BIOSKINS:
Beyond Darcy's law in deformable porous media

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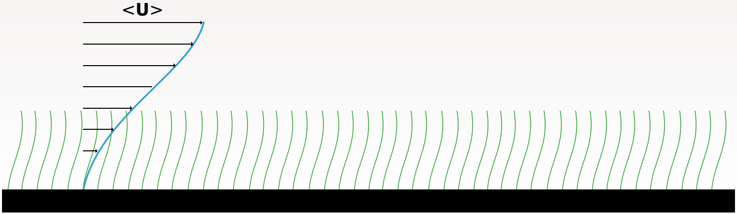
Overview

Plan

Typical poro-elastic layer



Parameters:

- rigidity: restoring force vs hydrodynamic load
- sparsity
- macroscopic Reynolds number: $\frac{hU}{\nu}$
- ...

For a certain range of parameters we observe a regime of large scale turbulence (honami). Our final goal is to capture this large scale turbulence through homogenized model.

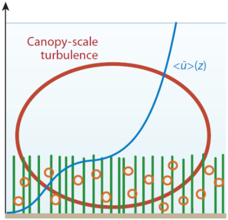
Regime of large scale turbulence

- flow instability
- flow-structure coupling, coherent eddies
- ℓ_{micro}, L_{macro}

$$\begin{cases} u = \bar{u} + u' \\ v = \bar{v} + v' \end{cases} \quad (1)$$

Vertical momentum transfert

- $u' > 0$, $v' < 0$: sweep
- $u' < 0$, $v' > 0$: ejection



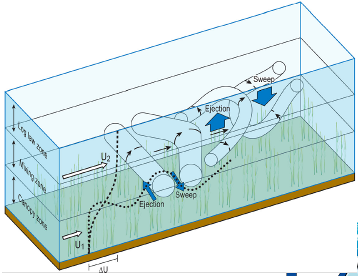
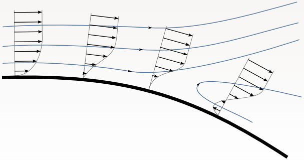


Figure: 3D aspect.

Ref: [Ghisalberti and Nepf, 2002], [Marjoribanks et al., 2017]

Applications



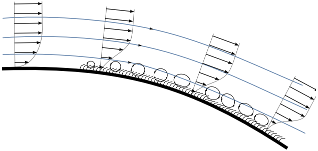


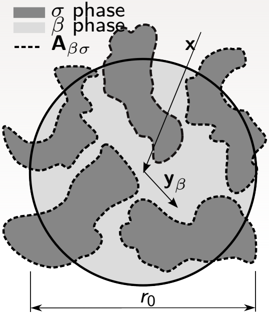
Figure: Innovative coating.

Figure: Field damage.

- sediment and nutrient transport
- vegetation management
- **passive flow control**

Plan

Macroscopic approach



Volume Averaged Navier-Stokes equations [VANS]

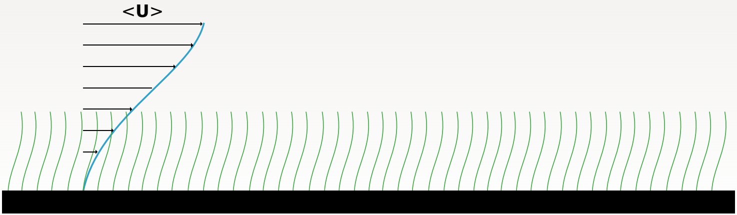
$$\frac{\partial \left\langle \mathbf{v}_\beta \right\rangle^\beta}{\partial t} + \left\langle \mathbf{v}_\beta \right\rangle^\beta \cdot \nabla \left\langle \mathbf{v}_\beta \right\rangle^\beta = -\frac{1}{\rho_\beta} \nabla \left\langle p_\beta \right\rangle^\beta + \nabla \cdot \frac{\mu_\beta}{\rho_\beta} \left[\nabla \left\langle \mathbf{v}_\beta \right\rangle^\beta + ^T \cdot \right]$$

$$+ \frac{1}{\rho_\beta} \underbrace{\mathbf{D}_{\beta\sigma}}_{\text{fluid-solid force}} - \nabla \cdot \underbrace{\left\langle \tilde{\mathbf{v}}_\beta \tilde{\mathbf{v}}_\beta \right\rangle^\beta}_{\text{subgrid scale stresses}} , \quad (2)$$

