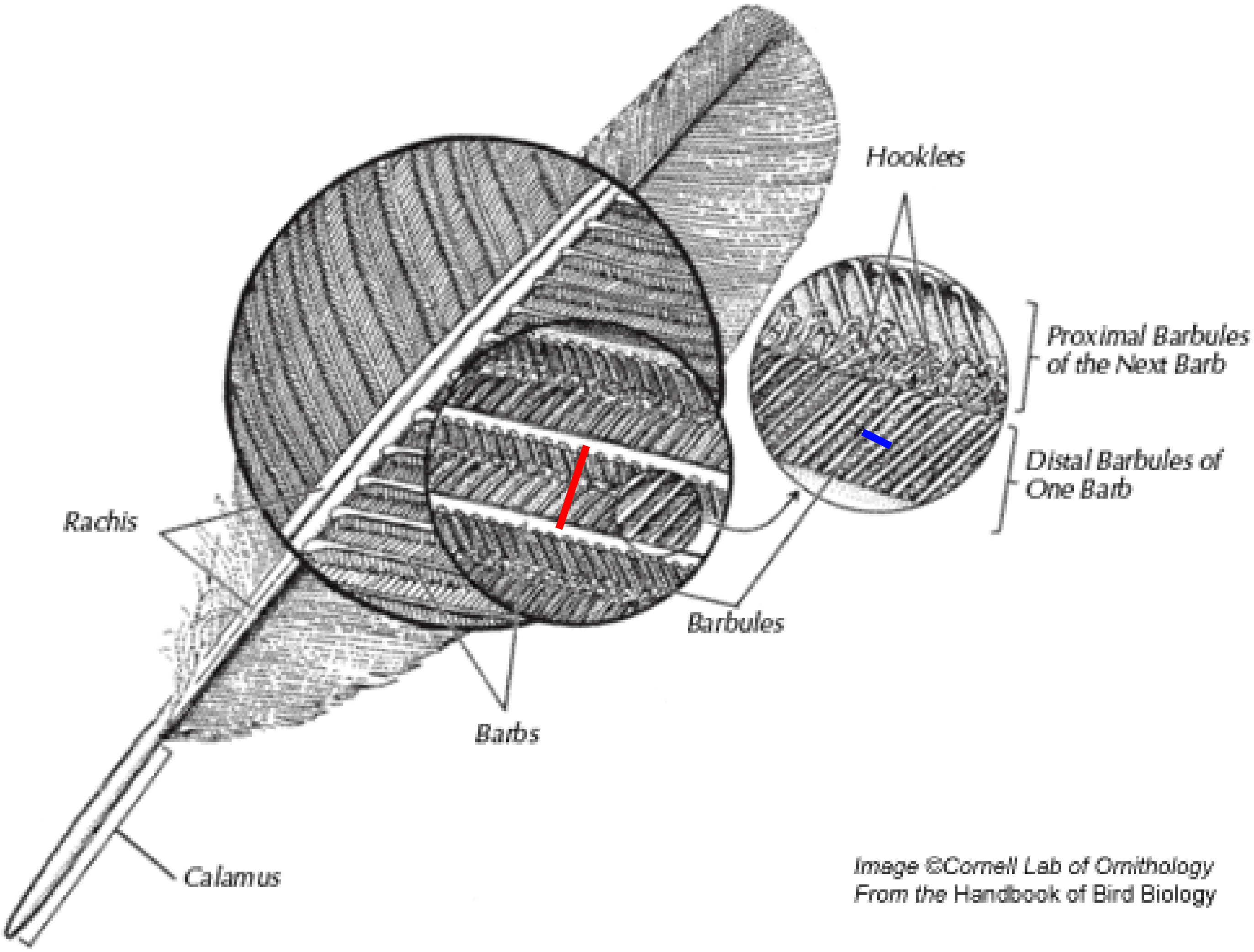
Superhydrophobicity





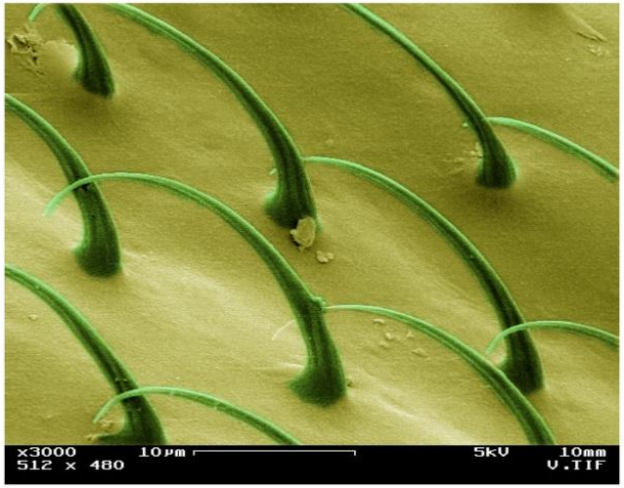


Bird feather

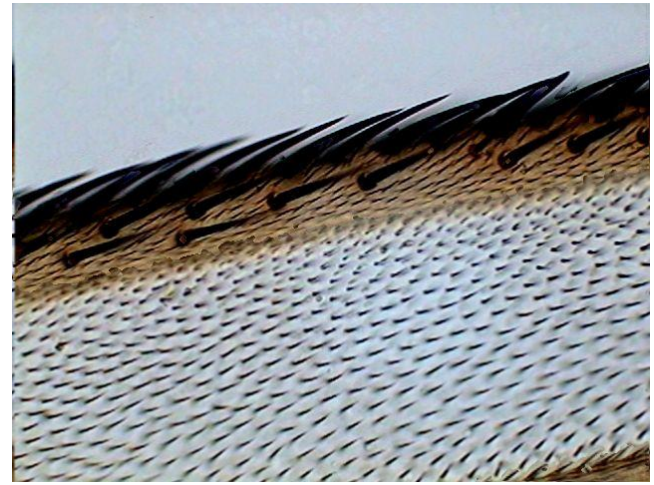


200 μm

10 μm



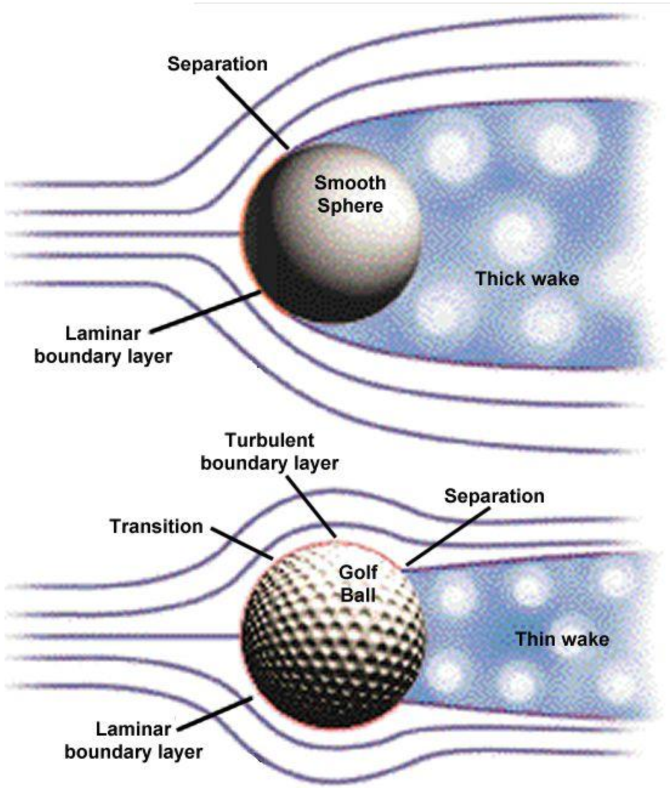
fly

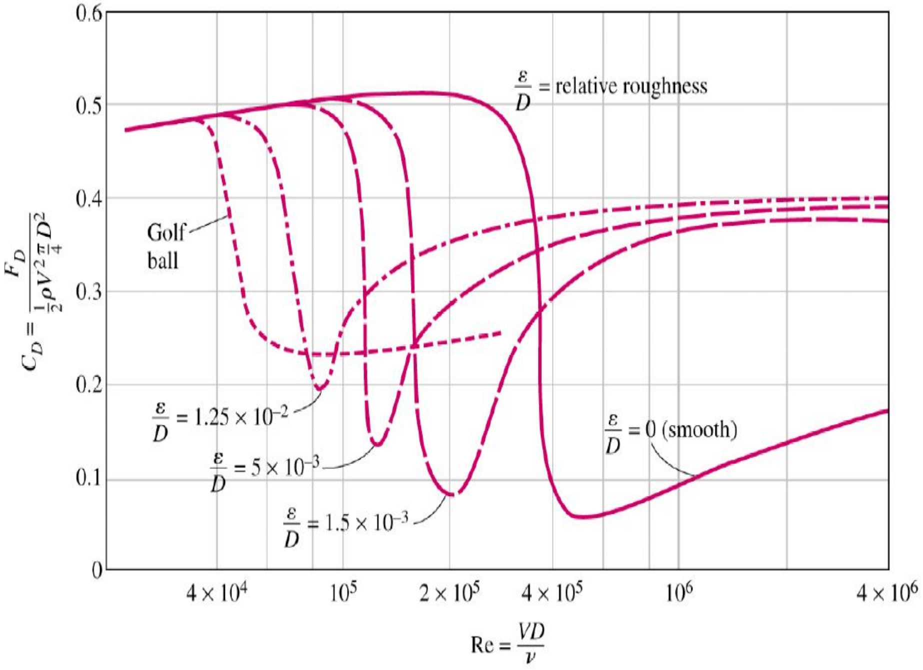

mosquito


river otter


skua

Can a poroelastic coating reduce drag “optimally” (thanks to its compliance) as opposed, i.e., to the (sub-optimal) pressure drag reduction of golf/tennis/baseball balls?





