

# Università degli Studi di Genova

## Scuola Politecnica

Corso di Laurea in Ingegneria delle Acque e della Difesa del Suolo



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## Strutture Lagrangiane Coerenti e barriere al trasporto: applicazione al Golfo di Trieste

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# Problemi di trasporto in un fluido

- Problema molto sentito in campo ambientale.
- Capacità di prevedere traiettoria agenti inquinanti.

Campo di  
velocità  
Euleriano



Descrizione  
Lagrangiana  
del moto

Ovvero traiettorie traccianti passivi sono soluzione sistema ODE.

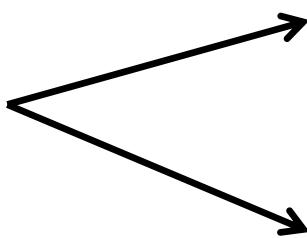
$$\dot{x} = v(x, t) \longrightarrow x(t; t_0, x_0) = x_0 + \int_{t_0}^{t_0+T} v(x(\tau; t_0, x_0), \tau) d\tau$$

Inconveniente: forte sensibilità alle condizioni iniziali



Necessario uno strumento per separare regioni a differente comportamento dinamico

Sistemi dinamici

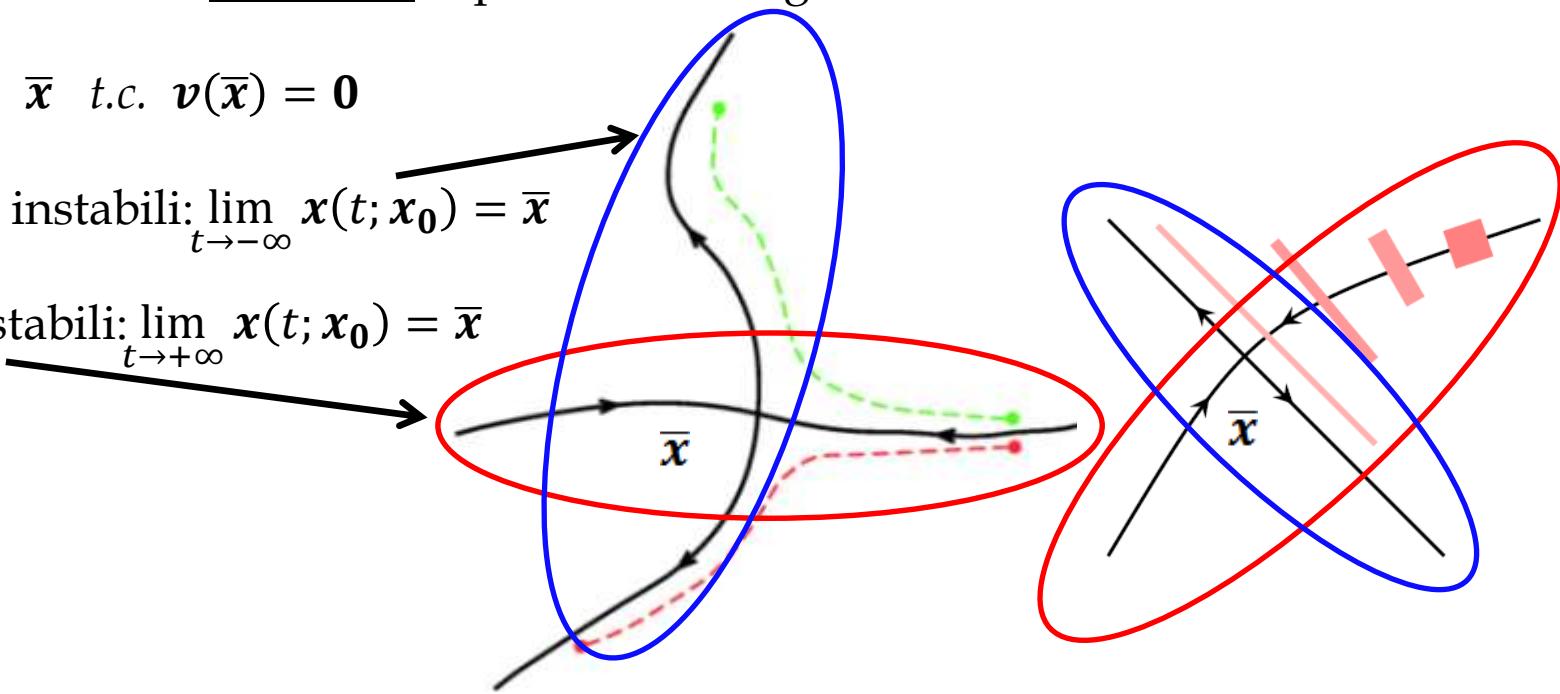


autonomi: non esplicitamente dipendenti dal tempo  $\rightarrow \dot{x} = v(x)$

non-autonomi: esplicitamente dipendenti dal tempo  $\rightarrow \dot{x} = v(x, t)$

Nei sistemi dinamici autonomi si possono distinguere:

- Punti fissi:  $\bar{x}$  t.c.  $v(\bar{x}) = 0$
- Traiettorie instabili:  $\lim_{t \rightarrow -\infty} x(t; x_0) = \bar{x}$
- Traiettorie stabili:  $\lim_{t \rightarrow +\infty} x(t; x_0) = \bar{x}$



Sistemi dinamici non autonomi:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{v}(\mathbf{x}(t; t_0, \mathbf{x}_0), t) \\ \mathbf{x}(t_0; t_0, \mathbf{x}_0) = \mathbf{x}_0 \end{cases}$$

Mappa di flusso:

$$\phi_{t_0}^{t_0+T}: D \rightarrow D: \mathbf{x}_0 \mapsto \phi_{t_0}^{t_0+T}(\mathbf{x}_0) = \mathbf{x}(t; t_0, \mathbf{x}_0), \quad D \subset \mathbb{R}^2$$

Nei sistemi dinamici non autonomi la nozione di traiettorie stabili ed instabili diviene ambigua



Separatrici:

Strutture Lagrangiane Coerenti

LCS chiamate così perché:

Lagrangiane



Inseguendo le particelle

Coerenti



Stabili, nel senso che conservano le loro caratteristiche nel tempo

Identificazione LCS

Finite-Time Lyapunov Exponents

Espansione di una coppia di particelle:

$$\delta \mathbf{x}(t_0) = \mathbf{y}(t_0) - \mathbf{x}(t_0) \quad \text{al tempo } t_0$$

linearizzazione  $\downarrow$   $T$ : tempo di integrazione

$$\max \|\delta \mathbf{x}(t_0 + T)\| = e^{\sigma_{t_0}^T(\mathbf{x})|T|} \|\delta \mathbf{x}(t_0)\| \quad \text{al tempo } t_0 + T$$

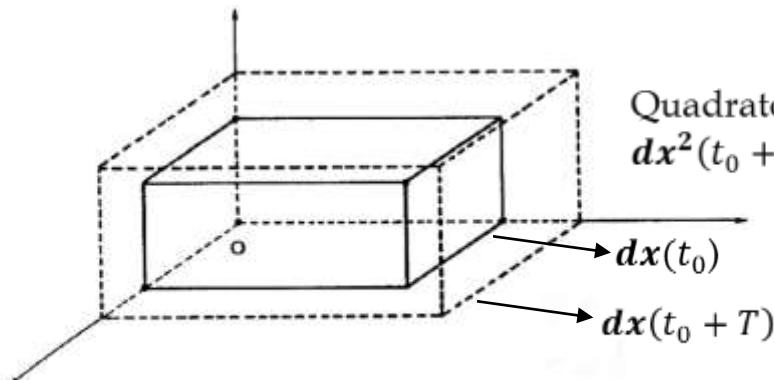


Il campo **scalare FTLE**  
vale in ogni punto

$$\sigma_{t_0}^T(\mathbf{x}) = \frac{1}{|T|} \log \sqrt{\lambda_{\max}(\mathbf{CG})}$$

Tensore di Cauchy-Green:

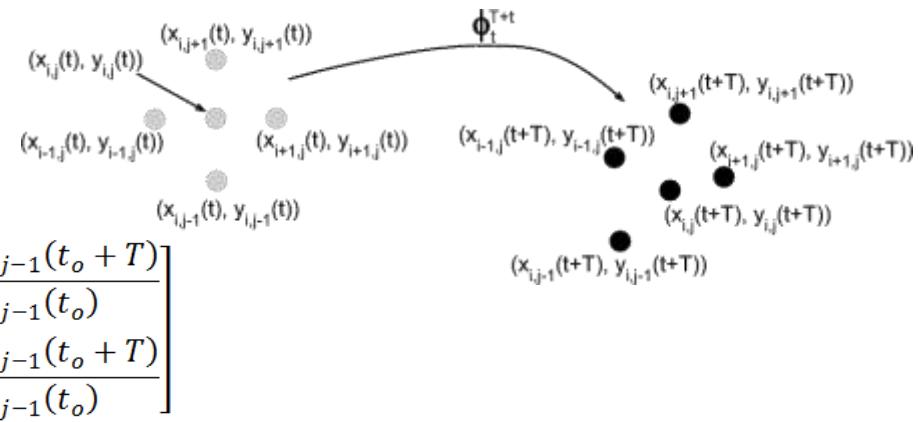
$$\mathbf{CG} = \frac{d\phi_{t_0}^{t_0+T}(\mathbf{x})^T}{d\mathbf{x}} \frac{d\phi_{t_0}^{t_0+T}(\mathbf{x})}{d\mathbf{x}}$$



