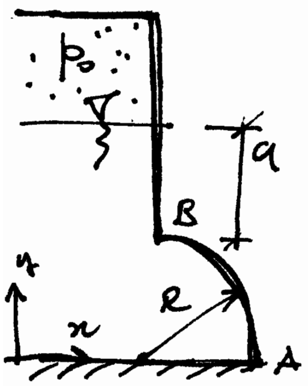
ESAME MECCANICA DEI FLUIDI 11 GIUGNO 2007

Esercizio 1



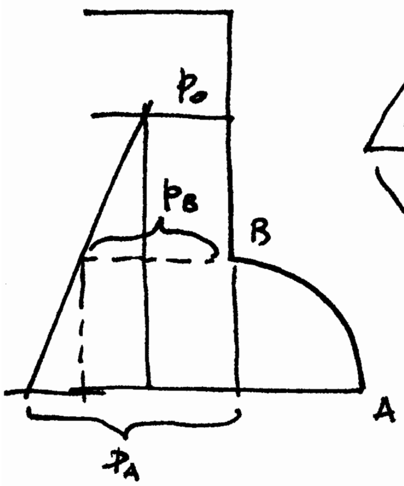
$a = 3\,m \quad R = 1\,m \quad \rho = 1000\ kg/m^3$

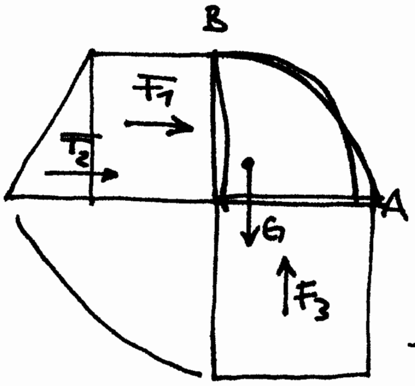
$\gamma = \rho \cdot g = 9810\ kg/m^2 s^2$

$p_B = p_0 + \gamma a = 9810 \cdot 3 + 15000 = 44430\ Pa$

$p_A - p_B = \gamma \cdot R = 9810 \cdot 1 = 9810\ Pa$

$F_1 = p_B \cdot R \cdot L = 44430 \cdot 1 \cdot 1 \overset{!}{=} 44430\ N$





$F_2 = \frac{(p_A - p_B)}{2} \cdot R \cdot L = 4905\ N$

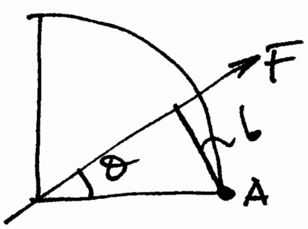
$G = \frac{\pi R^2}{4} \cdot \gamma = 9810\ \frac{\pi 1^2}{4} = 7700\ N$

$F_3 = p_A \cdot R \cdot L = (p_B + \gamma R) R = \overset{!}{=} (44430 + 9810) \cdot 1 = 54240\ N$

Quindi $F_x = F_1 + F_2 = 44430 + 4905 = 49335\ N$

$F_y = F_3 - G = 54240 - 7700 = 46540\ N$

$\theta = \operatorname{arctg} \frac{|F_y|}{|F_x|} \simeq 0.76\ rad \sim 43°$



$M_A = |F| \cdot b = F \cdot R \cdot \sin\theta$

$b = R \cdot \sin\theta \simeq 0.686\ m$

$|F| = \sqrt{F_x^2 + F_y^2} \simeq 67823\ N$

$M = |F| \cdot b = 46527\ Nm$

Esercizio 3

$$Fr_m = Fr_p \rightsquigarrow \frac{U_p}{\sqrt{g L_p}} = \frac{U_m}{\sqrt{g L_m}} \qquad \frac{L_m}{T_m}\frac{1}{\sqrt{L_m}} = \frac{L_p}{T_p}\frac{1}{\sqrt{L_p}}$$