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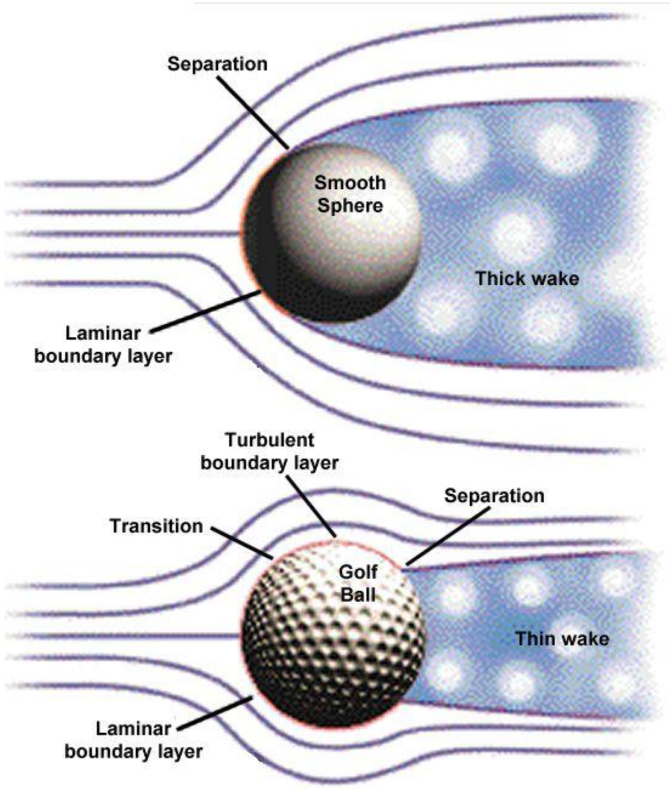
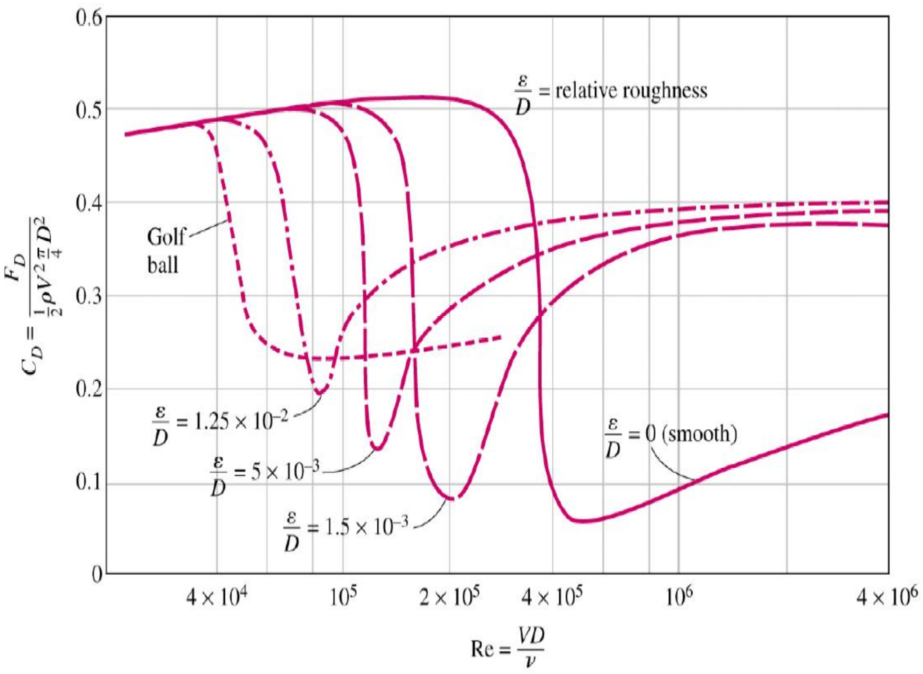
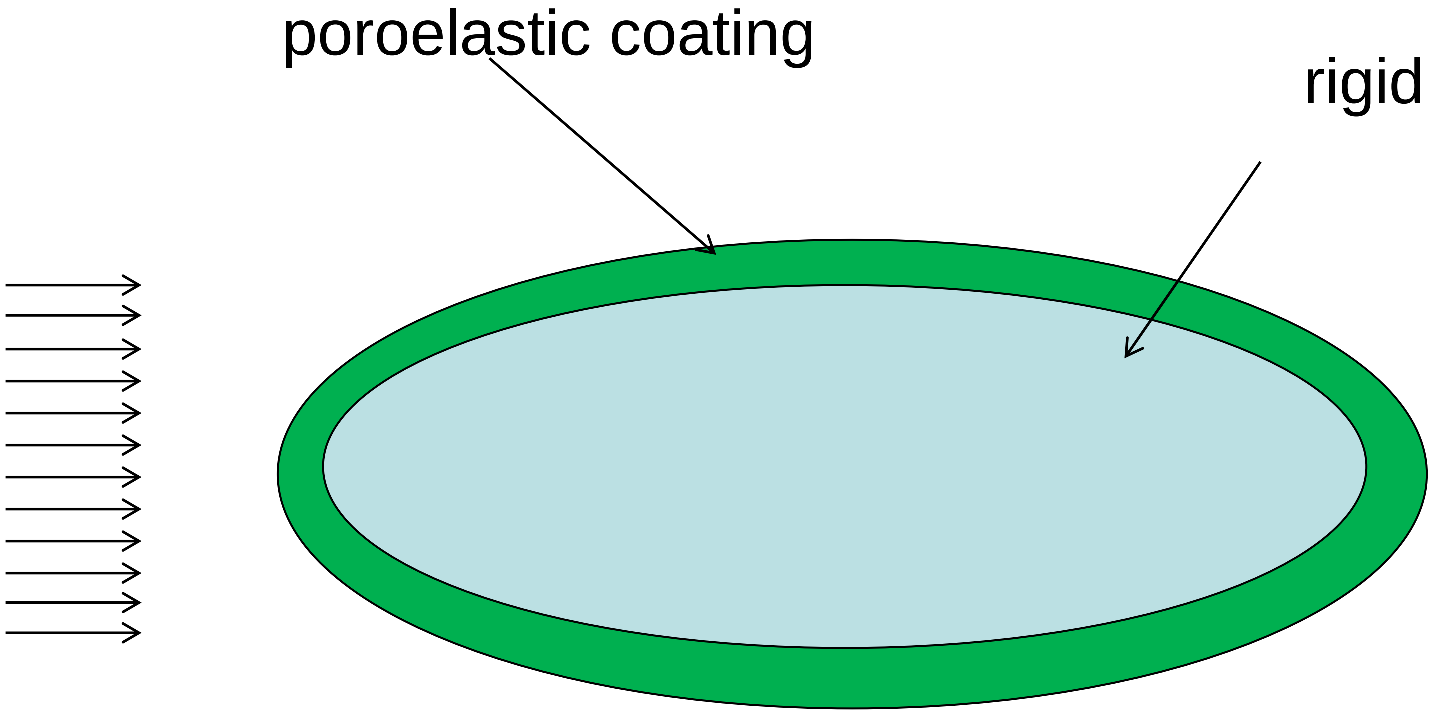
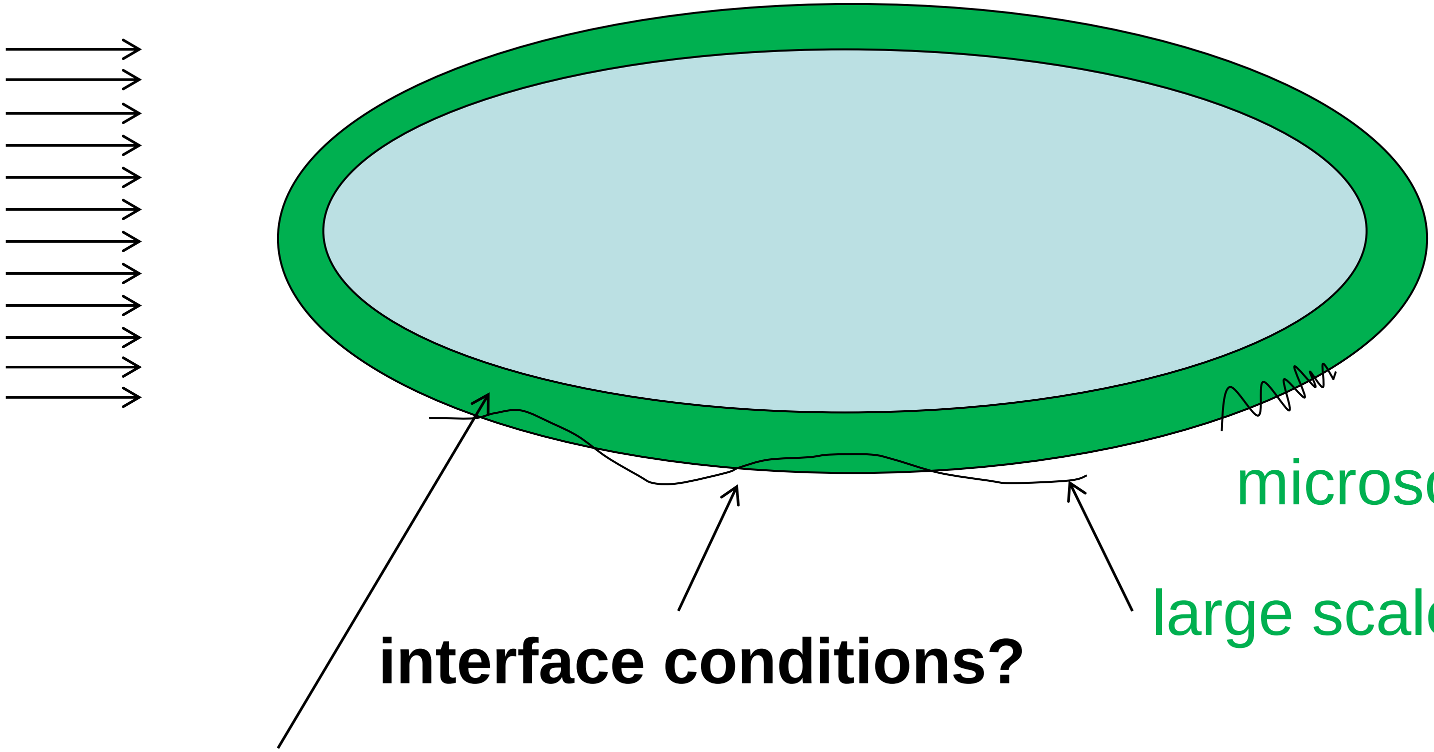




Transition delay/**lift**/**skin friction drag**/**wave drag**/**noise**/**…**



How to model a flow over a porous, flexible coating anchored onto a rigid substrate?



microscopic features

large scale compliance

interface conditions?

dynamic equations for the porous, elastic, anisotropic coating?

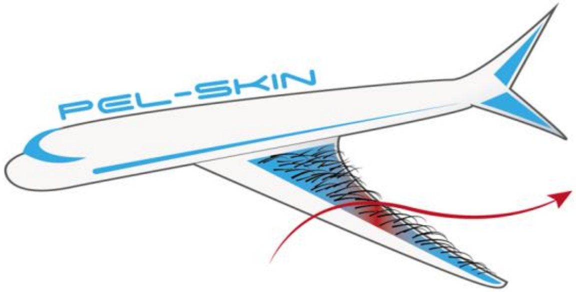
SMART MORPHING CENTRE, IMFT & Laplace

http://smartwing.org

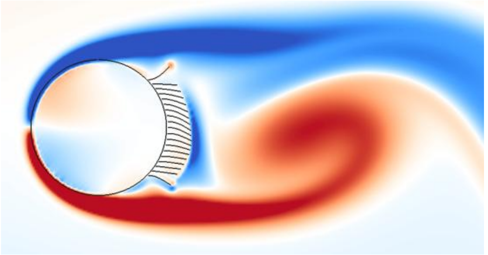
EMMAV + DYNAMORPH, sponsored by STAE-RTRA

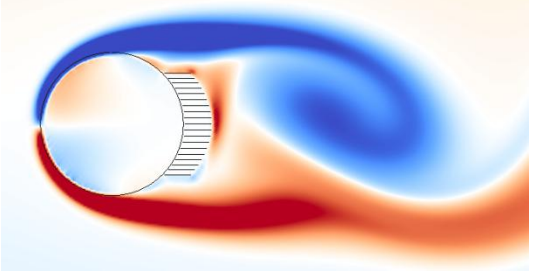
(SMA, EP, Piezo ...)

PelSKIN, ongoing EU project, TRL0

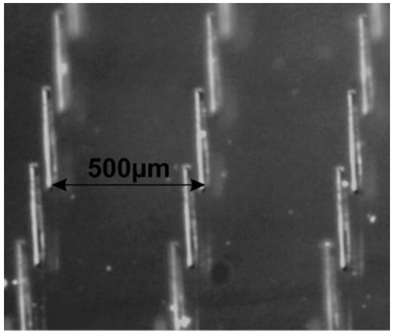


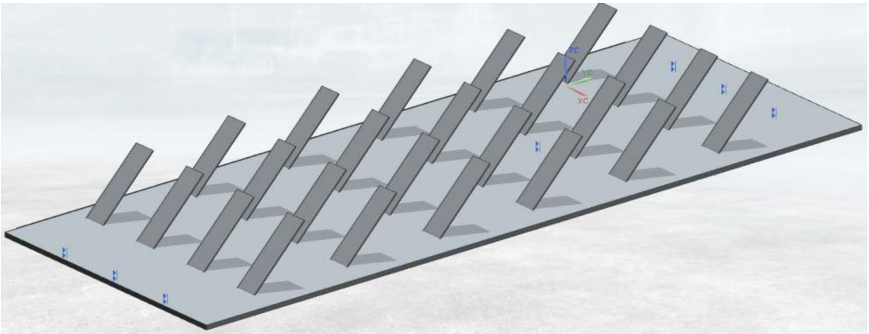
Simulations



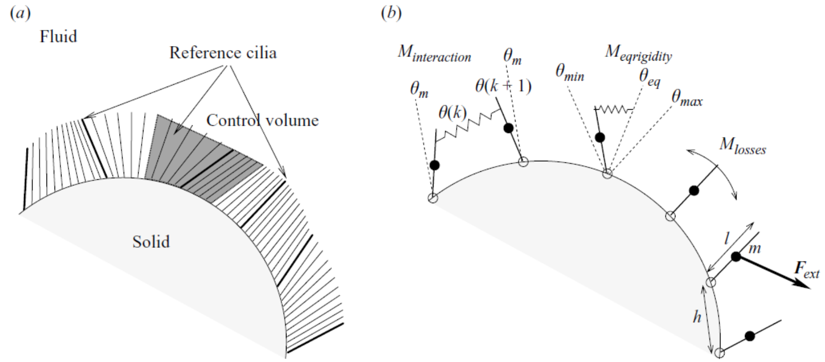


Experiments

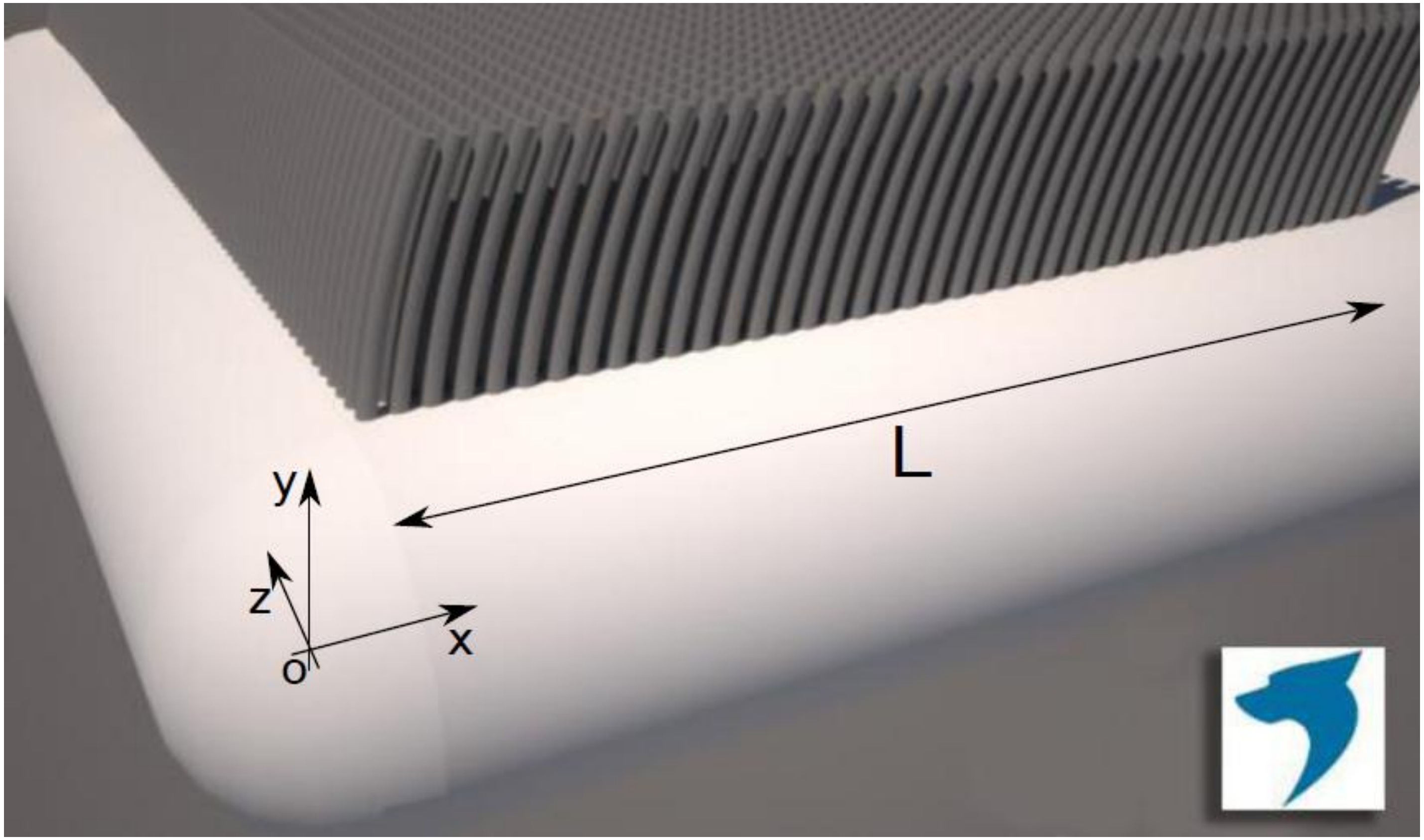




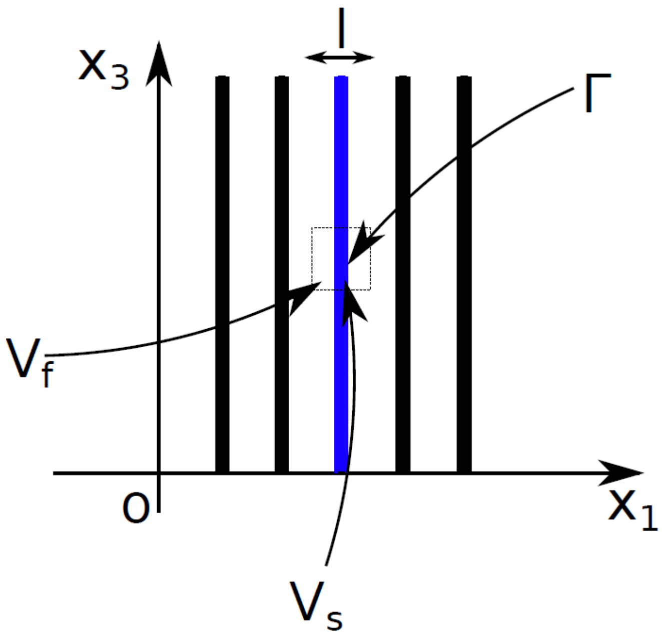
Theory

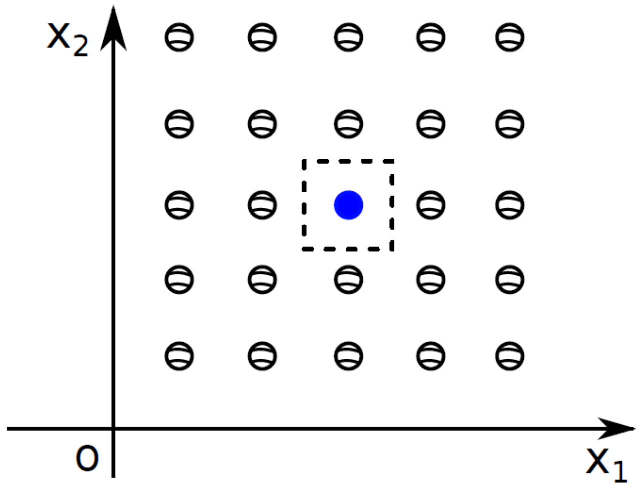


Homogenization theory for multiscale mechanics



$(x, y, z) = (x_1, x_2, x_3)$





Within dashed lines: *elementary cell* $V = V_s + V_f$

with Γ the fluid-solid interface

l 'microscopic' length scale

L 'macroscopic' length scale

$$\epsilon = \frac{l}{L} \ll 1$$

Fluid (on V_f)

$$\frac{\partial u_i}{\partial x_i} = 0,$$

$$\rho_f \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = \frac{\partial \Sigma_{ij}}{\partial x_j},$$

$$\Sigma_{ij} = -p\delta_{ij} + 2\mu\varepsilon_{ij}(\mathbf{u})$$

Solid (on V_s)

$$\rho_s \frac{\partial^2 v_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j},$$