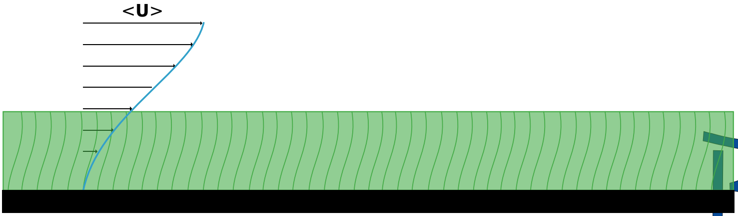
$$\nabla \cdot \left\langle \mathbf{v}_\beta \right\rangle^\beta = 0.$$

Refs: [Quintard and Whitaker, 1994],[Davit and Quintard, 2017]

Homogenization of the sparse solid phase



⇓ Rheology?



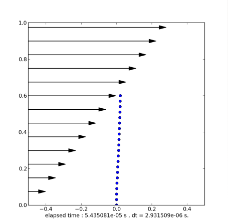
Simplest beam mechanics

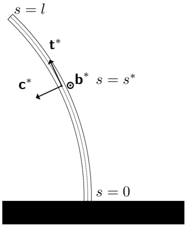
- ρ_σ density of the solid
- B bending rigidity
- I_{xx} second moment of stem area

[EULER]

$$I_{xx}\rho_\sigma \frac{d^2\theta}{dt^2}|_{s=s^*} = \frac{d}{ds}\left(B\frac{d\theta}{ds}\right)|_{s=s^*} + \mathbf{c}^* \cdot \int_{s^*}^{l} \mathbf{f}_{\beta\sigma}\, ds \quad (3)$$

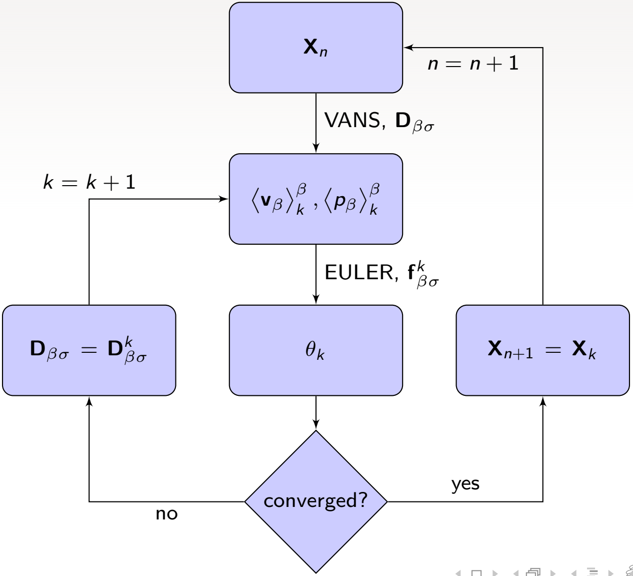
solver with variable time step for versatility





Macroscopic code

Let $\mathbf{X} = \left(\langle \mathbf{v}_\beta \rangle^\beta, \langle p_\beta \rangle^\beta, \theta \right)$. $\mathbf{D}_{\beta\sigma}$ and $\mathbf{f}_{\beta\sigma}$ are function of $\mathbf{X}$.



Plan

From Darcy's law to inertial, steady regime

$$\begin{cases} \langle \mathbf{v}_\beta \rangle^\beta = v\boldsymbol{\lambda} \\ \mathbf{D}_{\beta\sigma} = D_\lambda \boldsymbol{\lambda} + \mathbf{D}_\perp \end{cases} \text{, we write } D_\lambda = -\epsilon\mu K_D^{-1}(1 + F_\lambda)v \tag{4}$$

K_D is a scalar permeability $K_D^{-1} = \boldsymbol{\lambda} \cdot \mathbf{K}_D^{-1} \cdot \boldsymbol{\lambda}$.