Quadrato deformazione infinitesima  
 $dx^2(t_0 + T) = d\mathbf{x}(t_0) \cdot \mathbf{CG} d\mathbf{x}(t_0)$

Calcolo - 2 step principali:

1. traiettorie griglia particelle - soluzione ODE:  $\dot{x} = v(x, t)$ ;
2. calcolo Tensore  $CG$  alle differenze.



$$\left. \frac{d\phi_{t_o}^{t_o+T}(x)}{dx} \right|_{x_{i,j}} = \begin{bmatrix} \frac{x_{i+1,j}(t_o + T) - x_{i-1,j}(t_o + T)}{x_{i+1,j}(t_o) - x_{i-1,j}(t_o)} & \frac{x_{i,j+1}(t_o + T) - x_{i,j-1}(t_o + T)}{y_{i,j+1}(t_o) - y_{i,j-1}(t_o)} \\ \frac{x_{i+1,j}(t_o + T) - x_{i-1,j}(t_o + T)}{y_{i,j+1}(t_o) - y_{i,j-1}(t_o)} & \frac{y_{i,j+1}(t_o + T) - y_{i,j-1}(t_o + T)}{y_{i,j+1}(t_o) - y_{i,j-1}(t_o)} \end{bmatrix}$$

Strutture Lagrangiane Coerenti

→  
definizione

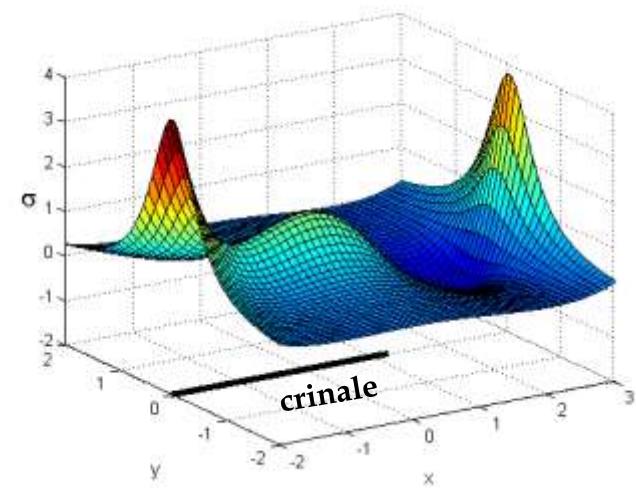
Crinali del campo FTLE

Barriere al trasporto: flusso attraverso queste strutture molto piccolo.

Parametri:

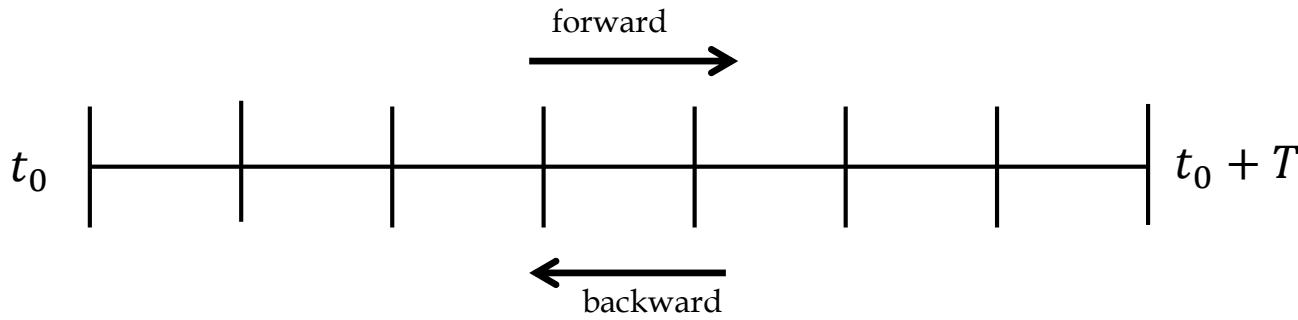
- $T ???$
- Numero particelle da inizializzare ???

Strutture dipendenti dal tempo!!!



Due tipi di LCS:

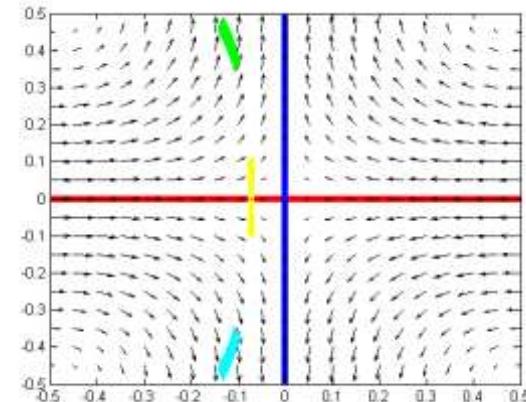
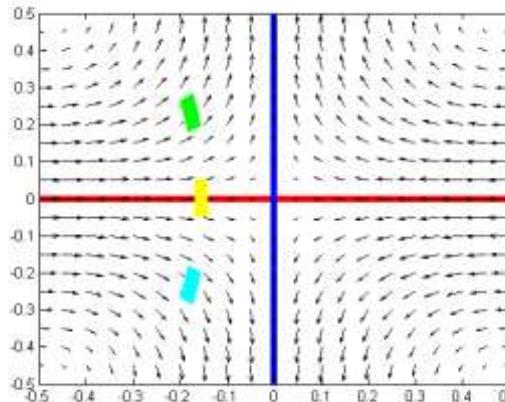
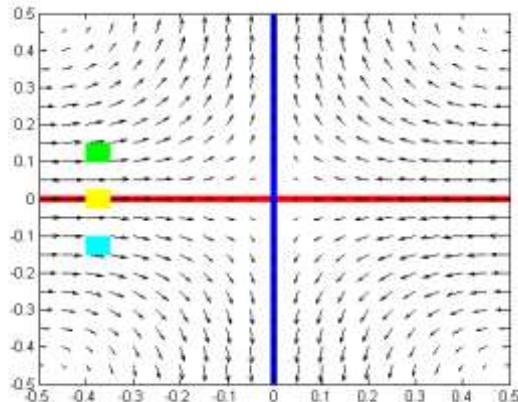
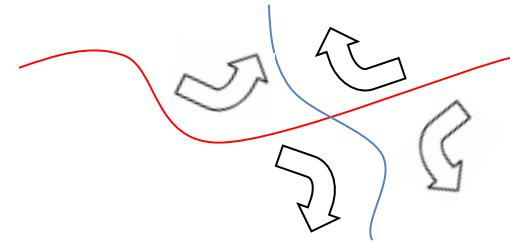
- Repulsive – integrazione forward  $\rightarrow$  espansione  $\rightarrow$  riferite a  $t_0$
- Attrattive – integrazione backward  $\rightarrow$  contrazione  $\rightarrow$  riferite a  $t_0 + T$



Come agiscono?

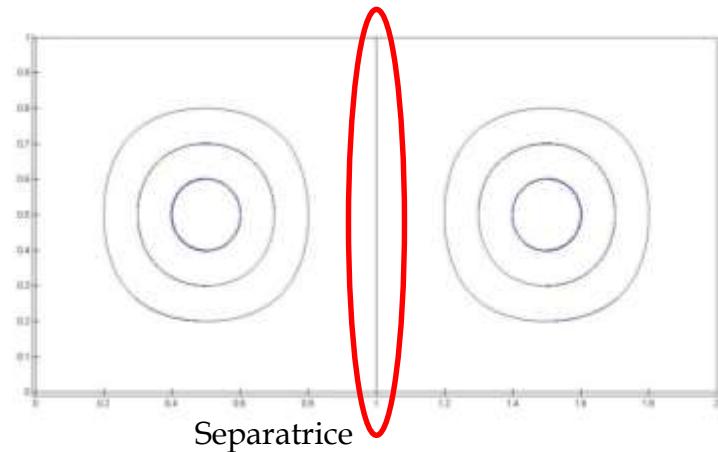
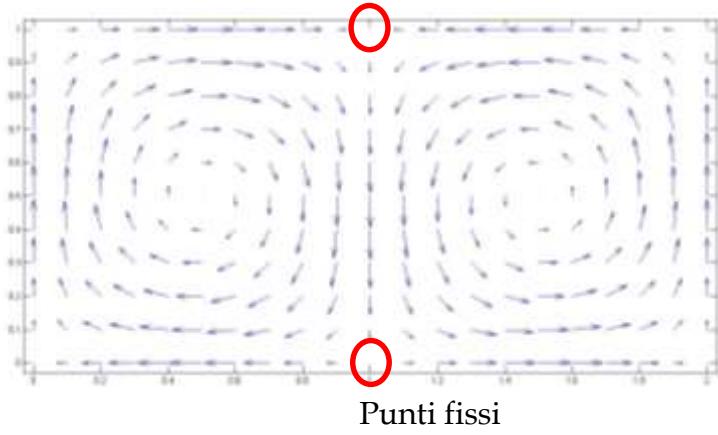
**Repulsive**: indicano direzioni di *inflowing*

**Attrattive**: indicano direzioni di *outflowing*



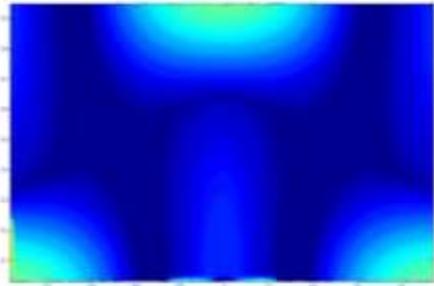
# Casi analitici

- Doppio vortice indipendente dal tempo definito su  $D = [0, 2] \times [0, 1]$   
Funzione di corrente:  $\psi = \sin(\pi x) \sin(\pi y)$