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}(\mathbf{v})$$

B.C. $\boxed{u_i = \frac{\partial v_i}{\partial t}}$ and $\boxed{\Sigma_{ij} n_j = \sigma_{ij} n_j}$ over Γ

plus V-periodicity

Order of magnitude estimates:

$$U = \frac{V}{T}$$

$$\rho_s \frac{V}{T^2} = E \frac{V}{L^2}, \qquad \rho_f \frac{U}{T} = \frac{P}{L}$$

Dimensionless variables (hat):

$$\boxed{\hat{t} = Tt = \frac{lt}{U}, \quad \hat{\mathbf{x}} = l\mathbf{x}, \quad \hat{p} = Pp = \rho_f \frac{UL}{T} p, \quad \hat{\mathbf{u}} = U\mathbf{u}, \quad \hat{\mathbf{v}} = UT\mathbf{v}}$$

After nondimensionalization of the equations (and dropping the hats):

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\epsilon}\frac{\partial p}{\partial x_i} + \frac{2}{\mathrm{Re}}\frac{\partial \varepsilon_{ij}(\mathbf{u})}{\partial x_j},$$

$$\boxed{\mathrm{Re} = (\rho_f U l)/\mu}$$

$$\epsilon^2 \frac{\partial^2 v_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j},$$

$$\frac{\rho_s}{\rho_f}\sigma_{ij} n_j = \epsilon \Sigma_{ij} n_j .$$

plus B.C. and periodicity.

Multiple scales:

$$
\begin{aligned}
x & \quad \rightarrow \quad \text{fast, microscopic variable} \\
x' = \epsilon x & \quad \rightarrow \quad \text{slow, macroscopic variable}
\end{aligned}
$$

Expansions:

$$f = f^{(0)} + \epsilon f^{(1)} + \epsilon^2 f^{(2)} + \ldots$$

with $f = \{u_i, v_i, p, \Sigma_{ij}, \sigma_{ij}\}$

Furthermore: $$\boxed{\frac{\partial}{\partial x_i} \rightarrow \frac{\partial}{\partial x_i} + \epsilon \frac{\partial}{\partial x'_i}}$$

The strain tensor (for either solid or fluid) is:

$$\varepsilon_{ij} + \epsilon\varepsilon'_{ij}$$

with

$$\varepsilon_{ij}(w) = \frac{1}{2}\left(\frac{\partial w_i}{\partial x_j} + \frac{\partial w_j}{\partial x_i}\right)$$

$$\varepsilon'_{ij}(w) = \frac{1}{2}\left(\frac{\partial w_i}{\partial x'_j} + \frac{\partial w_j}{\partial x'_i}\right).$$

Plugging the expansion into the governing equations and boundary conditions:

$$\mathcal{O}(\epsilon^0) \rightarrow \begin{cases} \dfrac{\partial u_i^{(0)}}{\partial x_i} = 0, \\ 0 = -\dfrac{\partial p^{(0)}}{\partial x_i}, \\ \dfrac{\partial \sigma_{ij}^{(0)}}{\partial x_j} = 0 \\ u_i^{(0)} = \dfrac{\partial v_i^{(0)}}{\partial t}, \\ \sigma_{ij}^{(0)} n_j = 0 \end{cases} \longrightarrow \boxed{p^{(0)} = p^{(0)}(\mathbf{x}', t)}$$

b.c.'s

Equation and boundary conditions on the solid stress tensor at leading order imply that

$$\sigma_{ij}^{(0)} = 0 \ \forall \ i,j \quad \text{i.e.} \quad C_{ijkl}(\varepsilon_{kl}(\mathbf{v}^{(0)})) = 0 \ \forall \ i,j$$

and thus $$\varepsilon_{kl}(\mathbf{v}^{(0)}) = 0 \ \forall \ k,l.$$

This yields: $$\boxed{\mathbf{v}^{(0)} = \mathbf{v}^{(0)}(\mathbf{x}', t)}$$

$$\mathcal{O}(\epsilon) \rightarrow \begin{cases} \dfrac{\partial u_i^{(1)}}{\partial x_i} + \dfrac{\partial u_i^{(0)}}{\partial x_i'} = 0, \\ \dfrac{\partial u_i^{(0)}}{\partial t} + \underline{u_j^{(0)} \dfrac{\partial u_i^{(0)}}{\partial x_j}} = \dfrac{\partial \Sigma_{ij}^{(0)}}{\partial x_j'} + \dfrac{\partial \Sigma_{ij}^{(1)}}{\partial x_j}, \\ 0 = \dfrac{\partial \sigma_{ij}^{(1)}}{\partial x_j} + \dfrac{\partial \sigma_{ij}^{(0)}}{\partial x_j'}, \\ \left.\begin{array}{l} u_i^{(1)} = \dfrac{\partial v_i^{(1)}}{\partial t}, \\ \dfrac{\rho_s}{\rho_f} \sigma_{ij}^{(1)} n_j = \Sigma_{ij}^{(0)} n_j = -p^{(0)} n_i \end{array}\right\} \text{b.c.'s} \end{cases}$$

$$\sigma_{ij}^{(0)} = C_{ijkl}(\varepsilon_{kl}(\mathbf{v}^{(0)})),$$

$$\sigma_{ij}^{(1)} = C_{ijkl}(\varepsilon_{kl}(\mathbf{v}^{(1)})) + C_{ijkl}(\varepsilon'_{kl}(\mathbf{v}^{(0)})),$$

$$\Sigma_{ij}^{(0)} = -p^{(0)}\delta_{ij},$$

$$\Sigma_{ij}^{(1)} = -p^{(1)}\delta_{ij} + \frac{2}{\mathrm{Re}}\varepsilon_{ij}(\mathbf{u}^{(0)}).$$

Equation and boundary conditions on the **solid** stress tensor at second order can be written:

$$\frac{\partial}{\partial x_j}\left\{C_{ijkl}\left[\varepsilon_{kl}(\mathbf{v}^{(1)})+\varepsilon'_{kl}(\mathbf{v}^{(0)})\right]\right\}=0 \quad \text{on } V_s,$$

$$\frac{\rho_s}{\rho_f}\left\{C_{ijkl}\left[\varepsilon_{kl}(\mathbf{v}^{(1)})+\varepsilon'_{kl}(\mathbf{v}^{(0)})\right]\right\}n_j=-p^{(0)}n_i \quad \text{on } \Gamma$$

plus V-periodicity. This is a linear differential form for $\mathbf{v}^{(1)}$ forced by $\mathbf{v}^{(0)}$ and $p^{(0)}$. We can thus write:

$$\boxed{\mathbf{v}^{(1)}(\mathbf{x},\mathbf{x}',t)=\chi_i^{pq}(\mathbf{x})\varepsilon'_{pq}(\mathbf{v}^{(0)})(\mathbf{x}',t)-\eta_i(\mathbf{x})p^{(0)}(\mathbf{x}',t)}$$

Equation and boundary conditions for the **fluid** after treating the convective term *a-la*-Oseen to linearize the equations

$$u_j^{(0)}\frac{\partial u_i^{(0)}}{\partial x_j} \approx U_j \frac{\partial u_i^{(0)}}{\partial x_j} \qquad \text{with} \qquad U_j := \frac{1}{V_{Tot}}\int_{V_{Tot}} \langle u_j^{(0)}\rangle \, dV$$

yields:

$$U_j \frac{\partial u_i^{(0)}}{\partial x_j} = -\frac{\partial p^{(1)}}{\partial x_i} - \frac{\partial p^{(0)}}{\partial x_i'} + \frac{\partial^2 u_i^{(0)}}{\partial x_j^2} \qquad \text{(if steady)}$$

$$\frac{\partial u_i^{(0)}}{\partial x_i} = 0$$

$$\boxed{u_i^{(0)} = -K_{ij}\frac{\partial p^{(0)}}{\partial x_j'}, \qquad p^{(1)} = -A_j \frac{\partial p^{(0)}}{\partial x_j'} + p_0^{(1)}(\mathbf{x}', t),}$$

Volume averaging for a quantity *g* defined over either V_s or V_f

$$\langle g \rangle = \frac{1}{V} \int_{V_f} g \, dV_x$$

$$\langle g \rangle = \frac{1}{V} \int_{V_s} g \, dV_x$$

To interchange differentiation and integration we use

Theorem (Spatial averaging theorem)

Let V_p $(p = s, f)$ be an elementary volume and $g_p(\mathbf{x}, t)$ a continuously differentiable function defined on V_p. In addition, we require that V_p is continuously differentiable in order to guarantee that the integral

$$\int_{V_p} g_p \, dV$$

is continuously differentiable. Then:

$$\langle \nabla g_p \rangle = \nabla \langle g_p \rangle + \int_{\partial V_p} g_p \cdot \mathbf{n} \, dS$$

which is a 3D version of Leibniz rule
(Marle, 1967; Whitaker, 1967)

System of equations after averaging:

$$
\begin{cases}
\dfrac{\partial \langle u_i^{(0)} \rangle}{\partial t} + (1-\vartheta)\dfrac{\rho_s}{\rho_f}\dfrac{\partial^2 v_i^{(0)}}{\partial t^2} = \dfrac{\partial}{\partial x'_j}\left[\mathcal{C}_{ijpq}\varepsilon'_{pq}(\mathbf{v}^{(0)}) - \alpha'_{ij}p^{(0)}\right], \\[2ex]
\dfrac{\partial \langle u_i^{(0)} \rangle}{\partial x'_i} = \langle \dfrac{\partial \chi_i^{pq}}{\partial x_i} \rangle \varepsilon'_{pq}(\dot{\mathbf{v}}^{(0)}) - \langle \dfrac{\partial \eta_i}{\partial x_i} \rangle \dot{p}^{(0)}, \\[2ex]
\langle u_i^{(0)} - \dot{v}_i^{(0)} \rangle(\cancel{\mathbf{x}}, \mathbf{x}', t) = -\int_0^t \langle K_{il} \rangle (\cancel{\mathbf{x}}, t-t') \left(\dfrac{\partial p^{(0)}}{\partial x'_l}(\mathbf{x}', t') + \ddot{v}_i^{(0)}(\mathbf{x}', t') \right) dt'
\end{cases}
$$

plus boundary conditions.

$$\vartheta = \frac{V_f}{V}, \qquad \mathbf{v}^{(0)} = \mathbf{v}^{(0)}(\mathbf{x}', t), \qquad p^{(0)} = p^{(0)}(\mathbf{x}', t)$$

$$\mathcal{C}_{ijpq} = F(\langle C_{ijpq} \rangle)$$

Resulting system of equations:

$$\begin{cases} \dfrac{\partial \langle u_i^{(0)} \rangle}{\partial t} + (1-\vartheta)\dfrac{\rho_s}{\rho_f}\dfrac{\partial^2 v_i^{(0)}}{\partial t^2} = \dfrac{\partial}{\partial x'_j}\left[\mathcal{C}_{ijpq}\varepsilon'_{pq}(\mathbf{v}^{(0)}) - \alpha'_{ij}p^{(0)}\right], \\[2ex] \dfrac{\partial \langle u_i^{(0)} \rangle}{\partial x'_i} = \langle \dfrac{\partial \chi_i^{pq}}{\partial x_i} \rangle \varepsilon'_{pq}(\dot{\mathbf{v}}^{(0)}) - \langle \dfrac{\partial \eta_i}{\partial x_i} \rangle \dot{p}^{(0)}, \\[2ex] \langle u_i^{(0)} - \dot{v}_i^{(0)} \rangle(\cancel{\mathbf{x}}, \mathbf{x}', t) = -\int_0^t \langle K_{il} \rangle(\cancel{\mathbf{x}}, t-t') \left(\dfrac{\partial p^{(0)}}{\partial x'_l}(\mathbf{x}', t') + \ddot{v}_i^{(0)}(\mathbf{x}', t') \right) dt' \end{cases}$$

$$\begin{cases} \dfrac{\partial}{\partial x_j}\left\{C_{ijkl}\left[\varepsilon_{kl}(\chi^{pq}) + \delta_{kp}\delta_{lq}\right]\right\} = 0, \\[2ex] \left\{C_{ijkl}\left[\varepsilon_{kl}(\chi^{pq}) + \delta_{kp}\delta_{lq}\right]\right\} n_j = 0, \end{cases} \qquad \begin{cases} \dfrac{\partial}{\partial x_j}\left[C_{ijkl}\ \varepsilon_{kl}(\eta)\right] = 0, \\[2ex] \left[C_{ijkl}\ \varepsilon_{kl}(\eta)\right] n_j = n_i, \end{cases}$$