Re$\ll 1$ Re~ 10

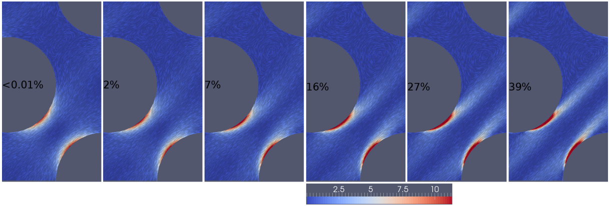
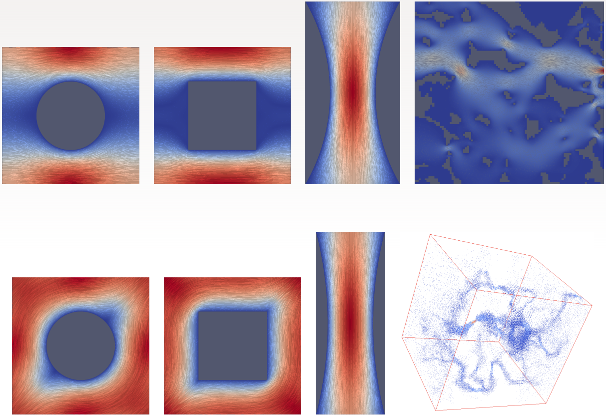


Figure: Normalized viscous dissipation

$$Re = \frac{\ell v}{\nu} \tag{5}$$

$\ell =?$

From Darcy's law to inertial, steady regime: geometries



short-cut	medium
DSA	disks in staggered arrangement
DRA	disks in regular arrangement
SSA	squares in staggered arrangement
SRA	squares in regular arrangement
CD1, CD2	convergent-divergent

Is Re_k the right scaling?

short-cut	medium
DSA	disks in staggered arrangement
DRA	disks in regular arrangement
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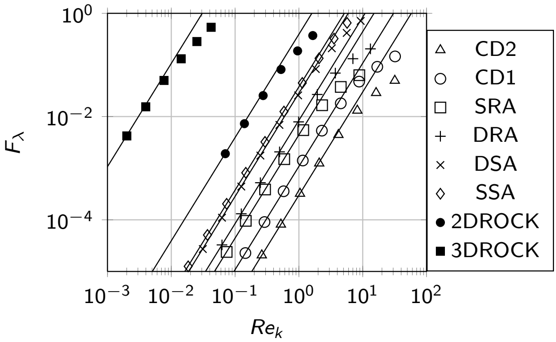
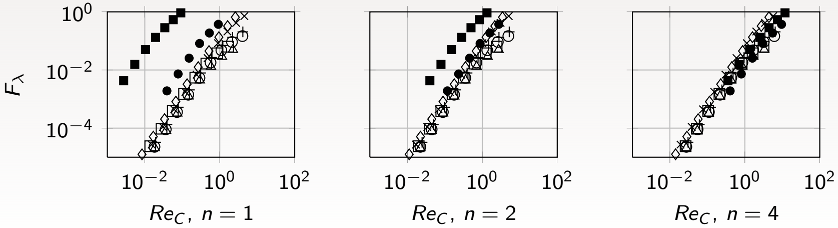
CD1, CD2	convergent-divergent



$$Re_k = \frac{v}{\nu}\sqrt{\frac{K_D}{\epsilon_\beta}} \quad (6)$$

New scaling

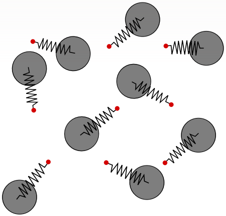


$$Re_C = \frac{v}{\nu}\sqrt{\frac{K_D}{\epsilon_\beta}}C_\lambda, \text{with} \begin{cases} C_\lambda = \zeta\left(\mathbf{v}_0^* \cdot \nabla^* \mathbf{v}_0^*\right) \\ \zeta\left(\mathbf{field}\right) = \left(\langle ||\mathbf{field}||^n \rangle^\beta\right)^{1/n} \end{cases} \quad (7)$$

medium	C_λ
CD2	0.10
CD1	0.19
SRA	0.36
DRA	0.49
DSA	0.62
SSA	0.77
2DROCK	5.77
3DROCK	176