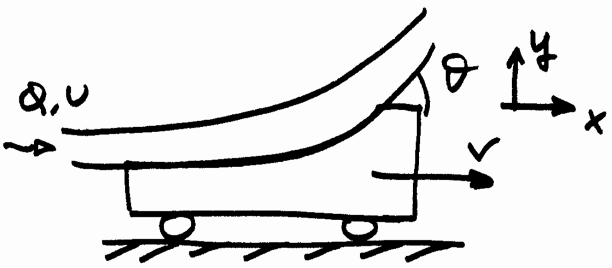
$$\rightsquigarrow \lambda^{1/2} = \tau$$

$$[Q] = \frac{L^3}{T} \rightsquigarrow \frac{Q_m}{Q_p} = \frac{L_m^3}{L_p^3}\frac{T_p}{T_m} = \lambda^3 \tau^{-1} = \frac{\lambda^3}{\lambda^{1/2}} = \lambda^{5/2} = \left(\frac{1}{20}\right)^{5/2} =$$

$$= 0.00056$$

$$Q_m = 0.00056 \cdot Q_p \simeq 2500 \cdot 0.00056 \simeq 1.4 \ m^3/s$$

Esercizio 4



$Q = 50$ l/s $\quad \rho = 1000$ kg/m³

$U = 20$ m/s $\quad s = 20$ m

Applico il Principio Quantità Moto lungo l'asse x

$$I_x = \phi \; ; \; G_x = \phi \qquad \Pi_x = -F_x$$

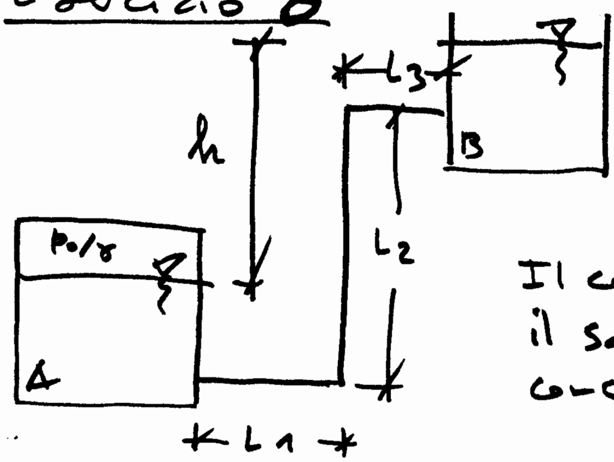
$$M_u^x = \rho Q U \cos\theta \quad ; \quad M_i^x = \rho Q U$$

$$\Rightarrow -F_x = \rho Q U \cos\theta - \rho Q U \rightsquigarrow F_x = \rho Q U (1-\cos\theta)$$

$$L = F \cdot s = F_x \cdot s = \rho Q U (1-\cos\theta) \cdot s =$$

$$= 1000 \cdot \frac{50}{1000} \cdot 20 (1-\cos\theta) \cdot 20 \cong 2680 \text{ Joule}$$

Esercizio 5



$L_1 = 3$ m $\qquad h = 10$ m

$L_2 = 7$ m $\qquad y_r = 0.1$ mm

$L_3 = 5$ m $\qquad Q = 45$ l/s

Il carico dovuto a p_0/γ deve vincere il salto h e le perdite concentrate e distribuite

$$\frac{p_0}{\gamma} = h + \frac{U^2}{2g}\left[\frac{(L_1+L_2+L_3)}{D}\lambda + \underset{\text{imbocco}}{0.5} + \underset{\text{gomiti}}{1 + 1} + \underset{\text{sbocco}}{1}\right]$$

$$U = \frac{Q}{\Omega} \qquad \Omega = \frac{\pi \cdot D^2}{4} = \frac{\pi \cdot 0.2^2}{4} = 0.0314\ m^2$$

$$\Rightarrow U = \frac{0.045}{0.0314} \cong 1.4\ m/s$$

$$Re = \frac{U \cdot D}{\nu} = \frac{1.4 \cdot 0.2}{10^{-6}} \simeq 286479 \qquad \varepsilon = \frac{y_r}{D} = 0.0005$$

$$\Rightarrow \lambda \simeq 0.018$$

quindi $$p_0 = \left\{h + \frac{U^2}{2g}\left[\frac{(L_1+L_2+L_3)\lambda}{D} + 0.5 + 1 + 1 + 1\right]\right\}\gamma = 103076\ Pa$$