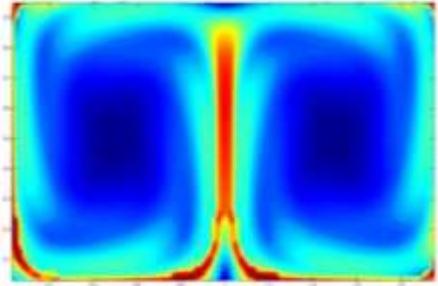


## LCS repulsive

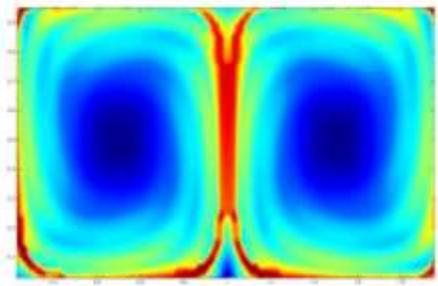
$T = 0.1$  s



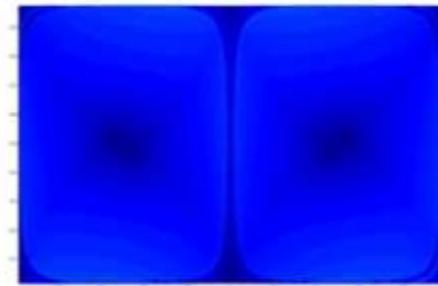
$T = 0.5$  s



$T = 0.8$  s



$T = 8$  s

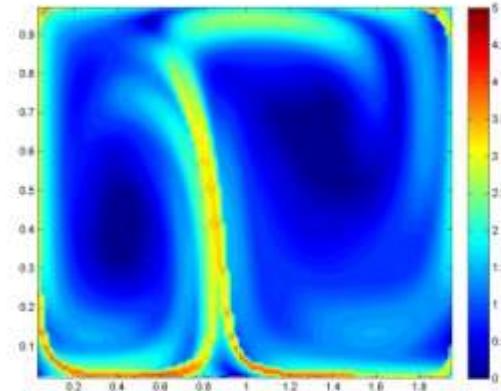


## 2. Doppio vortice dipendente dal tempo definito su $D = [0, 2] \times [0, 1]$

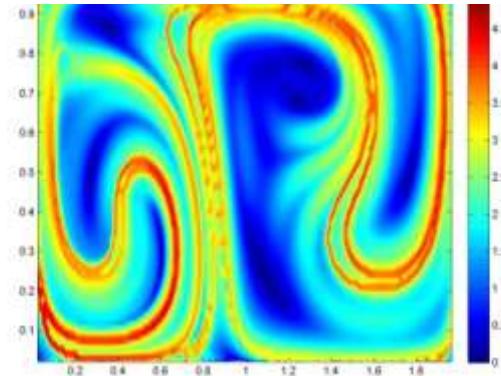
Funzione di corrente

$$\begin{aligned}\psi(x, y, t) &= A \sin(\pi f(x, t)) \sin(\pi y) \\ f(x, t) &= a(t)x^2 + b(t)x \\ a(t) &= \varepsilon \sin(\omega t) \\ b(t) &= 1 - 2\varepsilon \sin(\omega t)\end{aligned}$$

$T = 5$  s



$T = 15$  s

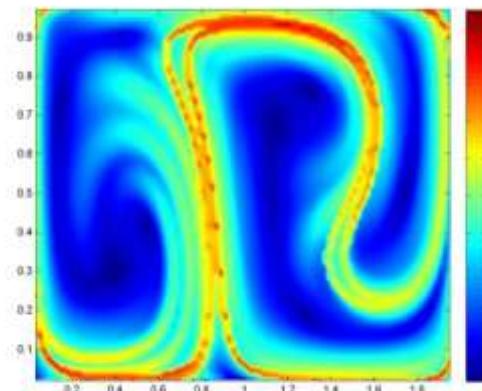


Parametri utilizzati

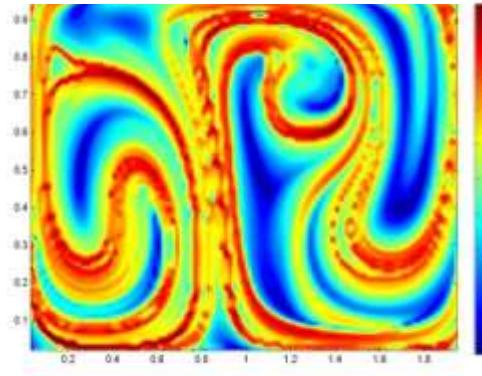
$$\begin{aligned}A &= 0.1 \\ \omega &= \frac{2\pi}{10} \\ \varepsilon &= 0.25\end{aligned}$$



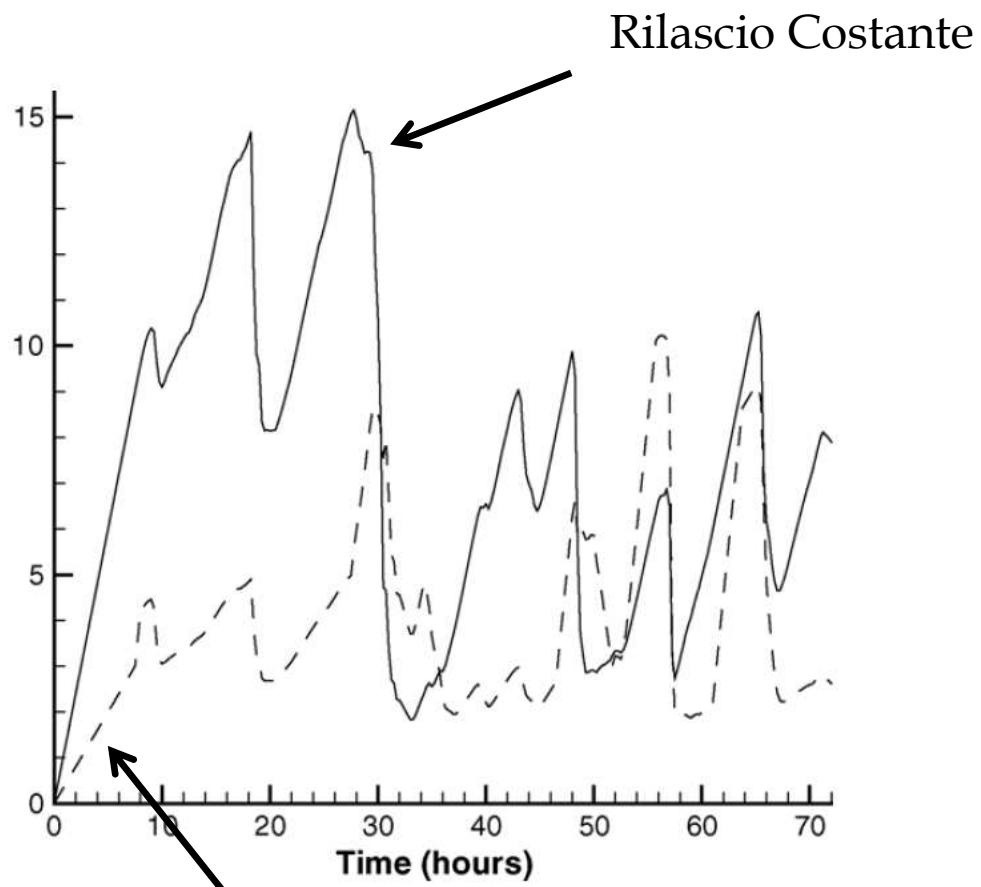
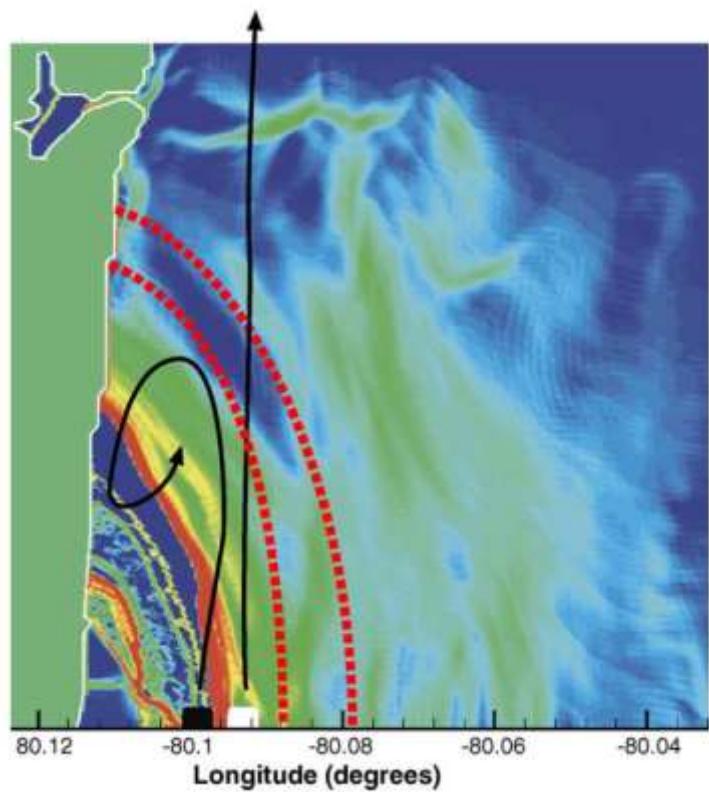
$T = 10$  s