Resulting system of equations:

$$
\begin{cases}
\dfrac{\partial\langle u_i^{(0)}\rangle}{\partial t} + (1-\vartheta)\dfrac{\rho_s}{\rho_f}\dfrac{\partial^2 v_i^{(0)}}{\partial t^2} = \dfrac{\partial}{\partial x'_j}\left[\mathcal{C}_{ijpq}\varepsilon'_{pq}(\mathbf{v}^{(0)}) - \alpha'_{ij}p^{(0)}\right], \\[2ex]
\dfrac{\partial\langle u_i^{(0)}\rangle}{\partial x'_i} = \langle\dfrac{\partial\chi_i^{pq}}{\partial x_i}\rangle\varepsilon'_{pq}(\dot{\mathbf{v}}^{(0)}) - \langle\dfrac{\partial\eta_i}{\partial x_i}\rangle\dot{p}^{(0)}, \\[2ex]
\langle u_i^{(0)} - \dot{v}_i^{(0)}\rangle(\cancel{\mathbf{x}},\mathbf{x}',t) = -\int_0^t \langle K_{il}\rangle(\cancel{\mathbf{x}}, t-t')\left(\dfrac{\partial p^{(0)}}{\partial x'_l}(\mathbf{x}',t') + \ddot{v}_i^{(0)}(\mathbf{x}',t')\right)dt'
\end{cases}
$$

$$
\begin{cases}
\dfrac{\partial K_{ij}}{\partial t} + \underline{U_k\dfrac{\partial K_{ij}}{\partial x_k}} \approx -\dfrac{\partial A_j}{\partial x_i} + \dfrac{1}{\mathrm{Re}}\dfrac{\partial^2 K_{ij}}{\partial x_k^2}, \\[2ex]
\dfrac{\partial K_{ij}}{\partial x_i} = 0, \\[2ex]
K_{ij}(\mathbf{x},t) = 0, \quad \text{over } \Gamma \\[2ex]
K_{ij}(\mathbf{x},0) = \delta_{ij},
\end{cases}
$$

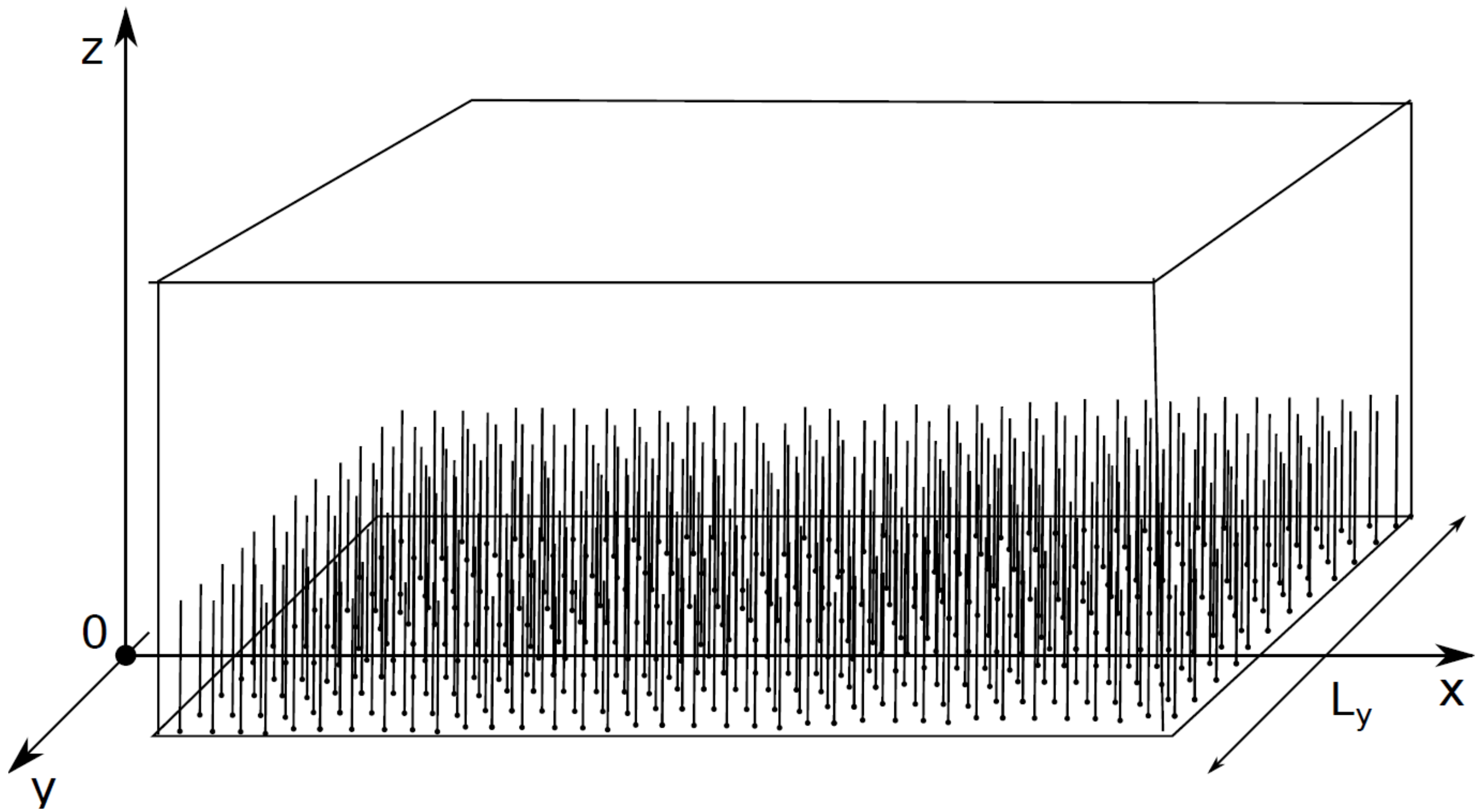
Dynamic permeability
with account of inertia (Oseen)

$$
\boxed{U_k := \frac{1}{V_{Tot}}\int_{V_{Tot}} \langle u_k^{(0)}\rangle\, dV}
$$

(iterations needed)

Simple example: RIGID system in steady state

The porous system considered is *transversely* isotropic



Darcy equation

$$\boxed{\langle u_i^{(0)} \rangle = -\mathcal{K}_{ij} \frac{\partial p^{(0)}}{\partial x'_j}} \quad \text{with} \quad \mathcal{K}_{ij} = \langle K_{ij} \rangle$$

with anisotropic permeability (in reality we have $K_{11} = K_{22}$ and K_{33}; the off-diagonal terms vanish.

$$\boxed{\frac{\partial K_{ij}}{\partial x_i} = 0, \qquad -\frac{\partial A_j}{\partial x_i} + \nabla^2 K_{ij} = -\delta_{ij}}$$

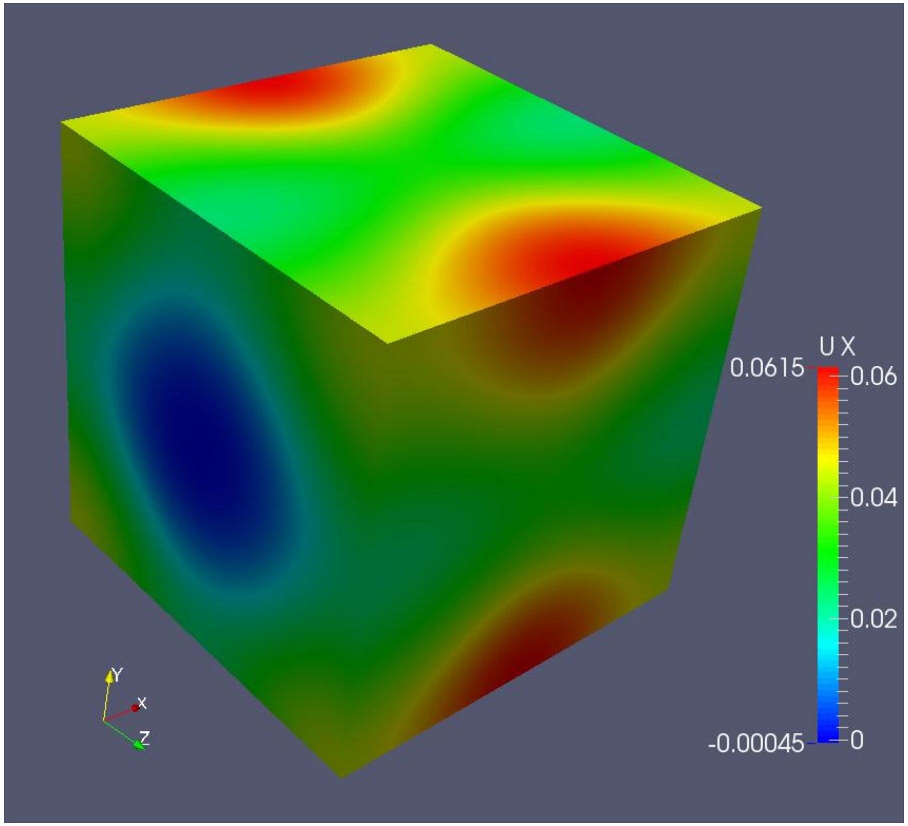
(small Re)

$$\left\{ \begin{array}{l} Re U_k \dfrac{\partial K_{ij}}{\partial x_k} \simeq -\dfrac{\partial A_j}{\partial x_i} + \dfrac{\partial^2 K_{ij}}{\partial x_k^2} + \delta_{ij} \\ \dfrac{\partial K_{ij}}{\partial x_i} = 0, \\ K_{ij}(\mathbf{x}, t) = 0 \quad on\ \Gamma, \end{array} \right.$$

(large Re)

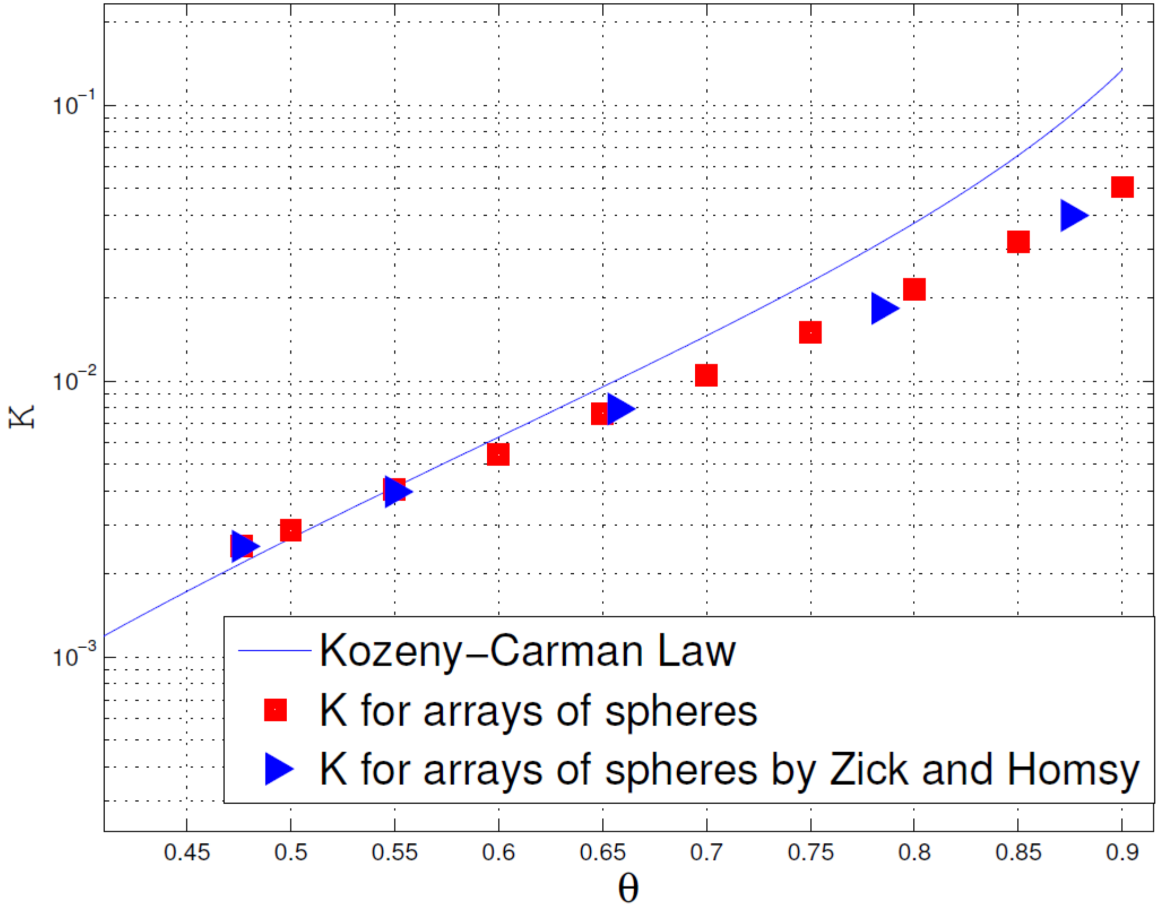
$$U_k := \frac{1}{V_{Tot}} \int_{V_{Tot}} \langle u_k^{(0)} \rangle \, dV$$

Small Re (isotropic case)

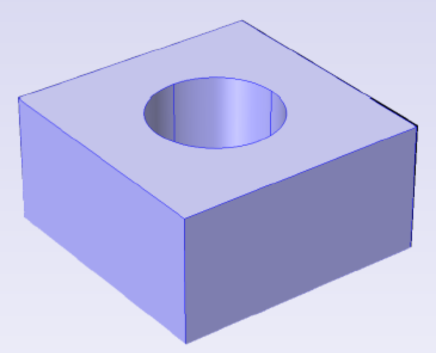


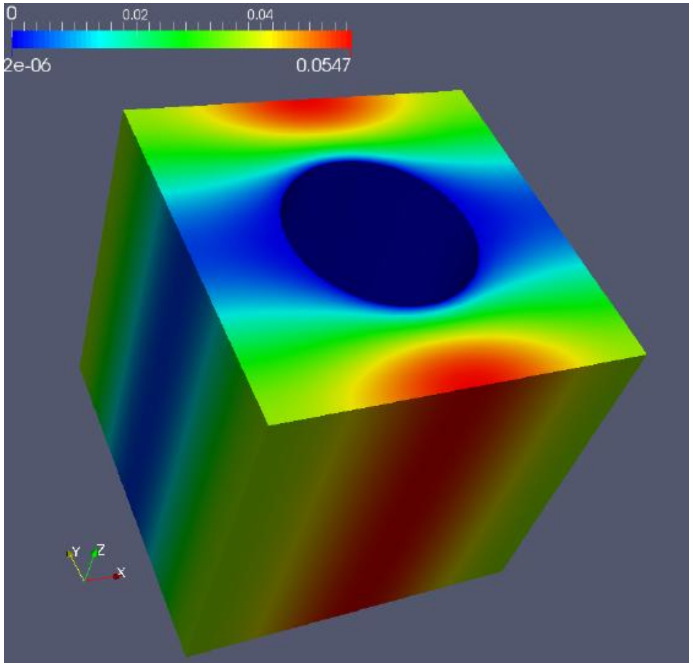
$$K_{ij} = K\delta_{ij}$$

Small Re (isotropic case)