Esercizio 5

$$[P] = [\gamma Q H] = \frac{M}{L^3}\frac{L}{T^2}\cdot\frac{L^3}{T}\cdot L = \frac{ML^2}{T^3} = \rho^\alpha U^\beta D^\gamma$$

$$ML^2T^{-3} = \underbrace{M^\alpha L^{-3\alpha}}_{\rho}\ \underbrace{L^\beta T^{-\beta}}_{U}\ \underbrace{L^\gamma}_{D}$$

$$\Rightarrow \begin{cases} 1 = \alpha \\ 2 = -3\alpha + \beta + \gamma \\ -3 = -\beta \end{cases} \quad \begin{cases} \alpha = 1 \\ \beta = 3 \\ \gamma = 2 \end{cases} \qquad \Pi_0 = \frac{P}{\rho U^3 D^2}$$

Esercizio 7

$$\underline{U} = (3.5x^2 + 7t)\,i + (7.2x - 3t)\,j$$

Il moto non è stazionario in quanto $\frac{\partial U}{\partial t} \neq \emptyset$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 7 + (3.5x^2+7t)\cdot 7x + (7.2x-3t)\,\emptyset$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -3 + (3.5x^2+7t)\cdot 7.2 + (7.2x-3t)\,\emptyset$$

$a_x = 7 + 24.5x^3 + 49tx$ $a_y = -3 + 25.5x^2 + 50.4t$

$a_x(5,7) = 3069.5 + 247.5t$ $a_y(5,7) = 634.5 + 50.4t$

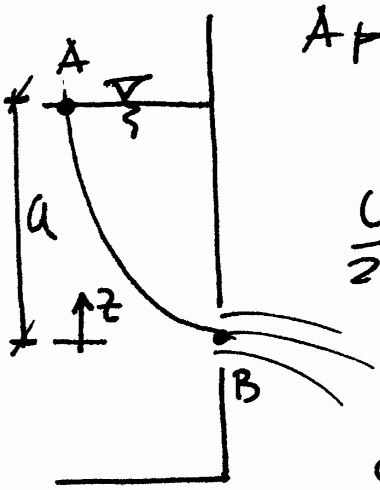
Il moto non è comprimibile, infatti $\nabla \cdot \underline{u} \neq \emptyset$

$$\frac{\partial u}{\partial x} = 7x \quad \frac{\partial v}{\partial y} = 7.2 \quad \nabla \cdot \underline{u} = 7x + 7.2 \neq \emptyset$$

$$\mathbb{E} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} \end{bmatrix} = \begin{bmatrix} 7.5x & 3.6 \\ 3.6 & \emptyset \end{bmatrix}$$

$$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 7.2$$

Esercizio 8



Applico Bernoulli tra il p.to A e il p.to B

$$H_A = H_B$$

$$\frac{U_A^2}{2g} + \frac{p_A}{\gamma} + z_A = \frac{U_B^2}{2g} + \frac{p_B}{\gamma} + z_B$$

$$\emptyset + \frac{p_{atm}}{\gamma} + a = \frac{U_B^2}{2g} + \frac{p_{atm}}{\gamma} + \emptyset$$

$$\Rightarrow U_B = \sqrt{2ga}$$

$$Q = \Omega \cdot U_B \cdot C_Q = C_Q \cdot \pi \cdot R^2 \sqrt{2ga}$$

$a = 3\,m$; $C_Q = 0.75$; $R = 0.3\,m$

$$Q = 0.75 \cdot \pi \cdot 0.3^2 \sqrt{2 \cdot 9.81 \cdot 3} \cong 1.626\ m^3/s$$