$T = 20$  s



## Esempi in Letteratura



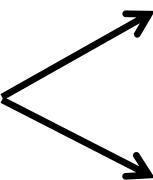
# Il caso del Golfo di Trieste

Campo di velocità della superficie del mare



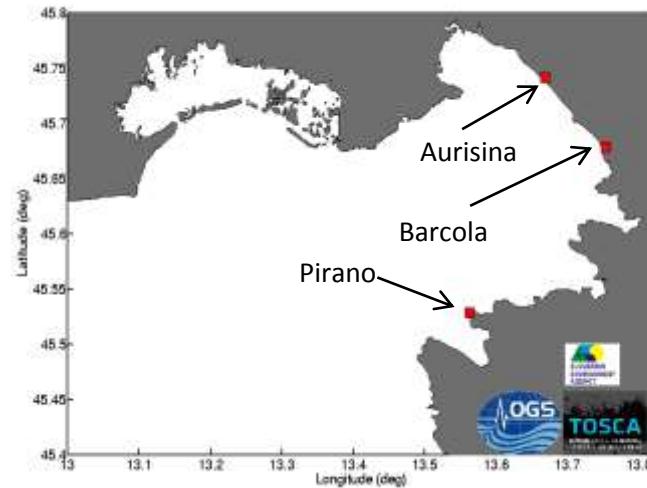
Radar CODAR  
(*Coastal Ocean Dynamics Applications Radar*)

Dati utilizzati fanno parte progetto TOSCA

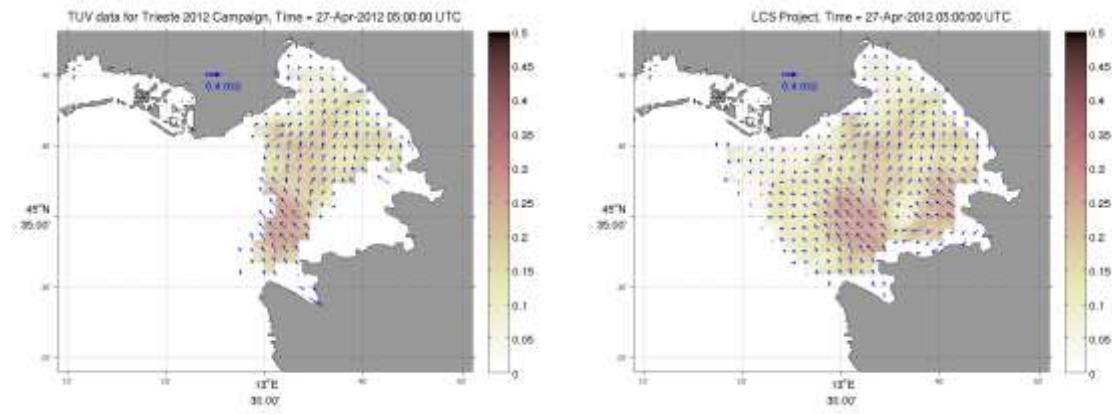


Incidenti in mare:  
fuoriuscita di petrolio

Operazioni ricerca e salvataggio in mare



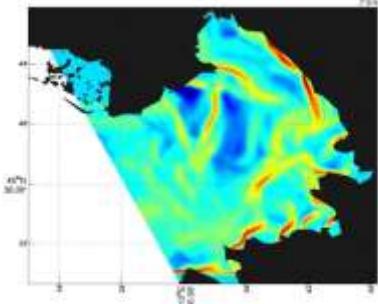
- Misure effettuate tra 23 e 30 aprile 2012.
- Risoluzione spaziale 1500 m.
- Risoluzione temporale 1 h.
- Talvolta «buchi di velocità».



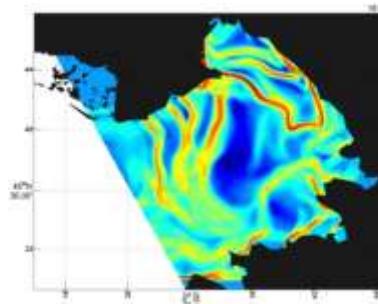
# Tempo di integrazione $T$

Se  $T$  aumenta vengono evidenziate un maggior numero di strutture e con una definizione migliore

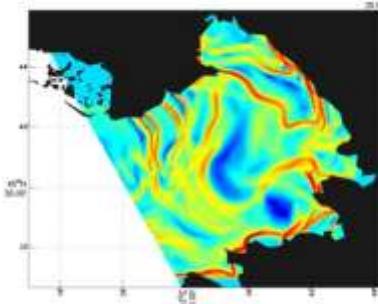
$T = 7 \text{ h}$



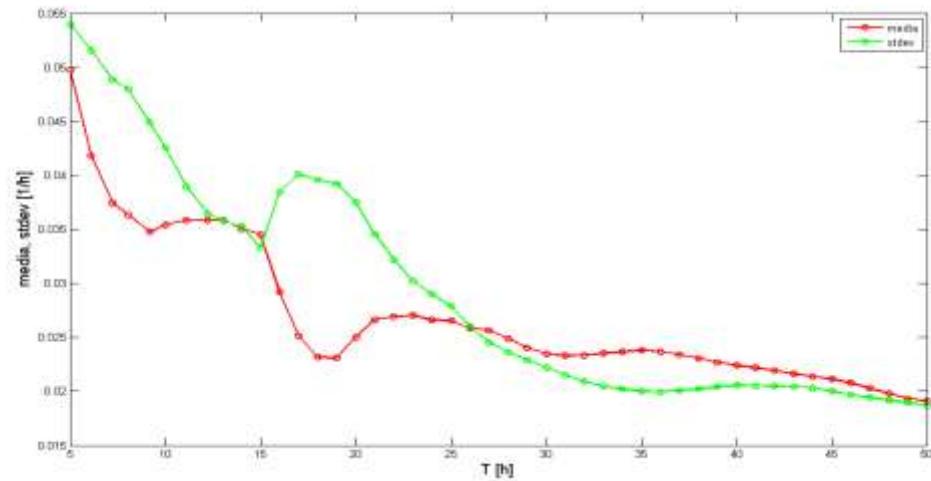
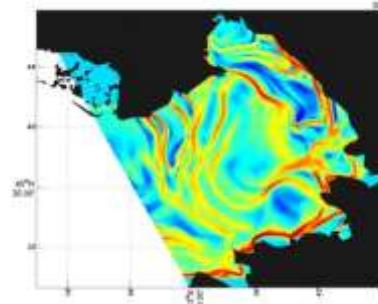
$T = 15 \text{ h}$



$T = 25 \text{ h}$



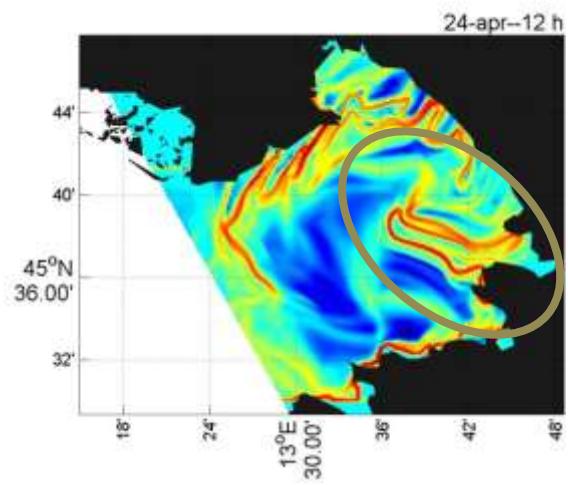
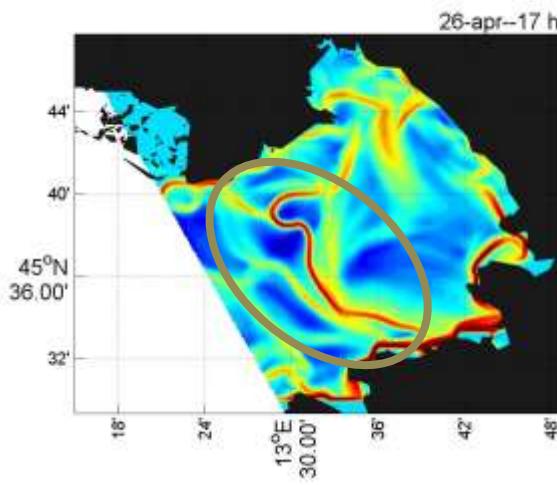
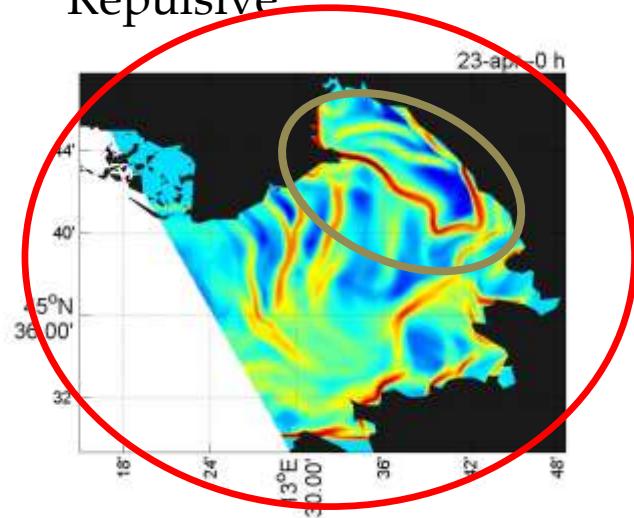
$T = 35 \text{ h}$