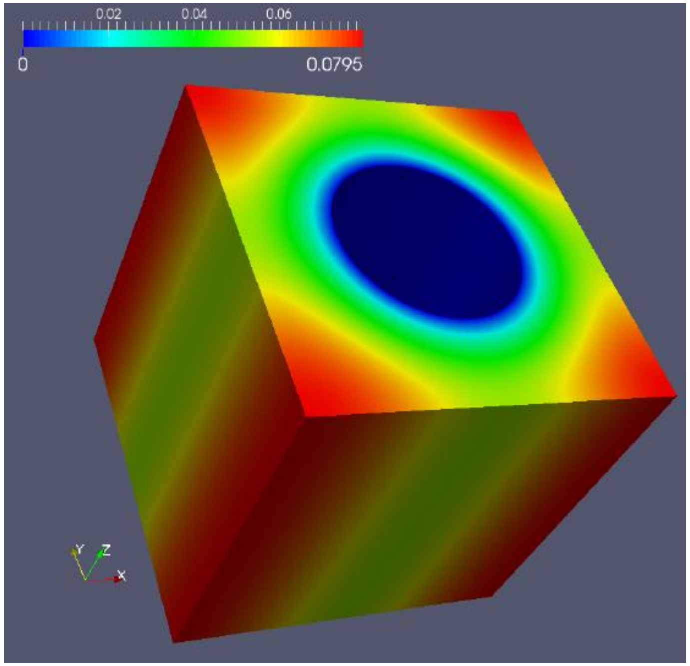


Small Re (transversely isotropic case)



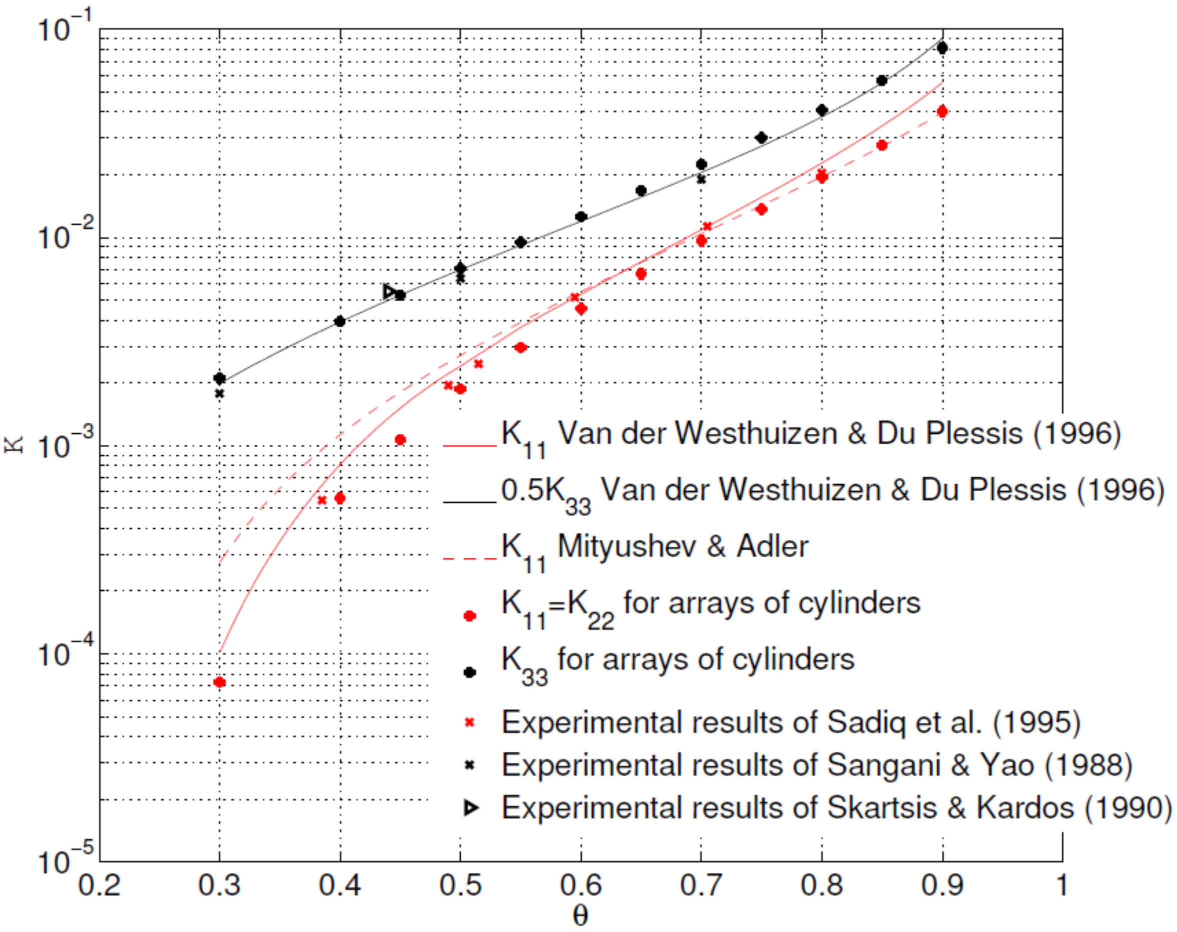


$$K_{11} = K_{22}$$



$$K_{33}$$

Small Re (transversely isotropic case)



Large Re (transversely isotropic case)

$$Re U_k \frac{\partial K_{ij}}{\partial x_k} \simeq -\frac{\partial A_j}{\partial x_i} + \frac{\partial^2 K_{ij}}{\partial x_k^2} + \delta_{ij}$$

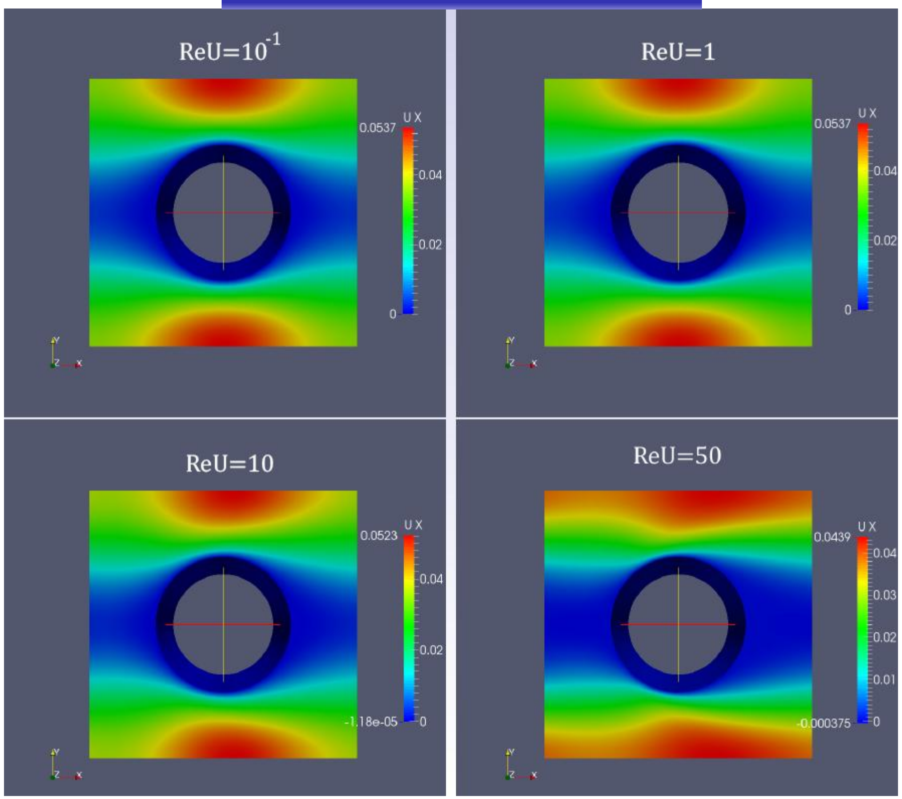
$$\frac{\partial K_{ij}}{\partial x_i} = 0,$$

$$K_{ij}(\mathbf{x}, t) = 0 \quad on\ \Gamma,$$

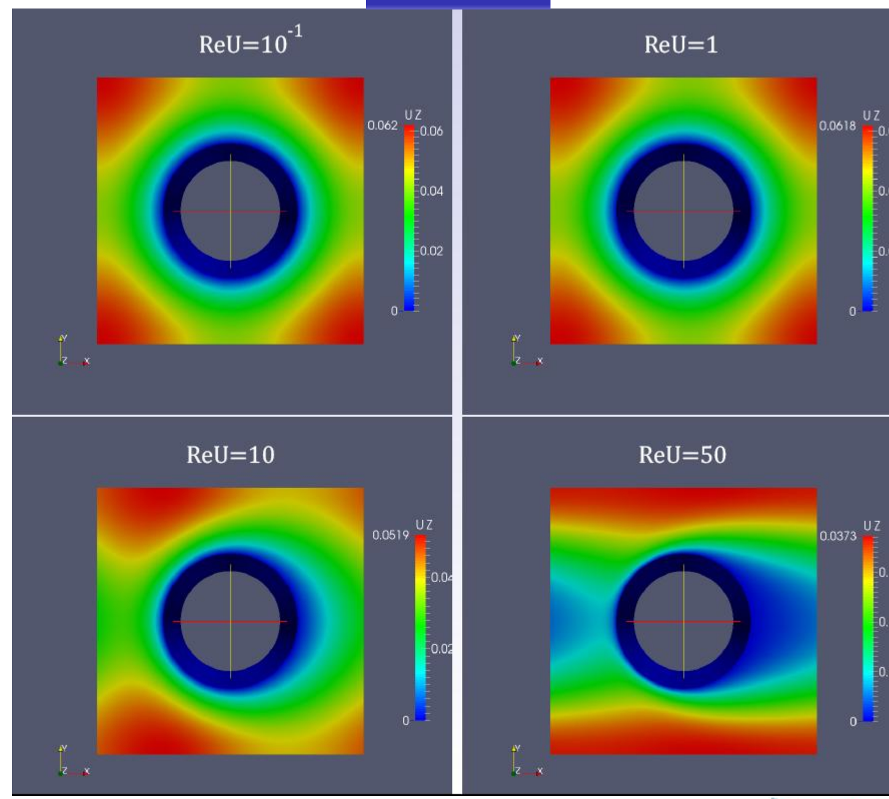
1. Test the iterative procedure and find K(θ, Re)
2. Validate against DNS (which captures the flow in the space within filaments)