[Pauthenet et al., under publication]

Pore scale fluid-structure interaction



Refs: [Bigot et al., 2014]

Parameters

- $m^* = \frac{\rho_\sigma}{\rho_\beta}$
- $f^* = \frac{\sqrt{k/m_\sigma}}{U/d}$
- $Re_d = \frac{dU}{\nu}$

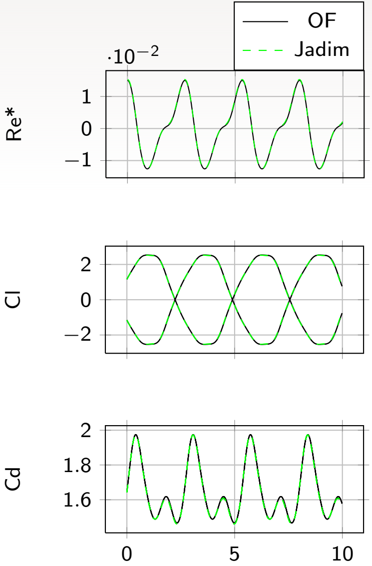
We add two parameters in the dataset for the fluid-solid force $\mathbf{D}_{\beta\sigma}$.

Numerical method

- large displacement → immersed boundary method
- paralellized code

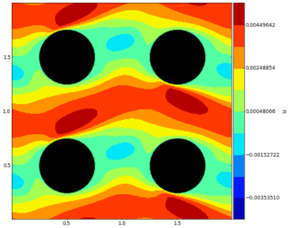
→ JADIM

Rigid case



Comparison with OpenFOAM (OF).
$Re = \frac{dv}{\nu} \sim 100$

$$\begin{cases} Re* = \dfrac{Re - \langle Re \rangle}{\langle Re_{ref} \rangle} \\ (C_d, C_l) = \dfrac{\mathbf{f}}{\frac{1}{2}\rho_\beta S U^2} \end{cases} \quad (8)$$



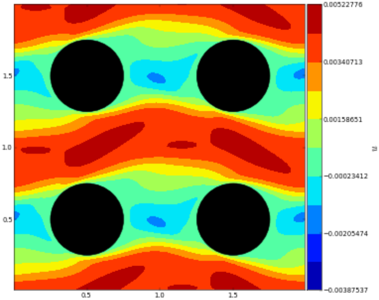
Validation and goals

Validation cases

- Periodic cell with elastic cylinders (different IBM)
- Early stage of wake behind a cylinder (experiments) [Bouard and Coutanceau, 1980]

Goals

- $\mathbf{D}_{\beta\sigma}(Re_d, m^*, f^*)$ from DNS
- $\ell_{macro} \gg \ell_{micro}$?
- $\tau_{macro} \gg \tau_{micro}$?
- size of a representative volume



Conclusion

Key points

Macroscopic code

- coupling of the VANS with EULER
- fluid-solid force from a dataset
- subgrid-scale stresses?

Structure

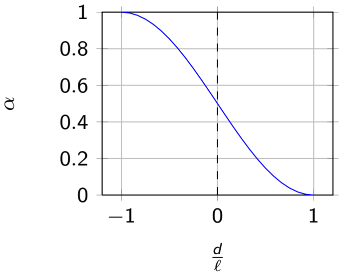
- no rheology law
- simplest individual beam mechanics
- reduce the number of degrees of freedom?

Fluid-solid force

- weak inertia regime
- unsteady flow in elastic media
- implement simple solid-solid contacts?

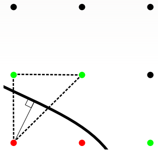
Immersed Boundary Method: direct forcing

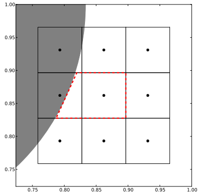
- $\mathbf{f}_\sigma$ imposes velocity in solid phase
- Pseudo Poisson equation for mass conservation
- Stress on solid obstacle from $\mathbf{f}_\sigma$



a different method (Ghost cell + cut-cell)

- Dirichlet for predicted velocity
- Neuman for pressure correction





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