COMPITO MECCANICA DEI FLUIDI I

23 Gennaio 2006

Esercizio 1

$$\underline{V} = \overset{u}{(1+2.5x+y)}i + \overset{v}{(-0.5-1.5x-2.5y)}j$$

1. Si in quanto non appare il tempo e quindi $\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = 0$

2. $a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \phi + (1+2.5x+y)\cdot 2.5 + (-0.5-1.5x-2.5y)(1)$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \phi + (1+2.5x+y)(-1.5) + (-0.5-1.5x-2.5y)(-2.5)$$

$$a_x = 2 + 4.75x \ \ m/s^2; \quad a_y = -0.25 + 4.75y \ \ m/s^2$$

3. $\frac{1}{V}\frac{dV}{dt} = \varepsilon_{xx} + \varepsilon_{yy} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2.5 - 2.5 = \phi$

Il fluido è incomprimibile

4. Punto di ristagno $\underline{V} = \phi \Rightarrow \begin{cases} u=0 \\ v=0 \end{cases}$

$$\begin{cases} u = 1+2.5x+y = 0 \\ v = -0.5-1.5x-2.5y = 0 \end{cases} \Rightarrow \begin{cases} y = -1-2.5x \\ -0.5-1.5x-2.5(-1-2.5x) = 0 \end{cases}$$

$$\begin{cases} y = -1-2.5x \\ 4.75x+2 = 0 \end{cases} \qquad \begin{cases} x = -2/4.75 = -0.42 \text{ m} \\ y = -1.-2.5\cdot(-0.42) = 0.05 \text{ m} \end{cases}$$

5. $\varepsilon_{xx} = \frac{\partial u}{\partial x} = 2.5 \ 1/s \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y} = -2.5 \ 1/s$

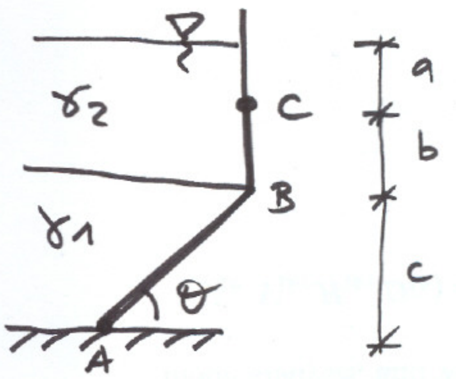
$$\varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{1}{2}(1-1.5) = -0.25 \ 1/s$$

Se $\underline{V} = (1+y)\hat{i} + (-0.5-1.5x)\hat{j}$ le traiettorie e le linee di corrente coincidono

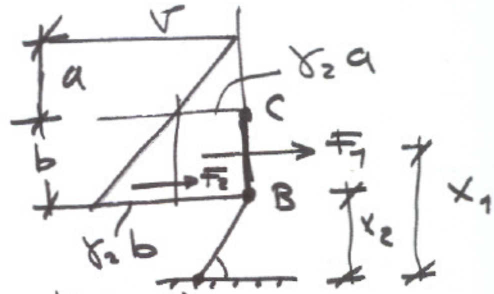
$$\frac{dx}{u} = \frac{dy}{v} \rightsquigarrow \frac{dx}{1+y} = \frac{dy}{-0.5-1.5x} \rightsquigarrow (0.5+1.5x)\,dx = (1+y)\,dy$$

$$0.5\,x + \frac{1.5}{2}x^2 = -y - y^2/2 + \text{costante}$$

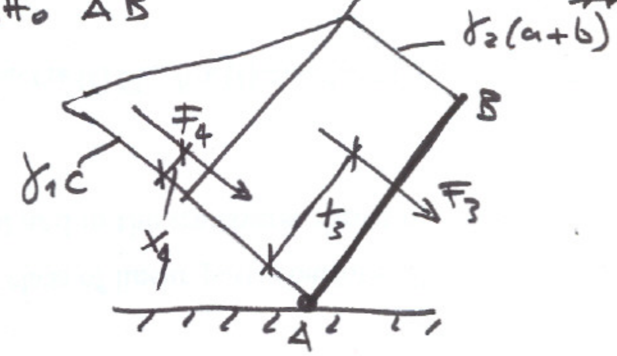
Esercizio 2



Tratto CB



Tratto AB



$$F_1 = \gamma_2 \cdot a \cdot b \cdot L = 800 \cdot 9.81 \cdot 1 \cdot 1 \cdot 1 = 15696\,N; \quad x_1 = \frac{b}{2} + c = 3\,m$$

$$F_2 = \gamma_2 \cdot b \cdot \frac{b}{2} \cdot L = 800 \cdot 9.81 \cdot 2 \cdot 1 \cdot 1 = 15696\,N; \quad x_2 = c + b/3 = 2.67\,m$$

$$F_3 = \gamma_2 (a+b) \cdot \frac{c}{\sin\theta} \cdot L = 800 \cdot 9.81\,(2+1)\frac{2}{\sqrt{2}/2} \cdot 1 = 66592\,N; \quad x_3 = \frac{c}{2\sin\theta} = 1.414\,m$$

$$F_4 = \gamma_1 \cdot c \cdot \frac{1}{2} \cdot \frac{c}{\sin\theta} \cdot L = 1100 \cdot 9.81 \cdot 2 \cdot \frac{1}{2} \cdot \frac{2}{\sqrt{2}/2} \cdot 1 = 30521\,N; \quad x_4 = \frac{c}{3\sin\theta} = 0.943\,m$$

$$M_A = F_1 \cdot x_1 + F_2 \cdot x_2 + F_3 \cdot x_3 + F_4 \cdot x_4 =$$

$$= 47088 + 41856 + 94176 + 28776 = 211896 \quad Nm$$

Esercizio 4

$P_p = 3\,kW \qquad \Delta H_z = 35\,m \qquad \Delta H_{perdite} = 8\,m$

$$P = \gamma Q H \rightsquigarrow P_p = \gamma Q \Delta H_{TOT} = \gamma Q (\Delta H_z + \Delta H_p)$$

$$\rightsquigarrow Q = \frac{P_p}{\gamma \Delta H_{Tot}} = \frac{3000}{9810 \cdot 43} \cong 7\ l/s$$

Esercizio 5

Impiegando il teorema di Bernoulli

$$\frac{\partial H}{\partial s} = 0 \;\rightsquigarrow\; H_1 = H_2 \qquad z_1 + \frac{p_1}{\gamma} + \frac{U_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{U_2^2}{2g}$$

dall'eq. di continuità $Q = U_1 \Omega_1 = U_2 \Omega_2$

Imponendo $z_1 = z_2$

$$\frac{p_1 - p_2}{\gamma} = \frac{U_2^2 - U_1^2}{2g} = \frac{Q^2\left(\frac{1}{\Omega_1^2} - \frac{1}{\Omega^2}\right)}{2g}$$

Esercizio 6

Applicando il Principio della Quantità di Moto

$$I + M_u - M_i = \Pi + G$$

Stazionario

$G_x = 0 \qquad I_x = 0$

$G_y = 0 \qquad I_y = 0$

Lungo x

$M_u = -\rho Q U \cdot \cos\theta$

$M_i = \rho Q U$

$\Pi_x = -F_x$

$$-\rho Q U \cos\theta - \rho Q U = -F_x$$

$$F_x = \rho Q U (1 + \cos\theta) = 14 \cdot 30 \left(1 + \frac{\sqrt{2}}{2}\right) \cong 717 \text{ N}$$

La forza di attrito dovrà essere eguale e contraria alla forza F_x