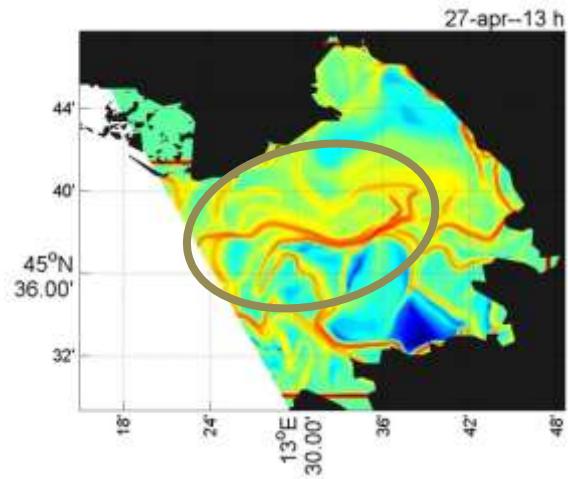
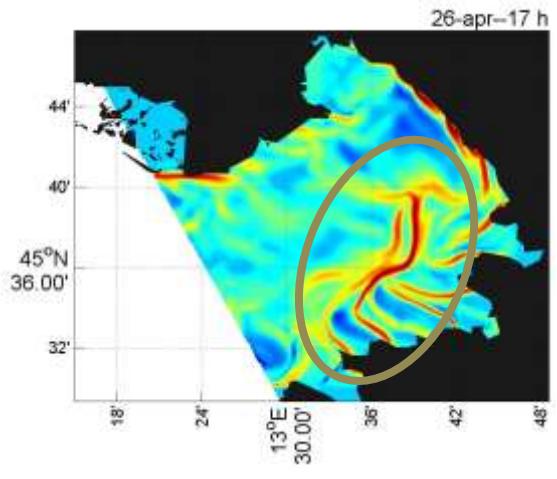
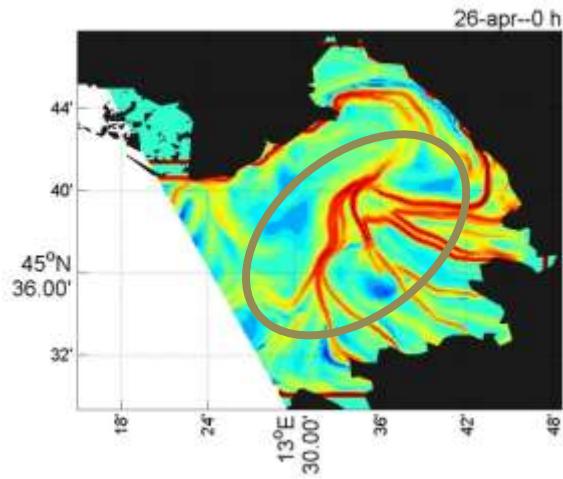
$$\sigma_{t_0}^T(x) = \frac{1}{|T|} \log \sqrt{\lambda_{\max}(CG)}$$

E' possibile individuare chiaramente alcune LCS

Repulsive

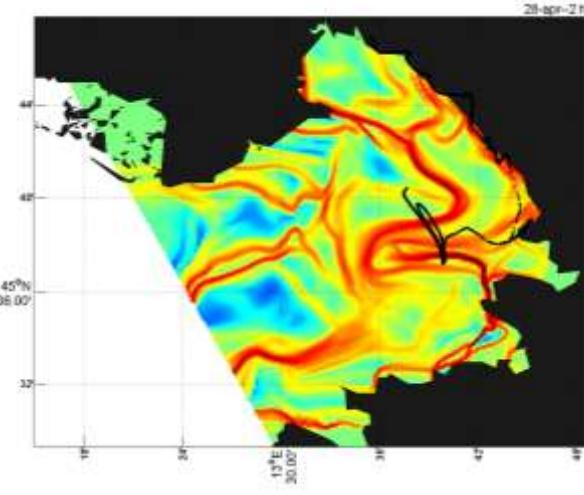
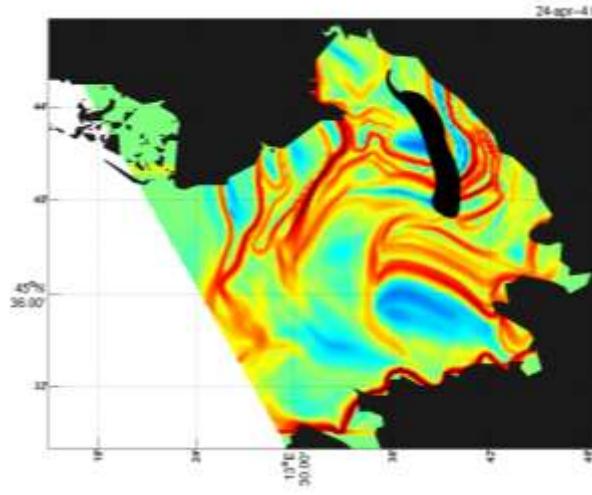
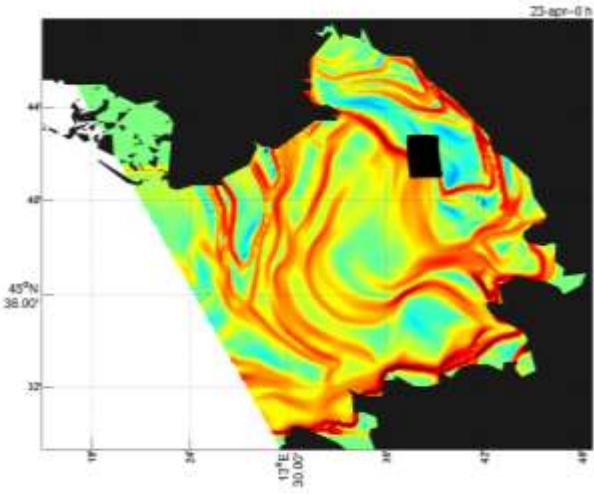


Attrattive

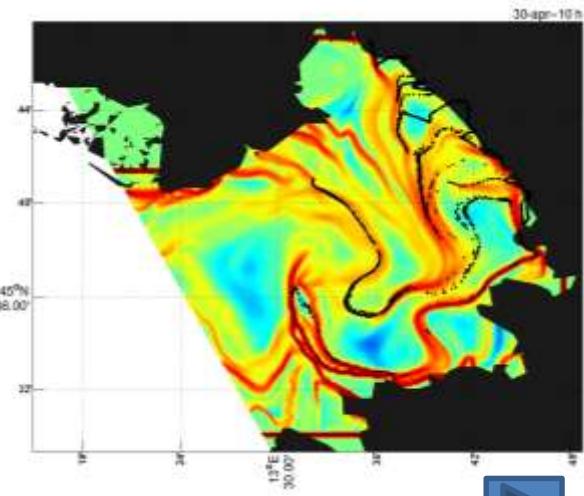
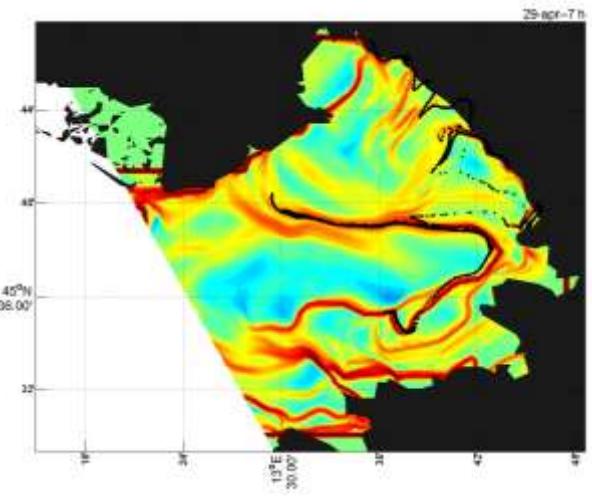
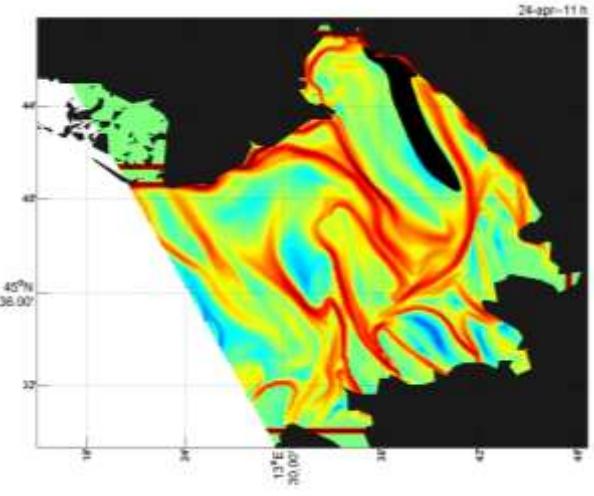