Effect of inertia

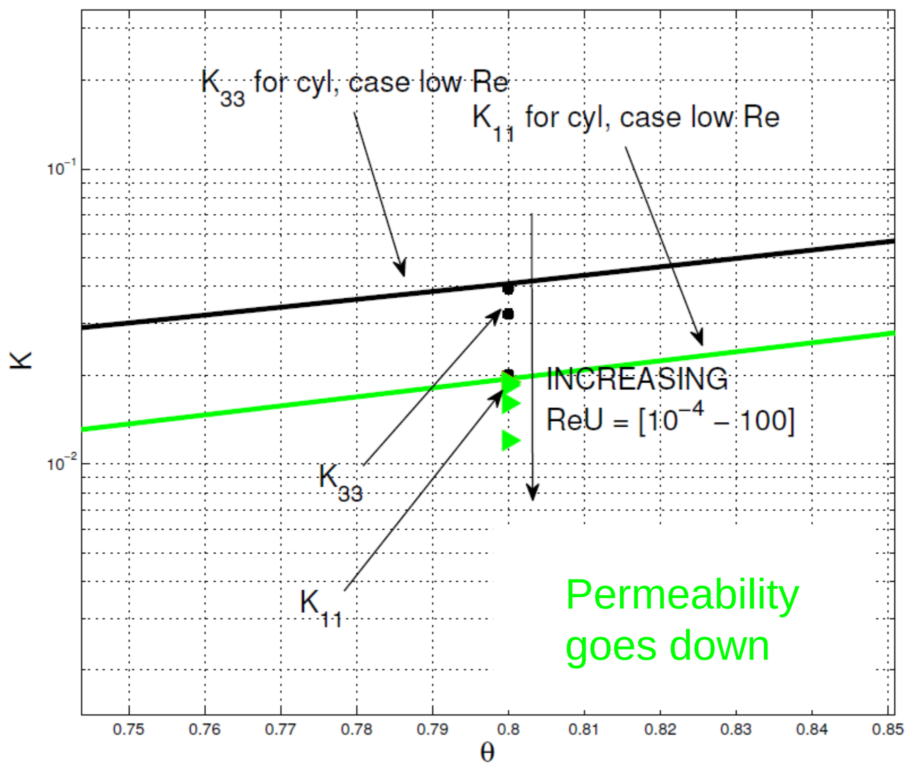
$$K_{11} = K_{22}$$

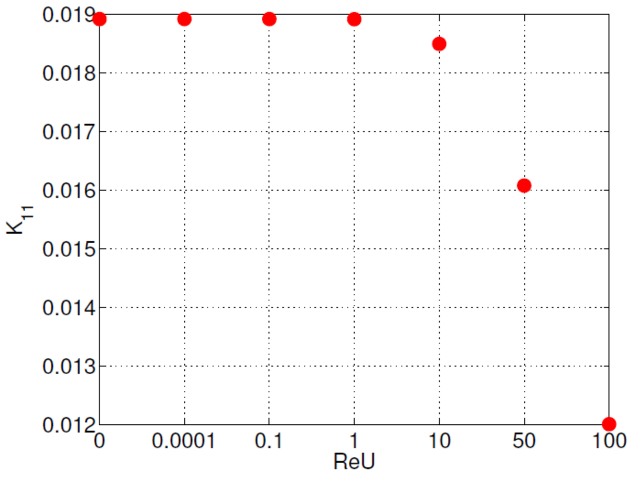


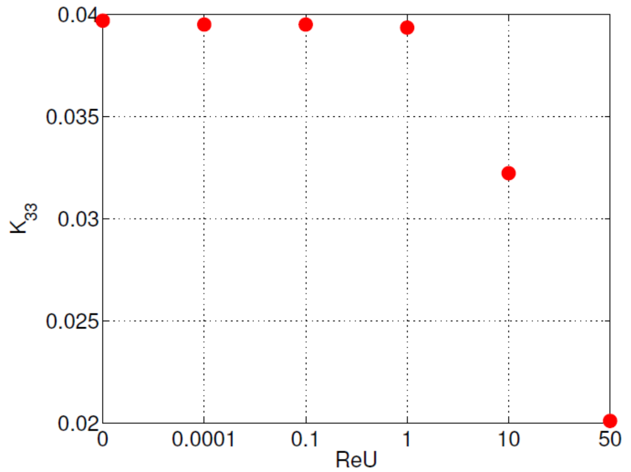
$$K_{33}$$



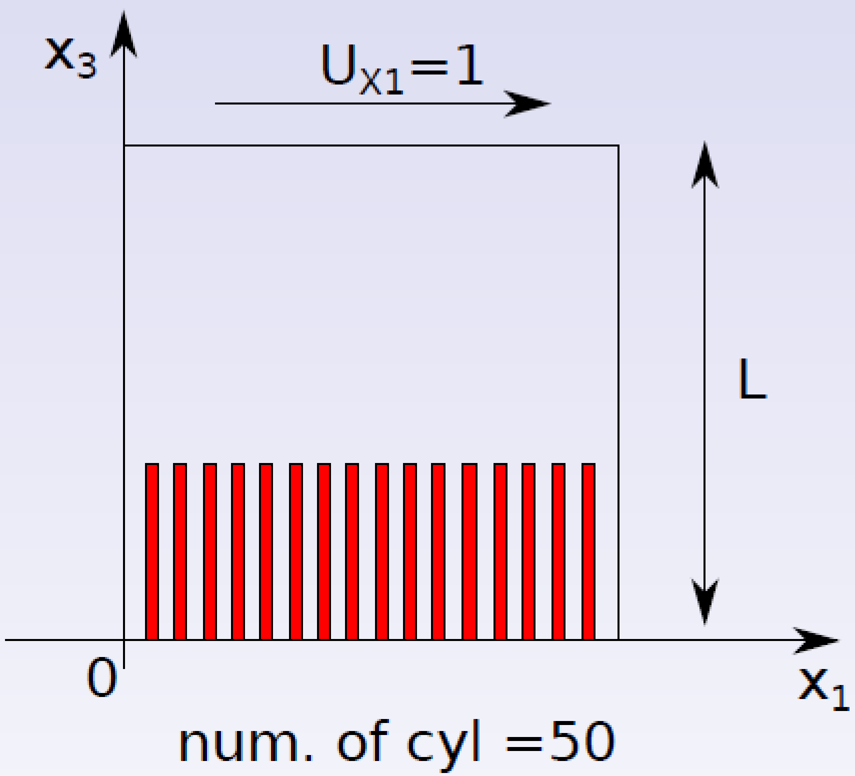
Effect of inertia (after averaging)





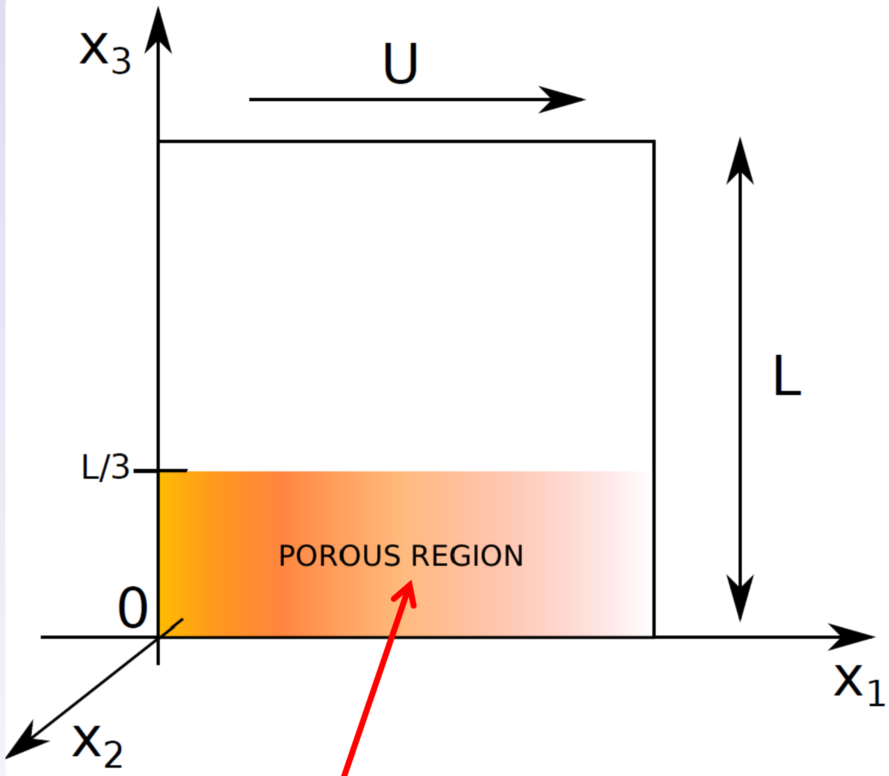


Case studied

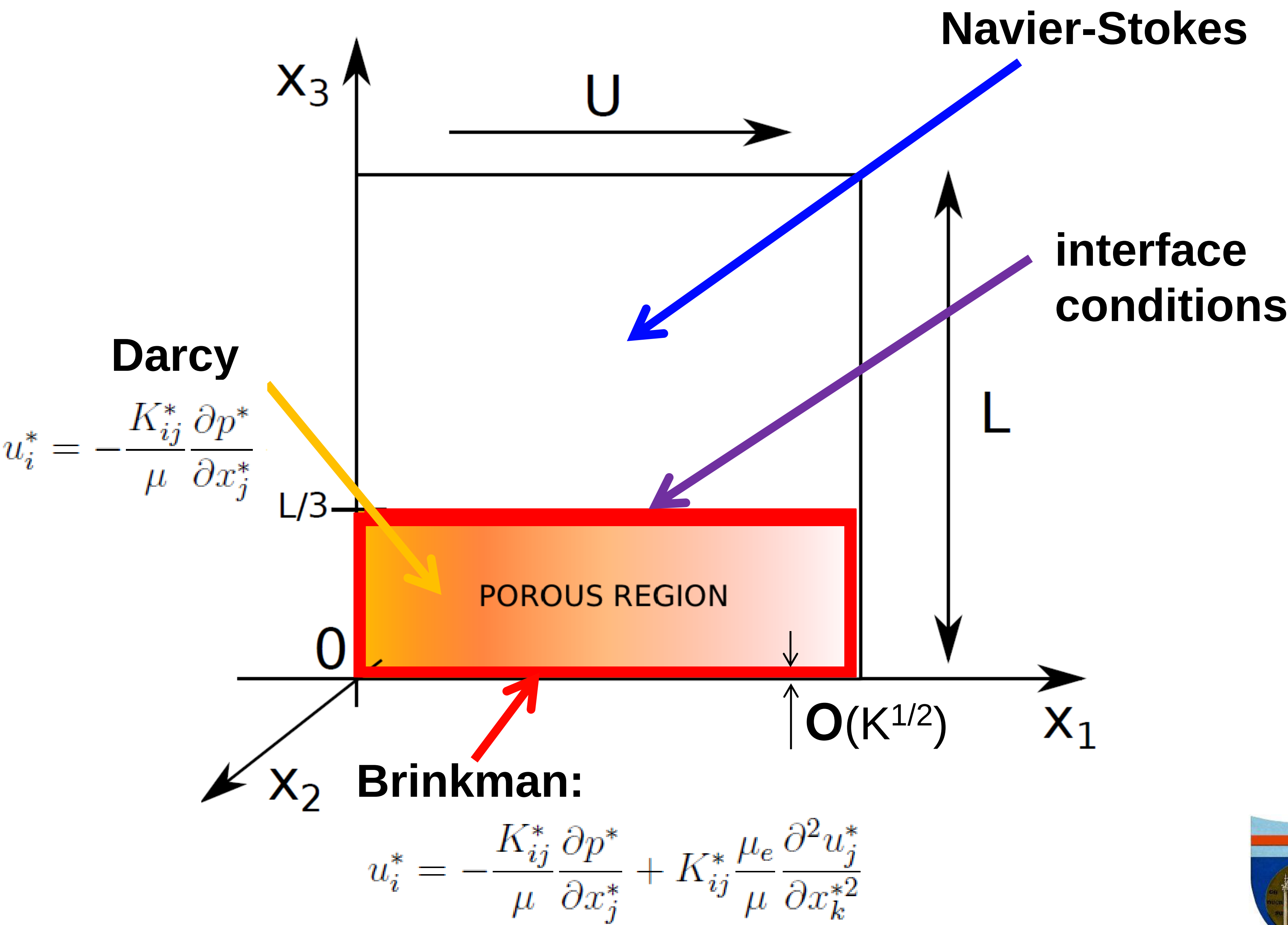


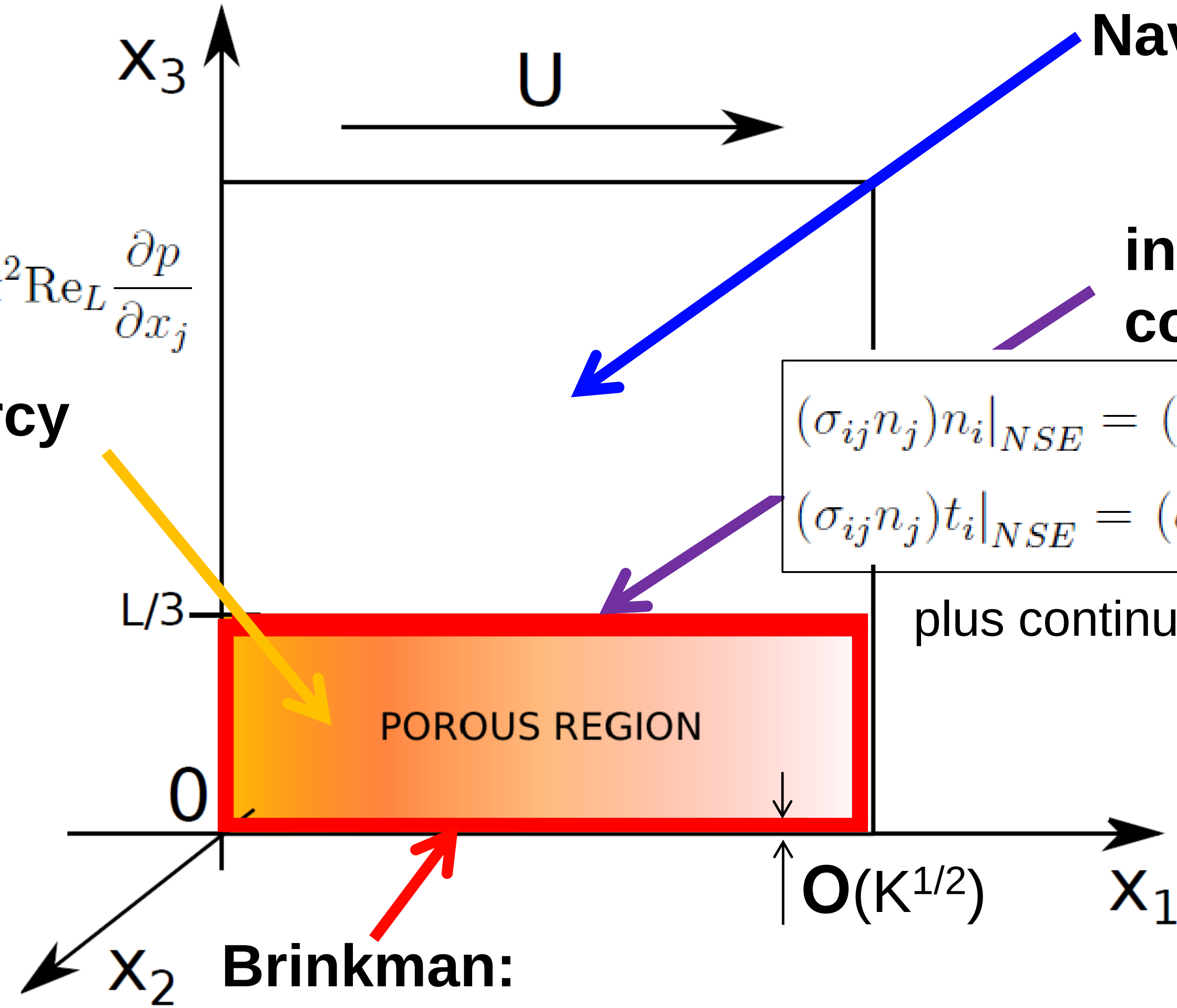
$\epsilon = 0.02$

$\boxed{Re = 2}$



Darcy
(and the boundaries?)