# Come agiscono nei confronti di nuvole di particelle?

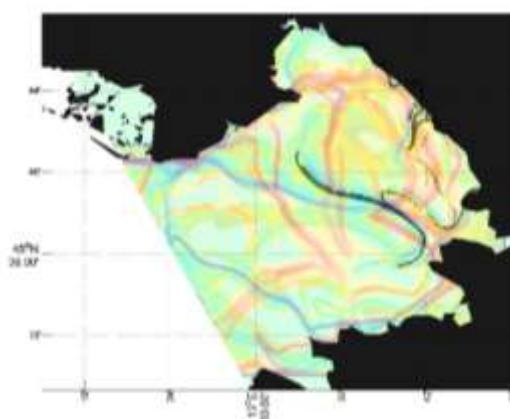
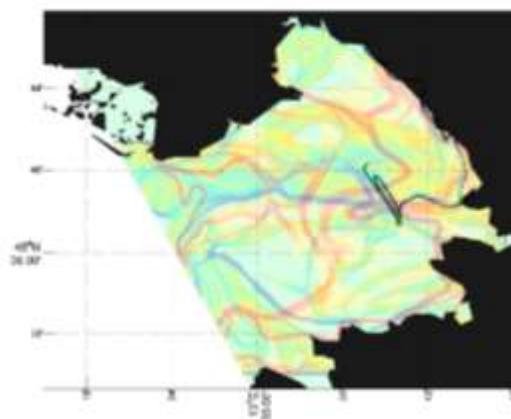
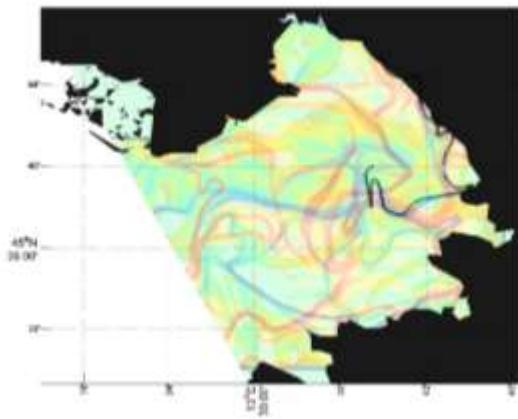
LCS repulsive



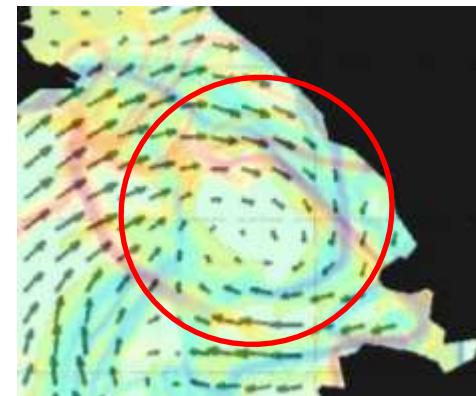
LCS attrattive



# Sovrapposizione



Permette di rintracciare vortici o strutture simili.



Nel progetto TOSCA sono anche stati rilasciati dei drifter



Rilascio drifter CODE  
*Coastal Ocean  
Dynamics Experiment*

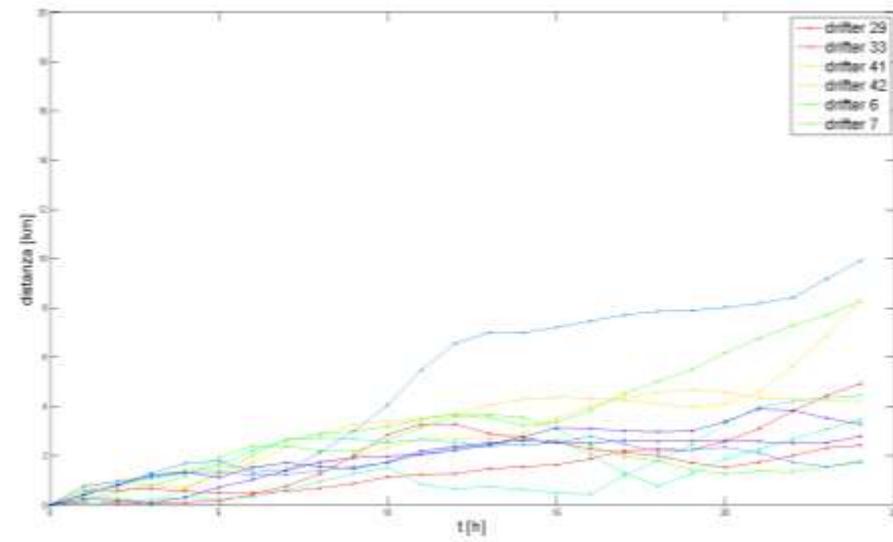
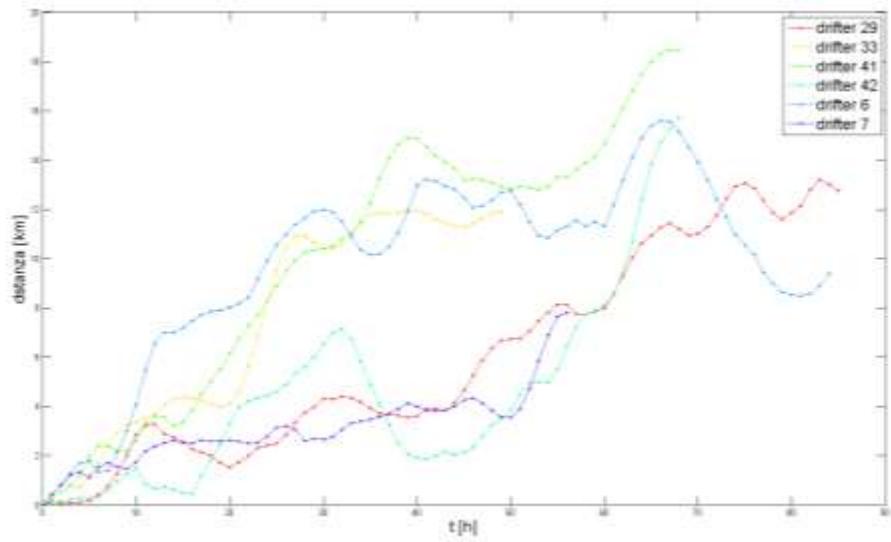


Confronto con  
simulazione

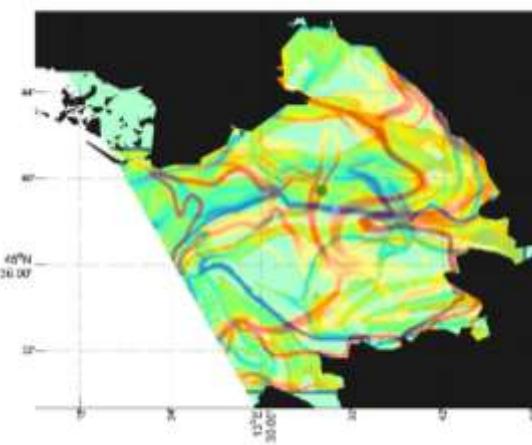
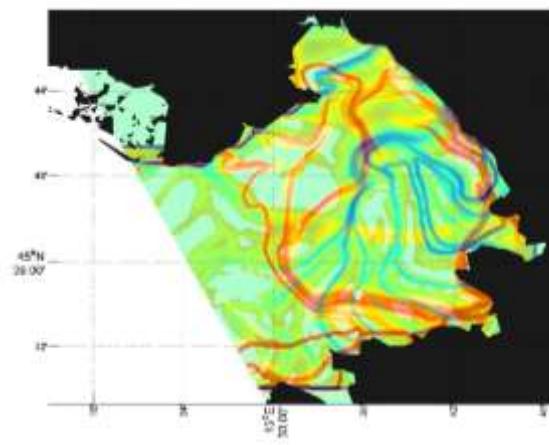
Causa caoticità moto,  
distanza fra drifter reale  
e simulato aumenta