Navier-Stokes
interface conditions
Darcy
$u_i^* = -\frac{K_{ij}^*}{\mu}\frac{\partial p^*}{\partial x_j^*}$
x_3
U
L
L/3
0
POROUS REGION
$O(K^{1/2})$
x_1
x_2
Brinkman:
$u_i^* = -\frac{K_{ij}^*}{\mu}\frac{\partial p^*}{\partial x_j^*} + K_{ij}^*\frac{\mu_e}{\mu}\frac{\partial^2 u_j^*}{\partial x_k^{*2}}$



$$u_i = -\mathcal{K}_{ij}\epsilon^2 \mathrm{Re}_L \frac{\partial p}{\partial x_j}$$

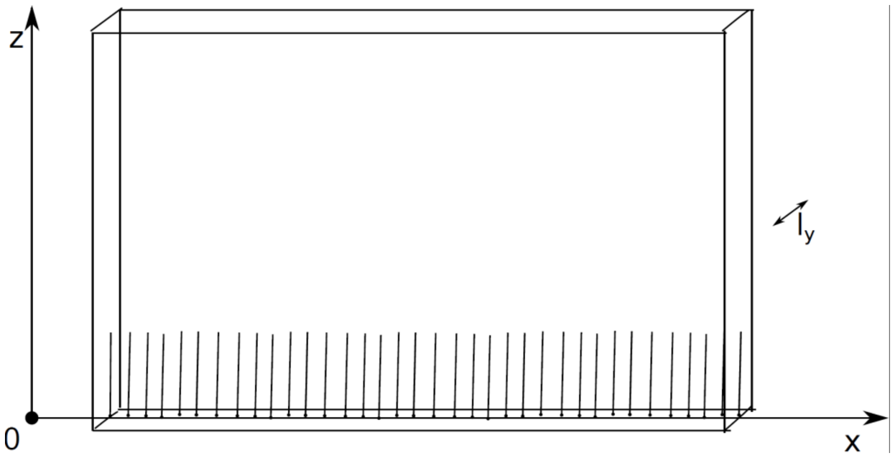
$$(\sigma_{ij}n_j)n_i|_{NSE} = (\sigma_{ij}n_j)n_i|_{BRINK}$$

$$(\sigma_{ij}n_j)t_i|_{NSE} = (\sigma_{ij}n_j)t_i|_{BRINK}$$

$$u_i = -\mathcal{K}_{ij}\epsilon^2 \mathrm{Re}_L \frac{\partial p}{\partial x_j} + \mathcal{K}_{ij}\epsilon^2 \frac{\mu_e}{\mu}\frac{\partial^2 u_j}{\partial x_k^2}$$

3D DNS (periodicity along y)

$\mathrm{Re}_L = 100$

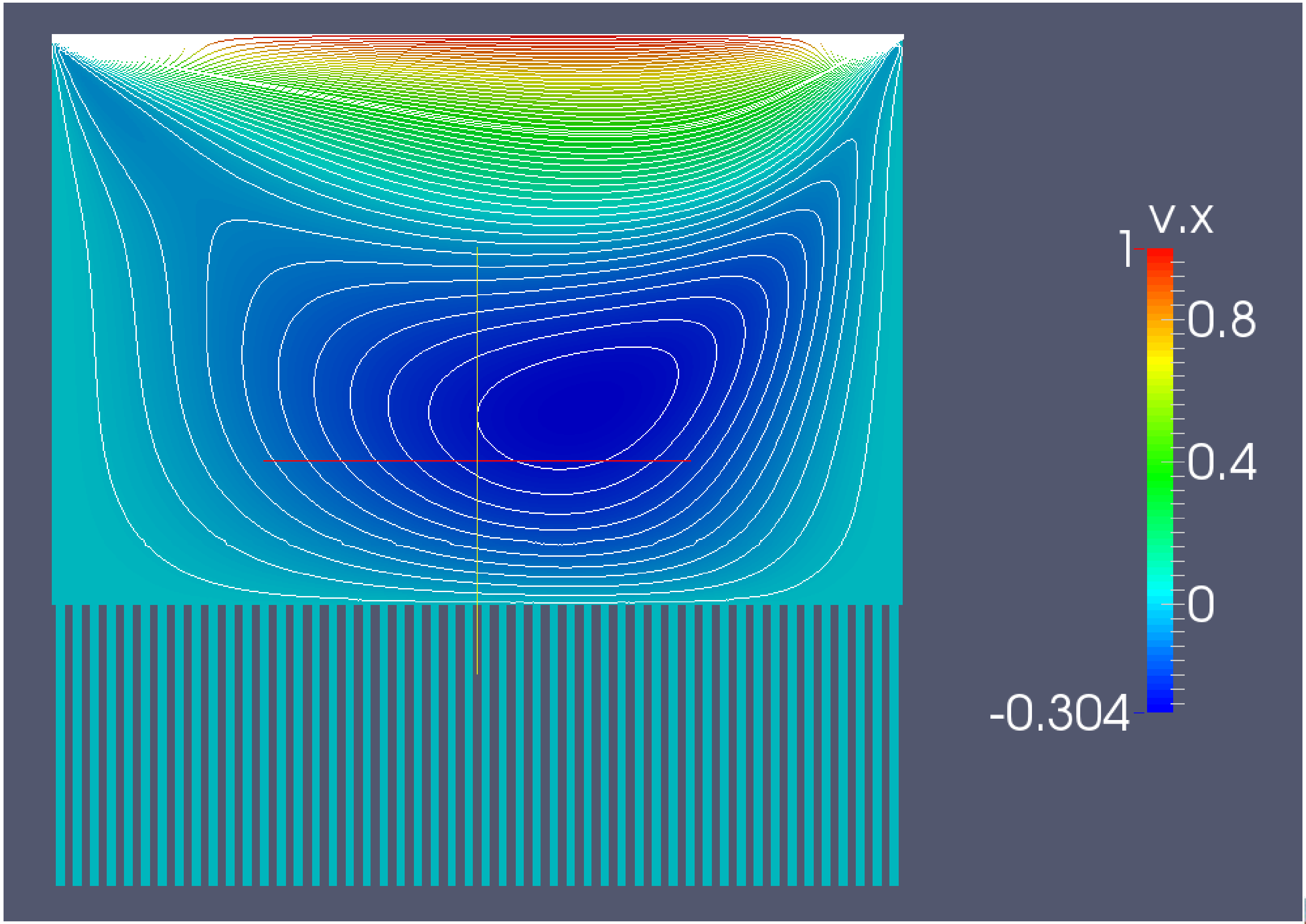


The DNS permits to compute $\mathcal{K}_{ij}$ which satisfies Darcy in the “bulk”; then, having established the value of $\mathcal{K}_{11}$ we can find the effective viscosity μ_e to use in the *Brinkman filter*

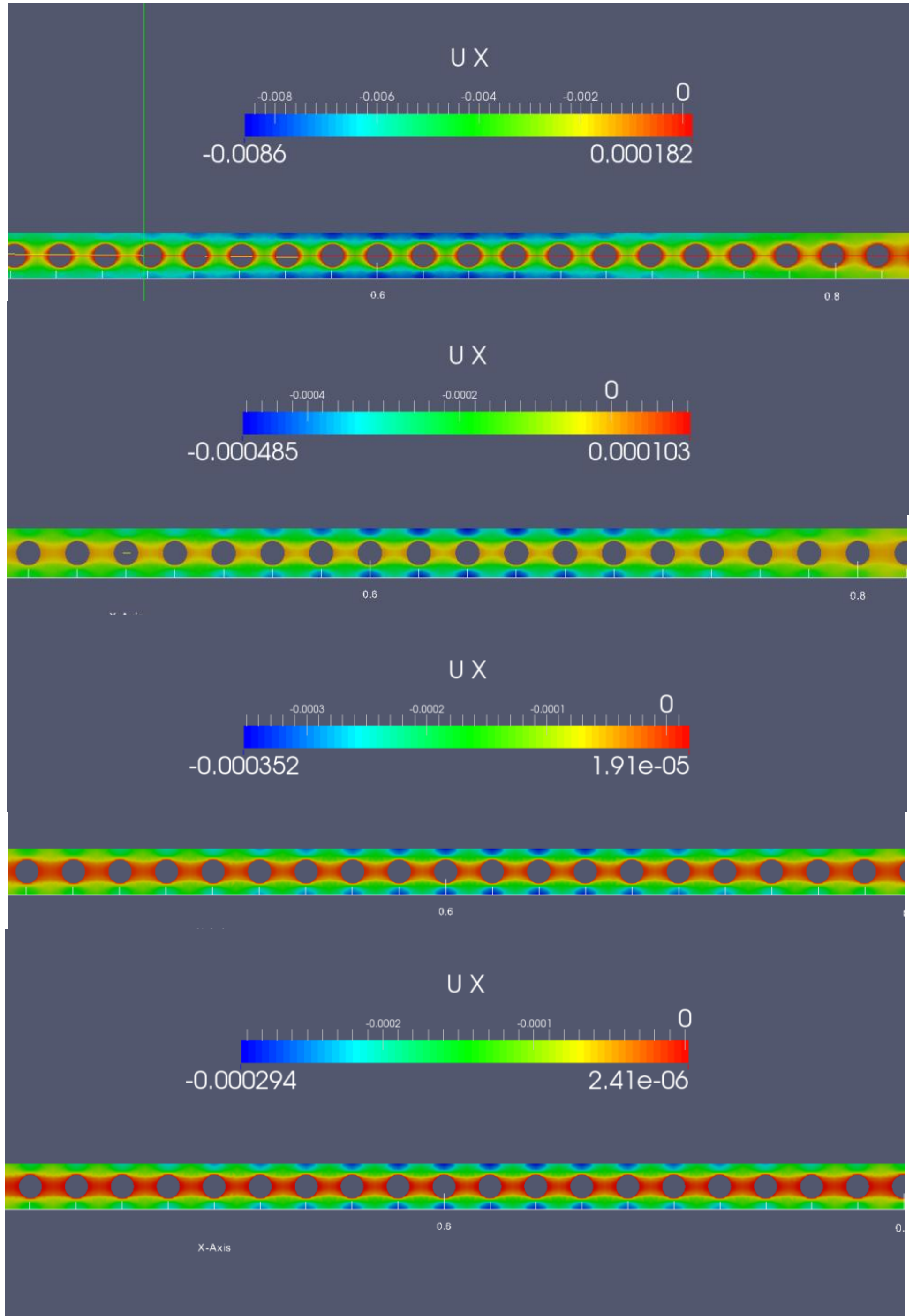
$$\boxed{\frac{\mu_e}{\mu} = u_1 \left(\mathcal{K}_{11} \epsilon^2 \frac{\partial^2 u_1}{\partial x_k^2} \right)^{-1} + \left(\frac{\partial^2 u_1}{\partial x_k^2} \right)^{-1} \mathcal{K}_{11} \epsilon^2 \mathrm{Re}_L \frac{\partial p}{\partial x_1}}$$

3D DNS

$Re_L = 100$



3D DNS



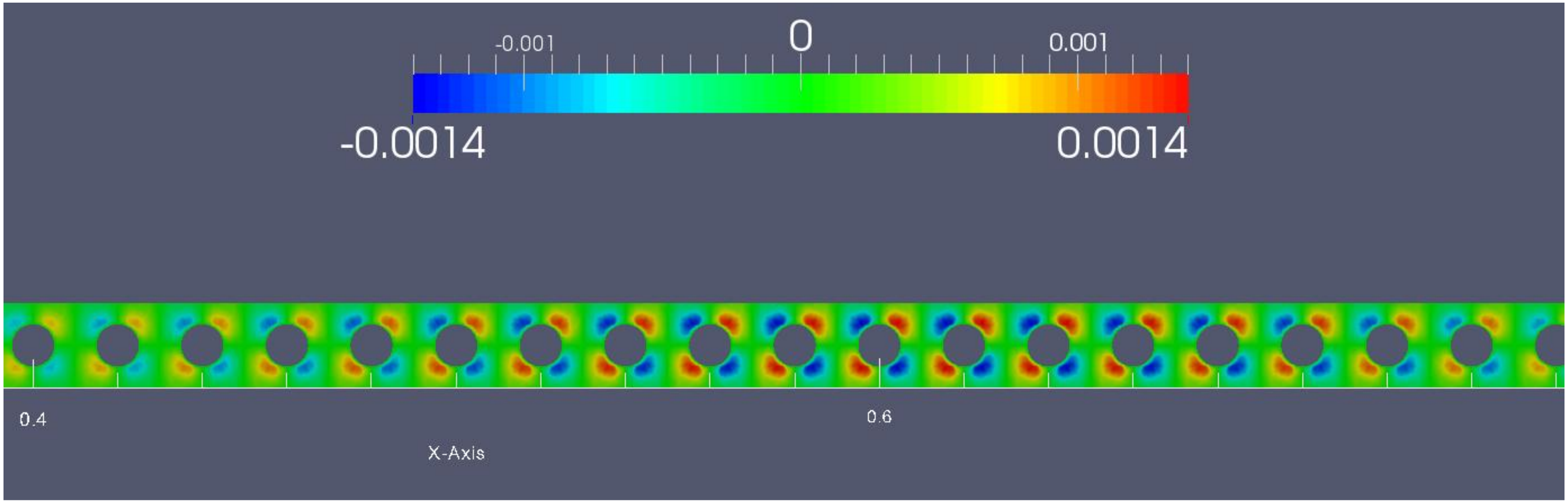
$x_3 = 0.333$ (interface)