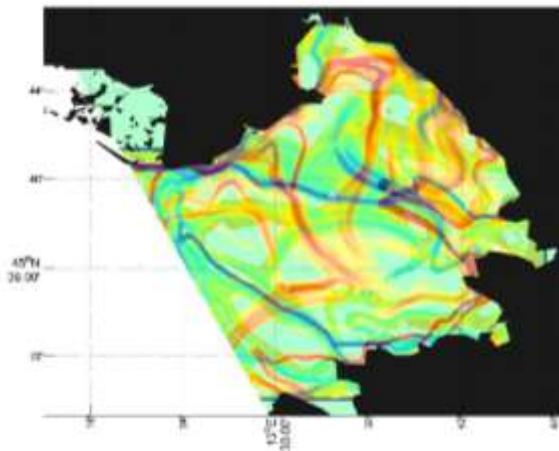
Si reinsemina ogni 24 h



# Drifter 42

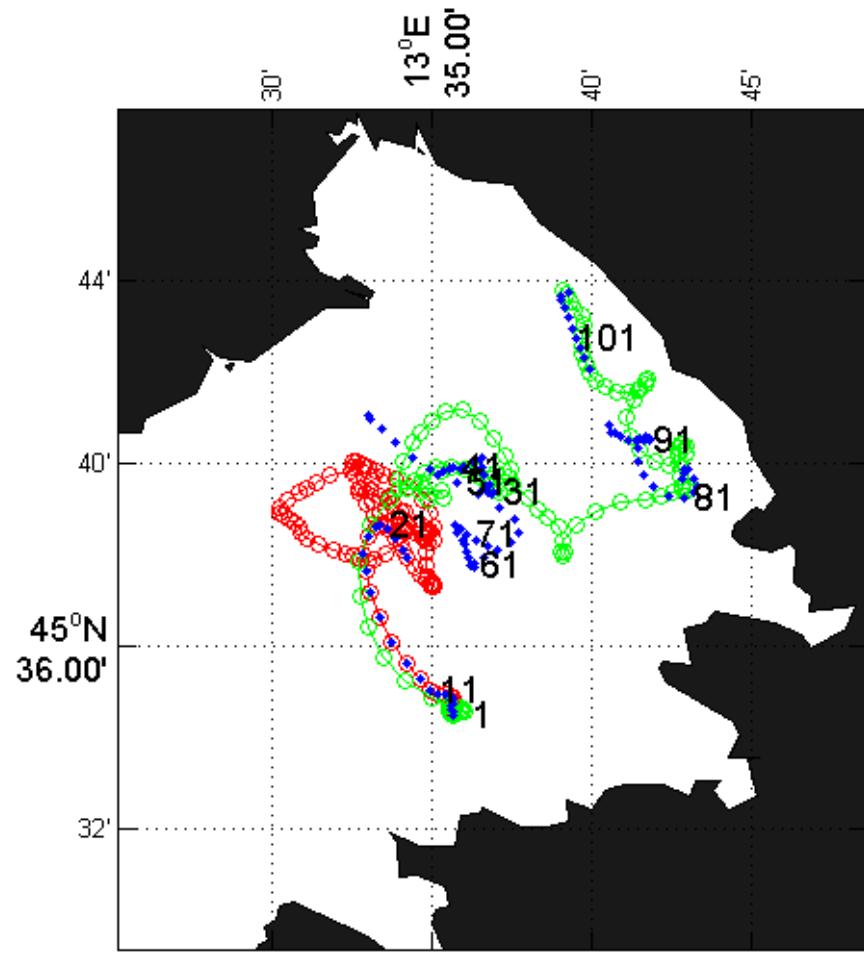


Il drifter 42 viene rilasciato su un punto iperbolico

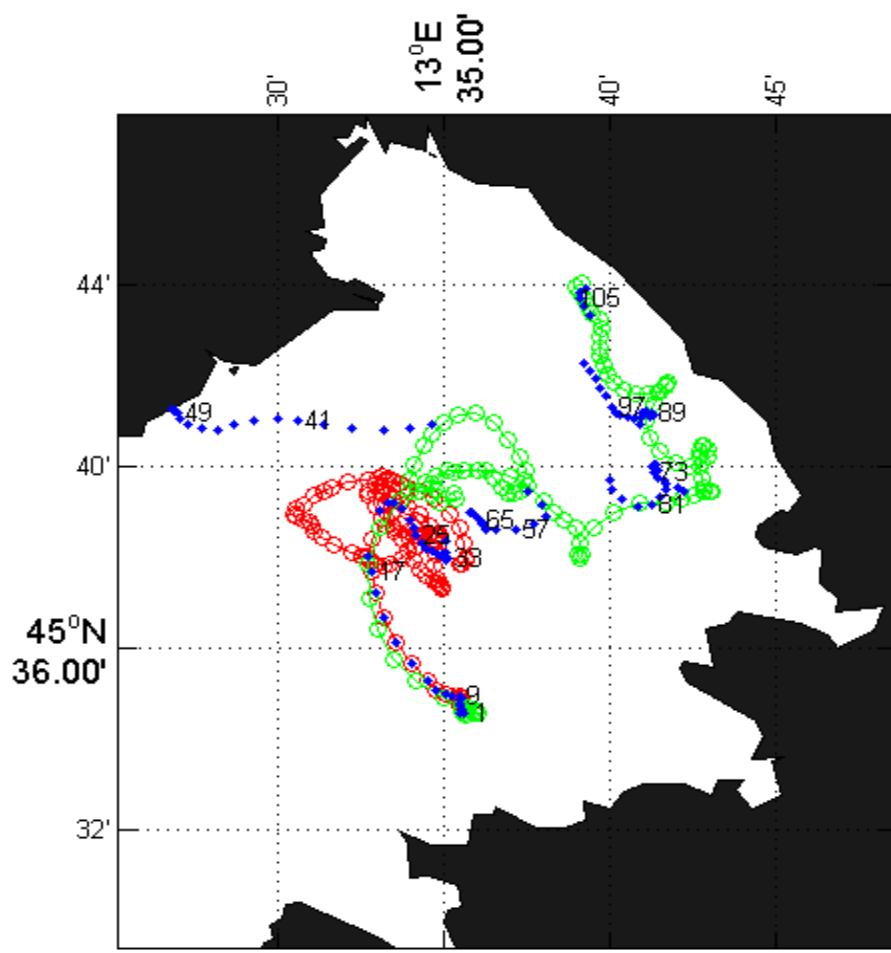


# Traiettorie

Reinseminato ogni 24 h

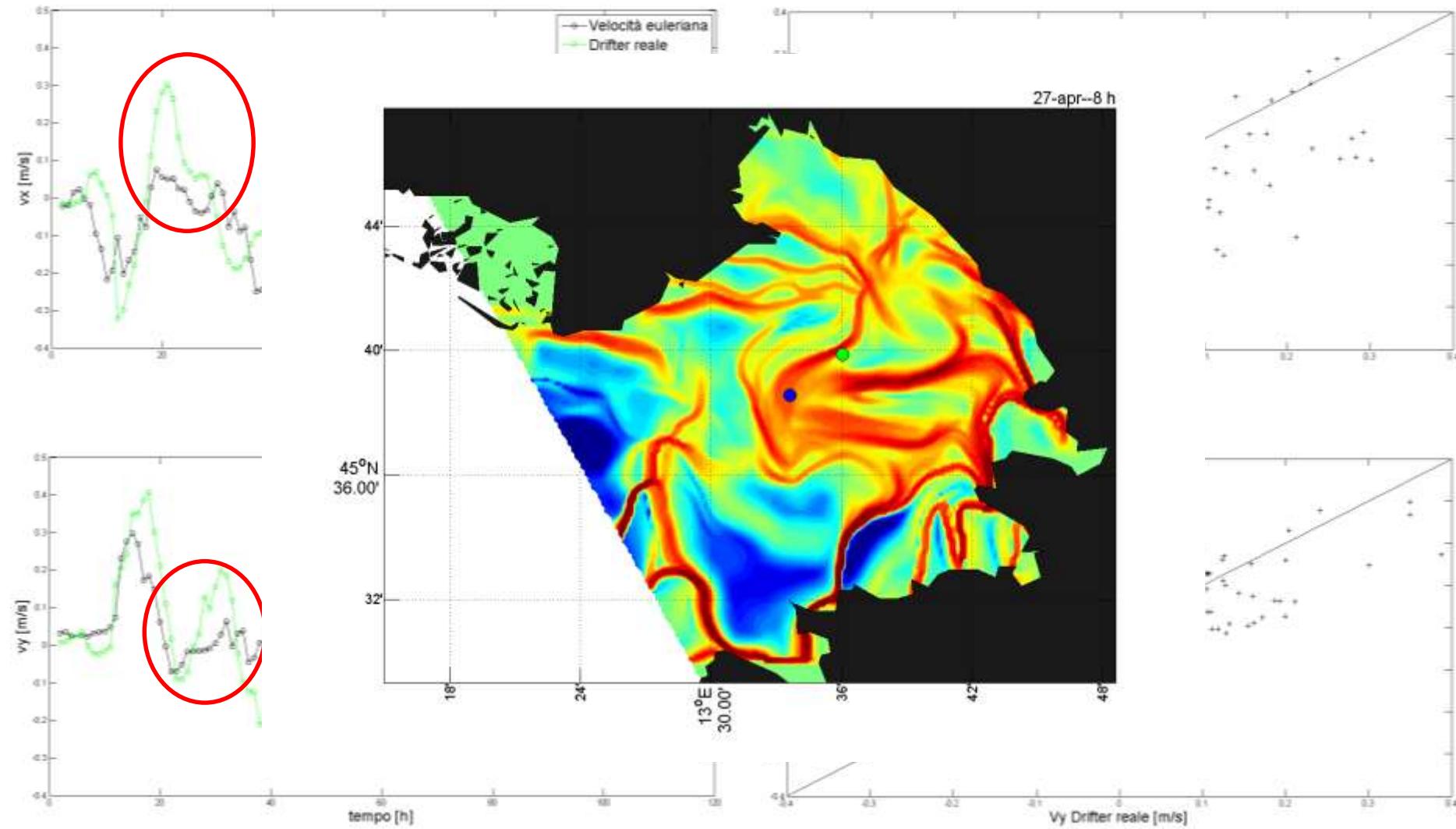


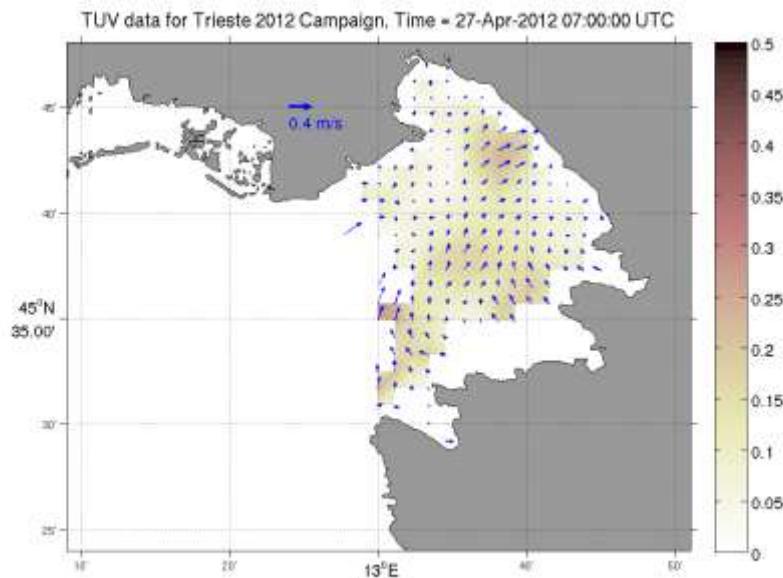
Reinseminato ogni 16 h



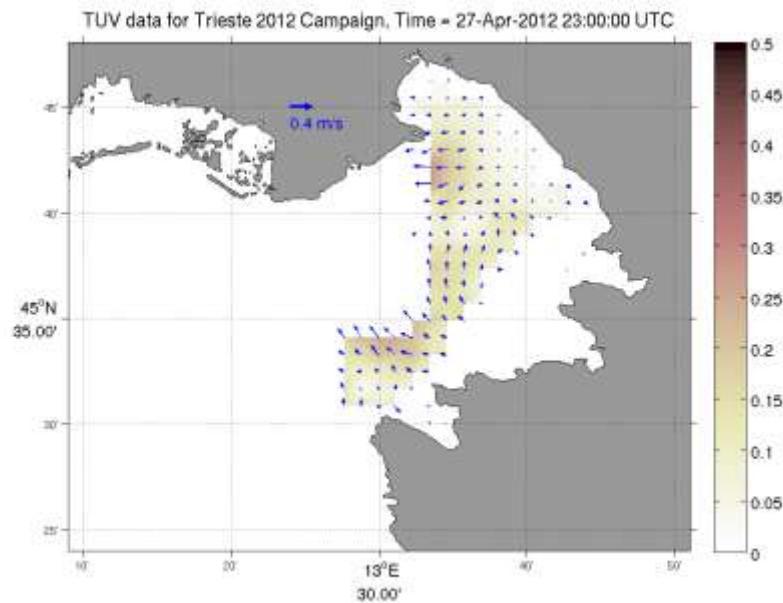
- drifter reale
- drifter simulato
- drifter simulato reinseminato

# Confronto velocità Euleriana – velocità Lagrangiana drifter reale

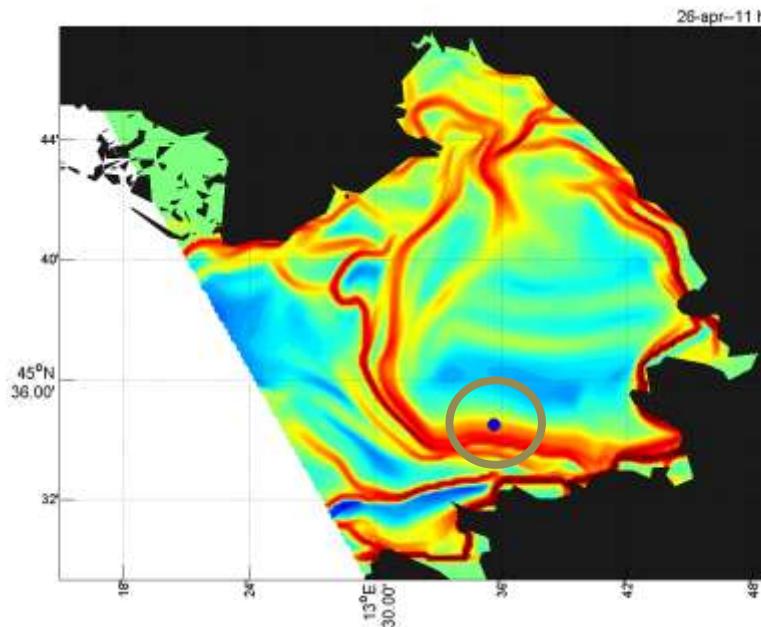




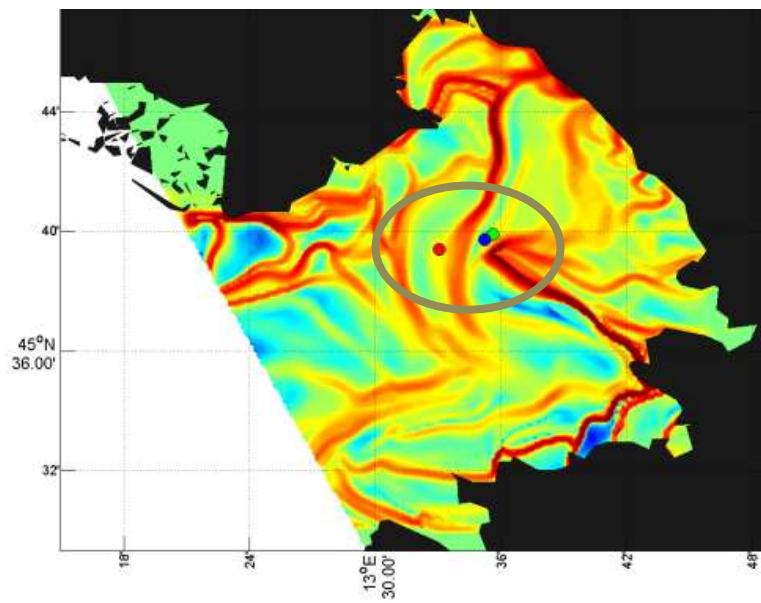
«Buchi» campo velocità



Alla separazione il campo misurato presenta una importante lacuna di dati.



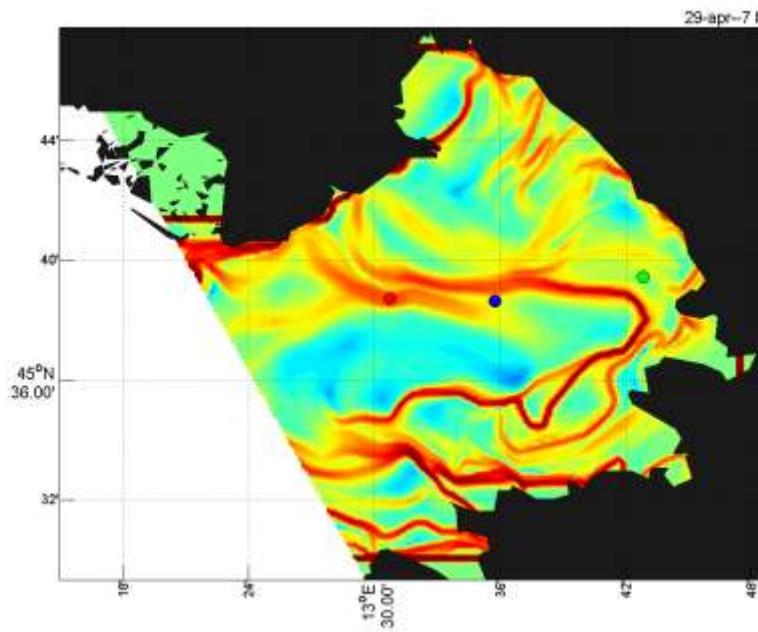
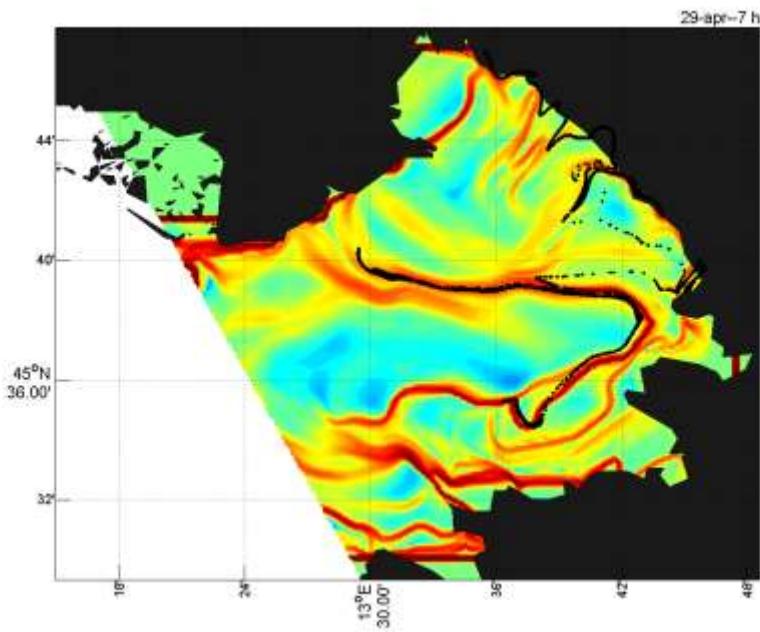
LCS repulsive



Drifter reale e simulato  
separati da LCS repulsiva

## LCS attrattive

La struttura attrattiva lungo cui il drifter reale e quello simulato si separano è la stessa lungo cui una nuvola di particelle subisce una forte deformazione da *stretching*.



# Conclusioni

L'analisi ha evidenziato come:

- nella simulazione di singoli traccianti passivi la corretta conoscenza del campo di velocità gioca un ruolo fondamentale così come l'interazione con il profilo della costa;
- comportamento nuvole di particelle può essere descritto da LCS;
- analisi delle traiettorie di *drifter* può essere condotta sulla base delle LCS;
- non è detto che tutte le LCS identificate siano effettivamente Lagrangiane e la recente teoria di *Haller* potrebbe fornire un metodo più efficace per ricercarle;
- possibilità di identificare LCS in campi di moto tridimensionali.

GRAZIE PER  
L'ATTENZIONE!