$x_3 = 0.3$

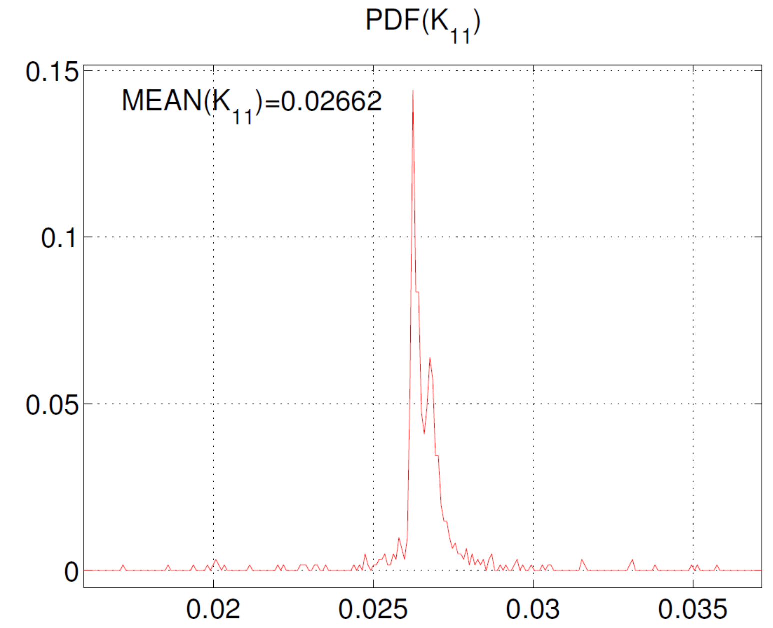
$x_3 = 0.2$

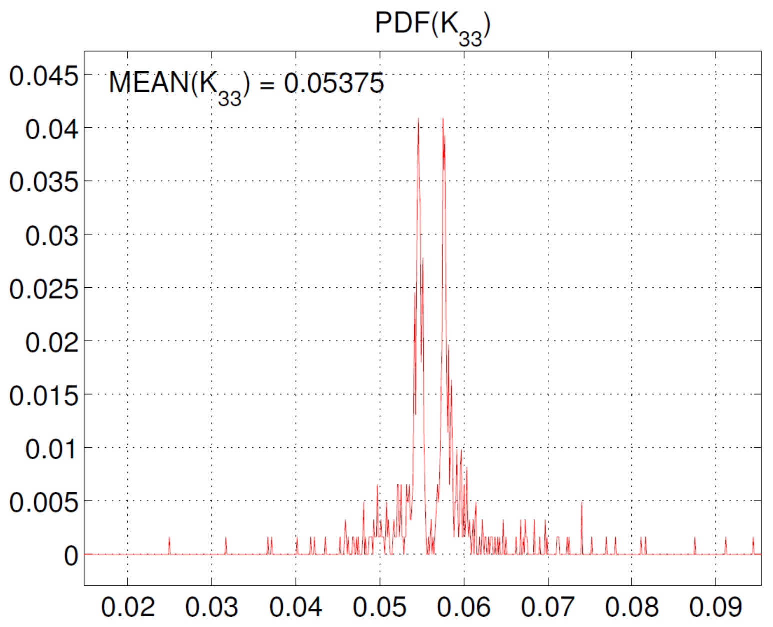
$x_3 = 0.1$

3D DNS Transverse velocity at the interface

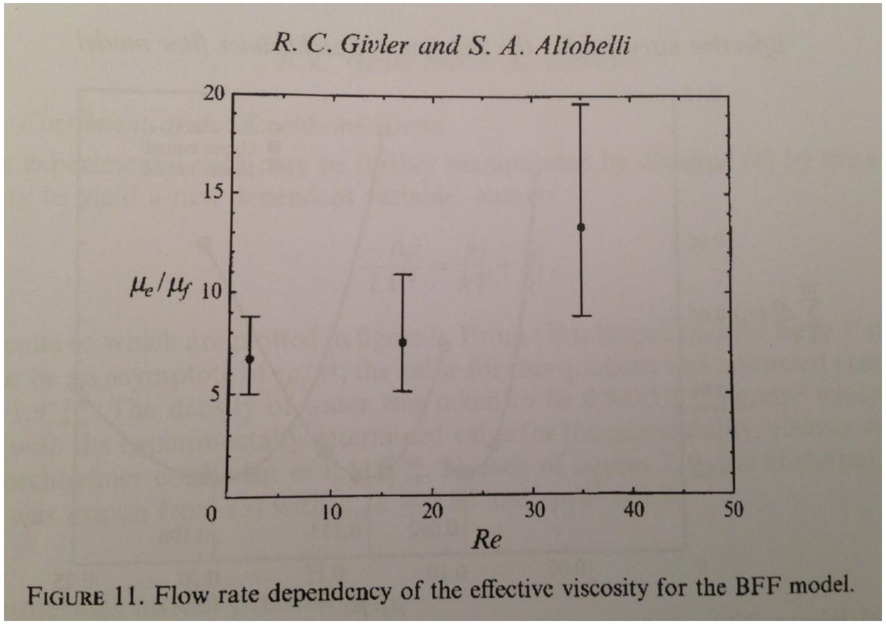


3D DNS

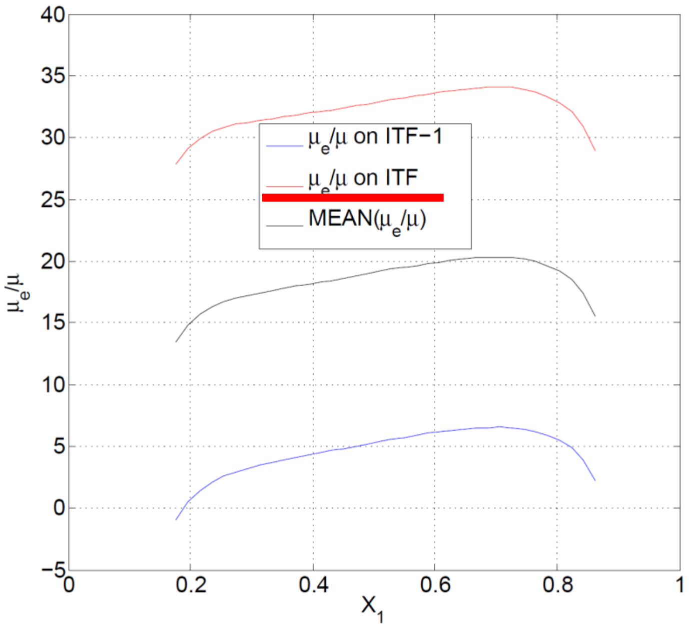

PDF(K_{11})
MEAN(K_{11})=0.02662
0.15
0.1
0.05
0
0.02
0.025
0.03
0.035


PDF(K_{33})
MEAN(K_{33}) = 0.05375
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09

3D DNS



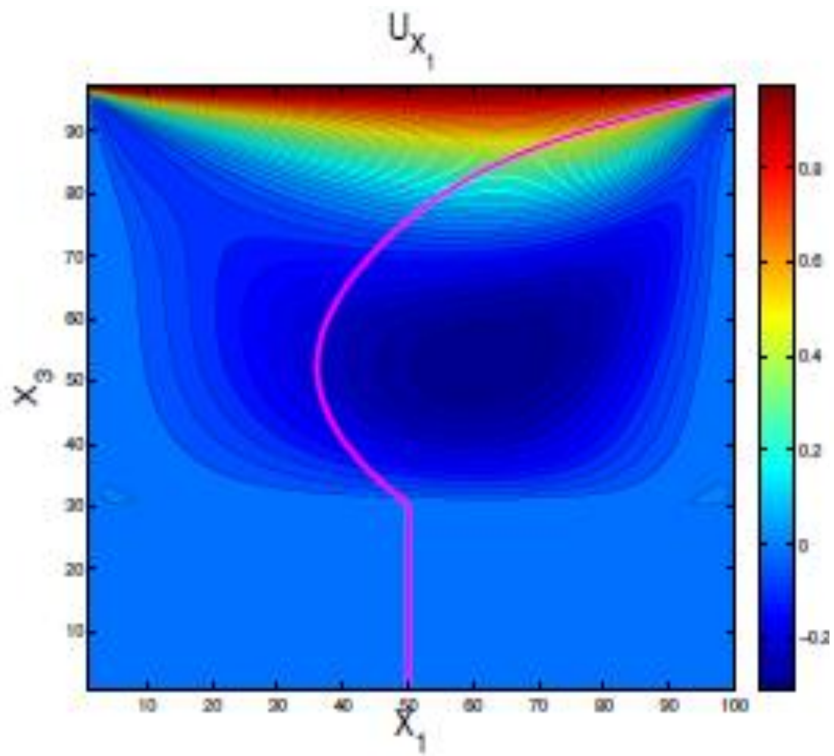
Givler & Altobelli
JFM 1994

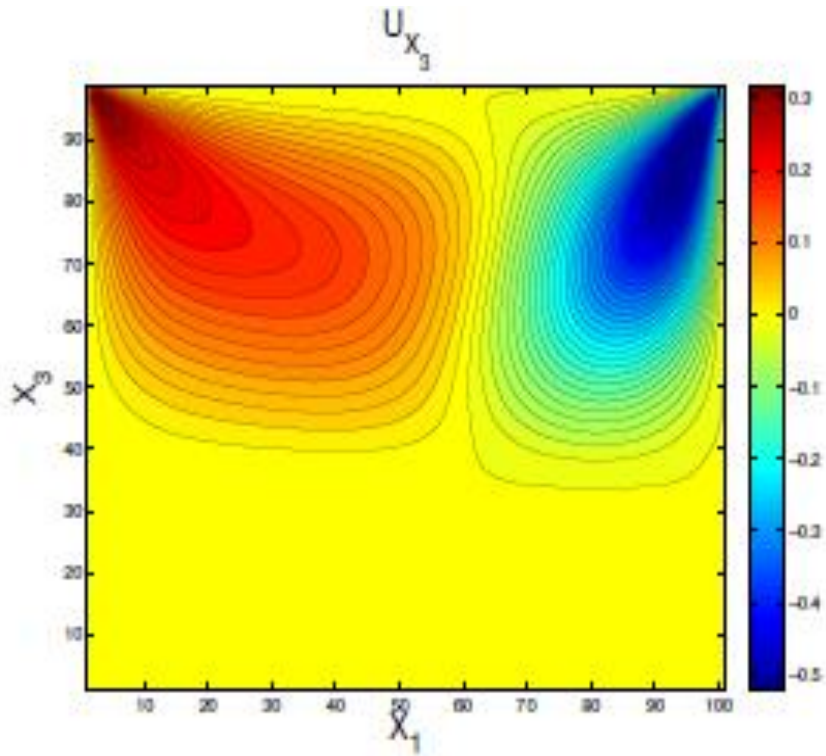


$Re_L = 100$

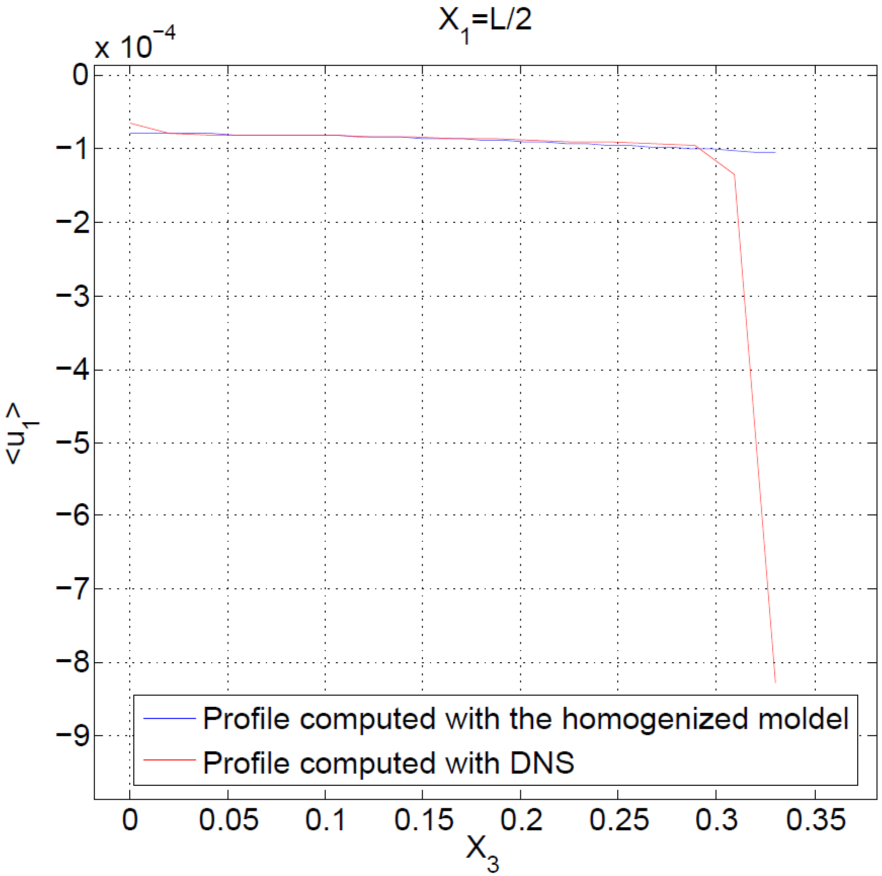
2D NS + Darcy in transversely isotropic medium, with inertia, $K_{ij} = K_{ij}(\theta, Re)$, no *Brinkman filter* at the boundary

$$\langle K_{11} \rangle = 1.9{\cdot}10^{-2}, \quad \langle K_{33} \rangle = 3.9{\cdot}10^{-2}$$





Agreement 3D DNS/model is acceptable, **not yet perfect** because of the "boundary layer" developing near the interface.



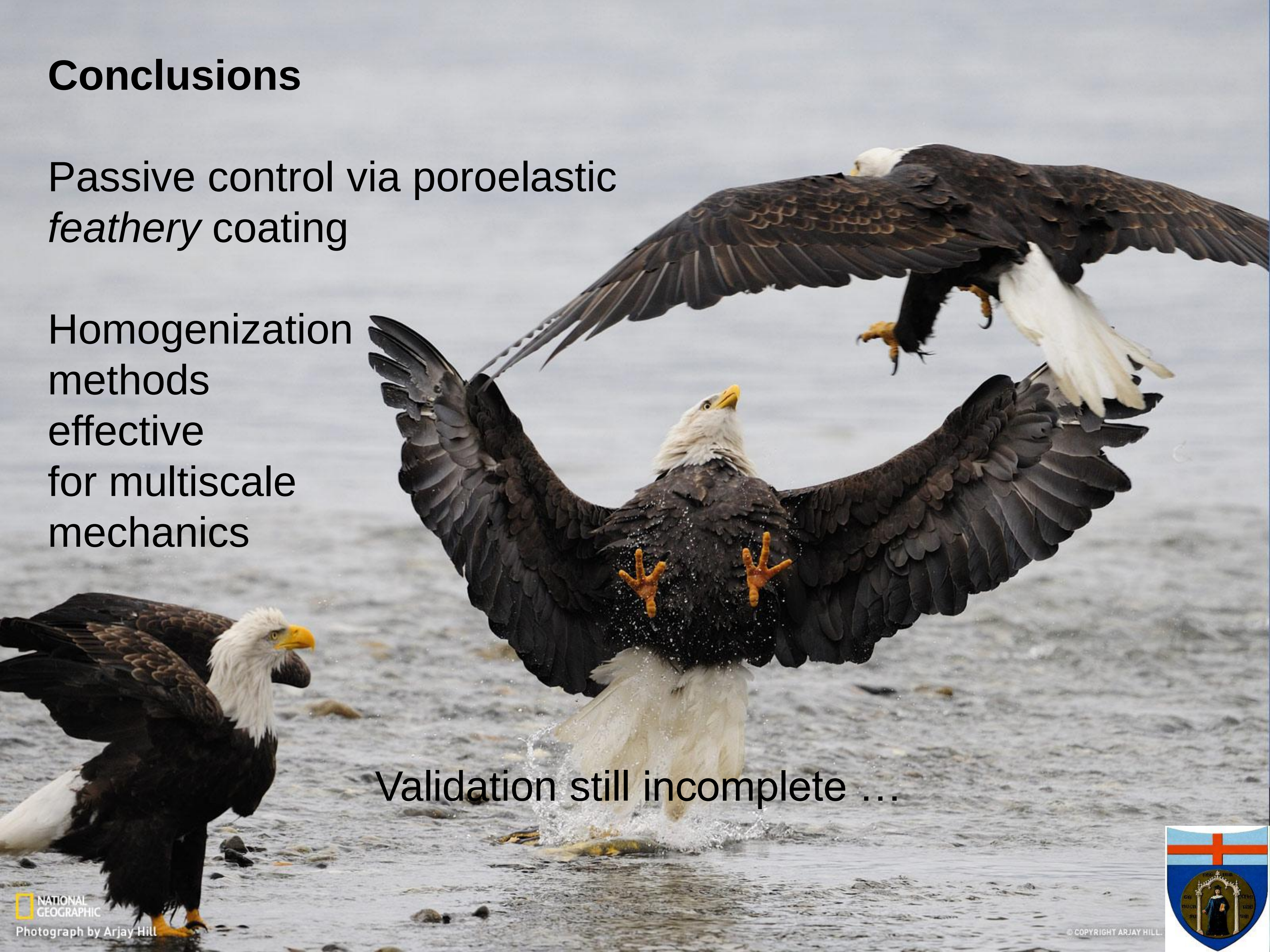
Need to implement Brinkman filter.

Conclusions

Passive control via poroelastic *feathery* coating

Homogenization methods effective for multiscale mechanics

Validation still incomplete …



Conclusions

Lots of interesting perspectives ahead!

THANK YOU!